Generalization of Grothendieck duality over Serre duality in [illegible] Cohen-Macauly schemes representing Calabi–Yau 3-fold on Bogomolov–Tian–Todorov Theorem

Deep Bhattacharjee (itsdeep@live.com)
https://orcid.org/0000-0003-0466-750X

Research Article

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Generalization of Grothendieck duality over Serre duality in deg_n $Cohen - Macauly$ schemes representing Calabi–Yau 3-fold on Bogomolov–Tian–Todorov Theorem

Deep Bhattacharjee[1]

Abstract:

Any induced isomorphism over the chain-complex of homology can be the abelian X defined for the functors of hypercohomology is a derived category for all the locally defined commutative algebra in coherent sheaves as $S(X)$. Regards to the schemes Ω going through the morphisms on field f relates to scheme Ω in fiber projections $*_Y : \mathbf{Spec}(f) \longleftarrow varaity(\rho)$: This closed domain can be extended over Serre via Relative Grothendieck where categorically expressed (C) the change to ***Base*** B viewing morphism $B \longrightarrow fixed\ point\ f^{\wedge}$ from $(C)_X \longrightarrow fixed\ point\ f^{\wedge}$ considering the class $\omega_{(C)_X}[deg_n] \exists\ n\ implies\ Cohen - Macauly\ degree$. $\mathbf{Spec}(f)$ on field f with deg_{finite} shows a smooth Serre functor $\Omega^{\mathrm{b}}_{\mathrm{coh}}(\boldsymbol{X})$ operating smoothly over field f. Thereby, applying over the Bogomolov–Tian–Todorov Theorem with Gorenstein formulations one can find Kähler moduli, Complex moduli, Calabi–Yau 3-fold having Euler characteristics $\mathcal{X} = -200$ in quintic $\mathbb{C}P^4$ over monoidal symmetric **K**–module. It's been shown that a Chern-Weil form exists in dimension $d/2$ over algebraically defined K–3 surfaces taking a $\ell - addic\ \acute{e}tale$ for elliptic curve $\mathbb{Q}$ representing Galois transformation for 2–form holomorphic having an isometry over Cartan matrix E_8 suffice $K_X \simeq \mathcal{O}_X$ for $X_c \subset \mathbb{P}^n$.

Keywords: Coherent Duality – Commutative Algebra – Kähler moduli - Poincaré duality – étale cohomology – Grothendieck-Riemann-Roch formulations – Hochschild cochain complex

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[1]***Email:*** itsdeep@live.com

I Grothendieck Generalization

In any projective plane $\mathbb{P}^4$ if the Hodge diamond is equal to $h^{2,1}$ for complex moduli spaces, then obstructions are restricted on any CY manifold on the $degree\ \eta$ of any quintic 3-fold equals $dim\big(H^1(X, \boldsymbol{B}X)\big)$.

Taking the Poincar$é$ duality for the i^{th} cohomology group where in the coherent sheaves of any complex manifold $\mathcal{M}$ the Serre duality is satisfied for any concerned integers $\ell\ \forall\ homology\ group$ as,

$$(\eta - \ell)\ satisfies\ isomorphism\ H_{i-\ell}(\mathcal{M})\ \cong \bigwedge^{\eta}(\boldsymbol{B}^*\boldsymbol{X})\ \forall\ \eta - forms$$

For any concerned canonical line bundle f_X a K$\ddot{a}$hler manifold satisfying the Kodaira dimensions for $(p,q) = (1,1)^+$ having the potential $i2^{-1}\partial\bar{\partial}_\rho$ for Hodge lemma $\partial\bar{\partial}\ \forall\ Serre\ \Omega_X^\eta$ equals,

$Linear - Hodge \star :\ \Omega^{n-p,n-q}(X)$ with its complex giving a $2n - form$ integrating over X as,

$$\int_X \theta \wedge_h\ \left(\Omega^{n-q,n-p}(X)\right)\ \bar{\theta}$$

Making the Grothendieck generalization an extension to the Cohen – Macaulay Schemes over the Serre duality on Smooth Schemes for category class $\omega_{(C)_X}[deg_n]\ \exists\ n\ implies\ Cohen - Macauly_{(C-M)}\ deg -$ a *dualizing sheaf* for field f with deg_{finite} can take the Gorenstein scheme over finite orders $\pi: X \longrightarrow \mathbf{Spec}(f)$ shows,

$$deg_0\ Ring \bigoplus_{complete\ \ intersection}^{k[x]/(x^2)} \textbf{dualizing sheaf}\ |_{C-M} \otimes \bigwedge F_{\frac{X}{Y}}\ \forall\ class\ \omega_{(C)_X}$$

Providing $X \subset \mathbb{P}^n$ for $Cohen - Macauly$ class $\omega_{(C)_X}$ the previously taken dualizing complex provides the space,

$$\int_{C-M} \exists\ smooth\ Serre\ functor\ holds\ over\ \Omega^{\mathrm{b}}_{\mathrm{coh}}(\boldsymbol{X})$$

Gives for *algebraic variety* $\epsilon\ and\ \epsilon^\star$ constructing the wedge $\wedge_h$ for Linear-Hodge θ and Conjugate Linear-Hodge $\bar{\theta}$ over schemes,

$$\int_X .$$

Implying an image from the domain X^d to the co-domain,

$$X^d \longmapsto X^d\omega^\bullet_{\mathbb{P}^n}$$

Thereby, extending image component $\omega^\bullet_{\mathbb{P}^n}$ for a $local\ sheaves\ S^\bullet$ a resolution can be obtained by $S^\bullet \longrightarrow \mathcal{O}_X$ where a sheaf extension can be observed over coherent sheaves X over ℓ integers for $\mathbf{Ext}^i\ \forall i\ integers\ in\ n > n_0$ as,

$$\Gamma\left(X, \mathcal{E}xt^i\big(\alpha, \alpha^0(n)\big)\right)\ \exists\ \alpha, \alpha^0 \in X$$

From the above relation, one can obtain $Calabi - Yau\ in\ \mathbb{P}^4$ for Quintic 3-fold satisfying Serre through $Hodge\ diamond\ h^{2,1}$ over,

$$H^2\left(X, \mathcal{O}_X \bigoplus_{Cohen-Macauly\ \ class}\right)$$

Hodge numbers corresponding CY 3-fold takes a conjugate structure over Euler characteristics $\mathcal{X} = -200$ in quintic $\mathbb{C}P^4$ as,

$$
\begin{array}{lll|l}
\mathit{Kähler\ moduli} & h^{1,1} = 1 & & \\
\mathit{Complex\ moduli} & h^{2,1} = 101 & & \mathcal{X} = 2(h^{1,1}, h^{2,1}) \\
CY\ 3-fold & h^{1,1}, h^{2,1} & &
\end{array}
$$

For the smooth complete intersections $\mathbf{Ext}^i\ \forall i\ integers\ in\ n > n_0$ taking the $2-form\ Kähler\ \mathcal{L}$ in order of $\mathcal{P}$ homogeneous polynomials, on $deg_d \ \exists\ \partial_i = \nabla_i$ and $deg_{\mathcal{J}} = \mathcal{F}$ represents,

$$
Chern\ class = \frac{(1+\mathcal{L})^{N+1}}{\prod_{d=1}^{\mathcal{P}}(1+deg_d J)}
$$

And the associated complex,

$$
\mathcal{O}(-\mathcal{F} - \nabla_1 - \nabla_2 - \nabla_3) \longrightarrow\ \square_{\mathcal{O}} \longrightarrow [\mathcal{J}_{\partial_1} \quad \mathcal{J}_{\partial_2} \quad \mathcal{J}_{\partial_3}] : \mathcal{O}
$$

Where $\square_{\mathcal{O}}$ takes,

$$
\xrightarrow{\begin{bmatrix} \partial_3 \\ -\partial_2 \\ \partial_1 \end{bmatrix}}
\begin{matrix} \mathcal{O}(-d-\nabla_1-\nabla_2) \\ \oplus \\ \mathcal{O}(-d-\nabla_1-\nabla_3) \\ \oplus \\ \mathcal{O}(-d-\nabla_1-\nabla_3) \end{matrix}
\xrightarrow{\begin{bmatrix} \partial_2 & \partial_3 & 0 \\ -\partial_1 & 0 & \partial_2 \\ 0 & -\partial_1 & \partial_2 \end{bmatrix}}
\begin{matrix} \mathcal{O}(-d-\nabla_1) \\ \oplus \\ \mathcal{O}(-d-\nabla_2) \\ \oplus \\ \mathcal{O}(-d-\nabla_3) \end{matrix}
$$

With the associated Hodge diamond,

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & & & 0 & \\
 & & & h^{1,1} & & & \\
1 & & h^{2,1} & & h^{2,1} & & 1 \\
 & & & h^{1,1} & & & \\
 & 0 & & & & 0 & \\
 & & 0 & & & & \\
 & & & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

II Bogomolov–Tian–Todorov Theorem

The unobstructed deformation theory as observed in BTT Theorem for non–singular $CY-n$ folds where n be taken as 3 for this paper, there are terminal singularities where for any bilateral constructions over normal and canonical singularities, there lies an open question regarding the nature of this unobstructedness. This would be given by the following conditions,

- Taking the derived sections of the tangent sheaf for a certain abelian, a differentially graded Lie algebra for any derived category takes a quasi-isomorphic form $\mathcal{Q}$ having a Hochschild cochain complex $\mathcal{HH}^{\bullet}(X_C/f)$ for the CY manifold X_C for a symmetric monoidal category over symmetric **K**–module spectra $sym^{\times}(\mathbf{K})$ representing $\mathcal{HH}^{\bullet}(X_C/f)$. Satisfying the induced morphisms $\zeta_c : f[n] \longrightarrow \mathcal{HH}^{\bullet}(X_C/f) \longrightarrow \mathcal{HH}^{\bullet}(X_C/f)$ for f^{th} homology group for **K**–module spectra to get the CY manifold X_C by,

$Note: Throughout\ this\ paper\ the\ K\ is\ taken\ as\ f\ for\ example\ K_X \simeq \mathcal{O}_X\ is\ written\ as\ f_X \simeq \mathcal{O}_X$

$$\mathcal{HH}_{\bullet}(X_C/f) \otimes_f f[n] \xrightarrow{\mathrm{id}\otimes\zeta_c} \mathcal{HH}^{\bullet}(X_C/f) \otimes \mathcal{HH}_{\bullet}(X_C/f)$$

$$\xrightarrow{for\ all\ restriction\ \ /promotion}$$

$$\mathcal{HH}_{\bullet}(X_C/f) \longrightarrow \mathcal{HH}_{\bullet}(X_C/f)$$

- Over any sheaf $\mathcal{S}$ concerning the CY manifold X_C any birational contraction for Y canonical singularities gives the pullback of $\mathcal{S}$ on functor $\mathcal{S}_{\times_{Hom\ (-,X)}} Hom(-,Y)$ for $\sigma : X \longrightarrow Y$. For true σ in noetherian schemes there's an inverse image σ^* where the action $\sigma^*\mathcal{S}$ any pointwise covering P_c related to the inclusion map $\mathcal{V} \longrightarrow P_c$ gives,

$\{\mathcal{V}_i\} \in P_c$ iff $\{(P_c)_{\mathrm{i}}\}$ makes a total cover over P_c $\forall$

$$\bigcup_i (P_c)_{\mathrm{i}} = P_c$$

Thus, any complex pullback with the compact functor of the higher image makes $R\sigma_!$ establishes the inverse image functor $R\sigma_*$ where for smooth f variety on $\mathbf{Spec}(f)$ the right adjoint functor on $\cap -orientation$ provides the map,

$$\mathcal{S} : X_c \longrightarrow {}^\wedge$$

Over the canonical sheaf on deg_n where for the $\ell - addic$ cohomology groups over the constant sheaves $\mathcal{S}^\wedge$ the schemes s satisfy the finite coefficients $\mathbb{Z}/\ell^f\mathbb{Z}$ for $field\ f$ giving,

$$H^i(s, \mathbb{Z}^\ell) = \xleftarrow{lim} H^i(s, \mathbb{Z}/\ell^f\mathbb{Z})$$

Thus, the sheaf when acting canonically on $\mathcal{S}$ for the category class of $\omega_{(C)_X}[deg_n]$, the smooth map $S : X_c \longrightarrow \mathbf{Spec}(f)$ for $\cap -orientation$ morphism in f variety on the ring $\sigma^! \cap$ takes the equivalence as,

$$\omega_{(C)_X}[deg_n] = \sigma^! f$$

Then the étale variety over the $Tate\ twist\ (t)$ for $\ell - addic$ sheaf $\mathbb{Q}_\ell$ gives,

$$H^i(s, \mathbb{Q}_\ell) = H^i(s, \mathbb{Z}_\ell) \otimes \mathbb{Q}_\ell$$

Where $\mathcal{S} : X_c \longrightarrow {}^\wedge$ takes the constant sheaf $\mathcal{S}^\wedge$ $\forall$ last − shriek $\ell - addic\ primes$ puts forward $\mathbb{Q}_\ell$ over the $Poincaré$ duality on $Tate\ twist\ (t)$,

$$H^i(X, \omega_n^N) \cong H^{2i-i}(X, \gamma)\ \exists\gamma\ satisfies\ \mathbb{Z}/\boldsymbol{n}\mathbb{Z}$$

Giving $\ell - addic$ Poincaré duality through,

$$H^n(X, \mathbb{Q}_\ell)^\wedge\ where\ \forall H^n(X)^\wedge\ there\ exists\ Hom(\mathbb{Q}_\ell \sigma_* \sigma^! \mathbb{Q}_\ell[-n])^\wedge$$

II.I Cohen-Macauly schemes and Gorenstein

For a complete intersection, CY 3-fold with $n = 3$ for in the arithmetically Cohen – Macauly when there's any R in Gorentein a Cohen – Macauly 3-fold for all canonical module ω_R there exists three relations for the non-degenerate I form,

- $X_c \subset \mathbb{P}^n$ generalizing to a hypersurface of deg_b $\forall$ complete intersection $\exists(b_1 \dots \dots b_{n-3})$ where the first generalization of CY takes through $\mathbb{P}^4$ for every $H^1(\mathcal{O}_X) : R$ is Gorenstein for a shift R' making the the $\ell - addic$ cohomology groups over the constant sheaves $\mathcal{S}^\wedge$ by,

[1] $Obtaining$ Complete Intersection $(b_1 \dots b_{n-3}) \subseteq \mathbb{P}^n$
[2] $Obtaining$ Cohen – Macauly $R = \mathcal{S}^\wedge/I$

[3] $\textit{Obtaining}$ Cohen – Macauly 3 – fold $\forall f_X \simeq \mathcal{O}_X$ when for deg_n CY gives $H^1(\mathcal{O}_X) \ldots\ldots, H^{n-1}(\mathcal{O}_X) = 0$

Getting $f_X = \mathcal{O}_X \longleftrightarrow -n - 1 + n - 3 + Reg_R = 0 \longleftrightarrow Reg_R = 4$

- A minimal resolution normalized indecomposable vector bundle of $Rank_2\ E_\delta$ taken over CY on quintic 3-fold for a resolution on $\mathcal{O}_{\mathbb{P}^n}$ the Grothendieck-Riemann-Roch formulations over a complete intersection CY 3-fold X_C $\forall X_C \subset \mathbb{P}^{\frac{r}{4}}$ we get,

  - $Chern\ class =$ $\begin{cases} c_1 = -2, \\ c_1 = -1, \end{cases}$ $\forall r\ over\ existance\ of\ E_\delta\ [except]\ on\ X_C -$ $16\ there\ exists$ $\begin{cases} c_1 = 0 \\ c_2 = 3 \end{cases}$

Thus from $Grothendieck - Riemann - Roch\ formulations -$

$$\mathcal{X}(E_\delta) = \frac{r}{6}c_1^3 - \frac{c_1c_2}{2} + \frac{c_1}{12}\left(12\left(\frac{r}{4} + 4\right) - 2r\right)$$

Then for co-dimension I smooth over sheaf S Artithmetically Cohen-Macaulay can be given over a sub-scheme S_V $\forall X_c \subset \mathbb{P}^n$ then the **ideal sheaf** given through,

$$(M^i)(S_V) = H_\star^i(I_{S_V}) = \bigoplus_{\frac{r}{4}\in\mathbb{Z}} H^i\left(X_c, I_{S_V}\left(\frac{r}{4}\right)\right)$$

Such that $-$
$(M^i)(S_V) =$ deficiency module
i^{th} cohomology module on S_V
$\forall (M^i)(S_V) = 0$ when $1 \leq i \leq r$

III Calabi–Yau Manifolds and Galois Representation

The proper existence of CY – metric can be found in Chern-Weil form for every non-negative grade of the second Chern number having an implicit CY form (X_C, ω) for every ≥ 0 $c_2(X_C) \wedge \omega^{n-2} \cap [\omega]^{n-2}$ $\forall$ $|\mathrm{Rm}| = 0, \mathbb{C}^n$ Euclidean space having forms,

$$Compact\ K\ddot{a}hler \equiv Ricci - flat\ CY \xrightarrow{metric} \sqrt{-1}\sum R_{i\bar{j}}dz^i \wedge d\,\bar{z}^j$$

$$Over\ existence\ of:$$

$$\int_{X_c} C_2(X_C) \wedge \omega^{n-2} = \varphi \int_{X_c} |\mathrm{Rm}|^2\ \mathbf{vol}\ \geq 0 \text{ for constant } \varphi$$

In In $\ell - addic\ \acute{e}tale$ any Galois representation for elliptic curve $\mathbb{Q}$ for CY reorientation X_c in $deg_{d>0}$ over algebraic cycles of $d/2$ dimensions taking $d + 1 = 2$ defines $K3$ surface in $\mathcal{L} - lattice$ for quadratic form $\langle , \rangle$ establishes 2-form holomorphic $\Psi_{(2)}$ isometry for Cartan matrix E_8 as,

$$L \coloneqq (\mathbb{Z}^{\oplus d}, -E_8\ \oplus -E_8 \oplus U \oplus U \oplus U) \text{ Satisfying} \begin{cases} \acute{e}tale\ form: H_{et}^2\left(X_{C_{\overline{\mathbb{Q}}}}, \mathbb{Q}_\ell\right) \\ Span: H^2(X_C, \mathbb{Z}) \otimes_{\mathbb{Z}} \mathbb{C} \\ Bilinear\ form: \begin{cases} \langle \Psi_{(2)}, \Psi_{(2)} \rangle = 0 \\ \langle \Psi_{(2)}, \overline{\Psi_{(2)}} \rangle > 0 \end{cases} \end{cases}$$

III.I Calabi–Yau 3-fold

Considering a vector bundle $\mathcal{V}_\times$ on $\mathcal{S} : X_c \longrightarrow$ ^ over the constant sheaves $\mathcal{S}^{\wedge}$ satisfying the finite coefficients $\mathbb{Z}/\ell^f\mathbb{Z}$ the schemes s where Grothendieck's convention for $CY\ n - fold$ generates,

$\mathbf{P}(\mathcal{S}(\mathcal{V}_\times))$	Through the fiber projections from Picard number P_j takes: $\gamma : \mathrm{P_j} \longrightarrow \mathbf{P}(\mathcal{O}_{\mathbf{P}_1}^{\oplus(\mathrm{n+1})})\ \exists X_c(s) \subseteq P_j$

A natural isomorphism can be observed in $CY\ n - fold$ where there exists a canonical singularity. However it's obvious that $n = 3$ is also a $non - singular$ structure without any terminal singularity over a multiplicity of $\left[\frac{n}{2}\right]$ where if $n = 3$ then the structure $X_c(\mu)$ is representable via the element of μ as $H^0(\omega_{P_2}^{-1})$ giving a natural isomorphism making an equivalence class over,

$$\begin{array}{ccc} \bigoplus_{i=0}^{n+1-p} \mathcal{S}^i(\mathcal{O}_{\mathbf{P}^1}(-1)) \otimes \mathcal{S}^{n+1-i}\left(\mathcal{O}_{\mathbf{P}^1}^{n-1} \oplus \mathcal{O}_{\mathbf{P}^1}(1)\right)(2_{\pi_*}) & \subseteq & H^0(\omega_{P_2}^{-1}) \\ \downarrow \cong & & \downarrow \cong \\ \bigoplus_{i=0}^{n+1-p} \mathcal{S}^i(\mathcal{O}_{\mathbf{P}^1}(-1)) \otimes \mathcal{S}^{n+1-i}\left(\mathcal{O}_{\mathbf{P}^1}^{n-1} \oplus \mathcal{O}_{\mathbf{P}^1}(1)\right)(2) & \subseteq & H^0\left(\mathcal{S}^{n+1}(\mathcal{O}_{\mathbf{P}^1}(-1))(2)\right) \end{array}$$

III.II Complete Intersections Calabi–Yau 3-fold

$CY - manifold\ X_c$ being considered over a broader generalizations for the multi-homogeneous polynomial $\mathcal{E}_p$ having $Rank\ k$ acting on the product of the projective space $\widetilde{\mathbb{P}^n}$ as,

$$\mathcal{E}_n on\ \widetilde{\mathbb{P}^n} = \begin{cases} \{\mathcal{E}_p\}_{p=1,\dots,\dots,k} & \exists X_c \subset \mathcal{E}_p \\ \widetilde{\mathbb{P}^n} = \mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_l} & \end{cases}$$

For the quintic spectrum $\mathcal{Q}_n^p$ through generalizations on $\mathcal{Q}_n^p \in \mathbb{Z}_{\geq 0}$ over p^{th} polynomial degree of $\mathcal{Q}_n^p \mid p = 1, \dots \dots k$ on the n^{th} factor of $\mathcal{E}_n \mid n = 1, \dots \dots, l$ there exist a configuration matrix,

$$X_c(C) = \begin{bmatrix} \mathcal{Q}_1^1 & \mathcal{Q}_2^2 & \cdots & \cdots & \mathcal{Q}_1^k \\ \mathcal{Q}_2^1 & \mathcal{Q}_2^2 & \cdots & \cdots & \mathcal{Q}_2^k \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ \mathcal{Q}_l^1 & \mathcal{Q}_l^2 & \cdots & \cdots & \mathcal{Q}_l^k \end{bmatrix}$$

Where CY 3-fold holds for the projective variety on $X_c \subset \mathcal{E}_p$ for the CY on $X_c(C)$ with the vanishing locus of $\{\mathcal{E}_p\}_{p=1,\dots,\dots,k}$ suffice a Complete Intersections CY 3-fold $CICY$ having conditions,

$X_c(C) \equiv CICY\ 3 - fold$

$$\textbf{for} \begin{cases} k = \sum_{p=1}^{l} \epsilon_p - 3, & (CI\ condition) \\ \epsilon_n + 1 = \sum_{p=1}^{k} \mathcal{Q}_n^p\ \ \forall n = 1, \dots, l\ , & (Adjunct\ Generalization) \end{cases}$$

Thus satisfying all the necessary conditions to suffice the Grothendieck Generalization acting over Serre duality under Bogomolov–Tian–Todorov Theorem representing the CICY 3-folds through $Cohen - Macauly$ schemes.

Results

Projective plane $\mathbb{P}^4$ with Hodge diamond $h^{2,1}$ having $K\ddot{a}hler$ potential $i2^{-1}\partial\bar{\partial}_{\rho}$ in $(p,q) = (1,1)^{+}$ form, the generalization of Grothendieck over Serre transits the Cohen – Macauly degree for category class $\omega_{(C)_X}[deg_n]$ where the existence of $dim\left(H^1(X, \boldsymbol{B}X)\right)$ quintic–3 fold can be found on any dualizing sheaf for finite Gorenstein scheme for the associated $Chern\ class = \frac{(1+\mathcal{L})^{N+1}}{\prod_{d=1}^{\mathcal{P}}(1+deg\ _d J)}$ of $\mathcal{P}$ homogeneous polynomials. Provided there's a pullback on the canonical singularities for every bilateral contractions:= $\ell - addic$ cohomology groups are satisfied over finite coefficients $\mathbb{Z}/\ell^f\mathbb{Z}$, giving the $e'tale$ for sheaf $\mathbb{Q}_\ell$ gives through $H^i(s, \mathbb{Z}_\ell) \otimes \mathbb{Q}_\ell$. Thereby taking the complete intersections for the quintic spectrum on deg_p polynomial $\mathcal{Q}_n^p \in \mathbb{Z}_{\geq 0}$ $CICY\ 3-$fold is satisfied for the unobstructed deformation having a Hochschild cochain complex representing Bogomolov–Tian–Todorov Theorem where any Galois representation for elliptic curve $\mathbb{Q}$ in $deg_{d>0}$ on CY results in algebraic cycles of $d/2$ dimensions. Any $non-singular$ structure over CY 3-fold associates a multiplicity of $\left[\frac{3}{2}\right]$ making an isomorphism over elements $H^0\left(\omega_{P_2}^{-1}\right)$ giving a natural isomorphism representing the associated conditions for this paper.

Delacration of Interest

The author has no conflicting interests related to this paper.

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