

Double Counting in Mystery: Journey of Intermediate Products in Multi-Country Trade

ONUR BIYIK (✉ biyik-onur@fuji.waseda.jp)

Waseda University

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Double Counting in Mystery: Journey of Intermediate Products in Multi-Country Trade¹

Onur Biyik²

Abstract

This study analyzes the production concept of the Value-Added (VA) chain by applying an alternative framework, as this concept is the main driver of the VA embodied in the intermediate export. VA supply chain spillover (upstream and downstream) of the production activities is investigated. The focal point of this study is to extend Global Value Chains (GVC) based on the multilevel and bilateral trade flow. In brief, there are two main contributions of this paper; the first begins with the originality of this work's theory which reveals the intermediate input journey among sector-country pairs, and the second is to introduce the optimal GVC participation with the knowledge spillover effect regarding vertical (upstream and downstream) integration. In other words, this paper found that different forms of the VA with traveled products (among countries) provide more precise (sectoral/regional) integration and optimal estimation method. This paper relies on constructed Global Trade Analysis Project Multi-Region Input-Output (GTAP-MRIO) and patent panel datasets. As a result of the analysis, if the product crosses the border only twice as the product returns home to its origin country, the export/import coefficient of the sectoral linkages, which boosts VA and then causes double-counting, is about 1.54 % in terms of (single) country-level data. More importantly, with regard to its contributions with the knowledge spillover effect, traveling products among countries as twice or infinity is to contribute by 1.6% or 154%. Lastly, its contribution to VA presents 0.8% to 72%.

Keywords: Double Counting; Global Value Chains; Value-Added; Technological Spillover.

JEL codes: E01, E16, F14, L14

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² PhD Program, Graduate School of Asia-Pacific Studies, Waseca University, 1-21-1 Nishiwaseda, Shinjuku, Tokyo, Japan, 165-0051 (biyik-onur@fuji.waseda.jp; onur.biyik@graduateinstitute.ch).

1. Introduction

Digitalization and product mobility create near-instantaneous globalization as multinational companies expand around the world due to the cost-effectiveness of one's location. More than half of the world's trade relies on intermediate parts and components. This means the participation of developing countries in the Global Value Chain (GVC) is a promising solution to escape heavy investment in capital-intensive industries, where they can instead improve their industrialization through specialization of single components and meet the global supply for local market needs³. Therefore, intermediate input trade among trade partners became more important as well as adding complexity to GVC participation.

This high volume of export/import among sector-country pairs raises a question about how we can measure "Made in Japan, who produced whom." One of the possible answers is foreign input reliance which takes account of the infinite sequence of all the inputs into all the inputs as known the Leontief inverse matrix (Baldwin & Freeman, 2021). However, it can be said that the contribution of the intermediate inputs to Value-Added (VA) seems to not be visible in an aggregated dataset because there is more than one-way of intermediate trade between partners and this integration is difficult to disaggregate to the level of companies or production stages (see Alfaro et al. (2019) and Li et al. (2019) for complex global supply chain). In other words, gross export at sectoral, bilateral, and multilateral levels (cross-countries/sectors) cannot be separated easily because when goods are exported and then it is impossible with available data sources to show where the goods are combined with other components and services.

In this regard, this study raises concern about the double-counting challenges and then introduces optimal GVC participation effect, the idea of where it comes from, and how to measure its contribution to VA under infinity production stages condition. Because unknown/Infinite intermediate product journeys among countries with reference to the subsequent location of the production stages cause bias estimation in VA studies due to the distribution of trade among partners (see the roundabout model in de Gortari's (2019) paper) in relation to the great unbundling (Baldwin, 2006). Moreover, Blaum (2018) shows that export-oriented firms are likely to be more dependent on imports (see Bernard et al. (2012) and Amiti et al. (2014)). This also causes overstated trade in VA regarding GVC involvement because aggregated sectoral analysis does not characterize the heterogeneity in export/import intensity across firms (Bems & Kikkawa, 2021). Scholars have investigated where VA was created (Koopman et al., 2014; Wang et al., 2013) and the upstream effect of VA (Fally, 2012; Antràs et al., 2012; Alfaro et al., 2019), as well as measuring the VA exchange rate (Bems & Johnson 2017), factor content (Trefler & Zhu, 2010), international inflation spillover (Auer et al., 2019), and bias estimate of trade in VA (Bems & Kikkawa, 2021; Kee & Tang, 2016).

Likewise, this study focuses on the production activity of VA embodied in intermediate and final export. Compared to the literature, the logic of this paper's method is similar to the approach of Koopman, et al. (2014), Johnson, (2018), Los et al., (2016), Wang et al. (2013), and Wang et al. (2017) articles, which provide a separated accounting framework of gross exports such as returns home (VS1*), foreign value-added (VS), and additional double-counting terms. Nonetheless, it is closely related to the papers of Kuboniwa (2016) and Koopman, et al. (2010) concerning double-counting.

Koopman et al. (2014) and Wang et al., (2013) extended the gross export decomposition methodology in Koopman et al. (2010) by differentiating the definitions of "domestic VA in exports" and "domestic content in exports", integrating different TiVA measures. Also, Wang et al. (2017) and Los et al. (2016) proposed

³ Developing nations join GVCs to become competitive, and then industrialize by densifying their participation.

two additional analytical frameworks such as the GDP decomposition framework and the production of the final goods decomposition framework. These studies are concerned with how the fragmentation of the production chain can be separated/decomposed. For example, intermediate export is re-exported and absorbed in both home and importing countries, as well as third countries separately. These methods can show where products end, or which country/sector receives these products in the GVC participation.

The limitations of these studies are that the double-counting estimations are portrayed based on two-way or three-way gross export (Kuboniwa, 2016; Koopman et al., 2014; Los et al., 2016) among trade partners. In addition, the GVC participation in a multi-country export content treats third partners as one unified country and then segments product parts in relation to Domestic Value-Added (DVA) and Foreign Value-Added (FVA). However, this paper shows that the aggregated third country in more than three-way trade leads to complex integration. That is because cross-border trade at a multilateral level is a part of a complex integrative process, and the GVC estimations thereafter heavily rely on strong assumptions of observing foreign/local goods in host/destination countries as documented by Koopman et al. (2014). This means that their complex integrations in VA estimation turn out to be deceptively simple mathematical equations that we cannot predict, hereinafter referred to as the "paradox of intertwined trade." In this regard, this paper is in line with Bems and Kikkawa (2021)⁴ to propose a different approach including vertical integration regarding production stages, and thus estimates optimal contribution to sectoral outcomes.

These infinity/unknown production stages of vertical integration between industry-country pairs inspire the idea of the (knowledge) spillover effect and its contribution to VA. Briefly, GVC can be one of the important channels of knowledge spillovers that drive up innovation/productivity because of (i) high intensity between foreign firms and domestic suppliers (World Bank (WB), 2020; Piermartini & Rubínová, 2021) and (ii) (efficient) production of the outsourced inputs of which foreign outsourcing firms will eventually be the consumers (Baldwin & Lopez, 2015). Productivity growth is higher for firms/sectors that are engaged in imports and exports (Baldwin & Yan, 2014). This is because participation in GVCs, particularly importing goods, is positively associated with imitation/innovation outcomes; thus, the international fragmentation of production may result in a conduit for international technology transfer (Keller, 2010). Thus, import relations within GVCs are used as a source of knowledge transfer, operating as potential information flow channels between sector-country pairs. This is consistent with the logic of this paper as the learning-by-exporting hypothesis that imports provide a channel of technology diffusion.

Existing studies show that engaging in very specific and more production stages through GVC results in a process that encourages innovation (Alfaro et al., 2019; de Gortari, 2019), as well as using foreign intermediate inputs increased plant productivity in Chile and Hungary (Kasahara & Rodrigue, 2008; Halpern et al. 2015). Moreover, scholars prove evidence that there is a high integration between patent inflows/outflows and value-added production (Zolas & Lybbert, 2022) such as GVC driving up international knowledge spillovers by 5% (Piermartini & Rubínová, 2021). To explain the knowledge spillover effect in our model, we combine sectoral network linkages (Leontief method) and knowledge flow coefficient, presenting the knowledge spillover effect among sector-country pairs through sectoral linkages. In other words, this paper merges the (knowledge) spillover effect with GVC participation through which we can estimate the production stage and its impact on VA. This is crucial because the role of GVC in international knowledge spillovers is essential for gauging the impact on economic growth.

⁴ They found that sectoral I-O tables cause bias estimation in VA (DVA and FVA) because of firm-level heterogeneities, especially undervalue for downward GVC participation (see Kee and Tang (2016) for firm-level evidence in China).

The following questions are proposed: What is the challenge of measuring double-counting in relation to two or more than a two-country model? What role does the knowledge spillover play in increased value-added production in the form of production process/stages in export (its contribution to VA)? To address theoretically and empirically the questions proposed in this paper, this study provides two examples (i) to solve double-counting in the two-country model regarding where double contributions come from and (ii) to explain the mystery of the double counting in the n-country model. Also, this paper in line with the literature presents vertical (upstream and downstream) integration by decomposing matrix of domestic plus (Intra-country) feedback effect, as well as a bilateral and multilateral spillover effect, separately.

First, it is possible to roughly estimate double-counting as hypothetical, assuming there are only two countries in the world and measuring the bias concerning where double contributions come from. The main assumption is if the intermediate product crosses the border twice as returning home to the product's origin. Identifying the double-counting based on the GVC participation, we use different assumptions to estimate Kuboniwa's (2016) model, presenting the new measurement of DVA (reducing return home part of the intermediate inputs) and FVA (reducing import of final goods including destination country's intermediate input) with the vertical integration coefficient. Second, the reality of (n-way) trade in (n-by-n) countries is to, however, present a complex integrative process in which we fail to present a clear transaction of intermediate trade. The literature lacks an n-way of intermediate export/import in multi-country; thus, I theoretically reveal intermediate trade flow in continuous integration of the GVC participation, the paradox of intertwined trade with trading partners (the distribution within and between nations).

Since the input-output dataset explains the last stage of production and provides precise information about sectoral and sector-country spatial linkages, the basic mapping of DVA and FVA can be directly obtained in Input-Output (I/O) data. There might be international knowledge spillovers⁵ that we do not capture with I/O data. This method can be enhanced by including additional information about technological diffusion with respect to a structured patent panel dataset. To estimate optimal VA and its spillover effect among sectors and sector-country pairs regarding vertical integration of the subsequent production stages (hereinafter referred to as traveling goods among countries), I use the constructed Global Trade Analysis Project version 10A Multi-Region Input-Output (GTAP-MRIO) and patent panel datasets with an assumption of intermediate product flow among countries as two or infinity times. In other words, the optimal measurement of the GVC participation regarding constructed GTAP-MRIO and patent panel datasets is to present more precise information than that contained in input-output data. In this regard, our study complements the broader literature on GVC-related measurements by estimating optimal contribution and knowledge flow within GVC involvement concerning the production/consumption process.

As a result of the analysis, if an intermediate product crosses the border only twice, the export/import coefficient of the sectoral linkages, which boosts VA and then causes double-counting in some conditions, is about 1.5 % regarding (single) country-level data. In reality of traveling goods in GVC, this paper in line with literature expects that firms/industries that increase production fragmentation/stages in GVCs will benefit with higher levels of innovation. Therefore, traveling products among countries with the technological spillover effect as twice or infinity is to contribute more by 1.6% or 154%. Lastly, this also applies to VA in a single country, it boosts VA more by 0.8% to 72%.

⁵ This paper is inspired by Piermartini and Rubínová (2021) who found that "...GVCs boost innovation on average by 5% both in developed and emerging economies..." and Li et al. (2019) and Zolas and Lybbert (2022) who highlight "...increasing the flow of intermediate and final goods and catalyzing knowledge spillovers across sectors and regions"

The rest of this paper is organized as follows: The second section explains the background of the Leontief method. The third section presents the VA in export content in which we theoretically introduce double-counting in mystery, as well as the GTAP-MRIO data and its estimation of sectoral linkages' contribution to sectoral outcomes. The fourth section portrays the knowledge spillover in GVC participation, through which patent panel datasets, econometric models, and knowledge flows are presented. After that, we combine two different results to calibrate the knowledge spillover effect. In the fifth section, potential policy paths are discussed and proposed in light of empirical results. The last section concludes this study. The appendix contains additional material on both theory and empirical analysis.

2. The Model

2.1. Basic Matrix

The assumption is that there would be $r, s=1, 2, \dots, N$ countries in which produce and input $r(i), s(j)=1, 2, \dots, n$ part and components. Gross production (X) can be divided into intermediate (AX) and final products (Y). Equation (1), which is displayed in a matrix format in equation (1.1), presents the basic input-output model.

$$X^r = A^{r1}X^1 + A^{r2}X^2 + \dots + A^{rn}X^n + Y^{r1} + Y^{r2} \dots Y^{rn} \quad (1)$$

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} + \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad (1.1)$$

Where, X^r, Y^r and $A(A^{rr} + A^{rs})$ are an (aggregated) N^n -by-1 output vector of Country r , an (aggregated) N^n -by-1 final demand vector of Country r , and an N^n -by- N^n (diagonal+off-diagonal) standard matrix of input-output coefficient showing the inputs from each sector meet in needed to produce a unit of output in each sector, respectively. Regarding the general equilibrium under the market-clearing condition with the Isard type of non-competitive multi-country input-output, we can then build the classical Leontief (1936) equation: $X = (I - A)^{-1}Y = BY$, where I is an identity matrix and $B (B^{rr} + B^{rs})$ is the well-known (global) Leontief (diagonal+off-diagonal) inverse matrix as presented in equation (2).

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \dots & B_{nn} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad (2)$$

At this stage, we can disaggregate the regional (D) and interregional feedback ($F=B-D$) inverse matrix. $D^{rr} = (I - A_{ij}^r)^{-1}$ represents local equilibrium (market clear) condition that each country satisfies (Johnson, 2018; Los et al., 2016; Miller & Blair, 2009). In other words, each regional domestic multiplier effect accounts for the matrix of a local economy, similar to the single country IO model. Therefore, we can disaggregate the interregional inverse matrix, $B = (I - A_{ij})^{-1}$, by introducing regional inverse matrix as documented by Los et al. (2016) and Wang et al., (2013).

$$= \begin{bmatrix} D_{11} & 0 & \dots & 0 \\ 0 & D_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & D_{nn} \end{bmatrix} + \begin{bmatrix} B_{11} - D_{11} & 0 & \dots & 0 \\ 0 & B_{22} - D_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & B_{nn} - D_{nn} \end{bmatrix} + \begin{bmatrix} 0 & B_{12} & \dots & B_{1n} \\ B_{21} & 0 & \dots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \dots & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad (3)$$

Further, we disaggregated the multi-country level into the two-country level and the feedback effect from the multi-country level by following Los et al. (2016) and Johnson (2018) with both types of the diagonal matrix of domestic input coefficient and off-diagonal matrix of imported input coefficient. The $L^{rs} \cong D^{rr}A^{rs}D^{ss}$ (see Wang et al. (2013) and Miller and Lahr (2001)) represents the matrix of the bilateral spillover effect. $L_{12} [\cong D_{11}A_{12}D_{22}]$ and $B_{12} - L_{12} [F_{12} \cong (B_{11}A_{12}B_{22}) - (D_{11}A_{12}D_{22})]$ decompose,

respectively, the matrix of the bilateral and feedback effect from multilateral spillover as written in equation (4)⁶.

$$= \begin{bmatrix} D_{11} & 0 & \dots & 0 \\ 0 & D_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & D_{nn} \end{bmatrix} + \begin{bmatrix} B_{11} - D_{11} & 0 & \dots & 0 \\ 0 & B_{22} - D_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & B_{nn} - D_{nn} \end{bmatrix} + \begin{bmatrix} 0 & L_{12} & \dots & L_{1n} \\ L_{21} & 0 & \dots & L_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n1} & L_{n2} & \dots & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & B_{12} - L_{12} & \dots & B_{1n} - L_{1n} \\ B_{21} - L_{21} & 0 & \dots & B_{2n} - L_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} - L_{n1} & B_{n2} - L_{n2} & \dots & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad \#(4)$$

To conclude this part, we decomposed the matrix $[B = B^{rr} + B^{rs}]$ of (i) domestic effect (D^{rr}) and (intra-country) feedback effect ($F^{rr} = B^{rr} - D^{rr}$) and (ii) multilateral (B^{rs}) and bilateral (L^{rs}) spillover effect as well as a feedback spillover effect from multilateral integration ($F = B^{rs} - L^{rs}$) in equation 4 (see BOX1 for the definition of integrations and Table B2 for empirical examples).

BOX 1: Definition

we define the direct effect, intra-country spillover effect (net direct effects), feedback effect, and multilateral spillover effect. This kind of explanation would help to understand the decomposition of multilateral, trilateral, and bilateral (VA) integration.

Direct effect: Japan exporting to ROW can contribute to the VA in its own exporting sectors

Intra-country spillover effect: the exporting industries in Japan using intermediate products from other Japanese industries can boost Japanese VA through Japanese domestic industrial linkages (supplier)

Feedback effect: ROW demanding import products from Japan would lead Japan to produce more products. So that Japan needs to import part end components from partners (through CES or spatial linkages). In short, (Japanese) processed intermediate and final good export to ROW would increase demand for import products from its partners and finally stimulate Japanese VA again as the logic would be $ROW \rightarrow Japan \rightarrow Africa \rightarrow Japan$

Trilateral spillover effect: Japan's export to Africa would cause/lead Japan to import intermediate input from ROW. This transaction would stimulate VA in ROW which would estimate (ROW) FVA in Japan. In other words, when the integrated matrix of two countries meets a third country good is to explain the trilingual integration (effect).

Multilateral spillover effect: the condition should be more than 3 countries. As for the complexity of the trade transmission channel, those export and import patterns are based on country spatial linkages (or Armington CES). As simple as if we disaggregate ROW like RCEP and others, trade interaction between Japan and Africa would substitute not only RCEP countries but also others, simplified as $Africa \rightarrow Japan \rightarrow RCEP (\rightarrow EU \rightarrow \dots)$

2.2. Value Added Decomposition of Final Demand

As for the total VA coefficient matrix, we employed Koopman et al. (2010) to calculate VA as vXY ; where v is the n-by-n diagonal of the VA portion and Y is the n-by-n diagonal of the final demand, representing the location/place in which VA is produced. In other words, the final demand in all previous equations regards the consumption location in which goods are consumed. To estimate the production location in which parts and components are produced, we can decompose (aggregated) final demand to be a function of the production location (see Johnson (2018)) as presented in equation (5).

$$T_v = \begin{bmatrix} v_1 & 0 & \dots & 0 \\ 0 & v_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_n \end{bmatrix} \begin{bmatrix} D_{11} & 0 & \dots & 0 \\ 0 & D_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & D_{nn} \end{bmatrix} + \begin{bmatrix} F_{11} & 0 & \dots & 0 \\ 0 & F_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & F_{nn} \end{bmatrix} + \begin{bmatrix} 0 & L_{12} & \dots & L_{1n} \\ L_{21} & 0 & \dots & L_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n1} & L_{n2} & \dots & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & F_{12} & \dots & F_{1n} \\ F_{21} & 0 & \dots & F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ F_{n1} & F_{n2} & \dots & 0 \end{bmatrix} \begin{bmatrix} Y_1 & 0 & \dots & 0 \\ 0 & Y_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Y_n \end{bmatrix}$$

⁶ Miller and Blair (2009) calculate the D and L based on intraregional effect (M_1), intraregional spillover effect (M_2), and intraregional feedback effect (M_3); thus, the D and L is equal to $M_3 M_2 M_1$.

$$\begin{aligned}
&= \left[\begin{array}{cccc} v_1 D_{11} Y_1 & 0 & \cdots & 0 \\ 0 & v_2 D_{22} Y_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_n D_{nn} Y_n \end{array} \right] + \left[\begin{array}{cccc} v_1 F_{11} Y_1 & 0 & \cdots & 0 \\ 0 & v_2 F_{22} Y_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_n F_{nn} Y_n \end{array} \right] + \left[\begin{array}{cccc} 0 & v_1 L_{12} Y_2 & \cdots & v_1 L_{1n} Y_n \\ v_2 L_{21} Y_1 & 0 & \cdots & v_2 L_{2n} Y_n \\ \vdots & \vdots & \ddots & \vdots \\ v_n L_{n1} Y_1 & v_n L_{n2} Y_2 & \cdots & 0 \end{array} \right] \\
&\quad + \left[\begin{array}{cccc} 0 & v_1 F_{12} Y_2 & \cdots & v_1 F_{1n} Y_n \\ v_2 F_{21} Y_1 & 0 & \cdots & v_2 F_{2n} Y_n \\ \vdots & \vdots & \ddots & \vdots \\ v_n F_{n1} Y_1 & v_n F_{n2} Y_2 & \cdots & 0 \end{array} \right] \quad \#(5)
\end{aligned}$$

Total final demand ($\sum_{r=1}^n(Y^r) = \sum_{r=1}^n Y^{rr} + \sum_{s \neq r} E^{rs}$) can separately present (i) the (aggregated) domestic final demand ($\sum_{r=1}^n Y^{rr}$) and (ii) (aggregated) gross export ($\sum_{s \neq r} E^{rs} = \sum_r E^r$) of each country in equation (5.1)⁷. Here the VA transaction between trade partners can be written,

$$\begin{aligned}
T_v &= \left[\begin{array}{cccc} v_1 D_{11} Y_{11} & 0 & \cdots & 0 \\ 0 & v_2 D_{22} Y_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_n D_{nn} Y_{nn} \end{array} \right] + \left[\begin{array}{cccc} v_1 F_{11} Y_{11} & 0 & \cdots & 0 \\ 0 & v_2 F_{22} Y_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_n F_{nn} Y_{nn} \end{array} \right] + \left[\begin{array}{cccc} 0 & v_1 L_{12} Y_{22} & \cdots & v_1 L_{1n} Y_{nn} \\ v_2 L_{21} Y_{11} & 0 & \cdots & v_2 L_{2n} Y_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ v_n L_{n1} Y_{11} & v_n L_{n2} Y_{22} & \cdots & 0 \end{array} \right] \\
&\quad + \left[\begin{array}{cccc} 0 & v_1 F_{12} Y_{22} & \cdots & v_1 F_{1n} Y_{nn} \\ v_2 F_{21} Y_{11} & 0 & \cdots & v_2 F_{2n} Y_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ v_n F_{n1} Y_{11} & v_n F_{n2} Y_{22} & \cdots & 0 \end{array} \right] \\
&\quad + \left[\begin{array}{cccc} v_1 D_{11} E_1 & 0 & \cdots & 0 \\ 0 & v_2 D_{22} E_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_r D_{nn} E_n \end{array} \right] + \left[\begin{array}{cccc} v_1 F_{11} E_1 & 0 & \cdots & 0 \\ 0 & v_2 F_{22} E_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_n F_{nn} E_n \end{array} \right] + \left[\begin{array}{cccc} 0 & v_1 L_{12} E_2 & \cdots & v_1 L_{1n} E_n \\ v_2 L_{21} E_1 & 0 & \cdots & v_2 L_{2n} E_n \\ \vdots & \vdots & \ddots & \vdots \\ v_n L_{n1} E_1 & v_r L_{n2} E_2 & \cdots & 0 \end{array} \right] \\
&\quad + \left[\begin{array}{cccc} 0 & v_1 F_{12} E_2 & \cdots & v_1 F_{1n} E_n \\ v_2 F_{21} E_1 & 0 & \cdots & v_2 F_{2n} E_n \\ \vdots & \vdots & \ddots & \vdots \\ v_n F_{n1} E_1 & v_n F_{n2} E_2 & \cdots & 0 \end{array} \right] \quad \#(5.1)
\end{aligned}$$

3. Value-Added in Export

This paper is interested in examining the production activity of the VA embodied in intermediate export. In this regard, we modified equation (5.1) by focusing on a monetary basis of only export $E_{k^*}^r$ ($\cong E_{k^*}^{rs} + E_{k^*}^{rt} + \dots E_{k^*}^{rn}$) and then created VA in export (T_v^E) as displayed in equation (5.2*).

$$T_v^E = \begin{bmatrix} v^r B^{rr} & v^r B^{rs} & \cdots & v^r B^{rn} \\ v^s B^{sr} & v^s B^{ss} & \cdots & v^s B^{sn} \\ \vdots & \vdots & \ddots & \vdots \\ v^n B^{nr} & v^n B^{ns} & \cdots & v^n B^{nn} \end{bmatrix} \begin{bmatrix} E_{k^*}^r & 0 & \cdots & 0 \\ 0 & E_{k^*}^s & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{k^*}^n \end{bmatrix} = \begin{bmatrix} v^r B^{rr} E_{k^*}^r & v^r B^{rs} E_{k^*}^s & \cdots & v^r B^{rn} E_{k^*}^n \\ v^s B^{sr} E_{k^*}^r & v^s B^{ss} E_{k^*}^s & \cdots & v^s B^{sn} E_{k^*}^n \\ \vdots & \vdots & \ddots & \vdots \\ v^n B^{nr} E_{k^*}^r & v^n B^{ns} E_{k^*}^s & \cdots & v^n B^{nn} E_{k^*}^n \end{bmatrix} \quad (5.1^*)$$

We follow Koopman et al. (2010; 2014) to identify gross export-related VA. The diagonal of the matrix stands for DVA, and the sum of columns (without diagonal) represents FVA. We start interpreting the two-country model setup as (Japan) r and (ROW) s and gradually extend this in the n-by-n sector-country pairs model. DVA can be written,

$$DVA^r = [v^r B^{rr} E^{rs}]$$

Briefly, this DVA^r explains that Japanese domestic VA relies on its industrial linkages through its export. Next step, we introduce VA embodied in intermediate export which leads to a more complex VA integration because of leading double counting through the given level of final demand perspective. As for this issue, we consistently decompose each country's final demand with their domestic dependence matrix and then

⁷ We disaggregate final demand such as domestic intermediate use, domestic final demand use, and export use. After that, this study further decomposes gross export into intermediate export (A^{rs}, X^s) and final export (Y^{rs}) (see Koopman et al. (2010; 2012) and Wang et al. (2013)). Export decomposition can be rearranged $X = A^d X + Y^d + A^m X + Y^m = A^d X + Y^d + E$. In matrix form: $X^r = A^{rr} X^r + Y^{rr} + [\sum_{s \neq r}^n A^{rs} X^s + \sum_{s \neq r}^n Y^{rs}]$. Let's rewrite based on equation (5.1): $X^r = [D^{rr} + F^{rr} + L^{rs} + F^{rs}] Y^{rr} + E^{rs} + E^{rt} + \dots + E^{rn}$. National and feedback can be, respectively, presented $X^r = X_B^r + X_F^r = [D^{rr} (Y^{rr} + E^{rs} + E^{rt} + \dots + E^{rn})] + [(F^{rr} + L^{rs} + F^{rs}) (Y^{rr} + E^{rs} + E^{rt} + \dots + E^{rn})]$.

use it as they are absorbed. This means we can separate/extract them by addressing/reducing the double-counting parts. Separated Intermediate export ($A^{rs} \cdot X^s$) and final export (Y^{rs}) can be written,

$$\begin{aligned} DVA^r &= v^r B^{rr} Y^{rs} + v^r B^{rr} (A^{rs} X^s) \\ &= v^r (D^{rr} + F^{rr}) Y^{rs} + v^r (D^{rr} + F^{rr}) (A^{rs} X^s) \end{aligned} \quad (6)$$

Condition 1: the main assumption in this paper relies on no productivity differences (or constant/fixed productivity)⁸ among sector-country pairs. This means that the value of goods, which travel among countries, can be the same for reexport and reimport. In other words, when intermediate input crosses the border at least twice, we ensure that the monetary base of the initial value is stable/fixed no matter how many times the products cross the borders.

PROPOSITION 1: $D^{rr} A^{rs} D^{ss}$, $B^{rr} A^{rs} B^{ss}$, and $F^{rr} A^{rs} F^{ss}$ can replace with L^{rs} , B^{rs} , and F^{rs} , respectively, under condition 1 and Walras' law⁹ (see Los et al. (2016) and Johnson (2018) for the logic of creating bilateral/multilateral (export) sectoral integration between r and s).

From now on, we can re-arrange the equation (6) based on where it is absorbed or further production stages. To find $v^r (B^{rr}) (A^{rs} X^s)$, we first write X^s which is equal to $BY^s = [B](Y^{ss} + E^{sr})$ (see equation 5.1). After that, we combine A^{rs} and X^s as $A^{rs} X^s$ which is equal to $A^{rs} [B^{ss} + B^{sr}] (Y^{ss} + Y^{sr} + [A^{sr} X^r])$. If we multiply $v^r (B^{rr})$ and $(A^{rs} X^s)$, we can get $v^r [(B^{rs} + B^{rr} A^{rs} B^{sr}) (Y^{ss} + Y^{sr} + [A^{sr} X^r])]$ ¹⁰ under proposition 1. Here we present where (or which country) absorbs the intermediate products in GVC participation as a form of final and semi-finished products if we extend the intermediate product function of future consumption.

$$\begin{aligned} DVA^r &= v^r B^{rr} Y^{rs} + v^r [(B^{rs} + B^{rr} A^{rs} B^{sr}) (Y^{ss} + Y^{sr} + [A^{sr} X^r])] \\ DVA^s &= v^s B^{ss} Y^{sr} + v^s [(B^{sr} + B^{ss} A^{sr} B^{rs}) (Y^{rr} + Y^{rs} + [A^{rs} X^s])] \\ DVA^r &= v^r (D^{rr} + F^{rr}) Y^{rs} + v^r (L^{rs} + F^{rs} + B^{rr} A^{rs} B^{sr}) (Y^{ss} + Y^{sr} + [A^{sr} X^r]) \end{aligned} \quad (6.1)$$

To show vertical integration in multilevel trade, we need to define complex integration between trade partners $B^{rr} A^{rs} B^{sr} (\cong [D^{rr} + F^{rr}] A^{rs} [B^{sr}])$. For example, when the local industrial linkages of the country (r) meet bilateral and multilateral effects ($D^{rr} A^{rs} B^{sr}$)¹¹ of s , or feedback effect of r meet bilateral and multilateral effects ($F^{rr} A^{rs} B^{sr}$)¹² of another country (s) (see BOX1). Vertical integration triggers spillovers along the value chain because upstream suppliers through relationship-specific interactions/investments affect the incentives of suppliers in downstream stages (see Alfaro et al. (2019) for more details) regarding sequential complements/substitutes in the production process.

PROPOSITION 2: products in r stimulated by s multilevel integration are to present downstreamness ($D^{rr} A^{rs} B^{sr} = \lambda^{rs}$). Products in s stimulated by demand from r multinational integration are to present upstreamness ($F^{rr} A^{rs} B^{sr} = \delta^{rs}$).

We rearrange the intermediate integration function which is presented in equation 6.2 under proposition 2.

$$\begin{aligned} DVA^r &= v^r (D^{rr} + F^{rr}) Y^{rs} + v^r (\lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs}) (Y^{ss} + Y^{sr} + [A^{sr} X^r]) \\ DVA^s &= v^s (D^{ss} + F^{ss}) Y^{sr} + v^s (\lambda^{sr} + L^{sr} + \delta^{sr} + F^{sr}) (Y^{rr} + Y^{rs} + [A^{rs} X^s]) \end{aligned} \quad (6.2)$$

⁸ Regarding increase/constant to return to scale relying on Cobb-Douglas (Leontief) Production Function, this should be perfectly matched.

⁹ $\int_{\Omega_{ij}} e_{ij}(\omega) d\omega = \int_{\Omega_{ji}} e_{ji}(\omega) d\omega, i \neq j$, where export revenue equal to import expenditure, respectively (Eaton, & Kortum, 2002).

¹⁰ National one: $X_B^s = D^{ss} \cdot Y^{ss} + D^{ss} Y^{sr} + D^{ss} A^{sr} X^r$, and if we multiply with A^{rs} then we can get $A^{rs} X_B^s$, which is equal to $(A^{rs} D^{ss} \cdot Y^{ss} + A^{rs} D^{ss} Y^{sr} + A^{rs} D^{ss} [A^{sr} X^r])$. Thus, $v^r D^{rr} (A^{rs} X_B^s)$ represents $v^r L^{rs} Y^{ss} + v^r L^{rs} Y^{sr} + v^r L^{rs} (A^{sr} X^r)$ under $L^{rs} (\cong D^{rr} A^{rs} D^{ss})$ (see footnote 6).

¹¹ $D^{rr} A^{rs} L^{sr} + D^{rr} A^{rs} F^{sr}$ explain that multinational integration from country s stimulate local country r integration, downstreamness

¹² $F^{rr} A^{rs} L^{sr} + F^{rr} A^{rs} F^{sr}$ explain that multinational integration from country r advance local country s integration, upstreamness

These equations prove that journey of intermediate trade creates a positive spillover effect as appeared ($\lambda^{rs} + \delta^{rs}$). To portray double counting, we estimate FVA under the two-country model, presenting backward linkage which stands for such as foreign goods contributing to the destination country's export regarding imported intermediate inputs.

$$\begin{aligned} FVA^r &= v^s B^{sr} E^{rs} = v^s B^{sr} Y^{rs} + v^s B^{sr} (A^{rs} \cdot X^s) = v^s (L^{sr} + F^{sr}) (Y^{rs} + [A^{rs} \cdot X^s]) \\ FVA^s &= v^r B^{rs} E^{sr} = v^r B^{rs} Y^{sr} + v^r B^{rs} (A^{sr} \cdot X^r) = v^r (L^{rs} + F^{rs}) (Y^{sr} + [A^{sr} \cdot X^r]) \end{aligned} \quad (6.3)$$

Total VA^r ($FVA^r + DVA^r$) can be written

$$\begin{aligned} VA^r &= v^r (D^{rr} + F^{rr}) Y^{rs} + v^r (\lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs}) (Y^{ss} + Y^{sr} + [A^{sr} X^r]) + v^s (L^{sr} + F^{sr}) (Y^{rs} + [A^{rs} \cdot X^s]) \\ VA^s &= v^s (D^{ss} + F^{ss}) Y^{sr} + v^s (\lambda^{sr} + L^{sr} + \delta^{sr} + F^{sr}) (Y^{rr} + Y^{rs} + [A^{rs} X^s]) + v^r (L^{rs} + F^{rs}) (Y^{sr} + [A^{sr} \cdot X^r]) \end{aligned} \quad (6.4)$$

We show double-counting regarding the sum of VA, $VA^r + VA^s = v^r (D^{rr} + F^{rr}) Y^{rs} + v^r (\lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs}) (Y^{ss} + Y^{sr} + [A^{sr} X^r]) + v^s (L^{sr} + F^{sr}) (Y^{rs} + [A^{rs} \cdot X^s]) + v^s (D^{ss} + F^{ss}) Y^{sr} + v^s (\lambda^{sr} + L^{sr} + \delta^{sr} + F^{sr}) (Y^{rr} + Y^{rs} + [A^{rs} X^s]) + v^r (L^{rs} + F^{rs}) (Y^{sr} + [A^{sr} \cdot X^r])$.

We can rewrite DVA as a method of absorbing in foreign and domestic countries (see Koopman et al., 2010; 2014). Equation (6.2) can be transferred,

$$DVA^r = v^r (D^{rr} + F^{rr}) Y^{rs} + v^r (\lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs}) [A^{rs} X^{ss} + A^{rs} X^{sr}] \quad (6.2^*)$$

Where (Y^{rs}) final goods in r export to s. ($A^{rs} X^{ss}$) intermediated absorbed in s, ($A^{rs} X^{sr}$) processed and exported back to r, as a form of products embodied in semi-finished $[A^{sr} X^r]$ or final products (Y^{sr}).

To summarize, we experience that r's intermediate export would absorb in s' domestic final demand use and intermediate gross export based on (multinational) sectoral integration (B^{rs}) processed and exported back to r if products (intermediate inputs) cross the border more than once, returning home as final or semi-finished goods (see equations 6.2; 6.2*).

3.1. Double-Counting in VA

This sub-section explains the supply chain trade in relation to cross-border production measures in line with Hummels et al., (2001) and Baldwin and Lopez (2015). The concern here is that the VA embodied in export/import might cross the borders multiple times (at least twice), causing measurement bias (Los et al., 2016). The extent to which double-counting drives up VA depends on the degree to which the country is a routed hub of good-in-process within GVCs. A growing recent literature about double-counting has been examined based on GVC's approaches (Borin & Mancini 2019; Johnson & Noguera, 2012; Koopman et al., 2014; Los et al., 2016). World Bank (WB)¹³ (2020) estimates the measurement of the double-counting which is about 2 %. Likewise, Kuboniwa estimates double-counting based on the trade balance as it is $DVA^r = v^r (B^{rr} E^{rs} - B^{rs} E^{sr}) = v^r B^{rr} E^{rs} - v^r B^{rs} E^{sr}$ where he reduces the (total) import goods and then weakens the export-import effect on DVA in r. In short, he evaluates that when s exports to r (E^{sr}), then this (E^{sr}) would be reexported to s.

We observe that when import final products reach r, they end their GVC participation in the export context. However, when r imports intermediate input and produce semi-final or final goods, after that these products in r are ready to export to s (or domestically consumed in r). In this sense, the production process starts from (origin of the) intermediate products in r that are exported to s and then finalize their production

¹³ Dataset available with Chapter 1 of the World Development Report at: <http://pubdocs.worldbank.org/en/834031570559525797/Chapter-1.zip>.

process to be either semi-finished products in r or final products in s which are ready exported to the origin of the product in r . Technically, intermediate products in r end up being either semi-finished products in r , which are ready to export to s , or final products in s , which are ready to export to r . Thus, we purpose an alternative of addressing the double-counting, that is, eliminating returning to the home part of intermediate import “ $(A^{sr}X^r)$ ” and final export “ (Y^{rs}) ” regarding r . Note that imported final products consumed (non-tradable goods) and imported raw products (without using them in the production process) are not exported.

PROPOSITION 3: this paper first reduces intermediate re-exports (returning home bias) from DVA and second, subtracts final products (containing host intermediate products) from FVA.

A. Domestic Value Added in Export

we can calculate the (net) DVA^r in the two-country model by extending equation (6.2*) under PROPOSITION 3 as seen in equation (7).

$$\begin{aligned} DVA^r &= v^r((D^{rr} + F^{rr})[Y^{rs}] + v^r(\lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs})[A^{rs}X^{ss} + A^{rs}X^{sr}] - (D^{rr} + F^{rr})[A^{rs}X^{sr}]) \\ &= v^r((D^{rr} + F^{rr})[Y^{rs}] + v^r(\lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs})[A^{rs}X^{ss}] + (\lambda^{rs} + \delta^{rs})[A^{rs}X^{sr}]) \end{aligned} \quad (7)$$

We show double-counting in the two-country model and address this by eliminating the home bias part. Follow up question is how we implement this method to the n-by-n country-sector model due to the third country effect as documented by Koopman et al., (2010; 2014) and Wang (2013). In other words, since there is more than one-way of trade, how do we track the effect of country r 's intermediate products absorbed in n countries which have trade integration with $n-1$ multi sector-country pairs? In this regard, we extend this model to more than two-way trade in n -country models by using equation (6) where we rearrange $v^r B^{rr} E^{rs}$ based on (third) partners and its trade condition among third (*countries*) l .

$$\begin{aligned} DVA^r &= v^r B^{rr} E^{rs} + v^r B^{rr} \sum_{l \neq r, s}^n E^{rl} \\ v^r B^{rr} E^{rs} &= v^r B^{rr} Y^{rs} + v^r \{\lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs}\} \left(A^{rs} X^{ss} + A^{rs} X^{sr} + \sum_{l \neq r, s}^n A^{rs} X^{sl} \right) \end{aligned} \quad (8)$$

Where final goods (Y^{rs}) in r export to s . ($A^{rs}X^{ss}$) intermediated is absorbed in s , ($A^{rs}X^{sr}$) is processed and exported back to r , and ($\sum_{l \neq r, s}^n A^{rs}X^{sl}$) is processed and exported to third countries, as we explained above. This process is straightforward due to a single country's relationship with n countries. Lastly, products ($\sum_{l \neq r, s}^n E^{rl}$) in r export to third trade partners. The question is, how do third countries distribute their export with partners (r and s) and within third countries in relation to complex/continuous production processes in multi-level integration? First, we define third countries DVA^l in export content. In the literature, third partners do not trade with each other, and instead, use aggregated third (single) countries. In this sense, we introduce ($n-2$) countries' trade with ($n-3$) countries as presented $\sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n (B^{ll} E^{lt})$ in equation (8.1).

$$\begin{aligned} DVA^l &= \sum_{l \neq r, s}^n v^l \sum_{l \neq r, s}^n X^l = \sum_{l \neq r, s}^n v^l \left(\sum_{l \neq r, s}^n (B^{ll} E^{lr}) + \sum_{l \neq r, s}^n (B^{ll} E^{ls}) + \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n (B^{ll} E^{lt}) \right), t \in l = \{t-1, t, \dots, n\} \\ &= \left(\sum_{l \neq r, s}^n v^l \right) \left(\sum_{l \neq r, s}^n (B^{ll} Y^{lr}) + \sum_{l \neq r, s}^n (B^{ll} A^{lr} X^r) + \sum_{l \neq r, s}^n (B^{ll} Y^{ls}) + \sum_{l \neq r, s}^n (B^{ll} A^{ls} X^s) + \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n B^{ll} Y^{lt} + \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n B^{ll} A^{lt} X^t \right) \end{aligned} \quad (8.1)$$

If we extend the $\sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n B^{ll} Y^{lt} + \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n B^{ll} A^{lt} X^t$, we also face continuous trade integration (see de Gortari (2019), Fally (2012), Alfaro et al., (2019), and Antras et al. (2012) who explained n of production

stages and vertical integration) as presented in equation (8.2)¹⁴. This means that intermediate goods travel among sector-country pairs and are eventually put into final goods that are shipped and consumed in x , (see BOX2).

$$\begin{aligned} & \left(\sum_{l \neq r, s}^n v^l \right) \left[\sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n B^{ll} Y^{lt} + \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n B^{ll} A^{lt} X^t \right] \\ &= \left(\sum_{l \neq r, s}^n v^l \right) \left(\sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n \left[B^{ll} Y^{lt} + B^{ll} A^{lt} X^{tt} + B^{ll} A^{lt} X^{tr} + B^{ll} A^{lt} X^{ts} + B^{ll} A^{lt} X^{tl} + \sum_{k \neq r, s, l, t}^n B^{ll} A^{lt} X^{tk} \right. \right. \\ & \left. \left. + \dots + \sum_{k \neq r, s, l, t}^n \dots \sum_{x \in n} B^{ll} A^{lx} X^{x*} \right] \right), \quad k, x \in t [= k-1, k, k+1, \dots, n] \#(8.2) \end{aligned}$$

BOX 2: Sector-Country Pairs in GVC Participation

To estimate the stage of traveling products, this paper modifies the integration of intermediate trade flow. As we defined r as a country ($r, s \in \mathcal{R}$), i as sector ($i, j \in \mathcal{J}$), and k as sector-country pairs $\{k[r(i), s(j)] \in \mathbb{K}\}$; thus, intermediate flow at sector-country pairs present $\mathbb{P}^n(k^n, k^{n-1}, \dots, k^1, r)$, where inputs are sold from k^n to the sequence $k^{n-1} \rightarrow k^{n-2} \rightarrow \dots \rightarrow k \rightarrow r$ (Japan). This stage of GVC participation in production activities can illustrate that intermediate flow at sector-country pair can explain how many (times) inputs are sold from k^n to r . For example, produced products in k^{n-1} are sold to k^{n-2} , so on and so forth, until the products arrive at k^1 and are put into final goods that are shipped and sold to consumers in r . To illustrate sector-country pairs in GVC participation at bilateral integration in the last two stages. VA can be written,

$$VA = \sum_{n=2}^{\infty} \dots \sum_{k \in \mathbb{K}} VA^n(k^n | k^2, k, r)$$

However, this becomes more complicated when intermediate inputs flow at multilateral integration (see equation 8.1). Theoretical frameworks of highly stylized sequential production do not characterize asymmetries across production stages (see Alfaro et al. (2019) for firm-level solutions). This is because the production activities of transactions among countries are not easily observed. However, de Gortari (2019) reports that this challenge can be solved by specializing input in the roundabout model (US, MEX, CAN).

Let's arrange the X^r with equation (8.1), (see Appendix A for more details)

$$DVA^r = v^r \left[B^{rr} Y^{rs} + B^{rr} A^{rs} X^s + B^{rr} \sum_{l \neq r, s}^n Y^{rl} + B^{rr} \sum_{l \neq r, s}^n A^{rl} X^{l*} + B^{rr} \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n A^{rl} X^{lt*} \right]$$

Where, $\sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n A^{rl} X^{lt*}$ that processed and exported to t (intermediate absorbed in (n-3) countries) form l countries (n-2). We can imply this disaggregation to (n-2) multi countries. In other words, if we consider third partners in multination integration (see equation (8.1)), we reach out to draw equation (8.3).

$$v^r B^{rr} \sum_{l \neq r, s}^n E^{rl} = v^r B^{rr} \sum_{l \neq r, s}^n Y^{rl} + v^r \left[\sum_{l \neq r, s}^n (\lambda^{rl} + L^{rl} + \delta^{rl} + F^{rl}) \right] \left[\sum_{l \neq r, s}^n A^{rl} X^{lr} + \sum_{l \neq r, s}^n A^{rl} X^{ls} + \sum_{l \neq r, s}^n A^{rl} X^{ll} + \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n A^{rl} X^{lt} \right] \quad (8.3)$$

The number of (t) countries' continued trade integration with others can be written

$$\begin{aligned} &= v^r B^{rr} \sum_{l \neq r, s}^n Y^{rl} + v^r \left[\sum_{l \neq r, s}^n (\lambda^{rl} + L^{rl} + \delta^{rl} + F^{rl}) \right] \left[\sum_{l \neq r, s}^n A^{rl} X^{lr} + \sum_{l \neq r, s}^n A^{rl} X^{ls} + \sum_{l \neq r, s}^n A^{rl} X^{ll} + \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n A^{rl} X^{lt*} + \dots \right. \\ & \left. + \sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n \sum_{k \neq r, s, l, t}^n \dots \sum_{x \in n} A^{lx} X^{x*} \right] \quad (8.4^*) \end{aligned}$$

Where, $\sum_{l \neq r, s}^n Y^{rl}$ that are final goods in r exported to third partners. $\sum_{l \neq r, s}^n A^{rl} X^{lr}$ that are processed and exported back to r form (l) third partners, $\sum_{l \neq r, s}^n A^{rl} X^{ls}$ that are processed and exported to s form l .

¹⁴ We prefer simple explanation of the integration B^{ll} , instead of drawing spillover effect $\lambda^{lr} + L^{lr} + \delta^{lr} + F^{lr} + \dots$ regarding intermediate product function of future consumption for only this equation.

However, there is a challenge of describing the $\sum_{l \neq r, s}^n A^{rl} X^{ll}$ which technically accounts for intermediate absorbed in l domestically (see equations 6.2 and 6.2*). The problem is how we observe the intermediate distribution among or within (1) countries. To address this multi countries' integration, we created $\sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n A^{rl} X^{lt}$ that explains trade in n-3 countries. In short, (imported) intermediate products are processed (to produce semi-final and final products) and then exported to r ($\sum_{l \neq r, s}^n A^{rl} X^{lr}$), s ($\sum_{l \neq r, s}^n A^{rl} X^{ls}$), l ($\sum_{l \neq r, s}^n A^{rl} X^{ll}$), as well as distrusted (within) its trade partners ($\sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n A^{rl} X^{lt}$) in t , presenting the paradox of intertwined trade in n-way of trade relations. That is because the number of (t) countries have trade integration with r , s , l , and $(t - 1)$ countries. In other words, this process continues until intermediate goods finalize their journey to be final goods that are shipped and sold to consumers in x as ($\sum_{l \neq r, s}^n \sum_{t \neq r, s, l}^n \sum_{k \neq r, s, l, k}^n \dots \sum_{x \in n} B^{ll} A^{lx} X^{x*}$).

In general, equation (8.2) shows that we cannot track intermediate export and the origin of goods. Mathematically speaking, we could not create multi-faceted intermediate trade equations that explain the "intermediate export/import" journey across sector-country pairs. Therefore, there is no way to address double counting in n-way of trade in multi-country (see Appendix A) but two-way of trade in two countries in the world (if intermediate crosses border only twice). In light of the limitation and challenge, we can build DVA by combining equations (8) and (8.3) (as can be interpreted *Japan* (r), *Africa* (s), and $\sum_{l \neq t}^n ROW$ (l)), as can be seen in equation (9).

$$DVA^r = v^r [D^{rr} + F^{rr}] \left(Y^{rs} + \sum_{l \neq r, s}^n Y^{rl} \right) + v^r \{ \lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs} \} \left(A^{rs} X^{ss} + A^{rs} X^{sr} + \sum_{l \neq r, s}^n A^{rs} X^{sl} \right) + v^r \left(\sum_{l \neq r, s}^n \{ \lambda^{rl} + L^{rl} + \delta^{rl} + F^{rl} \} \right) \left(\sum_{l \neq r, s}^n \left[A^{rl} X^{ll} + A^{rl} X^{lr} + A^{rl} X^{ls} + \sum_{t \neq r, s, l}^n A^{rl} X^{lt} \right] \right) \#(9)$$

PROOF:

we rewrite $v^r B^{rr} A^{rs} X^s$ and $v^r B^{rr} \sum_{l \neq r, s}^n A^{rl} X^l$, by applying equation (8.1) for multi-level interconnection. Note that traveling goods bring upstream and downstream effects from bilateral and multilateral integration.

$$v^r B^{rr} [A^{rs} X^s] = v^r [B^{rr}] \left(A^{rs} [B^{ss} + B^{sr}] \left[Y^{ss} + Y^{sr} + A^{sr} X^r + \sum_{l \neq r, s}^n Y^{sl} + \sum_{l \neq r, s}^n (A^{sl} X^l) \right] \right) \#(9.1) \\ \cong v^r \left(\{ \lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs} \} \left\{ A^{rs} X^{ss} + A^{rs} X^{sr} + \sum_{l \neq r, s}^n A^{rs} X^{sl} \right\} \right) \\ B^{rr} \sum_{l \neq r, s}^n A^{rl} X^l \cong v^r \left(\sum_{l \neq r, s}^n \{ \lambda^{rl} + L^{rl} + \delta^{rl} + F^{rl} \} \right) \left(\sum_{l \neq r, s}^n \left(Y^{ll} + Y^{lr} + A^{lr} X^r + Y^{ls} + A^{ls} X^s + \sum_{t \neq r, s, l}^n Y^{lt} + \sum_{t \neq r, s, l}^n A^{lr} X^{rt} \right) \right) \#(9.2)$$

B. Foreign Value Added in Export

Backward linkage illustrates to what degree foreign goods contribute to the destination country's export regarding imported intermediate inputs. FVA is calculated under the two countries model and then presents an extended version of the n-by-n sector-country model. Compared to DVA estimation in equation (7), we did subtract the $v^r B^{rs} (A^{sr} X^r)$ from r 's DVA, which appeared in s 's FVA in equation (6.4). Similarly, we exclude final export (Y^{rs}) which is out of the scope of reexporting bias but includes destination intermediate products regarding where double contributions come from under Proposition 3.

$$\begin{aligned}
FVA^r &= v^s B^{sr} E^{rs} = v^s (B^{sr})(Y^{rs} + [A^{rs}X^s]) \\
&= v^s B^{sr} Y^{rs} + v^s B^{sr} (A^{rs} X^s) - v^s B^{sr} Y^{rs} = v^s [(L^{sr} + F^{sr})(A^{rs} X^s)]
\end{aligned} \tag{10}$$

We rearrange FVA regarding more than a two-country model by applying the logic of equation (8.1). As for the consistency of the VA in total, we prefer to present FVA in multi countries without eliminating the double contribution part. Form of three-country, *Japan* (r), *Africa* (s), and $\sum_{t=3}^n ROW$ (l), can be written

$$\begin{aligned}
FVA^r &= v^s B^{sr} E^{rs} + \sum_{l \neq r,s}^n (v^l) \sum_{l \neq r,s}^n (B^{lr} E^{rl}) \\
&= v^s (L^{sr} + F^{sr})(Y^{rs} + A^{rs} X^s) + \left(\sum_{l \neq r,s}^n (v^l) \sum_{l \neq r,s}^n (L^{lr} + F^{lr}) \right) \left(\sum_{l \neq r,s}^n Y^{rl} + \sum_{l \neq r,s}^n A^{rl} X^{l^*} + \sum_{l \neq r,s}^n \sum_{t \neq r,s,l}^n A^{rl} X^{lt} \right) \#(11)
\end{aligned}$$

In short, going back to the main question in this work, is it clear to eliminate the return home part? If we extend three ways or more than three ways of trade integration, this will lead to much more complex trade integration in double-counting (see equations (8.1; 8.2; 8.4)). As a result, we found that the return home part becomes impossible to track and then eliminate. The possible reasons are (i) complex multi countries' integration ($\sum_{l \neq r,s}^n A^{rl} X^{l^*}$) and within trade partners paradox ($\sum_{l \neq r,s}^n \sum_{t \neq r,s,l}^n A^{rl} X^{lt^*}$), (ii) hidden/unknown traveling product (production stages) ($\sum_{l \neq r,s}^n \sum_{t \neq r,s,l}^n \sum_{k \neq r,s,l,k}^n \dots \sum_{x \in n} B^{ll} A^{lx} X^{x^*}$). Therefore, we purpose optimal double counting method regarding inverse matrix in equation (5.1) that explain sectoral linkages through which we can eliminate depend on how many times goods return to origin of home country.

3.2. Data Source: Input-Output (I/O) Database

To present GVC participation in an aggregated dataset, Input-Output (I/O) databases are most commonly used. There are major sources of the data available: The GTAP Database, the Institute of Developing Economies and Japan External Trade Organization (IDE-JETRO) Asian I/O Tables, the World Input-Output Database (WIOD), the Eora Global Supply Chain Database (Eora MRIO), and the Organization for Economic Co-operation and Development (OECD) Inter-Country Input-Output Tables (OCIO).

This paper developed a GTAP-MRIO table based on Peters et al. (2011) and Walmsley et al. (2014) and applied GTAP data due to the availability/inclusion of the African countries. However, this method can be easily applied to other IO tables concerning each data limitation and advantage. We used the GTAP-MRIO version 10A database, which was launched in 2020 and accounts for 65 sectors in each of the 141 countries/regions with the reference year 2014 (Carrico et al. 2020, pp.1-14). Analysis of the aggregated data relies on 3 regions (countries) and 4 sectors (see Table B1 and B2).

3.3. Result of the Estimate

Briefly, transmitting VA relying on diverse intermediate and final exports/imports across sector-country pairs would be a different impact. Japan's intermediate exports account for 79% of agriculture and 62% of manufacturing. The largest Japanese exporting partners are China (22%) and the USA (19%), followed by ASEAN (15%) and EU (12%) members. Japanese import highly depends on China and the US by 21.9% and 9.8%, respectively. Japanese trade with aggregated African countries presents approximately 2% of total Japanese trade over a 20-year period (Biyik, 2022b). Japan exports mainly manufacturing (high tech) goods while African countries export agricultural and raw (oil/materials) goods (Biyik, 2022a).

Here we investigate optimal double-counting coefficients as percent contribution to sectoral output. To estimate a single country level, our sample relies on the Japanese market and its interaction with Africa and

ROW. Table 1 shows what percent of the national/international sectoral linkages contribute to sectoral outcomes regarding inverse matrix in equation (5.1/5.1*) (see Table B2 for more details of the matrixes). It is common sense that domestic sectoral linkages ($\sum_{r=1}^3 \sum_{i=1}^4 D_i^{rr}$) plays an important role in its sectoral output. If these sectors have a high proxy of trade with their trade partners, the feedback effect from multilevel integration ($\sum_{r=1}^3 \sum_{i=1}^4 F_i^{rr}$) becomes an important supplier. For example, while Japanese manufacturing has a high volume of integration with other countries, agriculture in Japan has a low interaction with the world economy. Empirically, bilateral integration ($\sum_{r,s=1,1}^3 \sum_{i=1}^4 L_i^{rs}$) in the Japanese manufacturing boosts its sectoral output (VA) by 3.1% as well as feedback effect from multilevel integration ($\sum_{r,s=1,1}^3 \sum_{i=1}^4 F_i^{rs}$) by 0.6% (see Table 1). This means if the world GDP grows fast (2015-18), the most likely scenario is that the manufacturing industry in Japan is to get the most benefit and contribute its outputs by 3.7%.

Table 1: Contribution of Sectoral Linkages (Sourcing-Side) to Sectoral Outcome (%)

		$\sum_{r=1}^3 \sum_{i=1}^4 v_i^r$	$\sum_{r=1}^3 \sum_{i=1}^4 B_i^{rr}$	$\sum_{r,s=1,1}^3 \sum_{i=1}^4 B_i^{rs}$	$\sum_{r=1}^3 \sum_{i=1}^4 D_i^{rr}$	$\sum_{r=1}^3 \sum_{i=1}^4 F_i^{rr}$	$\sum_{r,s=1,1}^3 \sum_{i=1}^4 L_i^{rs}$	$\sum_{r,s=1,1}^3 \sum_{i=1}^4 F_i^{rs}$
Japan	AGR	0.380	99.82%	0.18%	99.7869%	0.0317%	0.1544%	0.0270%
	MN	0.230	99.59%	0.41%	99.4859%	0.1010%	0.3567%	0.0565%
	MANF	0.277	96.31%	3.69%	95.6490%	0.6629%	3.1278%	0.5603%
	SRV	0.594	98.73%	1.27%	98.5112%	0.2178%	1.0931%	0.1779%
Japan Average		0.482	98.46%	1.54%	98.1812%	0.2750%	1.3178%	0.2260%
Africa	AGR	0.714	99.29%	0.71%	99.2368%	0.0573%	0.6073%	0.0986%
	MN	0.620	86.62%	13.38%	85.8669%	0.7522%	9.1372%	4.2437%
	MANF	0.311	98.20%	1.80%	98.0210%	0.1804%	1.4086%	0.3900%
	SRV	0.562	97.64%	2.36%	97.4777%	0.1656%	1.7048%	0.6519%
ROW	AGR	0.473	87.11%	12.89%	86.9739%	0.1404%	12.7568%	0.1289%
	MN	0.383	56.66%	43.34%	56.2998%	0.3644%	42.9900%	0.3458%
	MANF	0.269	66.80%	33.20%	66.2941%	0.5048%	32.7256%	0.4754%
	SRV	0.555	75.90%	24.10%	75.5691%	0.3336%	23.7739%	0.3235%
All Average		0.459	84.73%	15.27%	84.4182%	0.3166%	14.6248%	0.6404%

Note: v is the proxy of VA in total output. This analysis relies on the Leontief inverse matrix (see equation 5.1)

Source: GTAP 10A MRIO Database, author's estimation.

However, another concern is that products crossing the border at least twice (or returning the product of origin country as hypothetical) boost VA extra through the sectoral linkages as bilateral ($\sum_{r=1}^3 \sum_{i=1}^4 v_i^r L_i^{rs}$) or multilateral ($\sum_{r=1}^3 \sum_{i=1}^4 v_i^r B_i^{rs}$) integration. In other words, if the intermediate product crosses the border, the double-counting concerning the back-and-forth intermediate goods trade is an asset of around 1.3% at the bilateral level or 1.5% at the multilateral level regarding country-level data¹⁵ as an average estimation of contributions (see Table 1). Statistically speaking, sectoral distribution of international sectoral linkages, which leads overestimate VA in some conditions, is to display around 1.5% if the (average) product crosses the border only twice. This result is also consistent with WB's (2020) estimation such as 1.7% in Germany. In light of existing empirical papers, when we imply Koopman et al. (2014) approach where a percent of (per country's) final and intermediate demand in total with the sectoral linkages results in decreasing bias measurement value in VA because the sectoral distribution of international sectoral linkages becomes very small coefficient¹⁶.

¹⁵ Estimation of VA relying on an aggregated group of countries (ROW) data shows a high coefficient value that causes overestimation bias (see Bems and Kikkawa (2021) for firm-level contribution). This result can change versa if the analysis relies on the Ghosh approach, see Table B4 in the appendix for more information.

¹⁶ However, GVC participation to pile up in the region due to the weak role of global governance (WTO) and regional FTAs. This significantly raises the concern about bias estimation in VA regarding double-counting consequences of the great unbundling.

4. Technological spillover in GVC participation

A growing recent literature has discussed the trade channels of knowledge transfer¹⁷ regarding purchases from abroad (imports) and sales abroad (exports) (Atkin et al., 2017; Buera & Oberfield, 2020). This is because suppliers and consumers in GVC often have a high degree of interdependence on the share of international production which requires face-to-face interactions and thus facilitates the transfer of know-how and tacit knowledge. Moreover, we also experience that multilateral/vertical integration creates the positive spillover effect (see equations 6.2 ;6.2*; 7).

This paper considers the knowledge spillovers and the relationship between patenting and GVC participation across sector-country pairs at bilateral and multilateral levels. As highlighted, firms that participated in GVCs are more productive¹⁸ (see Baldwin and Lopez-Gonzalez (2015) and Baldwin and Yan (2014) for more information); thus, intermediate products embodied in various sector-country goods travel among industries and thus stimulate VA more due to the (knowledge) spillover effect regarding bilateral and multilateral interaction (Zolas & Lybbert, 2022; Tajoli & Felice, 2018). This is because knowledge is embodied in traveling goods¹⁹ (Halpern, Koren, & Szeidl, 2015; Keller, 2010) and creates a positive impact on sectoral output. As a result, increased international patenting flows are associated with greater value-added production.

We created the knowledge flow model based on Bottazzi and Peri (2003).

$$\sum \eta_{ij}^{rs} = \sum C_j^s \prod_{s \neq r} IMP_{ij}^{rs} \quad (12)$$

Where η_{ij}^{rs} is knowledge flow in relation to distribution probability in export. IMP is the international knowledge spillovers embodied in trade goods and C is the other variables, which have an impact on knowledge flow in regions such as distance and languages.

PROPOSITION 4: Knowledge flow relying on the production process among sector-country pairs at multilateral integration is to display jointly marginal distributional impact.

The total effect relying on the production stages of continuum inputs stands for a constant variable, $\zeta_{r(i)s(j)}^{\dagger}$, as called unknown of traveling product among countries (see equation 8.2; 8.4). Thus, $\zeta_{r(i)s(j)}^{\dagger}$ (the distribution of the joint probability²⁰) formulates $1 + (\eta_{ij}^{rs})^1 + (\eta_{ij}^{rs})^2 + \dots + (\eta_{ij}^{rs})^{\dagger-1}$, where $\dagger - 1$ ²¹ account for how many times a product crosses a border (production stages) across sector-country pairs.

$$\zeta_{r(i)s(j)}^{\dagger} = \begin{cases} 1, & \sum \eta_{ij}^{rs} = 0 \\ \frac{1}{1 - \eta_{ij}^{rs}}, & 0 < \sum \eta_{ij}^{rs} < 1 \end{cases}, \quad \dagger \approx \infty \quad (12.1)$$

¹⁷ transmitted through (i) international trade in final, intermediate, and capital goods, (ii) FDI, (iii) migration of educated (i.e. scientists and engineers) people, (iv) publications in technical journals and scientific papers, (v) international mergers and acquisitions, and (vi) foreign technology payments such as consulting services and the financing of R&D (Auboin et al., 2021; Hall et al., 2010).

¹⁸ As Melitz's model (2003) indicates, firms with higher-than-average productivity (cutoff of survival) are likely to export (participating in GVC).

¹⁹ For example, developing countries can (i) access new knowledge by importing intermediate inputs and (ii) export their products to more technologically advanced countries, learning advanced technologies used in export markets (Atkin et al., 2017; Keller, 2021).

²⁰ See Baldwin and Yan (2014) who stated "...GVC initializers become more productive, and this better performance accumulates over time."

The jointly cumulative is line with the industrial value-added propagation length through vertical (forward and backward) integration in supplier and consumer (see Dietzenbacher et al. (2005) and Antràs et al. (2012) for details of the average propagation lengths)

²¹ Japanese VA embodied in its timber export to Thailand in which is used in a temple construction at one time.

$\zeta_{r(i)s(j)}^\dagger = 1$ refers to products that complete their GVC participation as domestically used by the destination country, with no relation to export content (or no effect on the production chain in export). Otherwise, this can be diverse among sector-country pairs (Eaton & Kortum, 1999; 2002).

To merge the knowledge flow with GVC participation²², we employed the logic of Trefler and Zhu (2010) and Koopman et al. (2010) where we modified equation (5.1*) by multiplying $\zeta_{r(i)s(j)}^\dagger$ (n-by-n diagonal matrix) to both sides, as presented in equation (5.1**). This function explains the coefficients of the knowledge flow in a location that makes the inherently sequential.

$$\zeta^\dagger T_v^E = \begin{bmatrix} \zeta_r^\dagger & 0 & \dots & 0 \\ 0 & \zeta_s^\dagger & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \zeta_n^\dagger \end{bmatrix} \begin{bmatrix} v^r B^{rr} E_{k^*}^r & v^r B^{rs} E_{k^*}^s & \dots & v^r B^{rn} E_{k^*}^n \\ v^s B^{sr} E_{k^*}^r & v^s B^{ss} E_{k^*}^s & \dots & v^s B^{sn} E_{k^*}^n \\ \vdots & \vdots & \ddots & \vdots \\ v^n B^{nr} E_{k^*}^r & v^n B^{ns} E_{k^*}^s & \dots & v^n B^{nn} E_{k^*}^n \end{bmatrix} = \begin{bmatrix} \zeta_r^\dagger v^r B^{rr} E_{k^*}^r & \zeta_r^\dagger v^r B^{rs} E_{k^*}^s & \dots & \zeta_r^\dagger v^r B^{rn} E_{k^*}^n \\ \zeta_s^\dagger v^s B^{sr} E_{k^*}^r & \zeta_s^\dagger v^s B^{ss} E_{k^*}^s & \dots & \zeta_s^\dagger v^s B^{sn} E_{k^*}^n \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_n^\dagger v^n B^{nr} E_{k^*}^r & \zeta_n^\dagger v^n B^{ns} E_{k^*}^s & \dots & \zeta_n^\dagger v^n B^{nn} E_{k^*}^n \end{bmatrix} \quad (5.1^{**})$$

This equation explains the location of the knowledge spillover and its relationship with production activities that has a direct positive impact on export; thus, this allows us to calibrate sector-country pairs-related technological influence and the impact of its contribution on VA in export.

The aim here is to use this global input-output matrix to determine the know-how dissemination²³ content of the intermediates used in the production stages. In this regard, we modified DVA and FVA regarding $\sum_{s \neq r} E_{f,i,j}^{rs} = [\sum_{s \neq r} \sum_{i=1}^n \sum_{j=1}^3 (Y_{i,f}^{rs}) + \sum_{s \neq r} \sum_{i,j=1,1}^n (A_{i,j}^{rs} X_j^s)]$ ²⁴ under proposition 4 (see equation 5.1**),

$$\begin{aligned} DVA^r &= v^r [D^{rr} + F^{rr}] \left(\sum_{s \neq r} \sum_{i,j=1}^n \sum_{f=1}^3 (\zeta_{r(i)s(j)}^\dagger) (Y_{i,f}^{rs}) + \sum_{l \neq r,s} \sum_{i,j=1}^n \sum_{f=1}^3 (\zeta_{r(i)l(i)}^\dagger) (Y_{i,f}^{rl}) \right) \\ &\quad + v^r \{ \lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs} \} \left(\sum_{i,j=1,1}^n (\zeta_{r(i)s(j)}^\dagger) [A_{i,j}^{rs} X_j^{ss}] + \sum_{i,j=1,1}^n (\zeta_{r(i)s(j)}^\dagger) [A_{i,j}^{rs} X_j^{sr}] \right) \\ &\quad + \sum_{l \neq r,s} \sum_{i,j=1,1}^n (\zeta_{r(i)s(j)}^\dagger) [A_{i,j}^{rs} X_j^{sl}] \\ &\quad + v^r \left(\sum_{l \neq r,s} \{ \lambda^{rl} + L^{rl} + \delta^{rl} + F^{rl} \} \right) \left(\sum_{l \neq r,s} \sum_{i,j=1,1}^n (\zeta_{r(i)l(i)}^\dagger) [A_{i,j}^{rl} X_j^{ll}] + \sum_{l \neq r,s} \sum_{i,j=1,1}^n (\zeta_{r(i)l(i)}^\dagger) [A_{i,j}^{rl} X_j^{lr}] \right) \\ &\quad + \sum_{l \neq r,s} \sum_{i,j=1,1}^n (\zeta_{r(i)l(i)}^\dagger) [A_{i,j}^{rl} X_j^{ls}] + \sum_{l \neq r,s} \sum_{t \neq r,s,l} \sum_{i,j=1,1}^n (\zeta_{r(i)l(i)}^\dagger) [A_{i,j}^{rl} X_j^{lt}] \quad \#(9^*) \\ FVA^r &= v^s (L^{sr} + F^{sr}) \left(\sum_{f=1}^3 \zeta_{s(j)r(i)}^\dagger (Y_f^{rs}) + \sum_{i,j=1,1}^n \zeta_{s(j)r(i)}^\dagger [A_{i,j}^{rs} X_j^s] \right) \\ &\quad + \left(\sum_{l \neq r,s} (v^l) \sum_{l \neq r,s} (L^{lr} + F^{lr}) \right) \left(\sum_{l \neq r,s} \sum_{f=1}^3 \zeta_{l(j)r(i)}^\dagger (Y_f^{rl}) + \sum_{l \neq r,s} \sum_{i,j=1,1}^n \zeta_{l(j)r(i)}^\dagger [A_{i,j}^{rl} X_j^{ls}] \right) \\ &\quad + \sum_{l \neq r,s} \sum_{t \neq r,s,l} \sum_{i,j=1,1}^n \zeta_{l(j)r(i)}^\dagger [A_{i,j}^{rl} X_j^{lt}] \quad \#(11^*) \end{aligned}$$

²² This logic is in line with Zolas and Lybbert (2022), who found "...increased international patenting inflows are associated with greater value-added production.", and Baldwin and Yan (2014), who reported "...the benefits of GVC participation extended across many industries."

²³ See, for example, Nishioka and Ripoll (2012) for measurement of productivity from imports of intermediates related to R&D-intensive industries using the international input-output table.

²⁴ This represent sum of the (aggregated) gross export in country r of (i) the final goods $\sum_{s \neq r} \sum_{i=1}^n \sum_{j=1}^3 (Y_{i,f}^{rs})$, which account for the sum row of households, governments, and investments, and (ii) intermediate products $\sum_{i,j=1,1}^n (A_{i,j}^{rs} X_j^s)$, which accounts for the sum row of n-by-n industries.

This study aims to complete the GVC participation with the technological spillover effect in relation to vertical (upstream and downstream) interaction in literature. The continuous trade and its spillover effects in the GVC involvement document that we can roughly estimate traveling intermediate products among countries regarding national/international sectoral linkages with its technological spillover effect. Note that this should be diverse and heterogenous among sector-country pairs (Eaton, & Kortum, 1999). For example, such electronic inputs in production stages cross the border more than agricultural inputs among sector-country pairs. This diversity of input products across sector-country pair result in different technological spillover effect on trade at bilateral and multilevel integration.

4.1. Data Source: Patent Panel Database

The number of patent citations is often used as an indicator of knowledge flow (Nabeshima et al., 2018). To estimate the knowledge flow coefficient, the patent citation data between 2001 and 2010 from PATSTAT and trade data at the International Standard Industrial Classification 4-digit level from UN Comtrade can be obtained. The variables of the distance, language, and border from CEPII GeoDist to quantify (Japanese) trade partners' geographic characteristics were obtained (see Table B3; B4). The knowledge flow classification of patents and industry classifications of trade goods were merged based on the concordance table as documented by Schmoch et al. (2003).

In the consistency of the technological spillover effect, trade data could be linked with the patent citation data. Also, this paper's logic is in line with Bottazzi and Peri (2003) who found that knowledge spillovers decay with distance. Thus, the dataset covers 14 East Asian countries because of data availability and trade flow (industrial networks) within the region²⁵ (see Nabeshima et al. (2018; 2017) for more information).

4.2. Econometric Model: Basic Specification

Our empirical models followed by Nabeshima et al. (2018) are determined. Solving for the log linearizing system in equation (12), we obtain the following approximate relationship between Δ patent citations and import values:

$$\ln\left(\sum \eta_{ij}^{rs}\right) = \beta_0 + \beta_1 IMP_{ij}^{rs} + \beta_2 Control_{rs} + \alpha_t + \chi_{ij} + \varepsilon_{rs} \quad (Rg. 1)$$

$$\ln\left(\sum \eta_{ij}^{rs}\right) = \exp[\beta_0 + \beta_1 IMP_{ij}^{rs} + \beta_2 Control_{rs} + \alpha_t + \chi_{ij}] + \varepsilon_{rs} \quad (Rg. 2)$$

Where, $\ln(\sum \eta_{r(i)s(j)})$ represents the log of the number of patent citations, which accounts for all applicants in country r to country s in industry ij in year t, as a percentage of knowledge flow. IMP_{ij}^{rs} represents bilateral trade flow of imports. Control variables present a log of Distance-based on each country's capital, as well as a dummy of the Language and Border. χ_{ij} captures sector-fixed effect and α_t captures year-fixed effect. Lastly, the term ε_{rs} is the error/disturbance term and contains unobserved factors. Patenting data are non-negative; thus, we employed Negative Binomial Regression (NBReg) and Poisson Pseudo-likelihood regression with Multiple Levels of fixed effects (PPML) because of the distribution of the number of patent citations (see Table B3; B4).

²⁵ The knowledge transfer more efficiently in a specific region (Piermartini and Rubínová, 2021). This is because there is unbundling (Baldwin, 2006) that has created complex and regionally intense GVC participation over the past few decades. For example, the world's manufacturing outputs rely on 38% in East-Asia, 19% in North-America, and 20% in Europe (Baldwin, 2018; Li et al., 2019).

4.3. Result of the Estimate

In general, multinational firms relocating their productions such as designing, producing, and assembling parts and components due to the most cost-effective location can/should exchange knowledge when their products meet border restrictions (Baldwin and Lopez-Gonzalez, 2015; WB, 2020). This expansion of GVC participation allows inter-firm information links between countries (Baldwin, 2018). The results of knowledge flow between nations promote productivity outcomes. Empirical studies show that international trade stimulates the cross-border flow of technology (Nabeshima et al. 2018).

Table 2 shows the knowledge flow embodied in imports among trade partners. To estimate a single country level, our sample relies on the Japanese market. Since the dependent variable is in logarithms, the estimated coefficients correspond to elasticities as presented with percentage changes. As a form of learning-by-exporting/importing, it results in an increase in productivity stemming. If imports increase by 1, the knowledge flow coefficient results in an increase between 0.018% and 0.643% for all East Asian countries. The mean of the coefficient for Japan presents 0.058%, as well as 0.126% in Agriculture, 0.072% in Mining, and 0.070% in Manufacturing (see Table 2). Here we can estimate knowledge flow ($\zeta_{r(i)s(j)}^{\dagger}$) regarding infinity production stages. That is, $\zeta_{r(i)s(j)}^{\dagger}$ accounts for $1 + (0.058)^1 + (0.058)^2 + \dots + (0.058)^{\infty}$.

Table 2: Pooled Estimates of $\ln(\sum \eta_{ij}^{rs})$ Citation/Knowledge Flow (%)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>Import</i>	0.000175*** (5.39e-05)	0.000239*** (4.58e-05)	0.000258*** (4.29e-05)	0.00643*** (0.00140)	0.000575*** (0.000204)	0.00126*** (0.000192)	0.000721* (0.000417)	0.000700*** (0.000172)
<i>Llnalpa</i>				5.364*** (0.0746)				
<i>Constant</i>	29.81*** (1.393)	33.75*** (1.029)	35.98*** (1.040)	39.66*** (2.539)	95.54*** (21.78)	104.1*** (5.776)	25.37** (9.995)	111.0*** (20.95)
<i>Estimator</i>	PPML	PPML	Poisson	NBReg	PPML	PPML	PPML	PPML
<i>Region</i>	All	All	All	All	Japan	Japan	Japan	Japan
<i>Sector</i>	All	All	All	All	All	Agri	Mining	Manufact
<i>Regression</i>	Rg.2	Rg.2	Rg.1	Rg.1	Rg.2	Rg.2	Rg.2	Rg.2
<i>Period</i>	2001-10	2001-10	2001-10		2001-10	2001-10	2001-10	2001-10
<i>ContVar</i>	✓	✓	✓	✓	✓	✓	✓	✓
<i>Year Dummy</i>	✓	✓						
<i>Sector Dmmy</i>	✓							
<i>Cluster-ID</i>	✓	✓	✓	✓	✓	✓	✓	✓
<i>Observations</i>	64,949	68,060	68,060	68,060	5,280	240	140	4,800
<i># Countries</i>	14	14	14	14	13	13	13	13
<i>Pseudo R2</i>	0.4952	0.2731		0.0271	0.6243	0.7732	0.8239	0.6587

Note:(1)***,**,* represent significance at 1%,5% and 10% level respectively; (2)standard errors in parentheses. NBReg and PPML stand for, respectively, Negative Binomial Regression and Poisson pseudo-likelihood regression with multiple levels of fixed effects. ContVar includes Distance, Language, and Border. Sector classified product codes of 1-2 for Agriculture, 9 for Mining, 3-8 and 10-44 for Manufacturing, and 5 for Service, see Appendix Table 1 in Nabeshima et al. (2017). We could not estimate citation in the service sector in Japan due to the “0” standard deviation (min and max 0). Cluster-ID stands for technological influence among the heterogenized sector-county pairs (Eaton, & Kortum, 2002); Thus, this applies to the spillover effect on heterogenous sector-country pairs as a contribution within a country and among countries.

Source: Patent Panel dataset, author’s estimation.

Knowledge embodied in import products can also advance VA through the spillover effect. Country importing/exporting commodities is directly influenced by technology depending on its absorption capacity (Biyik, 2022b; Halpern et al., 2015). Baldwin and Yan (2014) highlight that participating in GVCs can bring positive and significant gains in productivity for participating countries (Constantinescu et al., 2017). In this sense, this sub-section investigates the optimal contribution of the traveling product among countries with the knowledge spillover effect through sectoral linkages that boost extra sectoral output (VA) as we aimed to predict the optimal (traveling goods among countries) contribution interval due to the continuous integration in the multilateral trade.

This study merges the knowledge flow with sectoral linkages to calculate the knowledge spillover effect and its contribution to VA. Traveling goods among countries regarding production stages contribute more by 1.6% ($1.54 \times 1.058 = B\zeta$) to 154% ($1.54 \times 10000/9942$) if the product crosses the border twice and infinity times (see equations 8.2; 8.4), respectively. Its contribution to VA as twice or infinity (see equation 5.1**) result in boosting extra by 0.8% ($0.48 \times 1.54 \times 1.058 = vB\zeta$) to 72% ($0.48 \times 1.54 \times 10000/9942$), respectively, (see Table 1 and 2). We prefer to focus on average rather than sectoral contribution because aggregated sectoral data in I/O cause bias estimation due to the heterogeneity in export/import intensity across firms (Bems & Kikkawa, 2021).

To compare the result of this paper with the existing studies, the contribution of the study is consistent with theoretical and empirical results as Alfaro et al. (2019) found that “...*more productive firms should integrate more inputs*” in production stages. Moreover, Caliendo and Parro (2015) report that global supply chain shocks (no intermediates trade) result in decreasing (national) GDP lowered by 3% to 70% in some scenarios (Antràs & Chor, 2018). This paper in line with the literature documents that if countries face a shock, such as a labor shortage due to COVID-19, that affects VA rather than the production, gains from trade are most likely to diminish by 0.8% to 72%.

Overall, knowledge spillovers²⁶ through production stages in export can directly have a positive impact on economic growth (Romer, 1990) by promoting catch-up or convergence to this world technology frontier. This means that GVC participation is an important component of economic development/growth for developing countries. This is because imitation/adoption/knowledge transfer²⁷ most likely to advantage for developing nations. Thus, geographically diversified supply chains are not only resilient but also more innovative by learning new knowledge from distant partners because international research collaboration is effective in fostering innovation (Iino et al., 2021; Nabeshima et al., 2018).

5. Policy Discussion and Suggestions

Empirical evident highlight that the knowledge effect relying on diversifying products is more productive if intermediate products travel among countries due to cultural and capacity differences (Iino et al., 2021). It can be said that the less market/regional concentration, the greater the productivity/innovation and dynamic recovery; thus, the journey of the intermediate product among sector-country pairs contributes more if the imports and exports are diverse and resilient²⁸. As a result of the expansion of global value chains, inter-firm information links between countries will strengthen more (Baldwin, 2018). However, trade with less developed countries is concentrated in the export of primary products (United Nations Conference on Trade and Development, 2021); thus, the benefits from the traveling goods among countries and the consequences of the technological spillover effect become limited for these countries. Nevertheless, more regulation policies²⁹ under (strong role of) WTO encourage trade flows and Foreign Direct Investment (FDI), enhancing the knowledge spillover effect among these countries.

²⁶ Flow of goods lead productivity through such as research publications, engineering of products, and imitation.

²⁷ Imitation is easier than innovation, so developing nations likely to growth faster. However, there are in need (right) institutions and policies to underpin innovation and necessary accumulation of capital as a growth frontier.

²⁸ As documented in the literature, policies of the diversification of suppliers, consumers, and delivery channels, as well as flexibility, stockpiling, inventory, and buffer stocks (see Miroudot (2020) for more information) are some possible solutions for dynamic recovery from any (national/international) shocks. Otherwise, domestic shocks such as Tohoku (Biyik 2022b) will result in higher GDP costs.

²⁹ One is to reduce reciprocal tariff barriers and improve trade facilitation (i.e., support measures, data governance, cross border investment, and trade in services, see Auboin et al. (2021)) for all nations (no distortion) under the WTO, which promote trade flows/FDI (ADB & ESCAP, 2013; Hillberry & Zhang, 2018) and thus enhance firm profit/productivity (Pavcnik, 2002) for all nations.

Since 2018, the world has been facing many challenges such as COVID-19 and macro uncertainty. While imports are the source of (comparative) gains from international trade (due to the cheaper input and knowledge flow about production techniques), there is an import-related negative spillover effect such as cost-push inflation under macro-economic uncertainty/instability. How do we solve the distortion of the GVC participation due to the US-China trade war and the Russian-Ukrainian war? How does the GVC participation (or idiosyncratic shock in the multi-connected sector) help growth and recovery, as well as trade and global warming? These issues highlight the need for international cooperation and collaboration to address global issues and strengthen global governance mechanisms to operate effectively. In other words, to improve integration and GVC participation from a game-theoretical perspective, building greater global governance (such as WTO and UN) and cooperation will be a long-run solution for distortion of adjustment from the agglomeration of GVC in a specific region (Gereffi, Humphrey, & Sturgeon, 2005; Baldwin, 2018) and macro-economic uncertainty issues or global supply chain disruption such as the US-China trade war, and the Russian-Ukrainian war. This is because create a positive environment and reliable regulatory policies for firms stimulates/maximizes the positive effect of knowledge flow, as well as limiting the distortion of the negative effects³⁰.

Empirical evidence for these shocks with input-output linkages shows that macroeconomic volatility coming from sectoral idiosyncratic shocks is significantly large and spreads (positive/negative) cross-country transmission from the origin to all over the world (Baqae & Farhi, 2019; Barrot & Sauvagnat, 2016). This is because the geographical distribution of activities by multinational companies, which are in high interaction with domestic suppliers, bear the impact of positive/negative shocks on industrial production. Global fast growth from 2015-2018, for instance, had a positive impact on Japanese trade recovery from the Tohoku earthquake (Biyik, 2022b). This also applies to the spread of the negative influence from the case of the Global Financial Crisis (Acemoglu et al., 2015) and the Tohoku Earthquake (Carvalho et al., 2021) through sectoral linkage. These shocks drove Japan into more regional or even national production networks, diminishing intermediate input imports between 2008 to 2016. However, Japan has been diversifying its intermediate input since 2016 due to its new trade (agreement) policies such as RCEP. One concern is that regional FTA might lead to strong trade relationships in the production networks within specific regions/partners and then, however, can cause agglomeration in specific regions due to the cost minimization effects, leading to deeper links within regional blocks³¹.

In summary, managing market focus from a global perspective, instead of regional preferences, will be an important policy for long-term growth (due to the innovation effect). This also contributes to the diversity of goods as well as markets, which not only boosts its welfare/GDP but also improves recovery dynamism (Alfaro & Chen, 2012; Jain et al., 2017) from any negative shocks; thus, policies that encourage diversification of foreign suppliers' lower volatility by reducing the exposure to individual economies (Baldwin & Freeman, 2021). Also, building a positive environment, under which firms can extend their production fragmentations, leads to extending GVC participation among the different regions and improving African countries' participation in GVC, consequently, they may specialize in more technology-intensive sectors. This extension and greater global governance create jobs and improve skill/technical workers in developing countries due to the technology push growth (Romer, 1990); thus, this not only resolves face-to-face cost but also disentangle agglomeration of GVC participation. Lastly, the supply chain

³⁰ WTO provide framework of rules that eliminates distortion of negative spillover effects (see Auboin et al., 2021 for more details).

³¹ For example, de Gortari (2019) presented the roundabout model regarding the U.S.-Mexico trade relation because of NAFTA.

presents an opportunity for not only innovations but also a future where supply chains will be shaped by the need for environmental sustainability.

6. Conclusion

This study analyzes the VA embodied in the intermediate export and investigates the VA of the spillover (upstream and downstream) supply chain of the production activities in export. The focal point of this study is to extend GVC based on the multilevel and bilateral trade flow of the VA. There are two main contributions of this paper. First, the originality of this work theoretically reveals the intermediate input journey among sector-country pairs as completed GVC participation. Second, the coefficient of the knowledge spillover effect regarding vertical integration in the production process was introduced. In other words, this paper found that different forms of the VA with traveled products multiplier provide more precise (sectoral/regional) integration and optimal estimation method. To calculate the integration and contribution, this paper uses constructed GTAP-MRIO and patent panel datasets. As a result, the export/import coefficient of the sectoral linkages, which boosts VA and causes double-counting in some conditions, is about 1.5 % in terms of (single) country-level data if the product crosses the border only twice. In reality of traveling goods in GVC, this paper in line with literature expects that firms/sectors that increase production fragmentation/stages in GVC will benefit with a higher level of imitation/innovation. Therefore, traveling products with the technological spillover effect as twice or infinity is to contribute more by 1.6% or 154%. This also applies to VA in a country, it boosts VA more by 0.8% to 72%.

This paper in line with the literature highlight that geographically diversified supply chains is the most efficient tool not only to boost innovation due to learning new knowledge from distant partners but also to improve dynamic solutions from any (national/international) shocks. Thus, managing market focus from a global perspective will be an important policy for long-term growth due to the innovation effect.

The limitation is that high population/GDP countries have a low openness proxy, and the case of Japanese data does not address small (GDP) countries with high openness proxy. Moreover, the nature of the input-output table does not explain foreign-owned firms located in host countries due to the residence-based rather than ownership-based; thus, we cannot measure their participation in GVC activities if they distribute their product in only host countries.

References

- Acemoglu, D., Ozdaglar, A., & Tahbaz-Salehi, A. (2015). Systemic risk and stability in financial networks. *American Economic Review*, 105(2), 564-608.
- Alfaro, L., & Chen, M. X. (2012). Surviving the global financial crisis: Foreign ownership and establishment performance. *American Economic Journal: Economic Policy*, 4(3), 30-55.
- Alfaro, L., Chor, D., Antras, P., & Conconi, P. (2019). Internalizing global value chains: A firm-level analysis. *Journal of Political Economy*, 127(2), 508-559.
- Amiti, M., Itskhoki, O., Konings, J., 2014. Importers, exporters, and exchange rate disconnect. *Am. Econ. Rev.* 104, 1942–1978.
- Antràs, P., & Chor, D. (2018). On the measurement of upstreamness and downstreamness in global value chains. *World Trade Evolution: Growth, Productivity, and Employment*, 126-194.
- Antràs, P., Chor, D., Fally, T., & Hillberry, R. (2012). Measuring the upstreamness of production and trade flows. *American Economic Review*, 102(3), 412-416.

- Asian Development Bank (ADB) and the United Nations Economic and Social Commission for Asia and the Pacific (ESCAP). (2013). *Designing and implementing trade facilitation in Asia and the Pacific* (The third Edition ed.). Mandaluyong: ADB and UNESCAP. doi:SBN 978-92-9254-200-9
- Atkin, D., Khandelwal, A. K., & Osman, A. (2017). Exporting and firm performance: Evidence from a randomized experiment. *The Quarterly Journal of Economics*, 132(2), 551-615.
- Auboin, M., Koopman, R., & Xu, A. (2021). Trade and innovation policies: Coexistence and spillovers. *Journal of Policy Modeling*, 43(4), 844-872.
- Auer, R. A., Levchenko, A. A., & Sauré, P. (2019). International inflation spillovers through input linkages. *Review of Economics and Statistics*, 101(3), 507-521.
- Baqae, D. R., & Farhi, E. (2019). The macroeconomic impact of microeconomic shocks: Beyond Hulten's theorem. *Econometrica*, 87(4), 1155-1203.
- Baldwin, J. R., & Yan, B. (2014). *Global value chains and the productivity of Canadian manufacturing firms* Citeseer.
- Baldwin, R. (2006). *Globalisation: The Great Unbundling (s)*, S.I.], Economic council of Finland.
- Baldwin, R. (2018). The great convergence: *Information technology and the new globalization* () Harvard University Press.
- Baldwin, R., & Lopez-Gonzalez, J. (2015). Supply-chain trade: A portrait of global patterns and several testable hypotheses. *The World Economy*, 38(11), 1682-1721
- Baldwin, R., & Freeman, R. (2021). *Risks and Global Supply Chains: What We Know and What We Need to Know*, Annual Review of Economics, vol 14(1).
- Bems, R., & Kikkawa, A. K. (2021). Measuring trade in value added with firm-level data. *Journal of International Economics*, 129, 103434.
- Bems, R., & Johnson, R. C. (2017). Demand for value-added and value-added exchange rates. *American Economic Journal: Macroeconomics*, 9(4), 45-90.
- Bernard, A.B., Jensen, J.B., Redding, S.J., Schott, P.K., 2012. The empirics of firm heterogeneity and international trade. *Ann. Rev. Econ.* 4, 283–313.
- Blaum, J. (2018). Global firms in large devaluations. *Unpublished Manuscript, Brown University*,
- Biyik, O. (2022a). Japanese Automobile Industry in the AfJPEA: Economic Impacts of Deep Integration Policy. *Journal of the Graduate School of Asia-Pacific Studies No.43* (3), 17-33.
- Biyik, O. (2022b). The effect of Tohoku earthquake on Japan's bilateral trade distribution: Case study of African markets.
- Borin, A., & Mancini, M. (2019). Measuring what matters in global value chains and value-added trade. *World Bank Policy Research Working Paper*, (8804)
- Bottazzi, L., & Peri, G. (2003). Innovation and spillovers in regions: Evidence from European patent data. *European Economic Review*, 47(4), 687-710.
- Buera, F. J., & Oberfield, E. (2020). The global diffusion of ideas. *Econometrica*, 88(1), 83-114.
- Caliendo, L., & Parro, F. (2015). Estimates of the trade and welfare effects of NAFTA. *The Review of Economic Studies*, 82(1), 1-44.
- Carvalho, V. M., Nirei, M., Saito, Y. U., & Tahbaz-Salehi, A. (2021). Supply chain disruptions: Evidence from the great east japan earthquake. *The Quarterly Journal of Economics*, 136(2), 1255-1321.
- Constantinescu, C., Mattoo, A., & Ruta, M. (2017). *Trade developments in 2016: Policy uncertainty weighs on world trade* World Bank.
- De Gortari, A. (2019). *Disentangling Global Value Chains*, (NBER Working Paper)
- Dietzenbacher, E., Romero, I., Bosma, N.S. (2005). Using Average Propagation Lengths to Identify Production Chains in the Andalusian Economy. *Estudios de Economía Aplicada*, 23: 405-422.
- Eaton, J., & Kortum, S. (1999). International technology diffusion: Theory and measurement. *International Economic Review*, 40(3), 537-570.
- Eaton, J., & Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5), 1741-1779.
- Fally, T. (2012). Production staging: Measurement and facts. *Boulder, Colorado, University of Colorado Boulder, may*, 155-168.

- Gereffi, G., Humphrey, J., & Sturgeon, T. (2005). The governance of global value chains. *Review of International Political Economy*, 12(1), 78-104.
- Hall, B. H., Mairesse, J., & Mohnen, P. (2010). Measuring the returns to R&D. *Handbook of the economics of innovation* (pp. 1033-1082) Elsevier.
- Halpern, L., Koren, M., & Szeidl, A. (2015). Imported inputs and productivity. *American Economic Review*, 105(12), 3660-3703.
- Hillberry, R., & Zhang, X. (2018). Policy and performance in customs: Evaluating the trade facilitation agreement. *Review of International Economics*, 26(2), 438-480.
- Iino, T., Inoue, H., Saito, Y. U., & Todo, Y. (2021). How does the global network of research collaboration affect the quality of innovation? *The Japanese Economic Review*, 72(1), 5-48.
- Jain, V., Kumar, S., Soni, U., & Chandra, C. (2017). Supply chain resilience: Model development and empirical analysis. *International Journal of Production Research*, 55(22), 6779-6800.
- Johnson, R. C. (2018). Measuring global value chains. *Annual Review of Economics*, 10, 207-236.
- Johnson, R. C., & Noguera, G. (2012). Accounting for intermediates: Production sharing and trade in value-added. *Journal of International Economics*, 86(2), 224-236.
- Kasahara, H., & Rodrigue, J. (2008). Does the use of imported intermediates increase productivity? plant-level evidence. *Journal of Development Economics*, 87(1), 106-118
- Kee, H. L., & Tang, H. (2016). Domestic value added in exports: Theory and firm evidence from china. *American Economic Review*, 106(6), 1402-1436.
- Keller, W. (2010). International trade, foreign direct investment, and technology spillovers. *Handbook of the economics of innovation* (pp. 793-829) Elsevier.
- Keller, W. (2021). *Knowledge Spillovers, Trade, and Foreign Direct Investment*. 28739 (NBER Working Paper) doi:10.3386/w28739. Retrieved from <http://www.nber.org/papers/w28739>.
- Koopman, R., Powers, W., Wang, Z., & Wei, S. (2010). *Give credit where credit is due: Tracing value added in global production chains*. SSRN Electronic Journal, 16426 (NBER Working Paper) doi:10.2139/ssrn.1949669.
- Koopman, R., Wang, Z., & Wei, S. (2012). Estimating domestic content in exports when processing trade is pervasive. *Journal of Development Economics*, 99(1), 178-189.
- Koopman, R., Wang, Z., & Wei, S. (2014). Tracing value-added and double counting in gross exports. *The American Economic Review*, 104(2), 459-494. doi:10.1257/aer.104.2.459
- Kuboniwa, M. (2016). Decomposition of gross exports: An alternative accounting system. Paper presented at the Seoul, Republic of Korea. , *CESSA WP 2016-01*.
- Li, X., Meng, B., & Wang, Z. (2019). *Technological innovation, supply chain trade, and workers in a globalized world: Global value chains development report 2019* World trade organization.
- Los, B., Timmer, M. P., & de Vries, G. (2016). Tracing value-added and double counting in gross exports: Comment. *The American Economic Review*, 106(7), 1958-1966. doi:10.1257/aer.20140883
- Miller, R. E., & Blair, P. D. (2009). *Input-output analysis [electronic resource]: Foundations and extensions / Ronald E. miller and peter D. Blair* (2nd ed.). Cambridge: Cambridge University Press.
- Miller, R. E., & Lahr, M. L. (2001). A taxonomy of extractions. *Contributions to Economic Analysis*, 249, 407-441.
- Miroudot, S. (2020). Reshaping the policy debate on the implications of COVID-19 for global supply chains. *Journal of International Business Policy*, 3(4), 430-442.
- Nabeshima, K., Kashcheeva, M., & Kang, B. (2018). The impacts of export competition on international technology flows. *Applied Economics Letters*, 25(15), 1058-1061.
- Nabeshima, K., Kashcheeva, M., & Kang, B. (2017). *The Impact of Import Vs.Export Competition in Technology Flows between Countries*, IDE Discussion Papers 654, Institute of Developing Economies, Japan External Trade Organization(JETRO)
- Nishioka, S., & Ripoll, M. (2012). Productivity, trade and the R&D content of intermediate inputs. *European Economic Review*, 56(8), 1573-1592.
- Pavcnik, N. (2002). Trade liberalization, exit, and productivity improvements: Evidence from Chilean plants. *The Review of Economic Studies*, 69(1), 245-276.

- Peters, G. P., Andrew, R., & Lennox, J. (2011). Constructing an environmentally-extended multi-regional input-output table using the GTAP database. *Economic Systems Research*, 23(2), 131-152. doi:10.1080/09535314.2011.563234.
- Piermartini, R., & Rubínová, S. (2021). How much do global value chains boost innovation? *Canadian Journal of Economics/Revue Canadienne d'Économique*, 54(2), 892-922.
- Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5, Part 2), S71-S102
- Schmoch, U., Laville, F., Patel, P., & Frietsch, R. (2003). Linking technology areas to industrial sectors. *Final Report to the European Commission, DG Research*, 1(0), 100.
- Tajoli, L., & Felice, G. (2018). Global value chains participation and knowledge spillovers in developed and developing countries: An empirical investigation. *The European Journal of Development Research*, 30(3), 505-532.
- Trefler, D., & Zhu, S. C. (2010). The structure of factor content predictions. *Journal of International Economics*, 82(2), 195-207.
- United Nations Conference on trade and Development (UNCTAD). (2021). *State of Commodity Dependence 2021*. dio: 10.18356/9789210057790.
- Zolas, N., & Lybbert, T. (2022). How do Patents Shape Global Value Chains? International and Domestic Patenting and Value-Added Trade. In A. Taubman & J. Watal (Eds.), *Trade in knowledge: Intellectual property, trade, and development in a transformed global economy* Cambridge University Press pp. 471-497.
- Walmsley, T. L., Hertel, T., & Hummels, D. (2014). *Developing a GTAP-based multi-region, input-output framework for supply chain analysis* Edward Elgar Publishing. doi:10.4337/9781783472093.00008
- Wang, Z., Wei, S., & Zhu, K. (2013). No title. *Quantifying International Production Sharing at the Bilateral and Sector Levels*,
- Wang, Z., Wei, S., Yu, X., & Zhu, K. (2017). *Measures of participation in global value chains and global business cycles*. NBER Working Paper 23222. doi:10.3386/w23222
- Wang, Z., Wei, S., Yu, X., & Zhu, K. (2018). *Re-examining the effects of trading with china on local labor markets: A supply chain perspective*. NBER Working Paper 24886. doi:10.3386/w24886
- World Bank (WB). (2020). *World development report 2020: Trading for development in the age of global value chains* Washington, DC: World Bank. doi:10.1596/978-1-4648-1457-0

Appendix A: N-Way of Trade in Multi-Country Integrations.

The motivation here is to present integrated continuous trade between trade partners. We use three single countries and one aggregated country. We present the DVA in (country) r in relation to gross export.

$$\begin{aligned}
 DVA^r &= v^r \left[B^{rr} E^{rs} + B^{rr} E^{rl} + B^{rr} \sum_{t \neq r, s, l}^n E^{rt} \right] \tag{A1} \\
 &= v^r \left[B^{rr} Y^{rs} + B^{rr} A^{rs} X^s + B^{rr} Y^{rl} + B^{rr} A^{rl} X^l + B^{rr} \sum_{t \neq r, s, l}^n (Y^{rt} + A^{rt} X^t) + B^{rr} \sum_{t \neq r, s, l}^n \sum_{k \neq r, s, l, k}^n A^{rt} X^{tk} \right] \#(A1.1) \\
 v^r B^{rr} E^{rs} &= v^r \left[B^{rr} Y^{rs} + B^{rr} A^{rs} X^{ss} + B^{rr} A^{rs} X^{sr} + B^{rr} A^{rs} X^{sl} + B^{rr} \sum_{t \neq r, s, l}^n A^{rs} X^{st} \right] \\
 v^r B^{rr} E^{rl} &= v^r \left[B^{rr} Y^{rl} + B^{rr} A^{rl} X^{ll} + B^{rr} A^{rl} X^{lr} + B^{rr} A^{rl} X^{ls} + B^{rr} \sum_{t \neq r, s, l}^n A^{rl} X^{lt} \right] \\
 v^r B^{rr} \sum_{t \neq r, s, t}^n E^{rt} &= v^r B^{rr} \sum_{t \neq r, s, t}^n \left(Y^{rt} + A^{rt} X^{tt} + A^{rt} X^{tr} + A^{rt} X^{ts} + A^{rt} X^{tl} + \sum_{k \neq r, s, l, t, k}^n A^{rt} X^{tk} \right), k \in t [= k - 1, k, k + 1.. n]
 \end{aligned}$$

Where, $\sum_{k \neq r, s, l, k}^n A^{rt} X^{tk}$ that proceeds and export to k from multi (t) countries. This means that three-way trade in the multi-country also leads to another complicated integration as t (n-3) countries with k (n-4) countries, as well as r, s, l, and t. An integrated model in the last stage of consumption in x can be written

$$= v^r B^{rr} \sum_{t \neq r, s, l}^n \left(Y^{rt} + A^{rt} X^{tt} + A^{rt} X^{tr} + A^{rt} X^{ts} + A^{rt} X^{tl} + \sum_{k \neq r, s, l, t, k}^n A^{rt} X^{tk^*} + \dots + \sum_{k \neq r, s, l, k}^n \dots \sum_{x \in n} A^{lx} X^{x^*} \right),$$

$k, x \in t [= k-1, k, k+1 \dots x \dots n]$

Let's define DVA in terms of where products are absorbed.

$$= v^r \left[B^{rr} Y^{rs} + B^{rr} A^{rs} X^{ss} + B^{rr} A^{rs} X^{sr} + B^{rr} A^{rs} X^{sl} + B^{rr} \sum_{t \neq r, s, l}^n A^{rs} X^{st} + B^{rr} Y^{rl} + B^{rr} A^{rl} X^{ll} + B^{rr} A^{rl} X^{lr} + B^{rr} A^{rl} X^{ls} \right. \\ \left. + B^{rr} \sum_{t \neq r, s, l}^n A^{rl} X^{lt} + B^{rr} \sum_{t \neq r, s, l}^n \left(Y^{rt} + A^{rt} X^{tt} + A^{rt} X^{tr} + A^{rt} X^{ts} + A^{rt} X^{tl} + \sum_{k \neq r, s, l, t}^n A^{rt} X^{tk} \right) \right] \#(A1.2)$$

We did not consider the technological spillover effect. The interaction effect of the upstream and downstream (see equation 7.1) can be written

$$DVA^r = v^r [D^{rr} + F^{rr}] \left(Y^{rs} + Y^{rl} + \sum_{t \neq r, s, l}^n Y^{rt} \right) + \{ \lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs} \} \left[A^{rs} X^{ss} + A^{rs} X^{sr} + A^{rs} X^{sl} + \sum_{t \neq r, s, l}^n A^{rs} X^{st} \right] \\ + \{ \lambda^{rl} + L^{rl} + \delta^{rl} + F^{rl} \} \left[A^{rl} X^{ll} + A^{rl} X^{lr} + A^{rl} X^{ls} + \sum_{t \neq r, s, l}^n A^{rl} X^{lt} \right] \\ + \left(\sum_{t \neq r, s, l}^n \{ \lambda^{rt} + L^{rt} + \delta^{rt} + F^{rt} \} \right) \left[\sum_{t \neq r, s, l}^n \left(A^{rt} X^{tt} + A^{rt} X^{tr} + A^{rt} X^{ts} + A^{rt} X^{tl} \right. \right. \\ \left. \left. + \sum_{k \neq r, s, l, t}^n A^{rt} X^{tk} \right) \right] \#(A2)$$

Proof:

$B^{rr} A^{rs} X^s$, $B^{rr} A^{rl} X^l$, and $B^{rr} \sum_{t \neq r, s, l}^n A^{rt} X^t$, which leads to more complex integration problems (see Equation A2.3*), can be written.

$$v^r B^{rr} A^{rs} X^s = v^r [B^{rr}] A^{rs} [B^{ss} + B^{sr}] \left[Y^{ss} + Y^{sr} + A^{sr} X^r + Y^{sl} + A^{sl} X^l + \sum_{t \neq r, s, l}^n (Y^{st} + A^{st} X^t) \right] \quad (A2.1) \\ \cong \left(\{ \lambda^{rs} + L^{rs} + \delta^{rs} + F^{rs} \} \left[Y^{ss} + Y^{sr} + A^{sr} X^r + Y^{sl} + A^{sl} X^l + \sum_{t \neq r, s, l}^n (Y^{st} + A^{st} X^t) \right] \right)$$

$$v^r B^{rr} A^{rl} X^l \cong \left(\{ \lambda^{rl} + L^{rl} + \delta^{rl} + F^{rl} \} \left[Y^{ll} + Y^{lr} + A^{lr} X^r + Y^{ls} + A^{ls} X^s + \sum_{t \neq r, l, s}^n (Y^{lt} + A^{lt} X^t) \right] \right) \quad (A2.2)$$

$$v^r B^{rr} \sum_{t \neq r, s, l}^n A^{rt} X^t = v^r \left(\sum_{t \neq r, s, l}^n \{ \lambda^{rt} + L^{rt} + \delta^{rt} + F^{rt} \} \right) \left(\sum_{t \neq r, s, l}^n \left\{ Y^{tt} + Y^{tr} + A^{tr} X^r + Y^{ts} + A^{ts} X^s + \sum_{k \neq r, s, t}^n (Y^{tk} + A^{tk} X^k) \right\} \right), \\ k \in t = [k-1, k, k+1, \dots, n] \quad (A2.3)$$

Equation (A2.3) can be written in relation to continuous trade flow at multi-integration as

$$v^r B^{rr} \sum_{t \neq r, s, l}^n A^{rt} X^t = v^r \left(\sum_{t \neq r, s, l}^n \{ \lambda^{rt} + L^{rt} + \delta^{rt} + F^{rt} \} \right) \left(\sum_{t \neq r, s, l}^n \left\{ Y^{tt} + Y^{tr} + A^{tr} X^r + Y^{ts} + A^{ts} X^s + \sum_{k \neq r, s, t}^n (Y^{tk} + A^{tk} X^k) \right. \right. \\ \left. \left. + \dots + \sum_{k \neq r, s, t}^n \dots \sum_{x \in n} A^{lx} X^{x^*} \right\} \right), \quad x \in k = [k, k+1, \dots x \dots, n] \quad (A2.3^*)$$

Appendix B: Descriptive Statistic of the Two Datasets

Table B1: Sectoral and Regional Aggregation of GTAP-MRIO

No	Sector name	GTAP concordance
1	Agriculture (AGR)	Paddy rice (PDR); wheat (WHT); cereal grains, NEC (GRO); vegetables, fruit, nuts (V_F); oilseeds (OSD); sugar cane, sugar beet (C_B); Fish (FSH); sugar (SGR); plant-based fibers (PFB); Fish (FSH); vegetable oils and fats (VOL); dairy products (MIL); crops, NEC (OCR); bovine cattle, sheep and goats, horses (CTL); animal products, NEC (OAP); raw milk (RMK); wool, silkworm cocoons (WOL); forestry (FRS)
2	Mining (MN)	Coal (COA); oil (OIL); gas (GAS); Mineral products, NEC (NMM); Petroleum, coal products (P_C)
3	Manufacturing (MANF)	Metal products (FMP); manufacturers, NEC (OMF); Textiles (TEX); Motor vehicles and parts (MVH); transport equipment nec (OTN); machinery and equipment nec (OMG); bovine meat products (CMT); meat products, NEC (OMT); processed rice (PCR); ferrous metals (L_S); metals, NEC (NFM); food products, NEC (OFD); beverages and tobacco products (B_T)wearing apparel (WAP); leather products (LEA); Computer, electronic, and optical products (ELE); electrical equipment (EEQ); Wood products (LUM); paper products, publishing (PPP); Chemical products (CHM); basic pharmaceutical products (BPH); rubber and plastic products (RPP); bovine meat products (CMT); meat products, NEC (OMT); vegetable oils and fats (VOL); dairy products (MIL); processed rice (PCR); sugar (SGR); food products, NEC (OFD); beverages and tobacco products (B_T); Other extraction (formerly other manufacturing (omn) minerals, NEC) (OXT)
4	Services (SRV)	gas manufacture, distribution (GDT); Construction (CNS); Trade (TRD); accommodation, food, and service activities (AFS); Real estate activities (RSA); business services, NEC (OBS); insurance (formerly ISR) (INS); warehousing and support activities (WHS); Transport, NEC (OTP); communication (CMN); water transport (WTP); air transport (ATP); Financial services, NEC (OFI); Electricity (ELY); water (WTR); recreational and other services (ROS); public administration and defense (OSG); education (EDU); human health and social work activities (HHT); dwellings (DWE)
No	Region	GTAP concordance
1	Japan (JPN)	Japan (JPN)
2	Africa	Egypt, Arab Rep. (EGY), Morocco (MAR), Tunisia (TUN), Rest of North Africa (XNF), Burkina Faso (BFA), Cameroon (CMR), Côte d'Ivoire (CIV), Ghana (GHA), Nigeria (NGA), Senegal (SEN), Benin (BEN), Guinea (GIN), Togo (TGO), Rest of West Africa (XWF), Central Africa (XCF), Congo, Dem. Rep. (COD), Ethiopia (ETH), Kenya (KEN), Madagascar (MDG), Malawi (MWI), Mauritius (MUS), Mozambique (MOZ), Rwanda (RWA), Tanzania (TZA), Uganda (UGA), Zambia (ZMB), Zimbabwe (ZWE), Rest of East Africa (XEC), Botswana (BWA), Namibia (NAM), South Africa (ZAF), Rest of South African Customs Union (XSC)
3	ROW	Austria (AUT), Belgium (BEL), Cyprus (CYP), Czech Republic (CZE), Denmark (DNK), Estonia (EST), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), Hungary (HUN), Ireland (IRL), Italy (ITA), Latvia (LVA), Lithuania (LTU), Luxembourg (LUX), Malta (MLT), Netherlands (NLD), Poland (POL), Portugal (PRT), Slovakia (SVK), Slovenia (SVN), Spain (ESP), Sweden (SWE), Bulgaria (BGR), Croatia (HRV), Romania (ROU); Canada (CAN), Mexico (MEX), Chile (CHL), Peru (PER); Malaysia (MYS), Singapore (SGP), Brunei Darussalam (BRN), Vietnam (VNM), Cambodia (KHM), Indonesia (IDN), Lao PDR (LAO), Philippines (PHL), Thailand (THA), rest of Southeast Asia-Myanmar (MMR)-(XSE); Australia (AUS), New Zealand (NZL); India (IND); United States of America (USA); China (CHN); Hong Kong, SAR, China (HKG), Republic of Korea (KOR); Mongolia (MNG), Taiwan, China (TWN), rest of East Asia (XEA), United Kingdom (GBR), Switzerland (CHE), Norway (NOR), rest of EFTA (XEF), rest of Oceania (XOC), Bangladesh (BGD), Nepal (NPL), Pakistan (PAK), Sri Lanka (LKA), rest of South Asia (XSA), rest of North America (XNA), Argentina (ARG), Bolivia (BOL), Brazil (BRA), Colombia (COL), Ecuador (ECU), Paraguay (PRY), Uruguay (URY), Venezuela (VEN), rest of South America (XSM), Costa Rica (CRI), Guatemala (GTM), Honduras (HND), Nicaragua (NIC), Panama (PAN), El Salvador (SLV), rest of Central America (XCA), Dominican Republic (DOM), Jamaica (JAM), Puerto Rico (PRI), Trinidad and Tobago (TTO), rest of Caribbean (XCB), Albania (ALB), Belarus (BLR), Russian Federation (RUS), Ukraine (UKR), rest of East Europe (XEE), rest of Europe (XER), Kazakhstan (KAZ), Kyrgyzstan (KGZ), Tajikistan (TJK), rest of former Soviet Union (XSU), Armenia (ARM), Azerbaijan (AZE), Georgia (GEO), Bahrain (BHR), Iran, Islamic Rep. (IRN), Israel (ISR), Jordan (JOR), Kuwait (KWT), Oman (OMN), Qatar (QAT), Saudi Arabia (SAU), Turkey (TUR), United Arab Emirates (ARE), rest of Western Asia (XWS), rest of the world (XTW)

Source: Author's aggregation based on GTAP 10A MRIO Database

Table B2: Inverse Matrix of Interregional (rs) Leontief ($\sum_{r,s=1,1}^3 \sum_{ij=1,1}^4 [B_{ij}^{rr} + B_{ij}^{rs}]$)

		Japan				Africa				ROW			
		AGR	MN	MANF	SRV	AGR	MN	MANF	SRV	AGR	MN	MANF	SRV
Japan	AGR	1.14537	0.00083	0.03801	0.00683	0.00009	0.00010	0.00034	0.00017	0.00030	0.00021	0.00076	0.00021
	MN	0.06732	1.23701	0.07255	0.03998	0.00020	0.00030	0.00076	0.00047	0.00062	0.00123	0.00167	0.00064
	MANF	0.36652	0.02556	1.69346	0.16425	0.00341	0.00408	0.01410	0.00723	0.00913	0.00867	0.03103	0.00850
Africa	SRV	0.43732	0.11115	0.46357	1.42711	0.00133	0.00186	0.00502	0.00312	0.00331	0.00321	0.00997	0.00357
	AGR	0.00249	0.00086	0.00102	0.00024	1.12379	0.00951	0.14638	0.02922	0.00166	0.00095	0.00161	0.00048
	MN	0.00792	0.08119	0.01072	0.00595	0.01770	1.20736	0.05902	0.07407	0.00593	0.08089	0.01089	0.00631
ROW	MANF	0.00227	0.00510	0.00435	0.00109	0.08021	0.06121	1.28561	0.14632	0.00230	0.00554	0.00615	0.00203
	SRV	0.00282	0.01546	0.00379	0.00190	0.15137	0.21569	0.39856	1.31991	0.00261	0.01572	0.00510	0.00292
	AGR	0.06054	0.01256	0.03125	0.00740	0.02305	0.00844	0.05415	0.01677	1.29057	0.01878	0.11077	0.02773
ROW	MN	0.10395	1.03054	0.13919	0.07743	0.02150	0.07371	0.06416	0.06107	0.10019	1.68575	0.16534	0.10359
	MANF	0.20004	0.18519	0.27988	0.08423	0.10217	0.10886	0.34196	0.18326	0.41329	0.28170	1.96665	0.32728
	SRV	0.10342	0.23557	0.12694	0.06120	0.05130	0.07417	0.15574	0.11134	0.41190	0.36677	0.58983	1.52830

Source: GTAP-MRIO database, author's estimation.

Table B2.1: Inverse Matrix of Domestic (rr) and Bilateral Leontief ($\sum_{r,s=1,1}^3 \sum_{ij=1,1}^4 [D_{ij}^{rr} + L_{ij}^{rs}]$)

		Japan				Africa				ROW			
		AGR	MN	MANF	SRV	AGR	MN	MANF	SRV	AGR	MN	MANF	SRV
Japan	AGR	1.14528	0.00070	0.03789	0.00679	0.00004	0.00005	0.00020	0.00009	0.00030	0.00020	0.00075	0.00021
	MN	0.06708	1.23625	0.07224	0.03986	0.00010	0.00016	0.00044	0.00027	0.00062	0.00120	0.00165	0.00063
	MANF	0.36299	0.01998	1.68865	0.16267	0.00170	0.00213	0.00846	0.00411	0.00906	0.00835	0.03083	0.00843
	SRV	0.43614	0.10908	0.46198	1.42658	0.00076	0.00120	0.00317	0.00208	0.00328	0.00306	0.00990	0.00354
Africa	AGR	0.00224	0.00026	0.00072	0.00014	1.12369	0.00939	0.14604	0.02904	0.00165	0.00094	0.00159	0.00047
	MN	0.00263	0.03171	0.00363	0.00211	0.01650	1.20365	0.05538	0.07083	0.00583	0.08053	0.01062	0.00621
	MANF	0.00136	0.00167	0.00313	0.00061	0.07983	0.06066	1.28438	0.14558	0.00227	0.00548	0.00606	0.00200
	SRV	0.00146	0.00582	0.00199	0.00102	0.15096	0.21479	0.39728	1.31896	0.00258	0.01563	0.00501	0.00289
ROW	AGR	0.06029	0.01184	0.03091	0.00729	0.02296	0.00832	0.05381	0.01659	1.29024	0.01798	0.10986	0.02744
	MN	0.10296	1.02463	0.13784	0.07683	0.02107	0.07301	0.06256	0.06009	0.09873	1.67931	0.16150	0.10213
	MANF	0.19802	0.17658	0.27700	0.08316	0.10139	0.10770	0.33894	0.18154	0.41048	0.27241	1.95881	0.32464
	SRV	0.10230	0.22991	0.12539	0.06057	0.05090	0.07354	0.15425	0.11046	0.41045	0.36076	0.58597	1.52690

Source: GTAP-MRIO database, author's estimation.

Table B2.2: Inverse Matrix of Leontief Feedback Effect ($\sum_{r,s=1,1}^3 \sum_{ij=1,1}^4 [F_{ij}^{rr} + F_{ij}^{rs}]$)

		Japan				Africa				ROW			
		AGR	MN	MANF	SRV	AGR	MN	MANF	SRV	AGR	MN	MANF	SRV
Japan	AGR	0.00009	0.00013	0.00012	0.00004	0.00004	0.00005	0.00014	0.00008	0.00000	0.00001	0.00000	0.00000
	MN	0.00024	0.00077	0.00031	0.00012	0.00010	0.00014	0.00033	0.00019	0.00000	0.00002	0.00001	0.00000
	MANF	0.00353	0.00557	0.00481	0.00157	0.00171	0.00195	0.00564	0.00312	0.00008	0.00032	0.00019	0.00007
	SRV	0.00118	0.00207	0.00159	0.00054	0.00057	0.00066	0.00185	0.00104	0.00003	0.00014	0.00007	0.00003
Africa	AGR	0.00025	0.00059	0.00030	0.00010	0.00011	0.00012	0.00034	0.00018	0.00001	0.00001	0.00002	0.00001
	MN	0.00530	0.04948	0.00709	0.00384	0.00120	0.00371	0.00364	0.00324	0.00010	0.00036	0.00027	0.00010
	MANF	0.00091	0.00343	0.00122	0.00048	0.00038	0.00054	0.00123	0.00074	0.00003	0.00006	0.00009	0.00003
	SRV	0.00136	0.00964	0.00180	0.00088	0.00041	0.00090	0.00128	0.00094	0.00003	0.00009	0.00009	0.00003
ROW	AGR	0.00024	0.00072	0.00034	0.00011	0.00009	0.00012	0.00034	0.00019	0.00033	0.00080	0.00091	0.00029
	MN	0.00099	0.00590	0.00135	0.00060	0.00042	0.00070	0.00160	0.00098	0.00146	0.00645	0.00384	0.00147
	MANF	0.00202	0.00861	0.00288	0.00108	0.00079	0.00116	0.00303	0.00172	0.00281	0.00929	0.00784	0.00264
	SRV	0.00111	0.00566	0.00155	0.00063	0.00039	0.00063	0.00149	0.00088	0.00145	0.00600	0.00387	0.00140

Source: GTAP-MRIO database, author's estimation.

Table B3: Summary Statistics of Variables in the Panel Data

Variable	Obs	Mean	Std. Dev.	Min	Max	Country (14)
LCitation	68,060	126164.6	4383954	0	4.27E+08	BRN; CHN; HKG;
Import	68,060	146.4425	791.8076	0	37929.21	IDN; JPN; KHM;
LDist	68,060	7.6715	0.6163	5.7543	8.6642	KOR; LAO; MMR;
Language	68,060	0.116221	0.3204921	0	1	MYS; PHL; SGP;
Border	68,060	0.148413	0.3555117	0	1	THA; VNM

Source: Patent Panel dataset, author's estimation.

Table B4: Correlation Matrix of Variables in the Panel Data

	LCitation	Import	LDist	Language	Border
LCitation	1				
Import	0.055	1			
LDist	-0.028	-0.020	1		
Language	-0.010	0.101	-0.073	1	
Border	-0.012	0.037	-0.462	0.189	1

Source: Patent Panel dataset, author's estimation.

Table B5: Contribution of Sectoral Linkages (Selling-Side) to Sectoral Outcome (%)

		$\sum_{r=1}^3 \sum_{i=1}^4 B_i^{rr}$	$\sum_{r,s=1,1}^3 \sum_{i=1}^4 B_i^{rs}$	$\sum_{r=1}^3 \sum_{i=1}^4 D_i^{rr}$	$\sum_{r=1}^3 \sum_{i=1}^4 F_i^{rr}$	$\sum_{r,s=1,1}^3 \sum_{i=1}^4 L_i^{rs}$	$\sum_{r,s=1,1}^3 \sum_{i=1}^4 F_i^{rs}$
Japan	AGR	83.82%	16.18%	83.64%	0.18%	15.94%	0.24%
	MN	84.81%	15.19%	84.61%	0.20%	14.96%	0.23%
	MANF	72.66%	27.34%	72.35%	0.31%	26.93%	0.42%
	SRV	92.58%	7.42%	92.50%	0.08%	7.31%	0.11%
Japan Average		84.19%	15.81%	84.01%	0.18%	15.57%	0.24%
Africa	AGR	88.95%	11.05%	88.87%	0.08%	10.77%	0.28%
	MN	36.94%	63.06%	36.39%	0.55%	60.18%	2.88%
	MANF	78.69%	21.31%	78.53%	0.16%	20.71%	0.60%
	SRV	86.04%	13.96%	85.94%	0.10%	13.51%	0.45%
ROW	AGR	98.13%	1.87%	97.62%	0.51%	1.85%	0.02%
	MN	95.41%	4.59%	94.29%	1.12%	4.54%	0.05%
	MANF	98.03%	1.97%	97.48%	0.55%	1.95%	0.02%
	SRV	99.17%	0.83%	98.96%	0.21%	0.82%	0.01%
All Average		79.66%	20.34%	79.27%	0.39%	19.62%	0.72%

Note: This analysis relies on the Ghosh inverse matrix as a selling-side approach: $X = GF, G = (I - A)^{-1}$.

Source: GTAP 10A MRIO Database, author's estimation.