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Abstract The coronavirus disease 2019 (COVID-19) pandemic has fatalized 216 countries across the world and has claimed the lives of millions of people globally. Researches are being carried out worldwide by scientists to understand the nature of this catastrophic virus and find a potential vaccine for it. The most possible efforts have been taken to present this paper as a form of contribution to the understanding of this lethal virus in the first and second wave. This paper presents a unique technique for the methodical comparison of disastrous virus dissemination in two waves amid five most infested countries and the death rate of the virus in order to attain a clear view on the behaviour of the spread of the disease. For this study, the dataset of the number of deaths per day and the number of infected cases per day of the most affected countries, The United States of America, Brazil, Russia, India, and The United Kingdom have been considered in first and second wave. The correlation fractal dimension has been estimated for the prescribed datasets of COVID-19 and the rate of death has been compared based on the correlation fractal dimension estimate curve. The statistical tool, analysis of variance has also been used to support the performance of the proposed method. Further, the prediction of the daily death rate has been demonstrated through the autoregressive moving average model. In addition, this study also emphasis a feasible reconstruction of the death rate based on the fractal interpolation function. Subsequently, the normal probability plot is portrayed for the original data and the predicted data, derived through the fractal interpolation function to estimate the accuracy of the prediction. Finally, this paper neatly summarized with the comparison and prediction of epidemic curve of the first and second waves of COVID-19 pandemic to picturize the transmission rate in the both times.

Keywords Coronavirus Disease · Fractal Time Series · Correlation Dimension · Fractal Interpolation Function · Autoregressive Model · Prediction Analysis.

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1 Introduction

The world is in a state of havoc and turmoil due to the increasing fatality of the epidemic virus, called the Severe Acute Respiratory Syndrome Corona Virus 2 (SARS-CoV-2) which is the most prevalent coronavirus. The Coronavirus Disease 2019 (COVID-19) has put the world in a mode of panic which has led to a state of emergency that has profoundly affected the manner in which we see our reality and our day by day lives. This COVID-19 pandemic has presented both health and financial crisis across the world. Currently there is no medications to avert or to treat the infection. The World Health Organization (WHO) advises not to treat the infection with any self-medication. Also, WHO recommends prevention, controlling infections and adopting health policies as key health priorities in dealing with the disease. However, there are many clinical trials involving western and traditional medicines. According to a report given by WHO and other world eminent organizations in the arena of public health, as on 07^{th} August 2020, the total number of confirmed cases and deaths of coronavirus disease are 19,172,505 and 716,327 respectively. Exclusively in USA, Brazil, Russia, India and UK the total number of COVID-19 confirmed cases are significantly increasing and unfortunately reports indicate that the total number of death cases are also gradually increasing. The increasing number of infected cases and deaths of the pandemic demands a continuous data analysis, so that the dynamical behaviour of the virus can be understandable and it leads us towards the growth control of the epidemic.

Controlling the outbreak of COVID-19 infections requires systematic planning and strategies, so the researchers can utilize the Mathematical modelling for COVID-19 data analysis. In mathematical perspective, one can recognize the patterns and general mechanisms in the process of disease, which can assist to identify some of the basic structures that govern eruptions and epidemics. Recently, several researchers have focused on predictions based on some mathematical analysis of the exact number of COVID-19 cases. In particular, ARIMA model is applied on COVID-19 dataset to predict the spread and occurrence of the virus [1]. B. K. Mishra et al [2] described three quarantine models to handle the pandemic disease by considering several compartments called susceptible population, immigrant population, home isolation population, infectious population, hospital quarantine population, and recovered population in 2020. Kassa, S. M. et al [3] developed a mathematical model for analyzing the disease COVID-19. Bouchnita and Jebrane [4] designed a multi scale model which simulates the transmission dynamics of COVID-19. Also the authors demonstrated that the stability of SARS-CoV-2 on hard surfaces determines the number of events reached during the peak of the infection. Djilali and Ghanbari [5] estimated a predictive mathematical analysis for the epidemiological outbreak of coronavirus infection in South Africa, Turkey, and Brazil in 2020. Furthermore, the authors investigated the impact of isolation of affected individuals on the spread of COVID-19 disease. Fanelli and Piazza used the susceptible-infected-recovered-deaths (SIRD) model for forecasting COVID-19 spreads to Italy, China and France in 2020 [6]. Sun and Wang [7] developed a mathematical model to classify imported escapes and asymptomatic patients. Using the described model, the authors accomplished some stochastic simulations for the pandemic. Ayinde et al. [8] proposed some curve estimation models with an auto regressive model of order 1 to contemplate the pattern of the COVID-19 cases from Nigeria. Ghosal et al. Ghosal, S et al. [9] predicted a linear regression analysis for the number of deaths due to SARS-CoV-2. The qualitative evaluation has been made based on the implemented prevention and control interventions to control the epidemic of COVID-19 [10]. Creating models that can interpret the infection and the general trend parameters are useful for prediction purposes; it may be useful for future planning for viral respiratory illnesses by other countries that are at an early stage of the epidemic. Two models are developed by the authors one model for analyzing the spread during the fast phase and the other for interpreting the entire data set and those models reasonably agree with the data [11]. In order to analyze this harmful COVID-19 outbreak, a modified SEIRS model has been constructed to extend prediction on the current projections of the pandemic into three possible outcomes; death, recovery, and recovery with a possibility of resusceptibility [12]. A power law with Pareto exponent has also been modelled nearly equal to the exponent estimated directly from the distribution of confirmed cases across counties [13].

As the fractal and multifractal analysis have been applied to time series and experimental signals, the dimensional measures have mainly been used to analyze the change in the chaotic nature in different physiopathological conditions and to estimate the complexity in forecasting process [14, 15, 16, 17, 18]. Among all the nonlinear schemes, the correlation fractal dimension measurement is more accessible in dealing with diffusion data with more complexity in nature for prediction [19]–[28]. Due to an uncertainty in the spread of coronavirus, the empirical data of the number of infected cases and death cases is being analyzed based on the power law kinetics with fractal exponent which gives a best fit to the contemporary data than the traditional epidemiology method. Hence, the fractal based comparative study of COVID-19 has been provoked at different situations [29,30,31,32,33]. This study proposes the correlation fractal dimension and the autoregressive moving average model based comparative prediction analysis of COVID-19 death rate in the first and second wave.

The main goal of this study is to provide a technique for a systematic comparison of COVID-19 spread among five most infected countries and the death rate to understand the chaotic behavior of the disease transmission. We have considered the datasets of the number of deaths per day and the number of infections per day in the five most affected nations, namely The United States of America (USA), Brazil, Russia, India and The United Kingdom (UK). The correlation dimension is estimated for the representative datasets of daily infections, daily deaths, and daily death rate of COVID-19. Further, the mean differences of the actual datasets and its computed correlation dimension estimates are compared by the analysis of variance. At last, the comparative analysis are performed to depict the structure of the transmission and death rate of the corona virus in both phases.

The present work is organized as follow: Section 2 succinctly portrays the mathematical techniques of the correlation dimension, fractal interpolation and the necessary statistical methods. The dataset information is described in section 3. In Section 4, the estimation of the correlation dimension and the fractal interpolation representation for COVID-19 dataset are explored. The ARMA Processes and the parametric statistical technique one-way analysis of variance are also performed on the actual datasets and the calculated correlation dimension estimates in the same section. Additionally, the second wave analysis are done by comparative study and also discussed elaborately in Section 4. The concluding remarks of the research work are presented in Section 5.

2 Mathematical Methods

2.1 Renyi Entropy

One of the most important concepts in modern science is entropy and it is a measure to estimate the multiformity, unsureness or randomness of a system in the theory of information. If the value of the random variable is unsure one can use Shannon entropy which is a measure of the average information. Renyi entropy is Shannon's generalization entropy which is also used to define the correlation fractal dimension. The Renyi entropy of order q, $q \neq 1$ is positive real number [34], is defined by

$$S_q = \frac{1}{1-q} \log_2 \left(\sum_{i=1}^{N} p_i^q \right), \tag{1}$$

where $p_i \in [0,1]$ are the probabilities of the discrete random variable X which takes N possible values $x_1, x_2, \ldots, x_N$, that is $P_X(x_i) = p_i$ for $i = \{1, 2, \ldots, N\}$.

2.2 **Correlation Fractal Dimension**

Correlation Fractal Dimension is one of the generalized fractal measures which imparts the data about the kind of dynamics available and also describes the presence of chaotic dynamics in the time series [35,36]. The various methods for calculating the dimension are the Hausdorff dimension, the box-counting dimension and the information dimension are available to analyze the signals or time series, but the correlation dimension has a predominant scope of being computed immediately [37,38,39]. This section concisely presents the necessary materials of the correlation dimension.

Based on the Renyi entropy, the correlation dimension (correlation fractal dimension) of a fractal time series is defined by constructing the probability distribution as follows [19]–[28].

Let $x_1, x_2, \ldots, x_n$ be the points of a fractal time series and the total range of the time series is divided into N intervals (bins) such that

$$N = \frac{x_{max} - x_{min}}{r}$$

where x_{max} and x_{min} are the maximum & the minimum values of the time series for the corresponding experiment, respectively; and r is the uncertainty factor.

Define $p_i = \frac{N_i}{N}$, where N_i is the number of points of the time series lies in the i^{th} bin of length r.

Then, the correlation dimension (D_c) for the known probability p_i is defined by

$$D_c = \lim_{\varepsilon \to 0} \frac{S_2}{\log_2(1/\varepsilon)} = \lim_{\varepsilon \to 0} \frac{\log_2 \left(\sum_{i=1}^{N} p_i^2 \right)}{\log_2 \varepsilon}. \tag{2}$$

2.3 Fractal interpolation function

Fractal interpolation functions are generated by a special type of iterated function system (IFS) of affine transformations. This section concisely presents the necessary materials to construct the fractal interpolation functions, more rigorous treatment of the fractal interpolation is given in [40]–[45].

Let the sample data $\{(x_k, y_k) \in \mathbb{R}^2 : k \in \mathbb{N}_n\}$ with $x_1 < x_2 < \cdots < x_n$ be given and x_k's are not necessarily at equal distances. Let $I = [x_1, x_n]$, $I_k = [x_k, x_{k+1}]$ for $k \in \mathbb{N}_{n-1}$ and $L_k : I \to I_k$, $k \in \mathbb{N}_{n-1}$ be $(n-1)$ contractive homeomorphisms satisfying the endpoint conditions

$$L_k(x_1) = x_k,\ L_k(x_n) = x_{k+1}. \tag{3}$$

Let $r_k \in [0, 1), k \in \mathbb{N}_{n-1}$ and $X = I \times \mathbb{R}$. Let $n-1$ continuous functions $R_k : I \times \mathbb{R} \to \mathbb{R}$ obeys

$$R_k(x_1, y_1) = y_k, R_k(x_n, y_n) = y_{k+1}. \tag{4}$$

Moreover, R_k is a contraction with respect to the second variable. Define the functions $f_k : X \to I_k \times \mathbb{R}, k \in \mathbb{N}_{n-1}$, by $f_k(x, y) = (L_k(x), R_k(x, y))$. The iterated function system $\{X; f_k : k \in \mathbb{N}_{n-1}\}$ corresponds to the self-mapping F on $\mathcal{K}(X)$ defined by $F(A) = \bigcup_{k\in\mathbb{N}_{n-1}} f_k(A)$, for any $A \in \mathcal{K}(X)$. Then, the space $\mathcal{K}(X)$ is complete with respect to the Hausdorff metric, since X is complete metric space. Further, F is contraction on $\mathcal{K}(X)$ hence by the IFS theory, F has a unique fixed point G whose graph is a continuous function $g : I \to \mathbb{R}$ which satisfies $g(x_k) = y_k$ for $k \in \mathbb{N}_n$. The function g generated by the IFS $\{X; f_k : k \in \mathbb{N}_{n-1}\}$ is termed as fractal interpolation function corresponding to the data points $\{(x_k, y_k) \in \mathbb{R}^2 : k \in \mathbb{N}_n\}$.

This paper deals the prediction of COVID-19 data with the following IFS to generate the fractal interpolation functions: $\{[x_1, x_n] \times \mathbb{R}; w_k : k \in \mathbb{N}_n\}$, where

$$w_k \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_k & 0 \\ c_k & \alpha_k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} b_k \\ d_k \end{pmatrix} \tag{5}$$

and $\alpha_k \in (-1, 1)$ for all $k \in \mathbb{N}_n$. Here the free parameter α is called the vertical scaling factor which decides the pattern and fractal dimension of the fractal interpolation function. By Eq.(4) w_k maps the terminals of given data to the terminal of each subinterval. That is, w_k maps (x_1, y_1) to (x_{k-1}, y_{k-1}) and image of (x_n, y_n) under the mapping w_k is (x_k, y_k) for each k. Hence, the mapping w_k has following constructions.

$$w_k \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_{k-1} \\ y_{k-1} \end{pmatrix} \text{ and } w_k \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} x_k \\ y_k \end{pmatrix} \tag{6}$$

for all $i \in \mathbb{N}_n$. The image of lines parallel to the y-axis under the mapping w_i is the lines parallel to y-axis. Further, if the length of the line L is l, then the length of its image is α_k times multiple of l, for all $k \in \mathbb{N}_n$. That is, the ratio between the length of the lines L and $w_k(L)$ is $|\alpha_k|$. If α_k is a predefined variable in the system (5) with constrains (6), then it provides a unique solution for (5). Therefore, the constants a_k, b_k, c_k, d_k can be obtained uniquely as follows

$$\begin{aligned} a_k &= \frac{x_k - x_{k-1}}{x_n - x_1} \\ b_k &= \frac{x_n x_{k-1} - x_1 x_k}{x_n - x_1} \\ c_k &= \frac{(y_k - y_{k-1}) - \alpha_k(y_n - y_1)}{x_n - x_1} \\ d_k &= \frac{(x_n y_{k-1} - x_1 y_k) - \alpha_k(x_n y_1 - x_1 y_n)}{x_n - x_1}. \end{aligned} \tag{7}$$

If $\alpha_k = 0$ for all $k \in \mathbb{N}_n$, then it gives the classical piecewise linear interpolation function. The scaling factor α_k decides the fractal dimension of fractal interpolation function.

2.4 ARMA Processes

The Time Series analysis has an extensive range of applications in forecasting, spectral estimation and modelling of time domain system. The time series can be analyzed by means of time domain and frequency domain. Frequency domain approach is the study of non-parametric decay using a spectral function in which the time series is used in its different frequency components. Albeit, the time domain approach focuses on parametric models to investigate the future values of the time series as a parameter function of the present and past values. One of the foremost methods to investigate the time series is Auto Regressive Moving Average (ARMA) which provides an intimate description of a standard random process based on two polynomials such as Autoregression (AR) and Moving-Average (MA). Besides, the complex time series can be analyzed, modelled and forecasted in both the time and the frequency domain in detail by this superlative method. ARMA(p,q) notion indicates the model with p autoregressive terms AR(p) and q moving-average terms MA(q),

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \phi_t X_{t-i} + \sum_{i=1}^{p} \theta_t \varepsilon_{t-i} + \cdots. \tag{8}$$

where c is a constant, ε_t is a random variable and $\phi_1, \phi_2, \ldots, \phi_p$ and $\theta_1, \theta_2, \ldots, \theta_q$ are parameters of AR(p) and MA(q) respectively.

2.5 Statistical Method

One-Way Analysis of Variance (ANOVA) is a parametric statistical technique used to compare datasets based on the mean and the variance of the given datasets. In ANOVA, the inferences about means are made by analyzing the variance. If the observed differences are high, then it is considered to be statistically significant. The p-value can be determined by the F statistic in the analysis of variance between groups which is the probability of getting a result. In ANOVA test, if the p value is near zero, the null hypothesis is rejected and recommends the conclusion that at least one sample mean is significantly different from the other sample means. We can establish the fact that the significant differences between the dataset increases when the p-value approaches to zero.

3 Data Description

The empirical data have been accessed in the data repository of COVID-19 provided by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University [46] and Statistics & Research Coronavirus Pandemic (COVID-19) available in the database "Our World in Data"[47]. The daily infection count and the daily death count datasets of the five most affected countries have been selected and utilized in this research work as on 18^{th} January 2021 and the same information of the representative datasets are meticulously presented in Table 1 [48].

Table 1: Countries with the Most Number of COVID-19 Cases as on 18^{th} January 2021

Country	Total Infected Cases	Total Deaths
USA	2,44,38,935	4,06,162
Brazil	86,38,249	2,12,831
Russia	36,16,680	66,810
India	1,06,10,883	1,52,869
UK	35,15,796	93,469

4 Results and Discussion

In this study, the five most disease-ridden countries have been considered and the dataset of each country comprises a total of 100 observations which is end on the date 30^{th} September 2020 for exploring the first phase.

The day wise total number of reported cases is subject to change with the amount of testing that has been completed in a particular day, henceforth we have decided to intensely examine the total number of death cases and the ratio of death cases with the total number of infected cases per day. Especially in the fatality rate or the death rate, the number of identified deaths out of the total number of confirmed cases, is classified in terms of its correlation dimension estimate curve. The daily death rate of COVID-19 as a function of the number of days is elucidated in Fig. 1 (a), (c), (e), (g), (i), respectively, for USA, Brazil, Russia, India, UK and extremum in the daily death rate is marked with a reference mark. Also, the correlation dimension estimates as a function of the number of days for the death rate are plotted in Fig. 1 (b), (d), (g), (h), (j), respectively, for USA, Brazil, Russia, India and UK. Fig. 1 narrates that the peak of death rate of USA and Brazil occurred in the initial timeline and conspicuously reducing at the end. Though the fatality rate of Russia and India is significantly less in the onset period, getting contrary progress in the end phase.

Further, the actual dataset of the death rate of each country is compared with the death rate of the world which is exhibited in Fig. 2 (a). USA, (c). Brazil, (e). Russia, (g). India, (i) UK. Similarly, analogies of the correlation dimension estimate between the total death rate of each country and the total death rate of the world per day is displayed in Fig. 2 (b). USA, (d). Brazil, (f). Russia, (h). India, (i) UK. Comparison in Fig. 2 demonstrates that the daily death rate of UK is in an increased amount than the the daily death rate of the world from the beginning to end. It can be noted that, the daily death rate of USA and Brazil is fluctuating over the death rate of the world, whereas the death rate of Russia and India is lower than the death rate of the world except one point at the end.

The correlation dimension estimates for the number of infected cases per day, the number of deaths per day and the daily death rate are plotted in Fig. 3. Comparison in Fig. 3 narrates that, though the total number of infections of UK is less than the other four countries, the correlation dimension estimates of the daily infected cases and the daily death rate of UK is notably higher. The correlation dimension estimate curve of the daily death rate of USA is of concern, it stands behind UK but the total deaths of USA is at peak. In the onset phase, the correlation dimension estimate curve of the daily death rate of India is very low, later crossing Russia's curve around 100 bins which gradually increases to overtake the

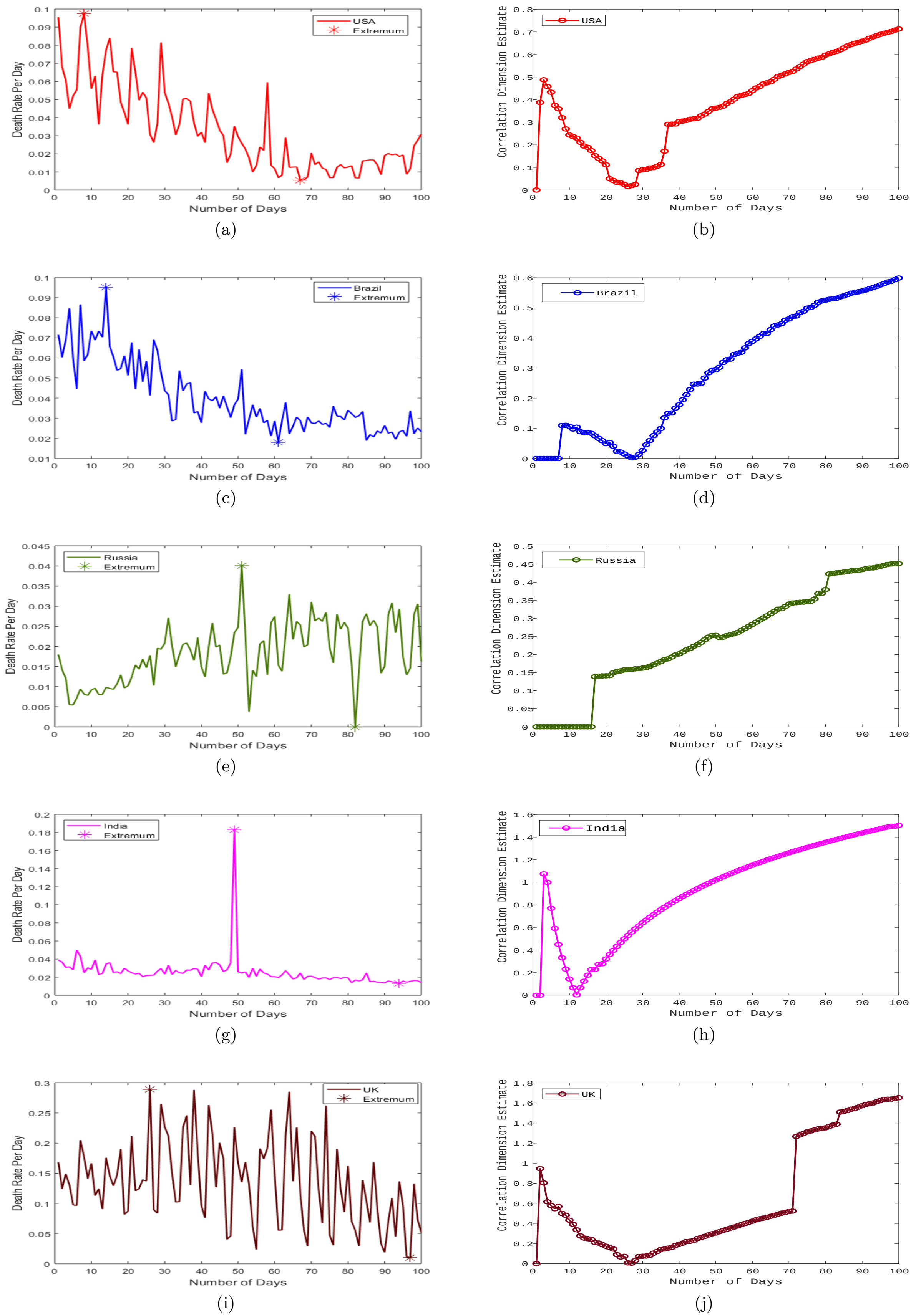

Fig. 1: Death Rate Versus Number of Days: (a). USA, (c). Brazil, (e). Russia, (g). India, (i) UK and the Corresponding Correlation Fractal Dimension Estimates Against the Number of Days

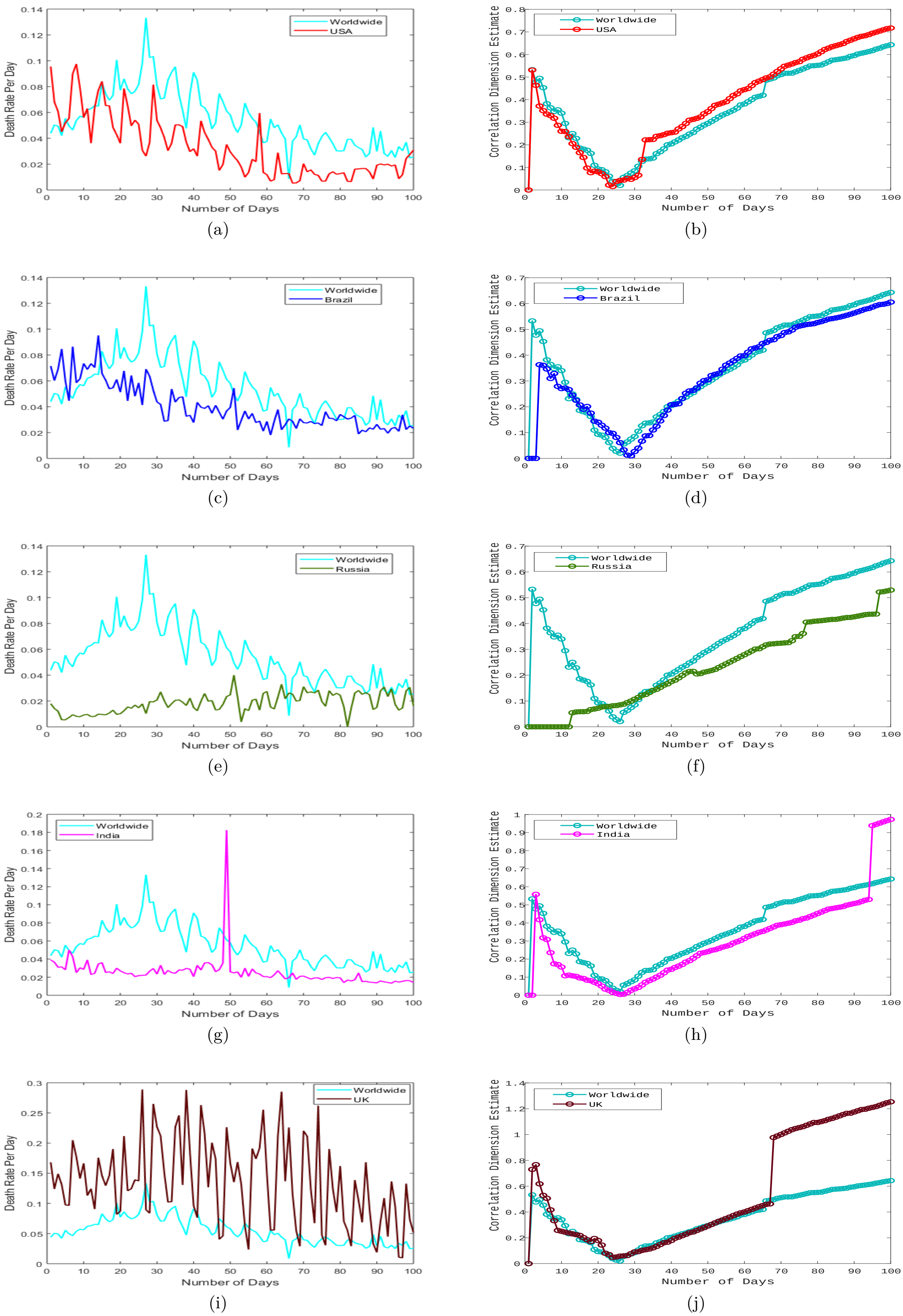

Fig. 2: Comparison Between the Worldwide Daily Death Rate and the Representative Countries' Daily Death Rate: (a). USA, (c). Brazil, (e). Russia, (g). India, (i) UK and Analogies of the Correlation Dimension Estimates Between Daily Death Rate of the Country and Worldwide Daily Death Rate: (b). USA, (d). Brazil, (f). Russia, (h). India, (j) UK

Brazil death rate curve. Computational results reveal that the death rate of UK significantly increases, so this study recommends that the necessary control measures to be taken by the public health authorities to stabilize the death rate of UK. The correlation fractal dimension for COVID-19 datasets of the five most diseased countries are computed and the gained results are tabulated in Table 2.

The mean differences of the datasets, the number of infections per day, the number of deaths per day and the daily death rate of the five countries USA, Brazil, Russia, India and UK are statistically tested by one-way analysis of variance. The statistical measures of one-way ANOVA is shown in Table.3. The box and whisker plot for COVID-19 datasets of daily infections, daily deaths, daily death rate and its correlation fractal dimension estimates among all five mostly affected countries are elucidated in Fig. 4. In box plot, the median value of daily infection of USA, India, Brazil, Russia, UK is 22738, 1943, 5177, 6386, 2984, respectively, which are marked in red line as in Fig. 4 (a). The one-way ANOVA test statistically substantiates the proposed procedure in this study, the p-value in Table.3 (a), (b) and (c) are near zero and in Table.3 (d), (e) and (f) also near to zero. Hence, one-way ANOVA suggests the conclusion that at least one sample mean is significantly different from the other sample means in actual datasets as well as the corresponding correlation fractal dimension estimates.

The periodograms in Fig. 5 sequentially demonstrate the power spectral density estimate for the original and the predicted data representing the daily death rate in USA, Brazil, Russia, India and UK. In Fig6, the autoregressive signal transformed from the original data and the predicted signal from the corresponding linear predictor coefficient are sketched against the number of days in terms of stem plot. Fig. 7 portrays the predicted data of the death rate along with the corresponding original death rate of COVID-19 of the represented countries. Autoregressive polynomial coefficient for predicted data is given in Table.4. A phenomenal time, the rapid rise of the death cases in several countries of the world is representing a major lacuna on what the future will carry for us. The top five countries together significantly account in the total infected cases and death cases. Fig. 6 and Fig. 7 illustrate a comparative analysis of death rate prediction. Countries around the world are working to flatten the death rate curve of this pandemic.

As a comparison in Fig. 7 elucidates that predicted death rate curves of USA and Brazil are set to peak in early time and gradually reducing the peak at the end. Though, the death rate curve of Russia, India is flatten throughout the prediction timeline, it is not seen a declining trend at the terminal. Hence, the public health authorities of Russia, India should cautiously maintain the stabilized action to uphold the current status of the death rate.

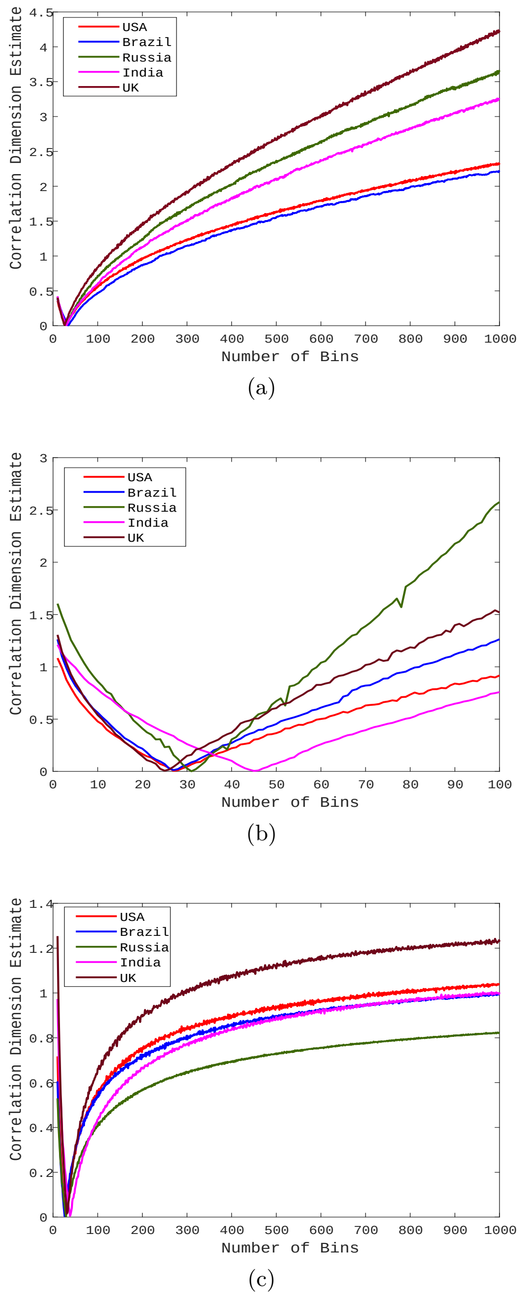

Fig. 3: Correlation Fractal Dimension Estimates of COVID-19: (a). Infected Cases Per Day, (b). Deaths Per Day, (c). Death Rate Per Day

Table 2: Correlation Fractal Dimension for the First Phase of COVID-19 Datasets

Country	Infected Cases Per Day	Deaths Per Day	Death Rate Per Day
USA	2.3163	1.0171	1.3026
Brazil	2.4918	4.7337	0.9665
Russia	3.5853	0.9429	0.7580
India	3.4152	1.4693	1.0801
UK	4.0864	1.7201	1.3026

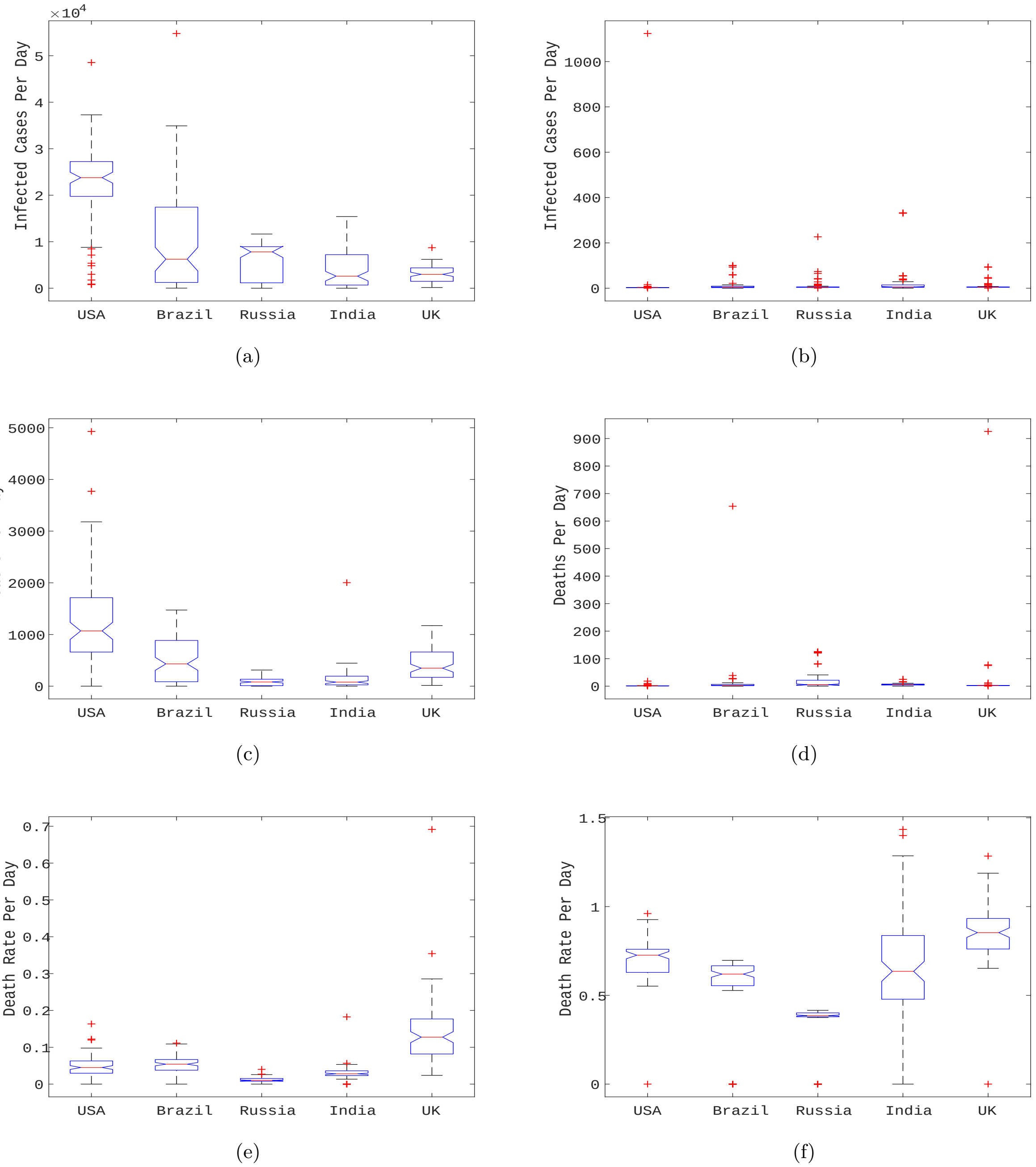

Fig. 4: Box Plot for Original COVID-19 Datasets: (a). Infected Cases Per Day, (c). Deaths Per Day, (e). Death Rate Per Day. Box Plot of the Correlation Fractal Dimension Estimate for the Datasets: (b). Infected Cases Per Day, (d). Deaths Per Day, (f). Death Rate Per Day

Table 3: One-Way ANOVA Table – Original COVID-19 Datasets: (a). Infected Cases Per Day, (b). Deaths Per Day, (c). Death Rate Per Day. Correlation Fractal Dimension Estimates: (d). Infected Cases Per Day, (e). Deaths Per Day, (f). Death Rate Per Day

ANOVA Table					
Source	**SS**	**df**	**MS**	**F**	**Prob > F**
(a) Infected Cases Per Day					
Columns	2.35743e+10	4	5.89358e+09	128.89	2.4571e-75
Error	2.26342e+10	495	4.57257e+07		
Total	4.62086e+10	499			
(b) Deaths Per Day					
Columns	7.77547e+07	4	19438678.7	89.14	5.07082e-57
Error	1.07942e+08	495	218063.7		
Total	1.85696e+08	499			
(c) Death Rate Per Day					
Columns	0.65621	4	0.16405	53.87	1.08402e-37
Error	1.50735	495	0.00305		
Total	2.16356	499			
(d) D_c Estimates of Infected Cases Per Day					
Columns	9527.47	4	2381.87	0.7	0.593
Error	1687131.86	495	3408.35		
Total	1696659.33	499			
(e)D_c Estimates of Death Rate Per Day					
Columns	33493.6	4	8373.39	2.43	0.0471
Error	1429702.7	415	3445.07		
Total	1463196.3	419			
(f) D_c Estimates of Death Rate Per Day					
Columns	14.7229	4	3.68073	122.68	1.13754e-72
Error	14.8516	495	0.03		
Total	29.5745	499			

Table 4: Autoregressive Moving Average Polynomial Coefficients for Prediction

Country	Autoregressive Moving Average Polynomial Coefficients				
	P_3	P_2	P_1	P_0	Error
USA	1	-0.4798	-0.4798	-0.5116	-4.0890e-04
Brazil	1	-0.4451	-0.4451	-0.5428	-2.5444e-04
Russia	1	-0.3856	-0.3856	-0.5251	-2.2326e-05
India	1	-0.4801	-0.4801	-0.5140	-5.9593e-04
UK	1	-0.4868	-0.4868	-0.5128	-0.0098
World	1	-0.3912	-0.3912	-0.5939	-1.6467e-04

The predicted death ratio of USA, Brazil, India, Russia and UK by the fractal interpolation function is exhibited in Fig. 8. Comparing the results of the fractal interpolation forecasting method, the predicted death rate curve of India follows a uniform pattern, but not has more fluctuation, whereas other countries have more oscillation in death rate. The probability plots for normal distribution of the original death rate and the predicted death rate of USA, Brazil, Russia, India and UK are depicted in Fig. 8. The normal probability plot generates each sample data point in death rate represented by + markers and also compared with the reference line theoretically given by the normal probability distribution. Basically, the reference line obtained by connecting the first and third quartiles of the sample data and extending to the extremum of the data. The sample data points appear along with the reference line provided the information that the sample data follows the normal distribution. Perhaps, the sample data does not possess the normal distribution, if those points are significantly deviated from the reference line. Comparisons illustrated in Fig. 8 elucidates that the predicted death rate by the fractal interpolation method obeys normal distribution, if the actual death rate follows normal distribution.

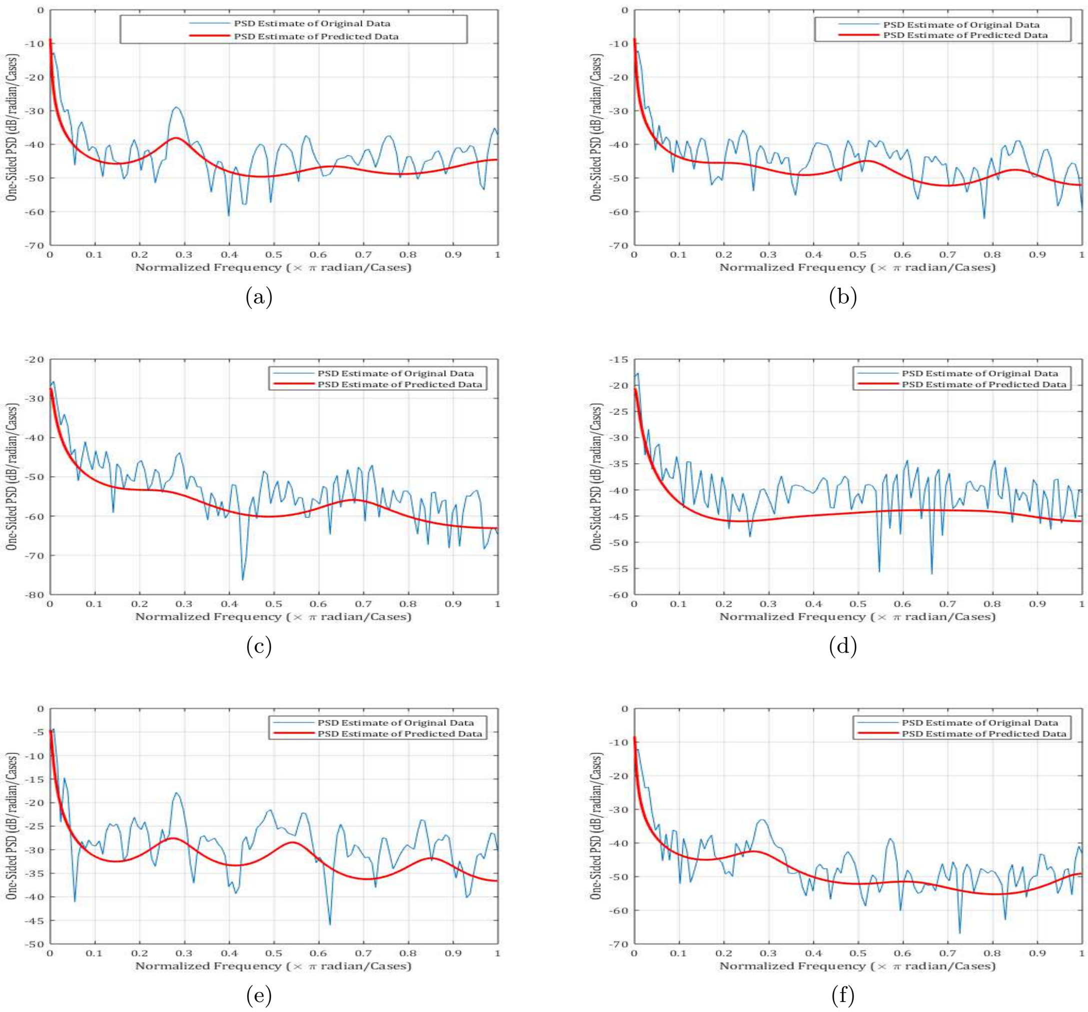

Fig. 5: Periodogram of Power Spectral Density Estimate for Original Data and Predicted Data: (a). USA, (b). Brazil, (c). Russia, (d). India, (e). UK, (f). World

4.1 Second Wave Analysis of COVID-19

In the first phase of COVID-19 pandemic, the massive public health interventions have been encountered across the world. Additionally, the most affected countries have executed stringent social measures based on local risk assessments, such as lockdown, mobility restrictions, compulsory face mask-wearing, telecommuting for non-essential community services, and virtual meeting in educational institutes as well as business sectors. However, the reduced COVID-19 positive count could undoubtedly resurge when the social measures are relaxed for economic and educational factors. As on 30^{th} September 2020, there were 7228526 (USA), 4810935 (Brazil), 1170799 (Russia), 6312584 (India), 455846 (UK), confirmed cases of COVID-19 infections reported in the database. It was collated publicly available officially confirmed cases of each country to construct the epidemic curve on January 18, 2021 in which 24078772 (USA), 8511770 (Brazil), 3552888 (Russia), 10581823 (India), 3443350 (UK), total number of confirmed cases of COVID-19 were reported. Moreover, the countries like Russia, UK, and USA are recording with the infected rate at the beginning of January 2021 comparatively higher than the infected rate of those detected from March to September, 2020. On the contrary, India's infected cases are lower than the other countries from the second week of January, 2021. It seems to be unpredictable that the attributes of the high severity of the first wave of pandemic in these countries.

The predicted death ratio of USA, Brazil, India, Russia and UK by the fractal interpolation function is

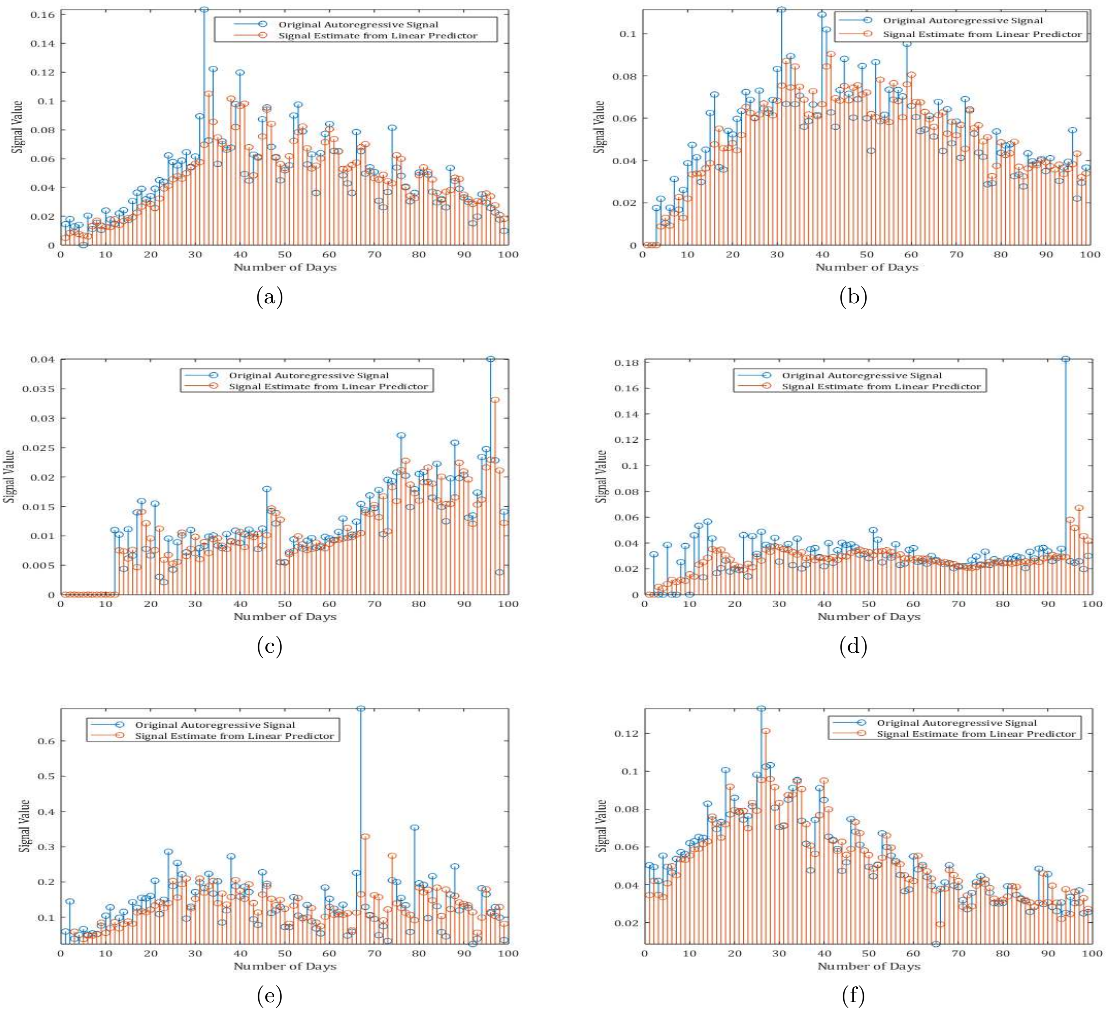

Fig. 6: Stem Plots of Original Autoregressive Signal and the Predicted Signal By Linear Predictor Against the Number of Days: (a). USA, (b). Brazil, (c). Russia, (d). India, (e). UK, (f). World

displayed in Fig. 9 based on the data recorded from 01^{st} October 2020 to 18^{th} January 2021. In [14], the authors have proposed a reconstruction method of the epidemic curves from the fractal interpolation function with constant vertical scaling factor. In particularly, vertical scaling factor is fixed as 0.1 for all the data constantly. The present study highlights a feasible reconstruction of the death rate based on the fractal interpolation function with the constant scaling factor. However, it is optimized the vertical scaling factor instead of choosing the constant value. Further, the correlation fractal dimension is estimated for the prescribed datasets of COVID-19 for the second phase as reported in the Table 5. Also, the correlation dimension estimates for the number of infected cases per day, the number of deaths per day and the daily death rate are plotted in Fig. 10. The Table 5 narrates the comparison that the fractal dimension of infected cases per day, number of deaths per day and daily death rate of Russia are 0.6942, 2.2627, 0.4834, respectively, which are higher than the other representative countries. The COVID-19 database of Russia marks the significant jump in the number of infected cases and the number of deaths in second wave of coronavirus compare to other countries.

5 Conclusion

The alarming and rapid rise of the rate of mortality due to the first and second wave of the COVID-19 in several countries of the world is bearing a major uncertainty on the forthcoming happenings. In this calamitous time of a contagious crisis, this study has provided an analytical insight on the comparative

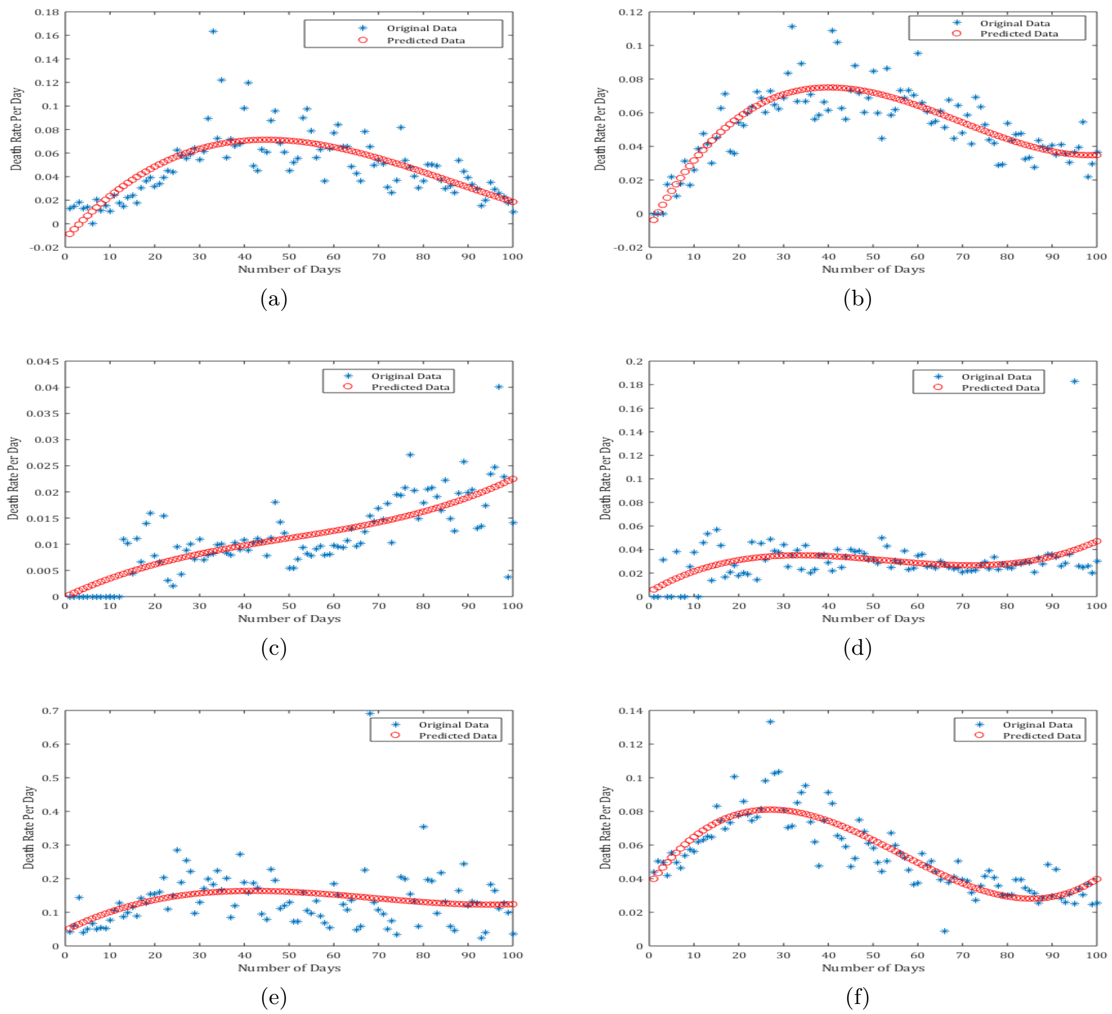

Fig. 7: Representation of Death Rate in Time Domain for Original COVID-19 Data and Predicted Data by ARMA Model Versus Number of Days: (a). USA, (b). Brazil, (c). Russia, (d). India, (e). UK, (f). World

Table 5: Correlation Fractal Dimension for Second Phase of COVID-19 Datasets

Country	**Infected Cases Per Day**	**Deaths Per Day**	**Death Rate Per Day**
USA	0.4646	0.9574	0.4311
Brazil	0.5312	1.3153	0.4163
Russia	0.6942	2.2627	0.4834
India	0.5277	1.4541	0.3743
UK	0.5193	1.2297	0.4613

analysis and prediction of the spread of the contagion in the first and second wave. This study has considered the datasets of the number of deaths and the number of infected cases per day of the five most affected countries for a comparative prediction analysis. The correlation fractal dimension is estimated for the prescribed datasets of COVID-19 in the first and second wave. Moreover, the comparative analysis of the death rates has been performed based on the correlation fractal dimension estimate curve. The performance of the proposed method has been supported by ANOVA statistical tool. Additionally, the prediction of the daily death rate has been demonstrated in two waves through the fractal interpolation function and autoregressive moving average model. This proposed method has successfully predicted the progress of this dreadful contagion and expressively implemented with the second wave

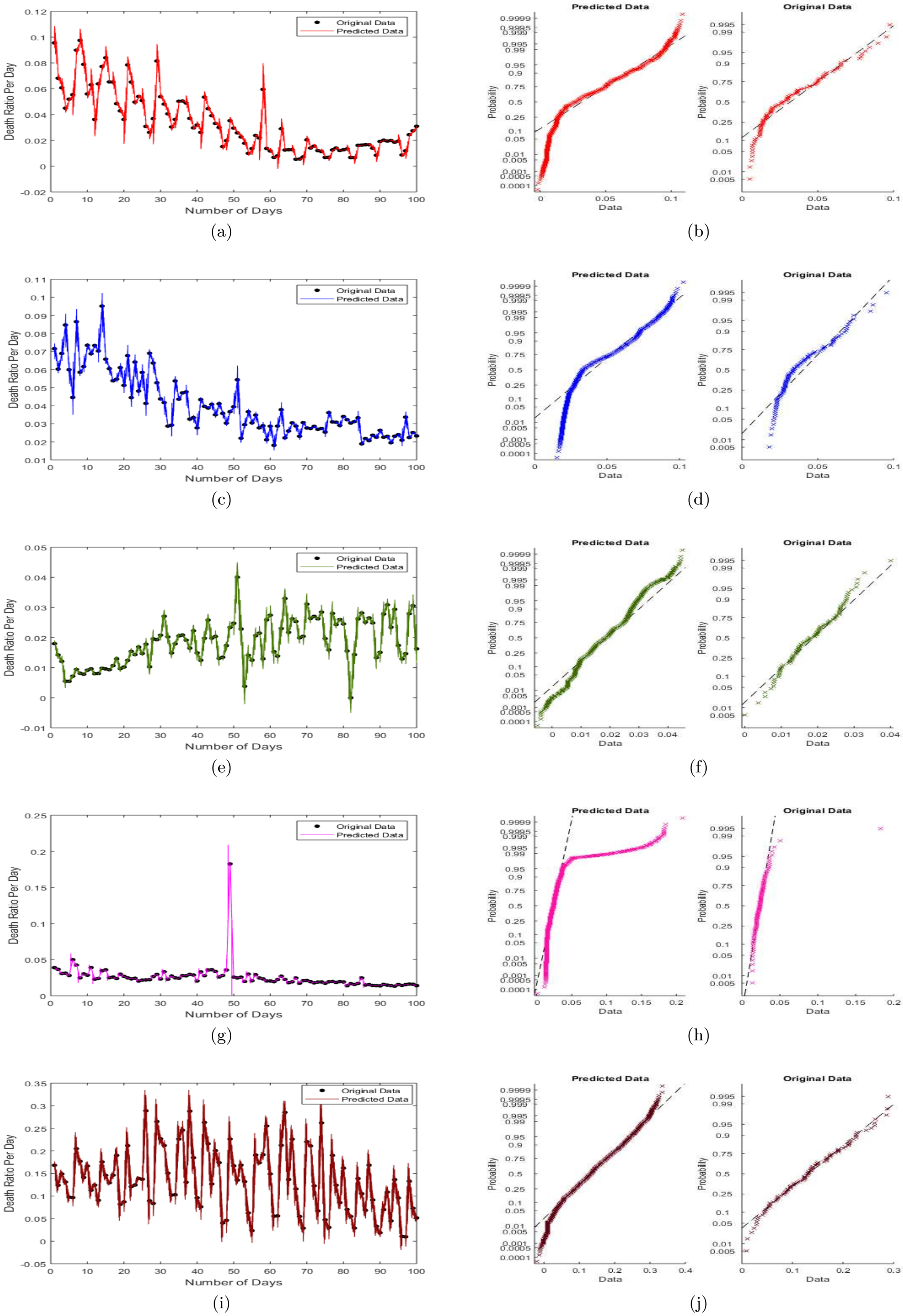

Fig. 8: Comparison Between the Original and Predicted Daily Death Rate of the Representative Countries by Fractal Interpolation Function: (a). USA, (c). Brazil, (e). Russia, (g). India, (i) UK and Analogies of the Probability Plot for Normal Distributions Between Predicted and Original Daily Death Rate: (b). USA, (d). Brazil, (f). Russia, (h). India, (j) UK

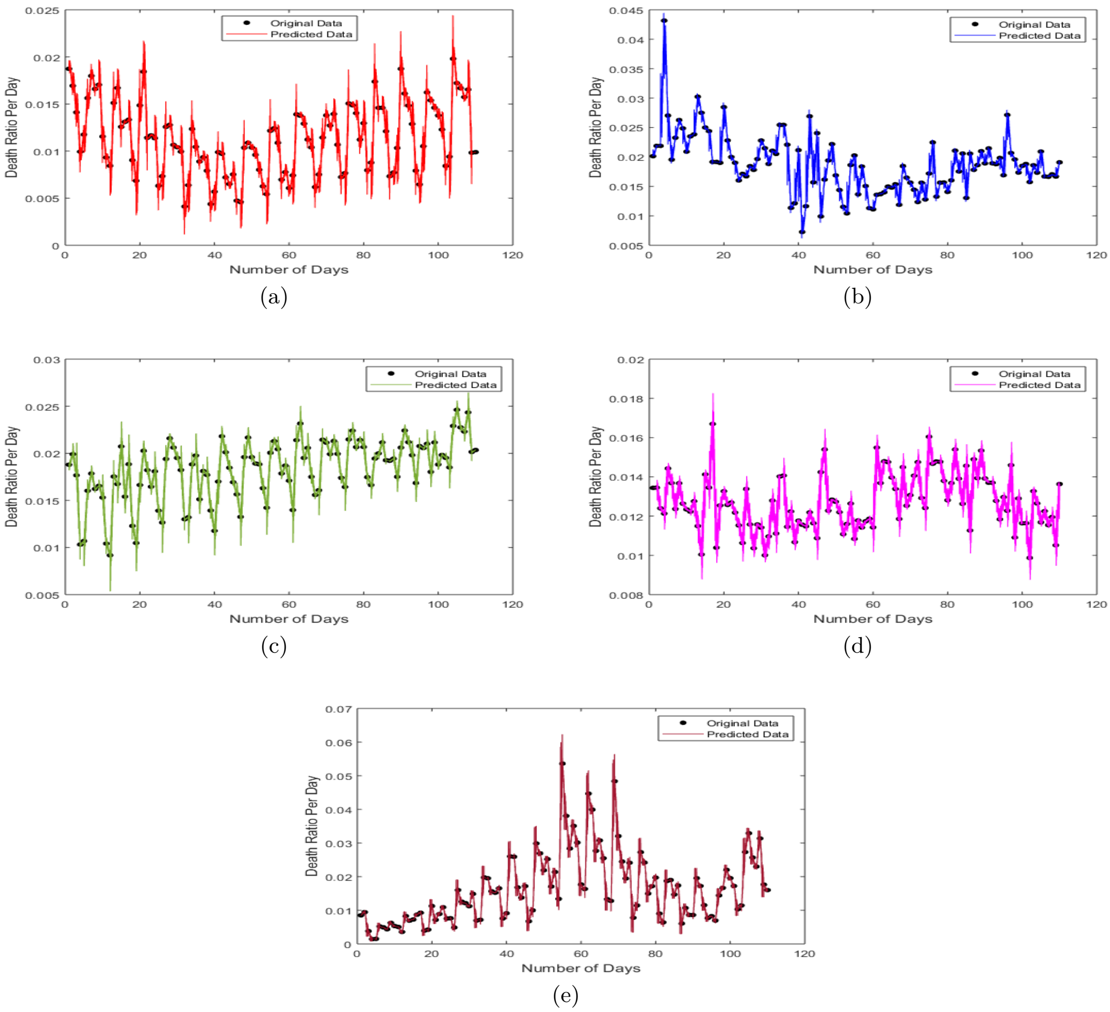

Fig. 9: Second Phase of COVID-19 – Comparison Between the Original and Predicted Daily Death Rate of the Representative Countries by Fractal Interpolation Function: (a). USA, (b). Brazil, (c). Russia, (d). India, (e). UK

analysis of COVID-19 by comparison based on the reported data provided by WHO and other prestigious organizations. It is observed generically from this study that, the prediction of the death rate is liable to change when an extensive amount of data is available with respect to the different phases and in such a scenario the rudiments of epidemic prevention, testing, tracking and treating must be concentrated severely.

Conflict of Interest

The authors declare that they have no conflict of interest.

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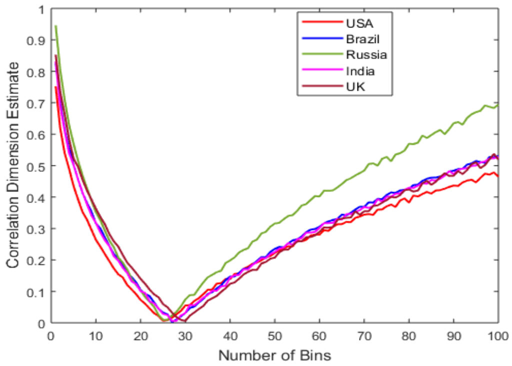

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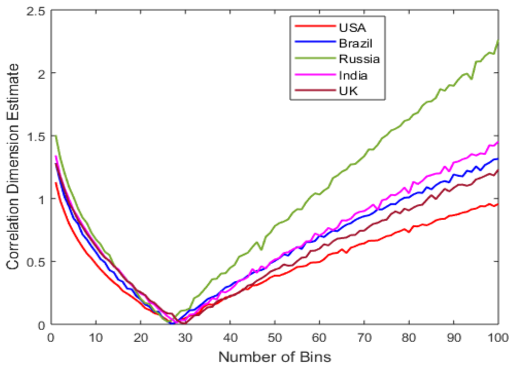

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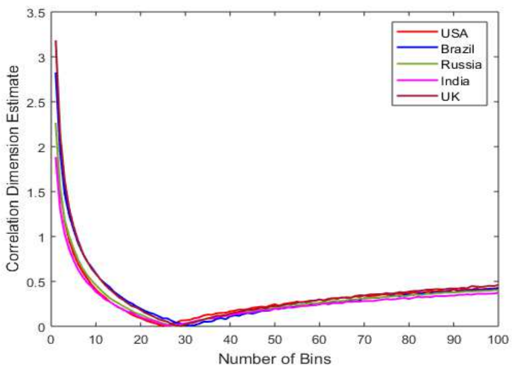

(c)

Fig. 10: Second Phase of COVID-19 – Correlation Fractal Dimension Estimates: (a). Infected Cases Per Day, (b). Deaths Per Day, (c). Death Rate Per Day

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Figures

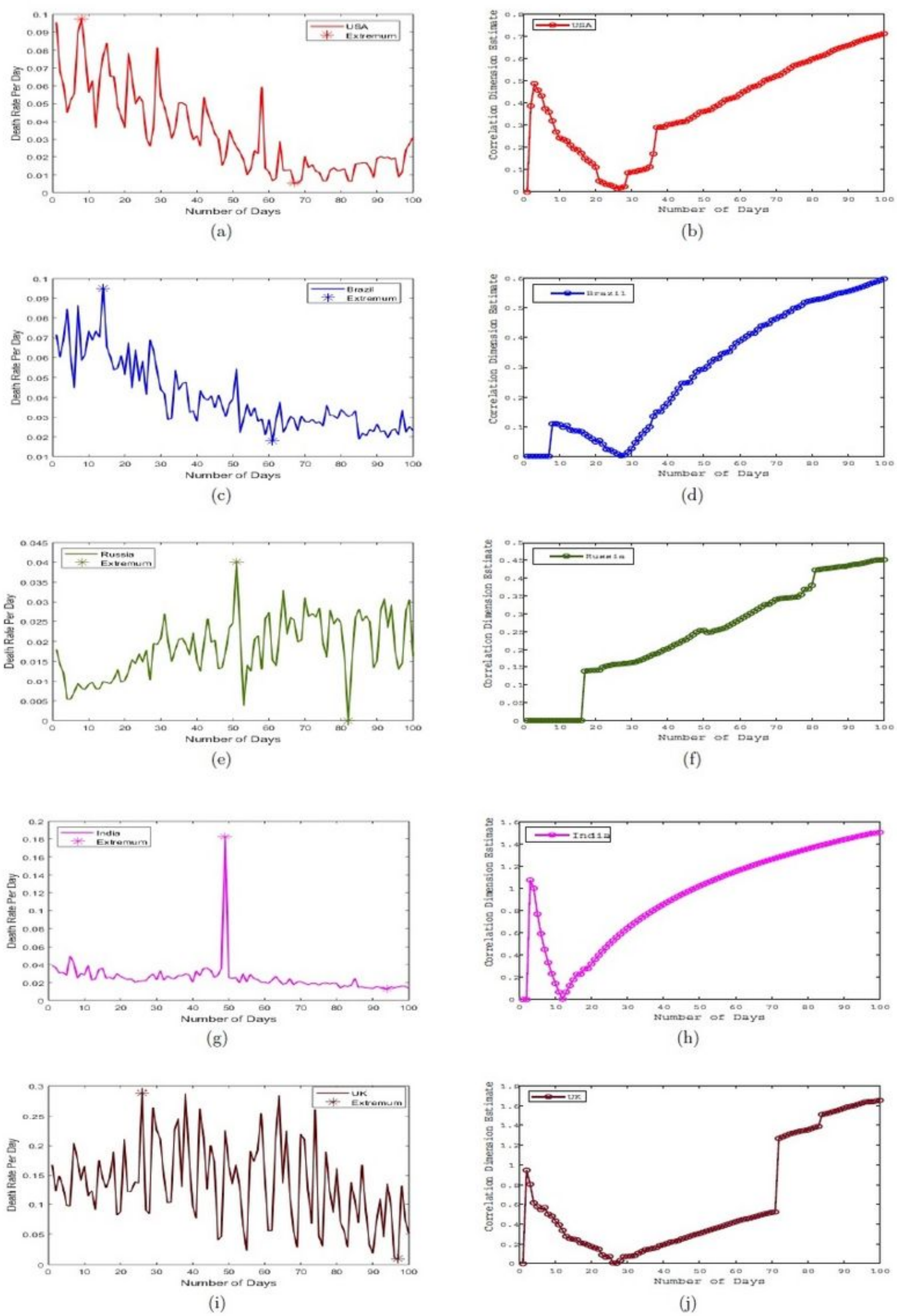


Figure 1

Death Rate Versus Number of Days: (a). USA, (c). Brazil, (e). Russia, (g). India, (i) UK and the Corresponding Correlation Fractal Dimension Estimates Against the Number of Days

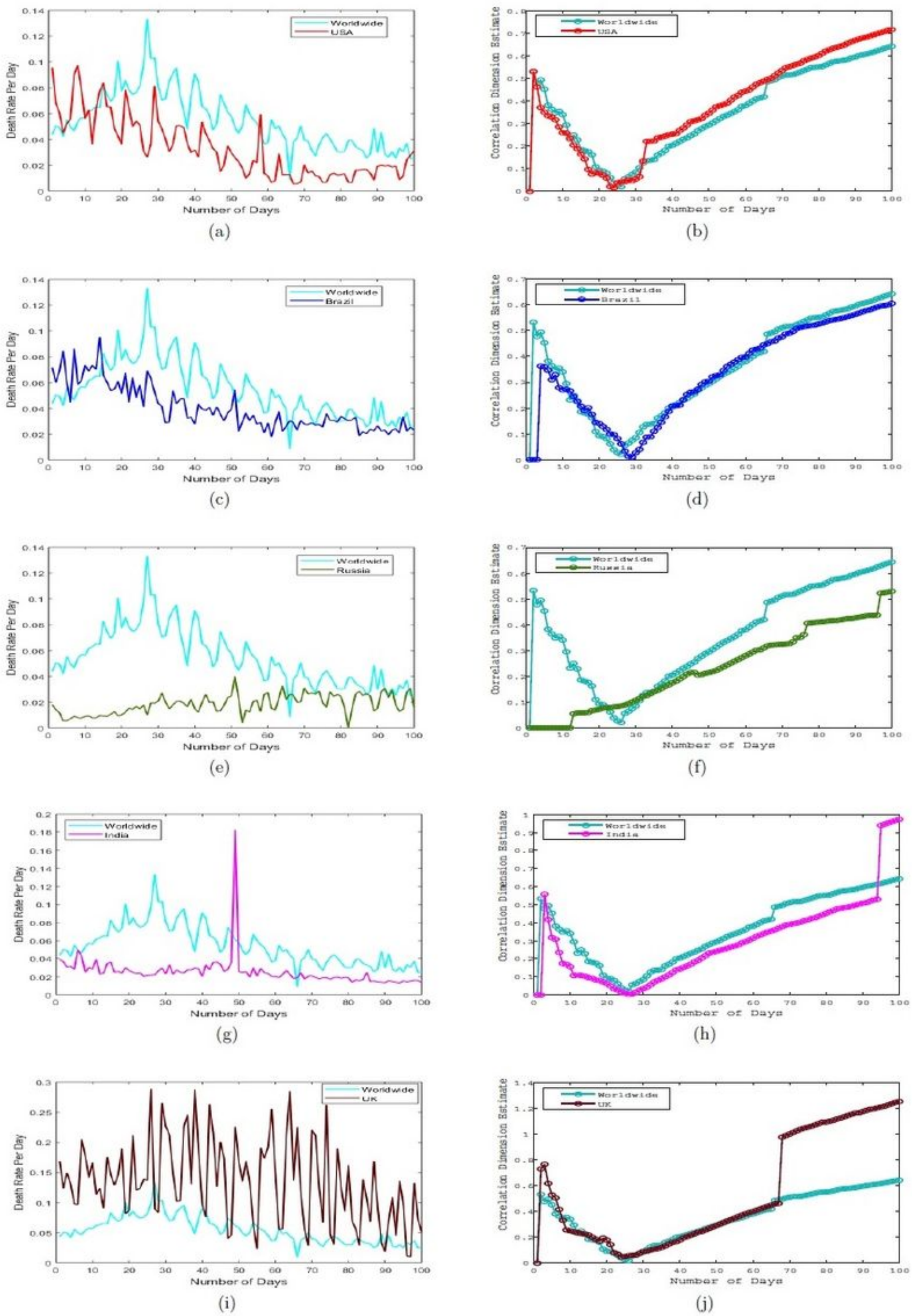


Figure 2

Comparison Between the Worldwide Daily Death Rate and the Representative Countries' Daily Death Rate: (a). USA, (c). Brazil, (e). Russia, (g). India, (i) UK and Analogies of the Correlation Dimension Estimates Between Daily Death Rate of the Country and Worldwide Daily Death Rate: (b). USA, (d). Brazil, (f). Russia, (h). India, (j) UK

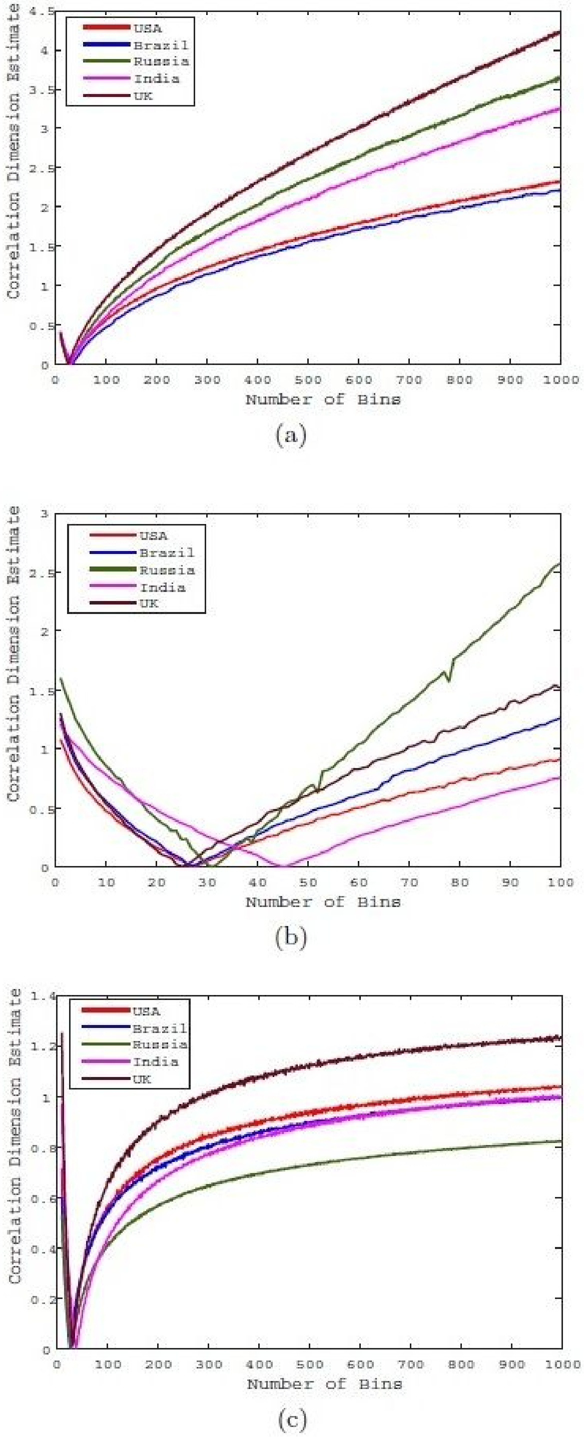


Figure 3

Correlation Fractal Dimension Estimates of COVID-19: (a). Infected Cases Per Day, (b). Deaths Per Day, (c). Death Rate Per Day

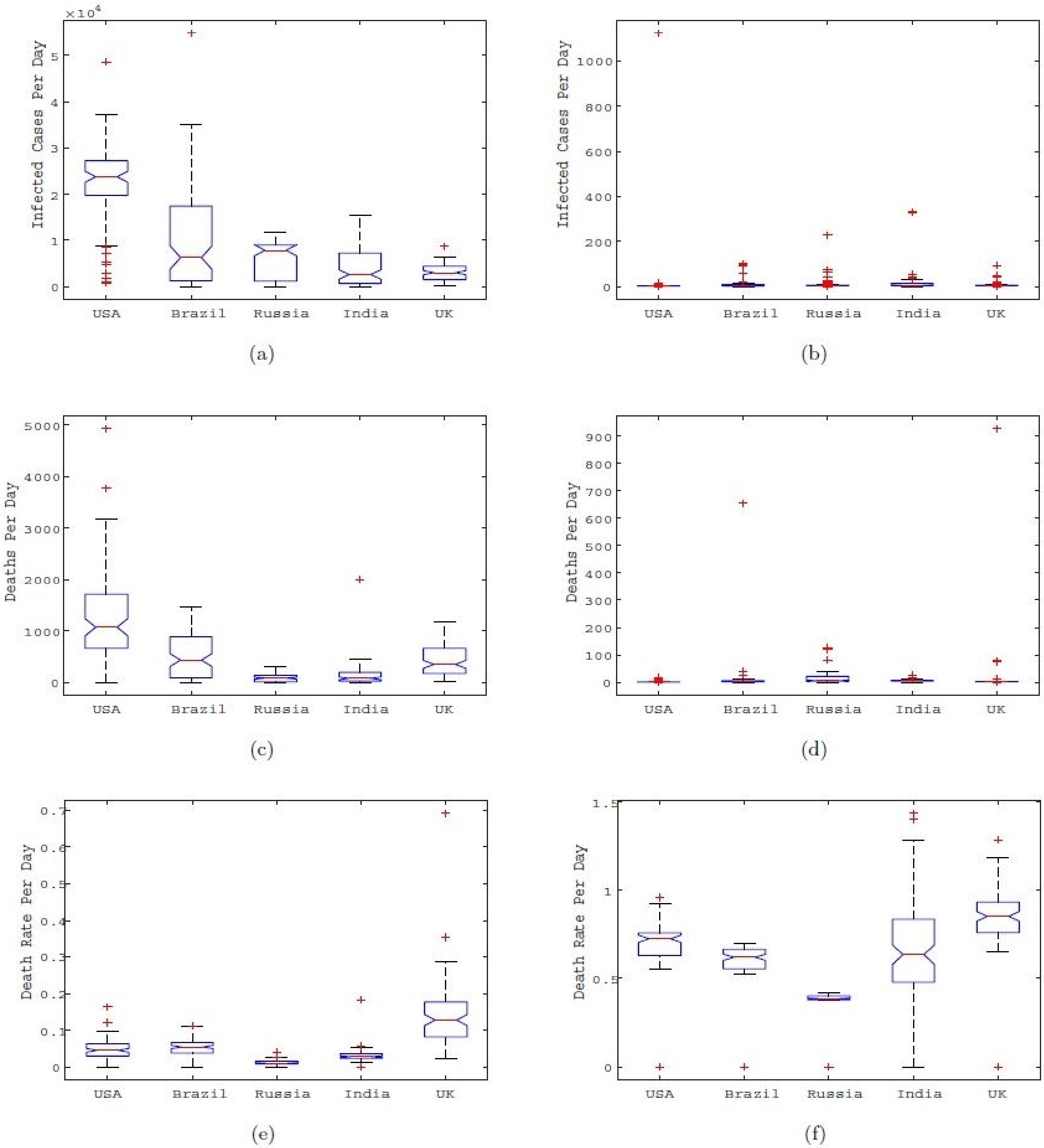
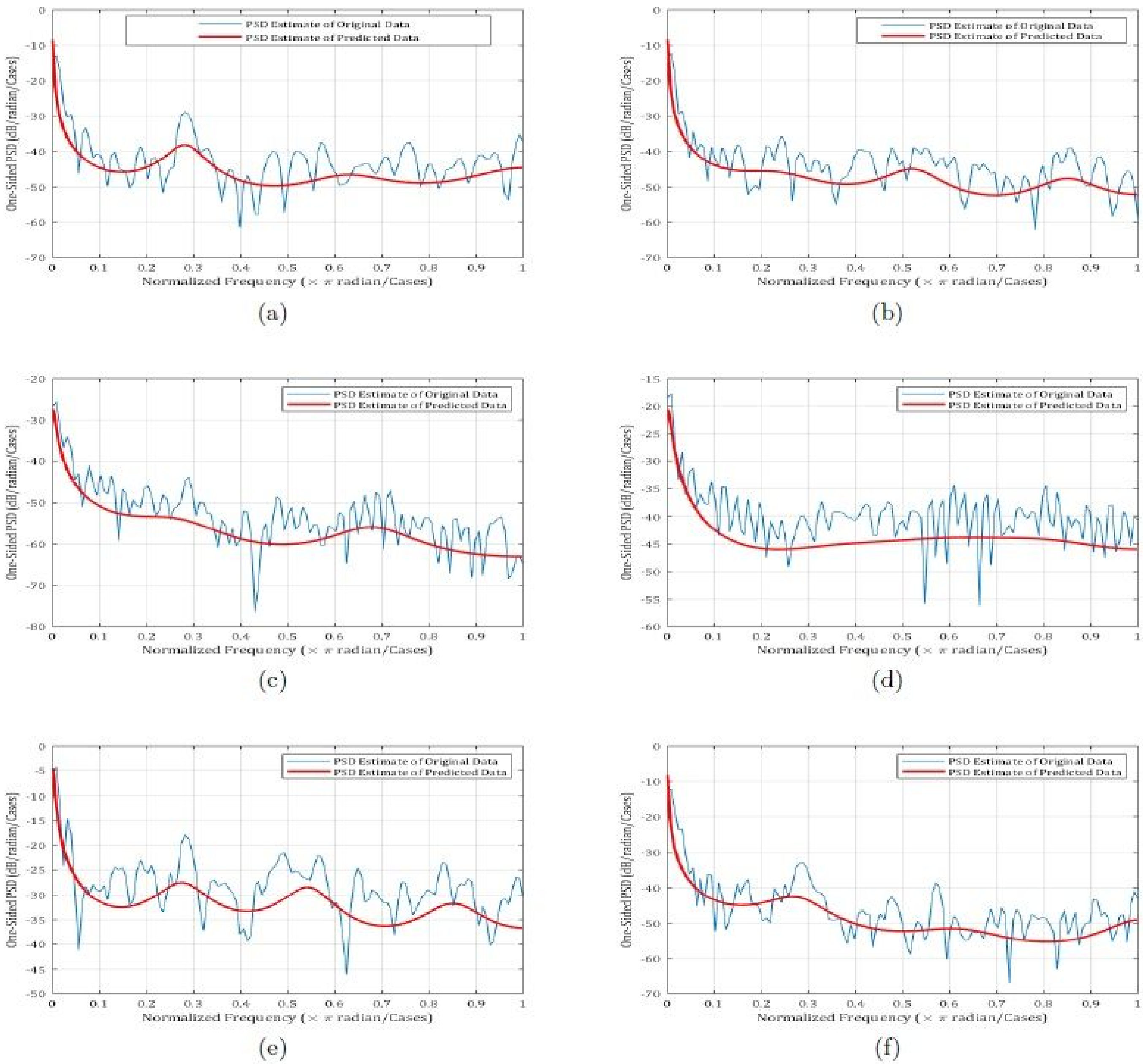


Figure 4

Box Plot for Original COVID-19 Datasets: (a). Infected Cases Per Day, (c). Deaths Per Day, (e). Death Rate Per Day. Box Plot of the Correlation Fractal Dimension Estimate for the Datasets: (b). Infected Cases Per Day, (d). Deaths Per Day, (f). Death Rate Per Day



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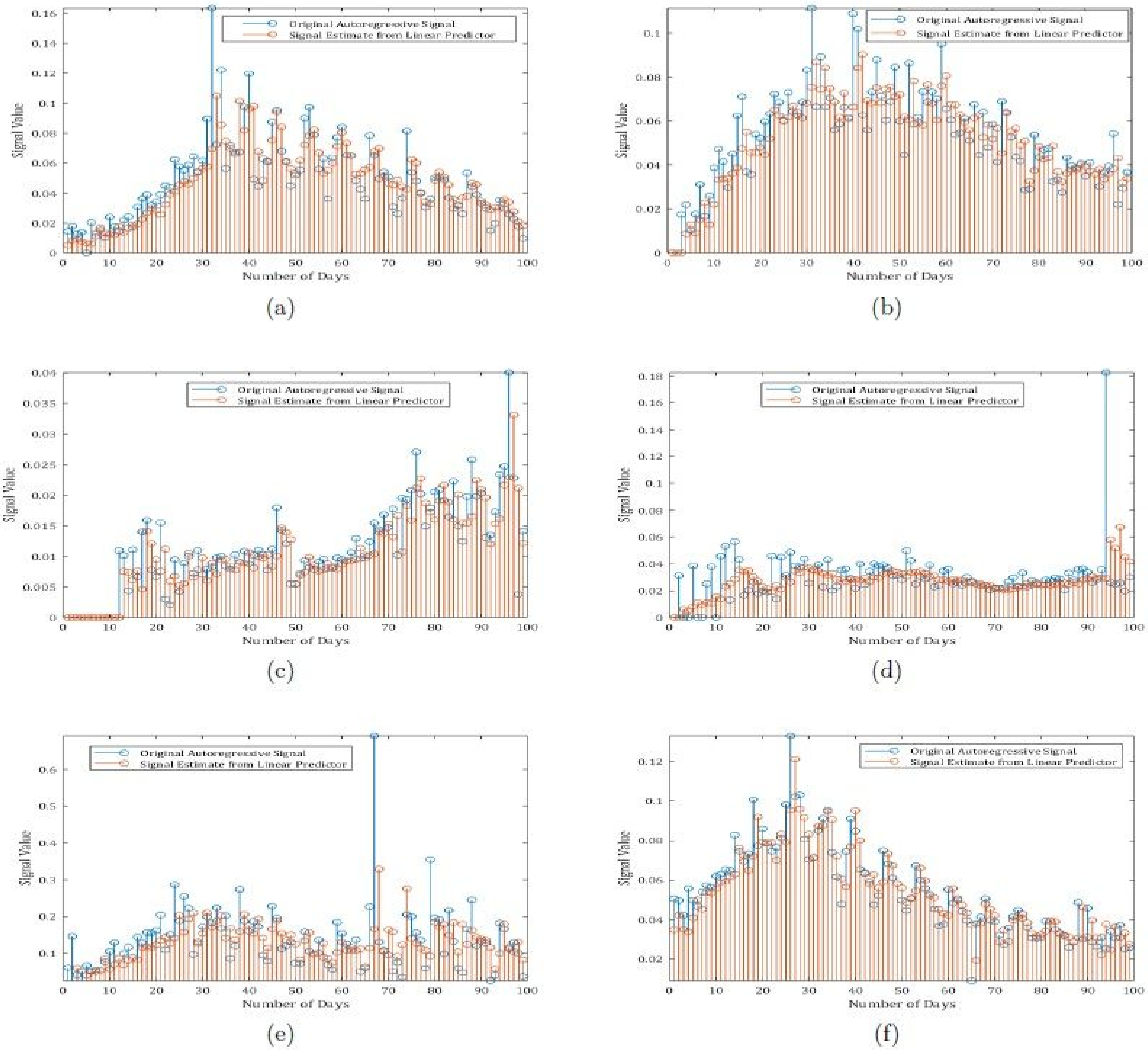

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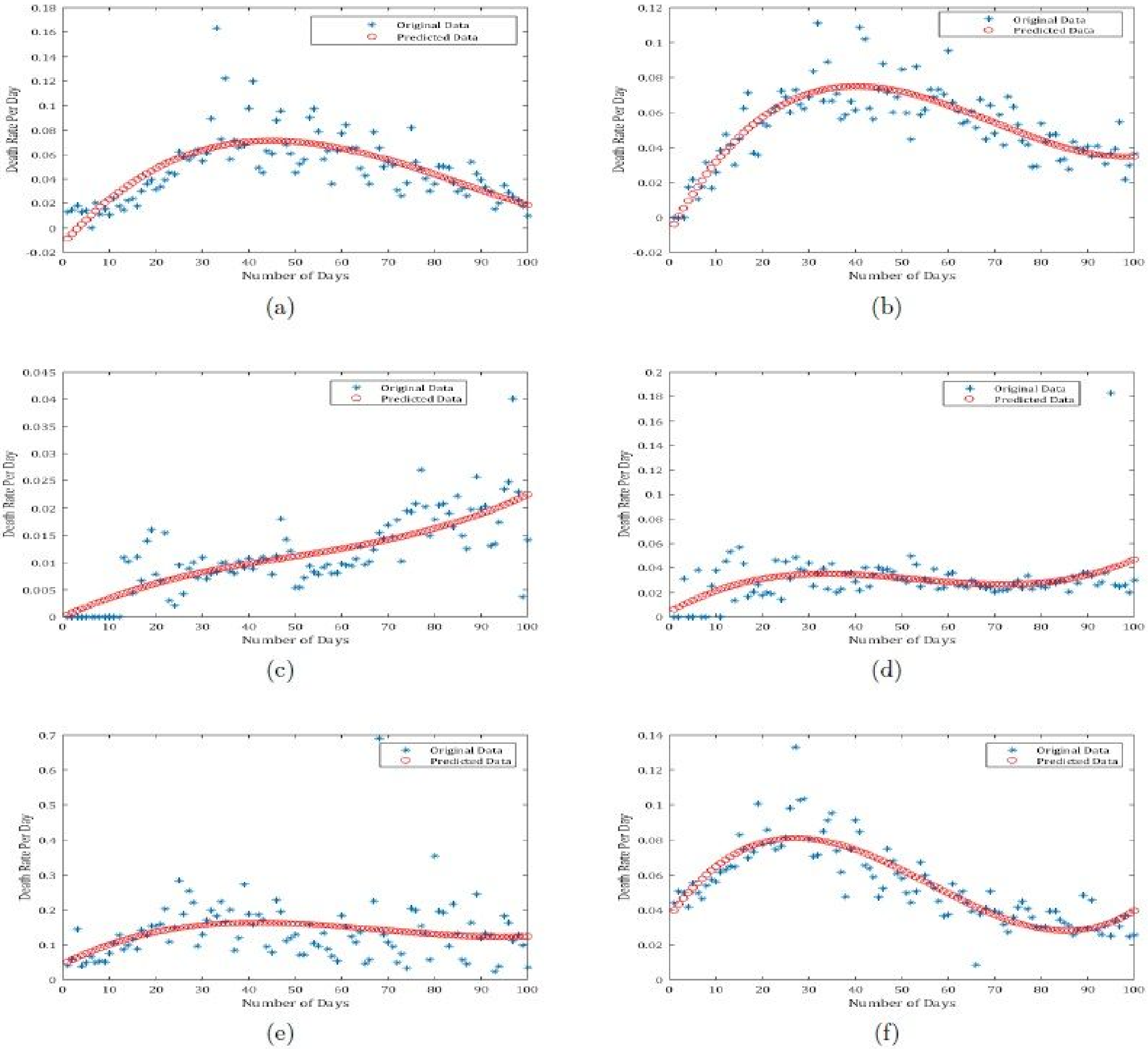

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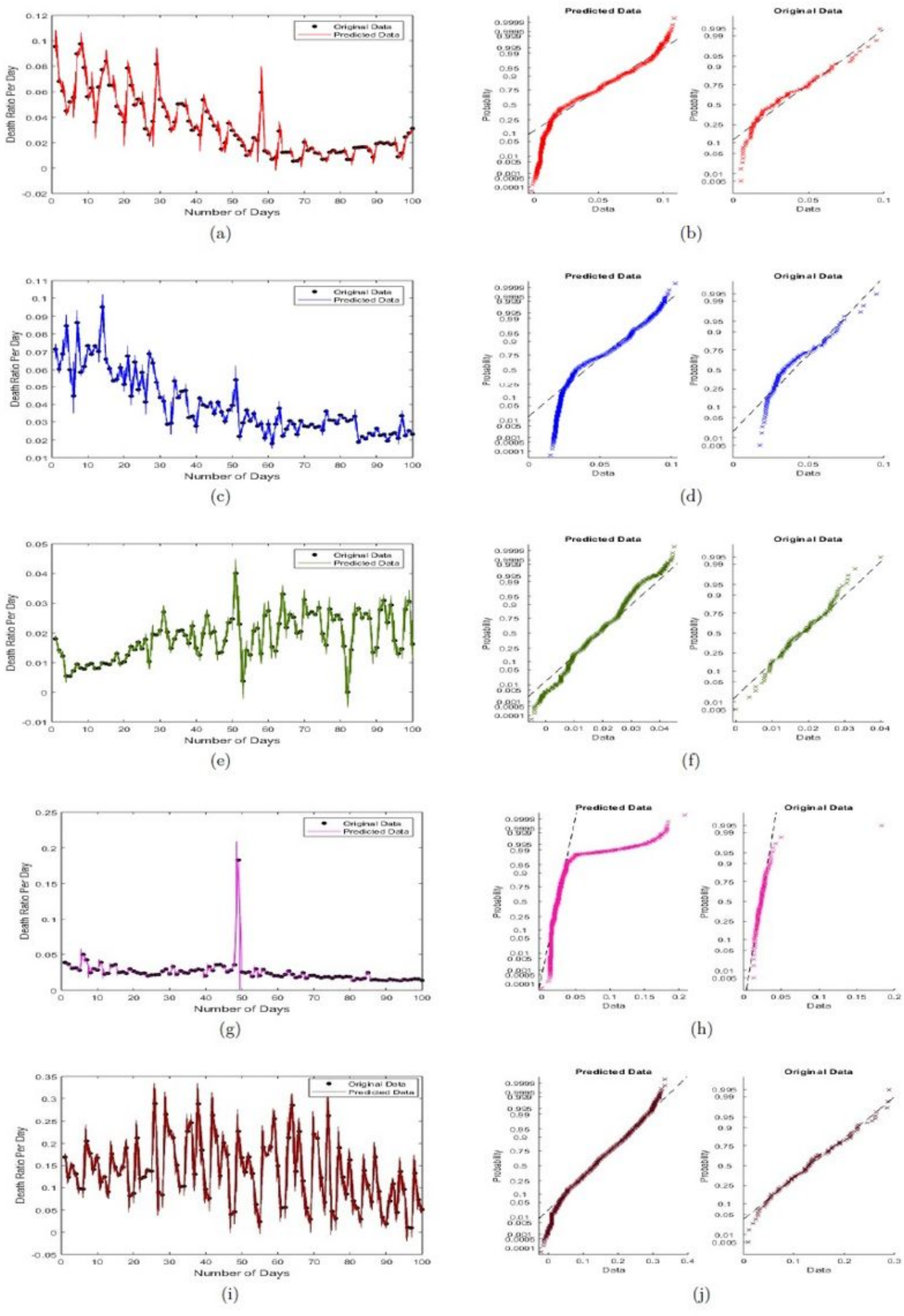
Representation of Death Rate in Time Domain for Original COVID-19 Data and Predicted Data by ARMA Model Versus Number of Days: (a). USA, (b). Brazil, (c). Russia, (d). India, (e). UK, (f). World



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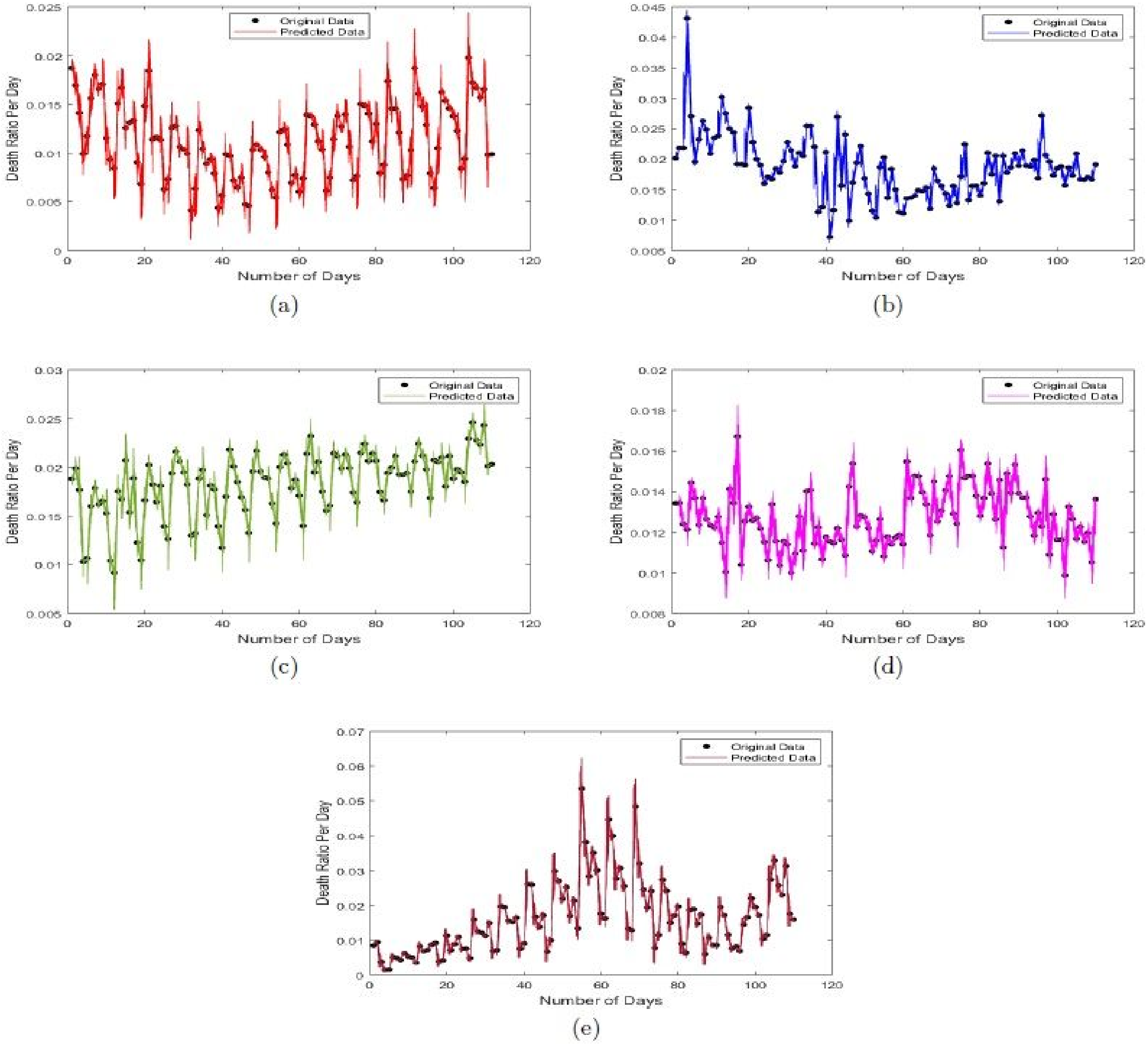
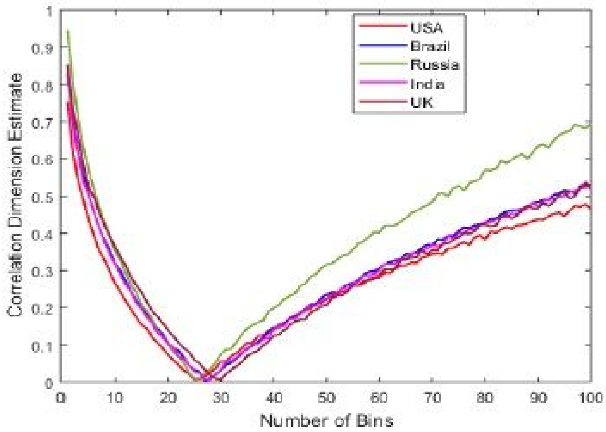
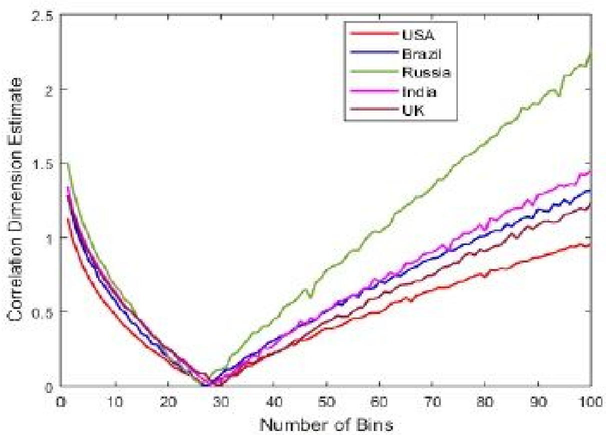


Figure 9

Second Phase of COVID-19 – Comparison Between the Original and Predicted Daily Death Rate of the Representative Countries by Fractal Interpolation Function: (a). USA, (b). Brazil, (c). Russia, (d). India, (e). UK



(a)



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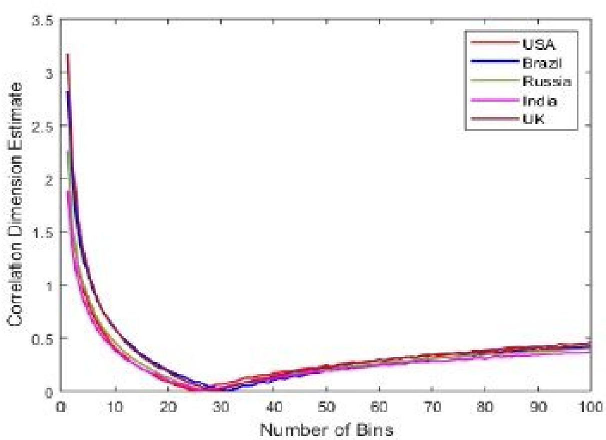

(c)

Figure 10

Second Phase of COVID-19 – Correlation Fractal Dimension Estimates: (a). Infected Cases Per Day, (b). Deaths Per Day, (c). Death Rate Per Day