Generalized Poincaré Conjecture via Alexander trick over C-isomorphism extension to h-cobordism on inclusion maps with associated Kan-complex

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Generalized Poincaré Conjecture via Alexander trick over C–isomorphism extension to h–cobordism on inclusion maps with associated Kan–complex

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The generalized norm of the Poincaré conjecture was initiated with the extension of Henri Poincaré's conjecture for n-dimensions. Steps have been taken thoroughly to prove the same from dimension-1 to dimension ≥ 5. Perelman closed the proof for dimension-3 thereby solving the millennium prize problem. Michael Freedman showed the validation in dimension-4 and Stephen Smale showed the validation in dimension ≥ 5. This paper would introduce a step-by-step proof taking Kan-fibration and Kan-complex being channeled from orders of Whitehead group, C-isomorphism, for any boundary manifold that implies a disjoint union among two n-dimensional manifolds for inclusion maps satisfying the vanishing torsion for a categorical correspondence on h-cobordism and Whitehead groups.

Poincaré conjecture – Kan-complex – C-isomorphism – Whitehead groups - Hausdorff space

Introduction

In 1904, Henri Poincaré made a revolutionary conjecture in the mathematics of geometric topology asserting the similar principle holding for a 3D space when any compact boundariless 2D surface is homeomorphic to a 2-sphere for a continuous deformation of loops ending at a point[1-4]. After giving his initial form of conjecture – Poincare later extended it to any dimensions such that there exists a continuous deformation for any n–dimensional compact manifold is homotopy equivalent to n – sphere only if there is homoeomorphism to that n – sphere.

His initial conjecture when takes the dimension n = 3 then through dimensional increment the generalization to the above paragraph hold.

Denoting the conjecture in equivariant form from dimensions 1 to 5 and even greater than 5 there exists 5 points[5],

[1] $For\ dimension - 1 -$ The 1-dimensional manifold that is closed compact and simply connected, homeomorphism exists in the circle.

[2] $For\ dimension - 2 -$ This holds for the ordinary sphere.

[3] $For\ dimension - 3 -$ Perelman closed this by proving the conjecture.

[4] $For\ dimension - 4 -$ Michael Freedman proved the validation of the conjecture.

[5] $For\ dimension\ \geq 5 -$ Stephen Smale showed the validation of the conjecture and this dimension $\geq 5 -$ The generalized version of this conjecture has been shown in this paper taking C – isomorphism and inclusion maps for Kan – fibration and Kan – complex taking Whitehead group for vanishing Torsion in

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simple Homotopic equivalence for ‘invariant in homotopy’, ‘invariant in topology’ and CW – complex over homotopic equivalent for all – Differential manifold, Topological manifold, piecewise Linear manifold thereby controlling the entire relation by establishing it through Alexander trick.

Perelman took the finest steps of Ricci flow with surgery which got recognized in 2006 as a solution to the millennium prize problem. Describing surgery in the simplest steps one can induce a relation[6-10], any Riemannian 3-manifold being simply connected for metric g_R there exists a curve shortening flow over time occurring in $Ricci$ curvature where the maximum time for the evolution of the structure is always less than infinity. Throughout this temporal evolution for a particular initiation time provided the final time is always less than infinity we find two relations,

[1] Any region with negative curvature expands

[2] Any region with positive curvature contracts making metric g_R converge to that via the associated Ricci flow such that this will shrink – develops singularity and splits apart called surgery or topological surgery.

WHITEHEAD GROUP

Any algebraic structure can be categorized as a groupoid ^ if any partial function $f_\star$ if it exists for any categorized subset $\mathcal{O}$ of $\mathcal{O}^{/}$ to $\mathcal{O}^{//}$ when being functioned over the parameter $\pi: \mathcal{O}^{/} \longrightarrow \mathcal{O}^{//}$ for every non-empty set ^ suffice[11-17],

$$(\wedge, f_\star)$$

The above being justified as $\mathbf{h}-\text{cobordism}$ for any continuous maps $f_\star^c: \mathcal{O}^{/} \longrightarrow \mathcal{O}^{//}$ and $f_\star^{-c}: \mathcal{O}^{//} \longrightarrow \mathcal{O}^{/}$ such that $f_\star^c \circ f_\star^{-c}$ makes a homotopy equivalence there exists a peculiar group[18-20],

$$\mathbf{Wh}\left(\sigma\left(\mathcal{O}^{//}\right)\right) \text{ implying Whitehead Group}$$

Taking $\mathbf{C}$ –isomorphism extension of $\mathbf{h}-\text{cobordism}$ satisfying a vanishing torsion T_0 for two n-dimensional manifolds M and N making an inclusion map[21-23],

$$\begin{cases} M \hookrightarrow W \;\cong \begin{cases} f_\star^c \; and \; f_\star^{-c} \\ \quad T_0 \end{cases} \\ N \hookrightarrow W \;\cong f_\star^c \; and \; f_\star^{-c} \end{cases} \text{for group relational equivalence } \mathbf{Wh} \;\longrightarrow\; \sigma(M) \cong \sigma(N)$$

This T_0 being ordered for any algebraic K – groups of higher varieties there exists a topological bias where the Poincaré conjecture can be proved for smooth varieties. Defining this Whitehead Torsion $T_0(f_\star)$for the same continuous maps $f_\star^c: \mathcal{O}^{/} \longrightarrow \mathcal{O}^{//}$ and $f_\star^{-c}: \mathcal{O}^{//} \longrightarrow \mathcal{O}^{/}$ we get Wall finiteness obstruction for any trivial ring in the Euclidean domain $\mathbb{Z}$ over a parameter of a continuous map or homotopy equivalence given $CW - construction\ X$ for any K – bounded attachments for the 0th K – Theory K^0 gives the module of symplectic class $[\omega]$ for Whitehead group $\mathbf{Wh}(\wedge)$ over any values of +1 and -1 suffice two relations[24],

[1] Trivial K – group 1st domain $- K^1\left(\mathbb{Z}(\wedge)\right)$

[2] Non – Trivial 0th domain $-K^0\left(\mathbb{Z}\left(\wedge(X)\right)\right)$ for $[\omega]_w \mathbf{Wh}(\wedge)$

Taking that X for a Hausdorff space the Euclidean domain $\mathbb{Z}$ is equivalent to the Kernel $\mathrm{Ker}(X)$ for any sequence α_w giving the reduced suspension in k^{th} order for any unit circle S^1 over a definitive relation,

$$\mathrm{Ker}(S^k X) = \begin{cases} \cong \mathbb{Z}, & \text{for } n < 0 \\ \cong X, & \text{for } S^1 \wedge X \end{cases}$$

Homotopic equivalence

Any isotopy defined over homotopy establishes the continuous homeomorphisms channelizing for any $\boldsymbol{cokernel}$ for rank f defines the quotient $\mathcal{O}^{//}/\,\mathbf{im}\,f$ over linear mapping extensions of $f_\star^c$ making a transformation ρ for the $\mathbf{Wh}\left(\sigma\left(\mathcal{O}^{//}\right)\right)$ for the trivial 1st domain of group ring ^[25-27],

$$\rho : \wedge \times \{\pm 1\} \longrightarrow K^1\big(\mathbb{Z}(\wedge)\big)$$

Deducing three properties,

$$T_0(f_\star^c) = T_0(f_\star^{-c}) \quad \Longrightarrow \quad \text{Invariance in homotopy}$$

$$T_0(f_\star^c) = 0 \quad \Longrightarrow \quad \text{Invariance in topology}$$

$$T_0(f_\star^c \circ f_\star^{-c}) \quad \Longrightarrow \quad \text{CW} - \text{complex over homotopy equivalence}$$

As mentioned that the vanishing torsion T_0 for any higher $K - Theory$ the k^{th} order is a standard simplex ∇^k where any horn Δ_n^k having the subcomplex structure suffice[31,32],

$$n^{th} \text{ order horn in simplex } \nabla^k \text{ for } k \geq 0$$

We get the Kan – fibration over simplex set X over 5 – relations with the last one satisfying this as a Kan – complex $\{\star\}$[28],

[1] $\nabla^k \xrightarrow{satisfies\ \ morp\ hism\ over} X$

$$\exists\ this\ simplex\ X \cong \mathcal{O}^{/}\ for\ homotopic\ equivalence \begin{cases} f_\star^c : \mathcal{O}^{/} \longrightarrow \mathcal{O}^{//} \\ f_\star^{-c} : \mathcal{O}^{//} \longrightarrow \mathcal{O}^{/} \end{cases} \forall\, f_\star^c \circ f_\star^{-c}$$

$$\Longrightarrow \text{Whitehead Group } \mathbf{Wh}\left(\sigma\left(\mathcal{O}^{//}\right)\right)$$

[2] $\mathbf{Img}\ (\mathrm{n}) - \nabla^{k-1} \longrightarrow \nabla^k$

[3] $\begin{cases} \Delta_n^k \longrightarrow \mathcal{O}^{/} \text{ for relation [1]taking parameter } m \\ \quad . \\ \Delta_n^k \longrightarrow \mathcal{O}^{//} \text{ for relation [1]taking parameter } \imath \\ \quad . \\ \Longrightarrow \begin{cases} \mathbf{Hom}_{\nabla^k}([\imath],[k]) \textit{ where } \imath \textit{ is inclusion} \\ \text{over: } \nabla^k \text{ for horn } \Delta_n^k \end{cases} \end{cases}$

[4] **Kan fibration** $- \begin{cases} m \circ \imath \\ f \circ m \end{cases}$

[5] **Kan complex** $- \begin{cases} m \longrightarrow \{\star\}_{\sim} \text{ over limitation } \sim \\ \forall \\ \boldsymbol{X} \longrightarrow \{\star\} \cong \mathcal{O}^{/} \longrightarrow \{\star\} \end{cases}$

Generalizations

This section would introduce the step-by-step approach for establishing the final results of the generalization on dimension ≥ 5 over correspondence, categorically established manifolds taking the account of inclusion maps thereby concluding the end via Alexander trick.

Thus T_0 over inclusion $\imath$ on relations $\begin{cases} M \hookrightarrow W \\ . \\ N \hookrightarrow W \end{cases} \xrightarrow{\textbf{vanishes for}}$

$$[\text{making correspondence}] \longrightarrow \begin{cases} \mathbf{h} - \text{cobordism} \\ \text{Whitehead Groups} \end{cases}$$

Thus T_0 over inclusion $\imath$ on relations $\begin{cases} . \quad \begin{matrix} M \hookrightarrow W \\ \\ N \hookrightarrow W \end{matrix} \xrightarrow{\textbf{vanishes for}} M \subset N \\ \quad . \\ \text{correspondance} \begin{cases} \mathbf{h} - \text{cobordism} \\ \text{Whitehead Groups} \end{cases} \end{cases}$

Then, it's quite clear that $\mathbf{C}$ –isomorphism extension of $\mathbf{h} -$ cobordism satisfying a vanishing torsion T_0 on $(\ell + 1) - \mathbf{dim}$ manifold M the boundary ∂ of W suffice,

$$\cong \partial W \Longrightarrow M^{\star} \bigcup_{\partial} N^{\star}$$

Thereby providing 2 – classes being equivariant to Poincaré conjecture for $\ell \geq 5$,

[1] Simply connected –W|M,N

[2] $\mathbf{C} -$ isomorphism –

$$\begin{cases} \mathbf{C} = \text{Differential manifold} \\ . \\ \mathbf{C} = \text{Topological manifold} \\ . \\ \mathbf{C} = \text{Piecewise linear manifold} \end{cases} \quad \exists \begin{cases} \mathbf{C} \text{ extension of h} - \text{cobordism} \\ . \\ \text{W} \xrightarrow{\textbf{isomorphic to}} \text{M} \times [0,1] \end{cases}$$

This $\mathbf{h} - \mathbf{cobordism}$ taken over a classifying space being over manifold M settles an isomorphism class on dim $\ell \geq 4$ for Whitehead group Wh$\big(\sigma(\hat{})\big)$. Relative to S^n over the satisfaction of,

$$\overleftarrow{M - homeomorphic} \; and \; \overleftarrow{M - homotopy\ equivalent}$$

The version of the Poincaré conjecture over the generalizations $n \geq 5$ (To avoid confusion – it is better to mention that both ℓ and n represent dimensions but are introduced differently just as a parameter for this paper).

Going back to the **Kan fibration** $- \begin{cases} m \circ i \\ f \circ m \end{cases}$ and noting the inclusion map i justifying $\begin{cases} M \hookrightarrow W \\ N \hookrightarrow W \end{cases}$ defining $M^\star \cup_\partial N^\star$ through disjoint discs $(d_- \cup d_+)$ for dimensions $(n-1)$ for homotopy equivalence – M and S^n,

$$(d_- \cup d_+) \cong S^{n-1} \times [0,1]$$

$\mathbf{h}$– cobordism being over W any homeomorphism exists on $f : S^{n-1} \times [0,1] \longrightarrow W$ giving Whitehead Wh$\big(\sigma(S^{n-1})\big)$ the Alexander trick can occupy S^{n-1} over extended homeomorphisms for $(d_- \cup d_+)$ with each disc in $n - \mathbf{dimensions}$ gives,

Generalized Poincaré conjecture – $\mathbf{dim}\, n \geq 4$ for $S^n \overset{\cong}{\rightarrow} M$ where the below table describes other dimensional generalizations[28].

Poincare Conjecture – Generalization Status

DIFF	**TOP**	**PL**	**Dim**
True	True	True	$\ell = 1, 2, 3^2$
?	True	?	$\ell = 4$
True	True	True	$\ell = 5, 6$
False	True	True	$\ell = 7$
Casewise True – Generally False	True	True	$\ell \geq 8$

Statement holds for generalized Poincaré conjecture –

Any homotopy sphere for category: Top, PL, Smooth is PL– isomorphic, diffeomorphic, homeomorphic to n– sphere.

To avoid confusion – it is better to mention that both ℓ and n represent dimensions but are introduced differently just as a parameter for this paper.

Results

Dimensional generalizations for $\ell = 1,\ 2,\ 3^2,\ 4,\ 5,\ 6,\ 7,\ \geq 8$ are shown at the end of the paper for 3-class of manifolds – Diff, Top, PL where the associated and the non-trivial ≥ 5 (been the purpose of this paper) is established in 5-steps – standard simplex and the horn, Hom-functor for the standard simplex satisfying isomorphisms over inclusions – the standard simplex for the horn making the way clear for Kan-fibration and Kan-complex for all justifications of C-isomorphism extensions to h-cobordism, vanishing torsions for invariance in homotopy, topology, and CW – complex over homotopy equivalence.

Declaration of interest

The author has no conflicting interests related to this paper.

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