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Babur M. Mirza (✉ bmmirza2002@yahoo.com)

Quaid-i-Azam University Islamabad: Quaid-i-Azam University

Research Article

Keywords: Galaxies, rotation dynamics, galactic mass, Tully-Fisher relation, galactic age

Posted Date: February 11th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-184903/v1>

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Relativistic Modifications in Galactic Rotation Curves under a Toroidal Galactic Field

Babur M. Mirza

Department of Mathematics, Quaid-i-Azam University, Islamabad 45320, Pakistan

Rotational dynamics of galaxies exhibits an increase beyond the Keplerian velocity which corresponds to a missing mass up to six times the dynamic mass in the observable universe. In this paper we show that the observed increase in galactic rotation velocities is a general relativistic effect resulting from the combined effect of toroidal magnetic energy density in galaxies and spacetime dragging due to the rotating compact mass in galactic center. The effect of magnetic energy density on spacetime vorticity is derived from Maxwell equations in axially symmetric spacetime where the dragging effects are shown to extend farther in the galactic disc via the toroidal field, modifying the rotational speed of the galactic matter. This is shown to lead to the diverse rotation curves of spiral galaxies, along with the Tully-Fisher relation for total galactic mass and maximum rotational velocity.

Keywords: Galaxies; rotation dynamics; galactic mass; Tully-Fisher relation; galactic age

Introduction.— Rotation curves of galaxies show a marked difference from the Keplerian velocity $\sqrt{2GM/r}$, for galactic mass M and radial distance r , which becomes increasingly significant over the galactic radius. This rotation curve discrepancy is now well established through a large number of astrophysical observations [1–4]. A main aspect in modeling galactic dynamics is the diversity exhibited by these rotation curves. Most of the spiral galaxies, like *NGC* 1566, have an almost flat rotation curve at large distances within the galactic disk, where as galaxies like *NGC* 1097 shows a marked rise in the rotational velocity. Satellite galaxies, like *M31*, even show a sharp steep in rotational speed over the galactic disk.

Despite the observed diversity of rotational patterns of different galaxies, certain features also emerge as similar in general galactic dynamics. For instance, apart from an increase over the Keplerian velocity at large distances, an almost direct proportionality exists between the galactic mass and the fourth power of the maximum rotational velocity. This Tully-

Fisher relation [5-9] is a general scaling relation with no significant dependence on other galactic properties like galaxy size or surface brightness, and has little intrinsic scatter [10-14]. A plausible model of galactic dynamics should explain these observed features of galactic motion with a minimum number of hypotheses.

In this paper, we give a model of galactic dynamics based on first principles, and deduce these features of galactic motion from directly observable galactic parameters, namely the galactic mass, radius, and the magnetic field. In this context the magnetic field generated in galaxies, mainly by accretion around galactic nuclei, provides the feedback mechanism for galactic rotation. Analysis of general relativistic Maxwell equations shows that toroidal magnetic field thus generated enhances spacetime vorticity. Such a coupling occurs when the central mass has non-zero angular momentum and an intrinsic magnetic field is present in the galactic center. Spacetime dragging is thereby extended to much larger distances under the magnetic pressure. We show that this enhanced spacetime drag corresponds directly to the velocity increase observed in the diverse behavior of galactic rotations.

The model presented here depends on the following main features of galactic structure: (1) A large proportion of galactic mass is contained in the region in and around the galactic center. In most spiral galaxies this region is fully formed as galactic bulge, and despite very different morphologies, masses, sizes, and gas fractions; matter in these galaxies can be generally divided into a disk and a central bulge or a nuclear mass. (2) Although most disk galaxies have a central bulge, pure disk galaxies contain a compact source at the galactic center. In such cases galactic dynamics is dominated by the compact source. In general, the existence of central compact gravitational sources is now well established in almost all galaxies [15-18]. (3) Galaxies have an intrinsic magnetic field, which we identify as toroidal field generated in the central region. In spiral galaxies the galactic magnetic field strength varies from $10 - 15\mu G$ within the galactic arm [19-21]. Observationally, the total magnetic field strength can be determined from the intensity of total synchrotron emission, assuming energy balance (equipartition) between magnetic fields and cosmic rays. The typical average equipartition strength for spiral galaxies is about $10 \mu G$. In general the galactic magnetic field has been observed to be of order of μG which extends across the galactic plane [22-24].

Velocity distribution.— The magnetic field around a rotating compact object can be derive from Maxwell equations in slowly rotating Kerr spacetime [25]. Using the standard spacetime coordinates, (r, θ, φ, t) , where the azimuthal angle φ is measured in the galactic

plane, the toroidal magnetic field in the galactic disk ($\theta = \pi/2$) is given by,

$$B(r) = \frac{B_0 \cos \xi}{ru_t}, 0 < \xi \leq \pi, \quad (1)$$

where $u_t = -\sqrt{1 - r_s/r}$, $r_s = 2GM_\bullet/c^2$ being the Schwarzschild radius, and M_c is the mass of the compact source in the galactic center. This field is coupled to the spacetime vorticity via $\xi = \varphi - \omega t$, where $\omega \neq 0$ is the Lense-Thirring dragging frequency. For a slowly rotating mass, $\omega(r) = 2J/r^3$, where J is the angular momentum of the central compact source mass M_c .

In the equatorial plane identified as the galactic plane, the total energy density due to the magnetic field interior to a circular orbit of radius r around the source at origin can be calculated by first integrating over the variable ξ , which gives a constant contribution absorbed in B_0 . Therefore, the magnetic energy density at any point r in the galactic disk is given by,

$$\epsilon_B = \int_0^r 4\pi r'^2 B^2 dr'. \quad (2)$$

Putting from equation (1) into equation (2), we obtain,

$$\epsilon_B = 4\pi B_0^2 [r + r_s \ln(r - r_s)], \quad r > r_s. \quad (3)$$

Equation (3) gives the energy density due to the magnetic field coupled to the spacetime dragging, also the total energy has contribution due to the Newtonian gravitational potential energy for the net mass M contained within the orbit. Correspondingly, in the outer region $r > r_s$, disk velocity has contribution v_0 due to the source at $r = r_s$, the velocity field v in the exterior due to the magnetic energy, and the Keplerian velocity due to the gravitational potential energy. The effective potential energy of gravity per unit mass is $E_G = GM/r$ where M is the total mass contained within the orbit. Defining the dimensionless magnetic energy density $E_B = \epsilon_B/B_0^2$, we equate the net potential energy ($E_B + E_G$) to the kinetic energy per unit mass. This gives for the orbital velocity at any point in the galactic disk for $r > r_s$,

$$v(r) = v_0 \pm \sqrt{r + r_s \ln(r - r_s) + \frac{2GM}{r}}. \quad (4)$$

For the zero of velocity v_0 , we have $v_0 = -\int_{v_1(0)}^{v_2(r_s)} dv$. Since $v = \sqrt{E_B}$, therefore the contribution from $r = 0$ to $r = r_s$ is $v_0 = -\sqrt{E_B} \big|_0^{r=r_s} = \sqrt{E_B(0)} - \sqrt{E_B(r_s)}$. Now since the magnetic field (1) changes sign across the Schwarzschild radius r_s , we have $E_B = 0$ at

FIG. 1: Rotation curves (solid plots) based on equation (4) for typical galaxies *NGC* 1097 (data points from [26]), *NGC* 1566 [27], *M31* [28], and *M33* [29]. The dashed plots represent the Keplerian velocity, where as the dotted plots correspond to the magnetic vorticity contribution $\sqrt{r + r_s \ln(r - r_s)}$.

$r = r_s$. Using (3) we have $v_0 = \sqrt{r + r_s \ln(r_s - r)}|_{r=0}$. We therefore obtain $v_0 = \sqrt{r_s \ln r_s}$, which determines the magnitude of v_0 in equation (4). Notice that the square-root sign in equation (4) is decided by co-rotating and counter-rotating motion in the galactic disk relative to the rotation of the central compact object.

A set of typical galactic curves is shown in Figure 1 for the disk region $r > r_s$, where the data points lie, within observational error, on the velocity curves derived from equation (4). In plotting the rotation curves for *NGC* 1097, *NGC* 1566, and *M31* galaxies, we have used $E_B = \epsilon_B/B_0^2$. Also the potential energy is scaled per *kpc* in the galactic disk. In the case of the Triangulum galaxy (*M33*) there is no central bulge, and so the Newtonian contribution to the velocity via GM is negligibly small. The galactic dynamics is thus dominated by the magnetic vorticity term with $B_0 = 0.18\mu G$. This produces the characteristic steepness in the rotation curve of *M33* galaxy, since $v(r)$ essentially corresponds to $\sqrt{B_0 r}$ in this case.

The sign of velocity in equation (4) can change for the negative square-root according to if v_0 is greater than or less than the square-root term $\sqrt{r + r_s \ln(r - r_s) + (2GM/r)}$, this can divide the galactic plane into two distinct regions rotating opposite to each other. Such

FIG. 2: Rotation curve for *NGC* 4826 derived from equation (4). The plot is shown in the absolute velocity measure, where points on lhs. of the velocity minimum have negative velocity hence represent opposite rotation relative to the points on rhs. of $v = 0$. This corresponds to the counter-rotating disks phenomenon observed in galaxies like *NGC* 4826. Data points are from [2].

phenomena has been observed in *NGC* 4826 and a number of other galaxies. As shown in Figure 2, the data points below radius of one *kpc* lying on the inner disk correspond to velocity in opposite direction to points lying on the scale larger than a *kpc*.

In galaxies with well formed spiral structure, such as barred galaxies, mass distribution differs from flat disk galaxies, and is not uniform over large radii. Thus for the Milky Way galaxy (Figure 3) we employ the mass distribution given approximately by $M \cos(0.86r)$ in the Newtonian potential. In the context of the present model, we point out that the barred structure of the galactic nucleus corresponds to the points of extrema of the magnetic field (1) that differ by an angle π in the galactic plane. Since spacetime dragging is largest at these regions, matter accumulation by accretion is larger at the poles than elsewhere around the central source mass. Once formed, these regions extend in space by the coupled effect of spacetime vorticity and the magnetic energy density build-up around the galactic center by charge accretion.

Tully-Fisher Relation.— The Tully-Fisher formula for mass-velocity relation can be de-

FIG. 3: Rotation curve (solid plot) for the Milky Way galaxy, with data points from [30]. The dash and dot plots represent the Keplerian and magnetic vorticity contributions, respectively.

duced directly from velocity distribution formula (4). In the Tully-Fisher equation maximum luminosity corresponds to maximum rotational speed in the galactic disk. Therefore, by the extremum condition $dv/dr|_{v=v_{\max}} = 0$, we have from equation (4),

$$r = \frac{2GM}{r} - \frac{2GM}{r^2}r_s, \text{ for } v = v_{\max}. \quad (5)$$

Since $r^2 \gg r_s$, for the galactic disk, the second term in equation (5) is negligible. Putting therefore in equation (4) $r = \sqrt{2GM}$ we find

$$(v_{\max} - v_0)^2 = 2\sqrt{2GM} + r_s \ln(\sqrt{2GM} - r_s). \quad (6)$$

By $\sqrt{2GM} > r_s$, the second term on rhs. of equation (6) is also negligible compared to the first, we then obtain the Tully-Fisher relation $M \sim v_{\max}^4$. Putting back the magnetic energy density we see that $v_{\max}^4 \sim 8B_0^2GM$, which shows that for the usual galactic parameters $B_0 \approx 10\mu G$ and total galactic mass of approximately $10^{10}M_\odot$, $v_{\max}$ is of the order of $10^2 km/s$, which agrees with the observed order of magnitude of galactic rotational velocities. We notice that when the Schwarzschild radius $r_s = 2GM_c/c^2$, where M_c is the mass of the compact source in galactic center, has magnitude comparable to $\sqrt{2GM}$, deviations from the Tully-Fisher law arise due to the second term on rhs. of equation (6).

FIG. 4: Deviations from the Tully-Fisher relation $\delta = \left| (v_{\max} - v_0)^2 - 2\sqrt{2GM} \right|^2$ as function of the central compact mass M_c for different values of the scaled galactic magnetic field. Magnitude of the field B_0 decreases by a unit μG from left to right from $5\mu G$ to $1\mu G$ for the plots. Here we choose the total galactic mass $M = 1$, and in gravitational units $c = 1 = G$.

In Figure 4 we give a plot based on equation (6) for deviation δ from the Tully-Fisher fourth power law. We see that $\delta \sim B_0^2$ and the predicted deviations from the Tully-Fisher relation become larger for larger magnetic field for a given compact mass, since it increases the maximum velocity in the galactic disk. Also, the deviation δ increases if the central compact source is more massive. But since a larger proportion of the total galactic mass M is then contained in the central compact source than in the luminous matter in the galactic disk, maximum velocity decreases due to the logarithmic term in equation (6) which depends on the difference of total mass M and the mass M_c . Since the deviation δ corresponds to $v_{\max}^4$ which in turn is proportional to maximum luminosity, this implies that even if the central compact source is more massive, deviation from Tully-Fisher relation for luminosity will still be small provided the galactic magnetic field is sufficiently high.

Galactic age and size.— Assuming that the primordial compact source with the toroidal field had moment of inertia $I \sim 10^{40} kg \cdot m^2$ and angular speed $\Omega \sim 10^4/s$, compatible with general relativistic equilibrium conditions [31], we have for the galactic mass to extend up

to the galactic radius of $10kpc$, the time $t \sim 2\pi/\omega = 2\pi r^3/I\Omega = 10^{17}s$ which agrees well with galactic age inferred from observation [32].

Since in this model spacetime vorticity coupled to the magnetic energy density determines radial spread of velocity, galactic radius can be derived, given the total angular momentum of the galaxy. To determine this we note that the effects of magnetic coupling to the spacetime vorticity extend only to a finite distance, since $\cos \xi$ in equation (1) has principal zero at $\xi = \pi/2$. Therefore, using $\omega \sim J/r^3$ and for a co-moving observer the angular speed $\Omega_0 = \varphi/t$, we find that the galactic radius is related to the total angular momentum by $\Omega_0 - J/r^3 = \pi/2t$. For sufficiently long period of time, we therefore have for the galactic radius $r \sim (J/\Omega_0)^{1/3}$. For the Milky Way galaxy, total angular momentum inferred from dynamics of double galaxies is approximately $10^{67}Js$ [33]. Given that the angular speed of the solar system is about $550km/s$, this gives for the radius of the Milky Way galaxy, $r \sim 2.6 \times 10^{20}m$, which is of the observed order of magnitude of $10^{20}m$ of galactic radius [34]. Notice that this corresponds to an outer ring of maximum acceleration, where as the region close to where $v = v_0$ must form an inner ring of increased acceleration due to the sudden change in velocity.

Conclusions.— In equation (6) we see that the contribution to the total galactic mass $\sim v_{\max}^4$, due to the magnetic vorticity is $8GM$ for the dominant term, whereas the second term is negligibly small. The Keplerian velocity alone however contributes only $2GM$ to $v_{\max}^4$ for the same radius $r = \sqrt{2GM}$. Given that the total galactic mass inferred from Keplerian velocity distribution is M , the rotational velocity (4) thus implies an additional mass of $6M$. This corresponds to the missing galactic mass inferred from rotational velocity curves.

Finally, we stress that in the linear approximation to Einstein field equations there is the gravitomagnetic field which couples to the magnetic field in particle dynamics around rotating magnetized objects [35]. The gravitomagnetic force however is too weak to derive orbital motion at large distances. Moreover, its effects decay as $1/r^3$, hence cannot support flat rotation curves. The effect we have discussed here is distinct from Lense-Thirring frame drag, since it results from a coupling of gravitational and electromagnetic fields. Moreover it is generally valid for any axially symmetric matter distribution endowed with a magnetic field structure. In astrophysical systems accretion provides an effective means of magnetic field generation around massive objects. Spacetime around a rotating compact object, while

remaining locally isomorphic to the Kerr spacetime, thus modifies the dynamics of matter by the mechanism discussed above. The formation of central region of sufficient mass density and magnetic field strength is essential for observed rotational dynamics of galaxies. In ellipticals, disk-like region formation in the galactic centers due to rotation have also been observed [36,37], indicating that the toroidal field coupled to the spacetime vorticity is also important in the evolution of spiral and elliptical galaxies. The above model also explains diverse rotation curves of dwarf galaxies [38], where the magnetic field generated by spacetime vorticity is comparable to that in disk galaxies, which cannot be otherwise accounted for by accretion and other physical mechanisms.

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Figures

Figure 1

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Figure 2

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Figure 3

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Figure 4

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