The Coase Conjecture When the Monopolist and Customers Have Different Discount Rates

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The Coase Conjecture When the Monopolist and Customers Have Different Discount Rates

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Abstract

One of the most famous and outstanding formalizations of the Coase Conjecture is that by Gul, Sonnenschein, & Wilson (1986). A peculiarity of their model—as well as nearly all other examinations of the Coase Conjecture, including that by Coase himself—is that it assumes that the monopolist and customers have the same discount rate. I re-examine their model, while relaxing this restriction. Gul, Sonnenschein, and Wilson show that, if the (common) discount rate of the monopolist and customers approaches one, then the Coase Conjecture follows. I show that one only needs the discount rate of the *customers* to approach one for this to be true. I also show a second result: If the customers' discount rate is fixed at a value less than one while the monopolist's discount rate approaches one, then the Coase Conjecture is guaranteed *not* to follow. (JEL: C7, C79, D7, D72)

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1. Introduction

One of the most surprising and unintuitive results of all economics is the Coase Conjecture. In brief, it examines the behavior of a monopolist who sells a product over several periods. As Coase (1972) notes, such a monopolist

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will naturally want to practice price discrimination—that is, to charge a high price in the initial period, wait for the highest-demanding customers to buy, then gradually drop the price in order to sell to lower- and lower-demanding customers. As Coase notes, however, the customers will anticipate this strategy, and accordingly, even the highest-demanding customers will be tempted to wait until future, lower-price periods to buy. As a consequence, the monopolist ends up competing against future versions of himself. As Coase notes, this may cause the monopolist to lose all bargaining power. That is, instead of practicing price discrimination, he immediately ("in the twinkling of an eye") sets price equal to marginal cost.

Gul, Sonnenschein, & Wilson (1986) (hereafter, GSW) construct a formal model of the Coase Conjecture. They prove that a sufficient condition for the Conjecture is that the discount rate of the customers and monopolist approaches one. Their model is very general and makes few assumptions. For instance, it makes no parametric assumptions about the customers' preferences, only, e.g., that the inverse demand curve is left continuous and everywhere finite.

However, their model does make one assumption that is fairly restrictive. This is that the monopolist and customers share the same discount rate. Nearly all theoretical examinations of the Coase Conjecture make the same assumption.[1] This includes, it appears, Coase himself. When he discusses (p. 148-9) whether an expected increase in demand will affect the monopolist's pricing decisions, he notes that this depends on several factors, including "*the* [my emphasis] rate of discount."

In this paper I re-examine GSW's model while allowing the discount rates of the monopolist and customers to differ. I examine conditions that produce, what I call, the *Coase Outcome*. By the latter phrase I mean that in the initial period ("in the twinkling of an eye") the monopolist sets price equal to (or arbitrarily close to) the reservation price of the lowest-demanding customer. As a consequence, all or nearly all the customers buy in the first period, and the monopolist—despite his wishes—does *not* practice price discrimination.

I prove two main propositions: (i) the Coase Outcome is still guaranteed to occur when only the customers' discount rate approaches one; and (ii) if the customers' discount rate is fixed at a level below one while the monopolist's rate approaches one, then the Coase Outcome is guaranteed *not* to occur.

Despite an extensive search, I am aware of only three theoretical examinations of the Coase Conjecture that allow the monopolist and customers to have different discount rates. These are articles by Sobel & Takahashi (1983), Fudenberg et al. (1985), and Guth & Ritzberger (1998).

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1. Such examinations include: Stokey (1981), Bulow (1982), Bond & Samuelson (1984), Kahn (1986), Bagnoli et al. (1989), Ausubel & Deneckere (1989), Fuchs & Skrzypacz (2010), Board & Pycia (2014), and Nava & Schiraldi (2019)

The latter two articles differ from GSW's (and my) setup in two significant ways. First, they make parametric assumptions about the preferences of the customers. Guth and Ritzeberger assume that customers' valuations are distributed uniformly, which makes the demand curve linear. Sobel and Takahashi assume that customers' valuations are distributed according to the cdf $F(v) = v^m$, for $m > 0$ and $0 \leq v \leq 1$. Second, the two models assume a finite number of periods, n. For some results the authors examine limiting outcomes as n approaches infinity. However, this still differs from GSW's setup, which allows trading to take place indefinitely.[2] Despite differences in our setups, the latter two articles derive results that are similar to my main results.[3]

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2. In the last section of Guth and Ritzberger's article, the researchers claim to derive results for the indefinite-trading case. However, to prove these results they frequently use results of their previous sections, where the number of periods is finite. For those cases, I do not believe the claims actually follow.

3. Sobel and Takahashi's Corollary considers a finite game of n periods but examines the limiting case as n approaches infinity. When one sets the customer's discount rate to one, then, for this Corollary, the only solution for $\hat{c}$ is zero, which, similar to my Proposition 2, implies that the monopolist sets initial price equal to zero. Sobel and Takahashi's Theorem 6 analyzes a game of n periods. As the authors note, the theorem implies that the initial price rises as the monopolist's discount factor rises. Given that the minimum possible initial price is zero, this implies, similar to my Proposition 3, that the initial price must be positive as the monopolist's discount factor approaches one. Guth and Ritzberger's equation (5) is derived from a game of n periods. The equation list's the monopolist's initial-period profits for the case in which n approaches infinity. As one can easily verify, if the customers' discount factor is one while the monopolist's discount factor is strictly less

Fudenberg et. al., like GSW, allow trading to take place indefinitely and do not make any parametric assumptions about customers' preferences. They claim, without proving, a Coase Conjecture- type result. Namely, "It can be shown that when the buyer's and seller's discount factors converge to 1, the seller's payoff converges to zero." Although for the rest of the article the authors allow the buyer's and seller's discount factors to differ, for the latter claim they apparently constrain them to be the same.

2. Assumptions and Definitions

The following is a description of the basic GSW model, except I have altered it so that the monopolist and the customers can have different discount rates. Further, in some instances, as I note, I adopt slightly stronger technical assumptions than GSW.

A monopoly producer (a male) sells a durable good to potential customers (all female) over a set of periods $i = 0, 1, 2, \ldots$. In each period he announces a price p_i. Customers decide to buy at that price or wait, possibly to buy at a different price or not to buy at all. Each customer buys at most one product.

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than one, then, similar to my Proposition 2, the monopolist's initial-period profit is zero. Meanwhile, if the customers' discount factor is strictly less than one while the monopolist's discount factor is one, then, similar to my Proposition 3, the monopolist's initial-period profit is positive. I show that the results of these authors extend to the case in which (i) customers have more general preferences and (ii) trading can take place indefinitely.

Customers are represented by the continuum $[0,1]$. Let q represent a generic customer and $f(q)$ represent her valuation of the good. Let $\delta_C < 1$ represent her discount rate. If she buys in period i, then her utility is $\delta_C^i[f(q) - p_i]$.

The monopolist has a discount rate of $\delta_M < 1$. His fixed and marginal costs are zero. I assume that he maximizes discounted profit (which equals discounted revenue, since costs are zero).

I assume that $f(q)$, the inverse demand curve, is continuous and strictly decreasing. This assumption is slightly stronger than that of GSW, who only assume that $f(q)$ is left-continuous and non-increasing. In addition, unlike GSW, I assume that $f(q)$ is differentiable everywhere.

I assume that the derivative of $f()$ is bounded—formally, that there exists a $k \in (0, \infty)$ such that for all $q \in [0,1]$, $-f'(q) < k$. GSW make a similar, yet weaker, "Lipshitzian" assumption. Specifically, they assume that there exists a finite k and a $\bar{q}$ such that for all $q > \bar{q}, -[f(q) - f(1)]/[q-1] < k$.

I assume that $f(1)$, the valuation of the least-demanding customer is greater than zero. (Recall that I assume that zero is the marginal cost of the monopolist.) In the literature on the Coase Conjecture, researchers often call this the "gap" case. An alternative assumption, that $f(1)$ is only *greater than or equal* to zero, is often called the "no gap" case. GSW, for some of their results assume the gap case, but for other results they assume the weaker, no-gap case.[4]

Like GSW, I look for subgame perfect Nash equilibria. Also, like GSW, I impose what I call a *non-atomistic* restriction on equilibria, which disallows any single customer too much influence on the outcome. Specifically, I (and GSW) assume the following: Suppose two nodes in the game tree (i) have the same histories in terms of prices and (ii) except for possibly a zero-measure set of customers, have the same histories in terms of which customers have and have not bought. Then in any equilibrium each player must adopt the same strategy at the two nodes. This means that a deviation by any single customer

4. In Groseclose (2020) I discuss justifications of the gap case. One of the most important may involve two, seemingly counterfactual, implications of the no-gap models. These are (i) that buying and selling must continue infinitely and (ii) that price changes must become infinitesimally small. Meanwhile, some markets seem to be exemplary cases of the Coase Conjecture framework—e.g. their producers really are monopolists, the customers buy at most one product, and the products do not change over time (e.g., due to technological improvements). Two examples seem to be books and movie DVDs. In the latter type markets price changes do not become infinitesimally small—e.g., they never become fractions of a penny—which they would if the no-gap models were literally true. Part of the problem, I believe, involves a more general point raised by Cramton (1991). This is that Coase Conjecture models often fail to account for transaction costs of bargaining—which might include the time and effort involved in changing a price. As a consequence, as I argue, if Coase Conjecture models did account for such transactions costs, then even the no-gap models would look much like gap models.

(who will be a zero-measure set) cannot, e.g., cause the producer to deviate from his equilibrium pricing strategy in future periods. For the remainder of the paper, when I write "equilibrium," I mean a subgame perfect Nash equilibrium that satisfies the latter restriction.

3. Main Results

3.1. Sufficient Conditions for the Coase Outcome

The first result is GSW's version of the Coase Conjecture.[5] It assumes that $\delta_C = \delta_M \equiv \delta$. It states that, as δ approaches one, the *Coase outcome* is guaranteed to occur.

> *Proposition 1 (GSW):* Let $\delta_C = \delta_M \equiv \delta$. Assume $\delta \in (0, 1)$. Then for all $\varepsilon > 0$, there exist $\bar{\delta}$ such that, if $\delta > \bar{\delta}$, then in any equilibrium $p_0 < f(1) + \varepsilon$.

As the next proposition shows, we can relax the assumption that the monopolist and customers share the same discount factor. It shows that, for the Coase Outcome to occur we only need the discount rate of the *customers* to approach one. In fact, the proposition actually shows an even stronger result.

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5. The proof of this proposition involves GSW's Theorems 1 and 3. Theorem 1 proves the case that, if $f(1) > 0$, then the equilibrium is unique and customers' strategies are stationary. Theorem 3 assumes that $f(1) \geq 0$ and that customers' strategies are stationary. It proves that under these assumptions the Coase Outcome must occur—that is, the initial price must be arbitrarily close (i.e. within ε) to $f(1)$ as δ approaches 1. Thus, combined, the two theorems imply that, assuming that $f(1) > 0$, the equilibrium is unique and the Coase Outcome must occur.

It shows that, as the customers' discount rate approaches one, the initial price does not just become arbitrarily close to $f(1)$; it equals $f(1)$.[6]

Proposition 2: Let δ_M be fixed at some value less than 1. Then there exists $\bar{\delta}_C$ such that for all $\delta_C \in (\bar{\delta}_C, 1)$, in any equilibrium, $p_0 = f(1)$.

If one wants to understand the intuition of Proposition 2, the first key step is to recall an important insight of GSW, formally stated as Lemma 6 in this paper's Appendix. This is that—although the rules of the game allow trading to take place indefinitely—in equilibrium the monopolist and customers trade over only a finite number of periods.[7] Lemma 6 says in some finite period—let us call it T—the monopolist *chooses* to end all buying and selling by setting price p_T equal to $f(1)$. Since $f(1)$ is the smallest reservation price of the

6. Much credit for my result belongs to GSW. First, as I note in the Appendix, to prove my Lemmas 1 and 4, I borrow heavily from GSW's proofs of their Lemmas 1 and 2. Second, at least under reasonable interpretations, GSW make a claim that is nearly identical my Proposition 2. Specifically, in footnote 5 the authors write "Only the coincidence of the consumers' discount factors is necessary for the analysis." I interpret this to mean that, like my paper, if GSW had allowed the monopolist and consumers to have different discount factors, then it would still be possible to prove the Coase Conjecture as long as the customers' discount rate approaches one. In footnote 14, which occurs just before GSW's Theorem 3, the authors note: "The theorem is stated for the case that $f(1) = 0$. If $f(1) > 0$, then the statement need not include the [assumption that consumers' strategies are stationary] and the price described by the [equilibrium strategies] will be less than $f(1) + \varepsilon$.". Thus, the two footnotes make a claim that is essentially my Proposition 2. However, it is not exactly my Proposition 2 because I prove a slightly stronger result. Namely, I prove that the initial price will *equal* $f(1)$, not just that it will be within ε of $f(1)$.

7. As GSW note, Fudenberg, Levine, and Tirole (1985), who prove essentially the same result in a different context, deserve much credit for the result.

customers, all customers buy at this price. When this happens, we say that *the market clears.*

Proposition 2 proves the following. Let T be the period in which the market clears in some equilibrium. Then, if δ_C is sufficiently high, then the monopolist necessarily improves his discounted revenue by clearing the market one period prior to T. Thus, no matter what period the monopolist chooses to clear the market, he can do better by clearing the market one period earlier. But this means that the monopolist is not maximizing under the supposed equilibrium. The only way for this not to be a logical contradiction is if $T = 0$—that is, if the market clears in the initial period, thus producing the Coase Outcome.

To see how Proposition 2 proves this, suppose that the Coase Outcome does not occur, thus implying that in equilibrium $T > 0$.

As shown by Lemmas 2 and 3 in the Appendix, for any equilibrium there will exist a set of numbers $\{q_i\}$ that separate the customers who prefer to buy in a period from those who do not. More specific, in any period i there will be a customer, whom we can define as q_{i+1}, who will be indifferent between buying and not buying. Further all customers $q < q_{i+1}$, if they have not bought already, will strictly prefer to buy, and all $q > q_{i+1}$ will strictly prefer not to buy. Let us define $q_0 = 0$. Then in equilibrium all $q \in [q_0, q_1)$ buy in period 0, all $q \in (q_1, q_2)$ buy in period 2, ..., and all $q \in (q_T, 1)$ buy in period T.

Now suppose that the monopolist deviates from the equilibrium by clearing the market one period earlier. That is, in period $T - 1$ he sets price equal to $f(1)$ (instead of p_{T-1} as the equilibrium requires).

His benefit from such a deviation is the following. Under the equilibrium, customers in $(q_T, 1)$ buy in period T at price $f(1)$, which produces a revenue of

$(1-q_T)f(1)$. This is represented by area B in Figure 1. But if the monopolist deviates in the above manner, then these customers still buy at the same price, $f(1)$; however, they do so one period earlier. Consequently, the deviation increases the monopolist's discounted revenue by

$$Benefit = (1-\delta_M)(1-q_T)f(1).$$

Meanwhile, the cost of the deviation is the loss in revenue from customers in (q_{T-1}, q_T). Whereas, under the equilibrium they pay price p_{T-1}, under the deviation they pay price $f(1)$. The monopolist's loss in revenue thus equals

$$Cost = (q_T - q_{T-1})[p_{T-1} - f(1)],$$

which is represented by area C in Figure 1.

Now consider the buying decision of customer q_T. By the definition of q_T, she is indifferent between (i) buying in period $T-1$ at price p_{T-1} and (ii) buying in period T at price $f(1)$. It thus follows that

$$f(q_T) - p_{T-1} = \delta_C[f(q_T) - f(1)],$$

which implies

$$p_{T-1} = (1-\delta_C)f(q_T) + \delta_C f(1). \tag{1}$$

Thus, p_{T-1} is a weighted average of $f(q_T)$ and $f(1)$, where δ_C is the weight on $f(1)$. This is the second key step in understanding the intuition of the proof. As the patience of the customers grows, p_{T-1} grows closer to $f(1)$, which causes the costs of the deviation to approach zero. Consequently, as the patience of the customers increases, the monopolist becomes more and more tempted to deviate in the prescribed manner—that is, to clear the market one period earlier. It follows that the only possible equilibrium is for the market to clear in period zero. Thus, in equilibrium the Coase Outcome must occur.[8]

3.2. Sufficient Conditions for the Coase Outcome Not to Occur

The next proposition, Proposition 3, examines the opposite case of Proposition 2. That is, it fixes the discount rate of the customers at a value between zero and one, while it lets the discount rate of the monopolist approach one. Perhaps unsurprisingly, the result of Proposition 3 is the opposite of

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8. The above analysis ignores the possibility that the benefit of the deviation might approach zero just as fast or faster than the cost approaches zero, thus causing the Coase Outcome *not* to occur. It turns out, however, that this is not the case. For a detailed explanation one should consult the proof in the Appendix, however the following provides a sketch of those details. Define the net benefit of the deviation as the benefit minus the costs. Thus, $Net\ Benefit = (1-\delta_M)(1-q_T)f(1) - (q_T - q_{T-1})[p_{T-1} - f(1)]$. Use (1) to substitute for p_{T-1}. This gives $Net\ Benefit = (1-\delta_M)(1-q_T)f(1) - (q_T - q_{T-1})(1-\delta_C)[f(q_T) - f(1)]$. Define s as the absolute value of the average slope between $f(1)$ and $f(q_T)$. Then $f(q_T) - f(1) = s(1-q_T)$. Substituting this into the former equation and simplifying gives $Net\ Benefit = (1-q_T)\{(1-\delta_M)f(1) - (q_T - q_{T-1})(1-\delta_C)s\}$. Since $q_T - q_{T-1} \leq 1$, $Net\ Benefit \geq (1-q_T)\{(1-\delta_M)f(1) - (1-\delta_C)s\}$. According to the postulated equilibrium $T > 0$. This means that some, but not all, customers purchase in period $T-1$, which means that $q_T \in (0,1)$, which in turn means that $1 - q_T > 0$. Finally, as δ_C grows close to one, the term in braces becomes positive. Thus, the net benefits of deviating necessarily become positive for large enough δ_C.

Proposition 2. It shows that, under these conditions, the Coase Outcome is guaranteed *not* to occur.

> *Proposition 3:* Let $\delta_C < 1$ be fixed at some value less than 1. Then there exists an $\varepsilon > 0$ and a $\bar{\delta}_M < 1$ such that for all $\delta_M \in (\bar{\delta}_M, 1)$, in any equilibrium $p_0 > f(1) + \varepsilon$.

The following is a non-technical summary of the proof of the proposition. The proposition shows that, under certain conditions, the Coase Outcome is guaranteed not to occur. That is, in any equilibrium the monopolist must charge a price above $f(1)$ in period zero. Accordingly, the market does *not* clear in the initial period.

The proposition actually proves an even stronger result. It shows that the monopolist's initial price is not just greater than $f(1)$, it is bounded away from $f(1)$—that is, it is not even "within ε of" $f(1)$. For simplicity, however, for this non-technical summary I focus on the weaker result—that the initial price is simply greater than $f(1)$.

To understand the result, suppose the opposite—that the Coase Outcome *does* occur and hence that in period zero the monopolist charges a price of $f(1)$. The proof shows that, if this occurs, then the monopolist can improve his discounted revenue by charging a slightly higher price, thus showing that the Coase Outcome strategy cannot be an equilibrium.

To illustrate this, the first step is to show that, if the monopolist charges a slightly higher price in period zero—call it $\tilde{p}$—then at least some customers still buy. Showing this is a little tricky, but as long δ_C is strictly less than one, it can be done. The intuition is the following: If the monopolist offers price $\tilde{p}$ in period zero, then the choice for customers is to buy at this price or wait until a future period to buy. As Lemma 1 shows, the monopolist will never charge a price below $f(1)$. Therefore, as long as the customer is not perfectly patient and as long as the monopolist sets $\tilde{p}$ close enough to $f(1)$, then the customer will prefer (i) buying at $\tilde{p}$ in period zero rather than (ii) buying at $f(1)$ in a future period.[9]

Accordingly, this implies the following: There exists a price, $\tilde{p}$, sufficiently close to $f(1)$ yet still greater than $f(1)$, such that if the monopolist offers it, then *some* customers will strictly prefer to buy at that price instead of waiting to buy in a future period. Let $[0, \tilde{q})$ be those customers. As the above reasoning shows, $\tilde{q}$ must be strictly larger than zero.

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9. To show this formally, in the proof I choose an arbitrary customer $q = \frac{1}{2}$ and show that the monopolist can indeed induce all customers in $[0, \frac{1}{2})$ to buy at a price above $f(1)$. (The proof will also work if I choose other customers, but it is convenient to choose $q = \frac{1}{2}$.) To do this, I construct a period-zero price, $\tilde{p}$, that would make the $q = \frac{1}{2}$ customer indifferent between (i) buying at price $\tilde{p}$ in period zero and (ii) waiting until period 1 to buy at price $f(1)$. It is easily shown that, to satisfy these conditions, $\tilde{p}$ must equal $(1 - \delta_C)f(\frac{1}{2}) + \delta_C f(1)$. (To see why this is true, note that if customer $q = \frac{1}{2}$ buys in period zero, then her utility is $f(\frac{1}{2}) - \tilde{p}$, which, after substituting for $\tilde{p}$, simplifies to $\delta_C[f(\frac{1}{2}) - f(1)]$. Meanwhile, if she waits until period one to buy at price $f(1)$, then her discounted utility is $\delta_C[f(\frac{1}{2}) - f(1)]$, which is the same as the former utility.) Now suppose that customer $q = \frac{1}{2}$ is offered price $\tilde{p}$ in period zero. If she rejects the offer, then according to Lemma 1, the monopolist will never offer a price below $f(1)$ in any future period. Accordingly, the highest utility she could receive is if the monopolist offered a price of $f(1)$ and did so in the very next period. Thus, if she is offered a price of $\tilde{p}$ she at least weakly prefers to accept this offer (as opposed to waiting to buy in a future period). Further, any customer $q < \frac{1}{2}$ *strictly* prefers to buy at $\tilde{p}$ as opposed to waiting until a future period to buy. Accordingly, if the monopolist offers a price of $\tilde{p}$ in period zero, then all customers in $[0, \tilde{q})$, where $\tilde{q} \geq \frac{1}{2}$, will buy.

It thus follows that, under this deviation, the monopolist receives a period-zero revenue of $\tilde{q}\tilde{p}$ from customers $[0, \tilde{q})$. Meanwhile, if he had played the Coase Outcome strategy, then his period-zero revenue from the same customers would have been $\tilde{q}f(1)$. Let us call the difference in these two revenues the monopolist's *Benefit* from the deviation. It equals:

$$Benefit = \tilde{q}[\tilde{p} - f(1)].$$

Now suppose that, under the deviation strategy, the monopolist sets the next-period price at $f(1)$. If so, then in that period all the remaining customers buy. (These customers are $(\tilde{q}, 1)$, plus possibly customers $q = \tilde{q}$ and $q = 1$, depending upon their buying decisions under indifference.) Consequently, the revenue from those customers is $(1 - \tilde{q})f(1)$. Meanwhile, under the Coase Outcome strategy the revenue that the monopolist receives from the same customers is $(1 - \tilde{q})f(1)$—i.e. the exact same amount that he receives from the deviation strategy. However, under the Coase Outcome strategy the monopolist receives the revenue one period earlier than he would receive it under the deviation strategy. Accordingly, because of discounting, the deviation strategy has a *Cost* of $(1 - \delta_M)(1 - \tilde{q})f(1)$, which is the decrease in discounted revenue from those customers. Define the *Net Benefit* from the deviation strategy as the $Benefit$ minus the $Cost$. Thus, it equals

$$Net\ Benefit = \tilde{q}[\tilde{p} - f(1)] - (1 - \delta_M)(1 - \tilde{q})f(1).$$

Finally, recall that we specifically constructed $\tilde{p}$ and $\tilde{q}$ so that (i) $\tilde{p} > f(1)$ and (ii) $\tilde{q} \in (0, 1)$. Because of these facts, as δ_M becomes close enough to one, the monopolist's net gain from the deviation will necessarily be positive. As a consequence, the Coase Outcome cannot occur in equilibrium.

4. Discussion I: Propositions 1 and 2

Given GSW's result (Proposition 1), I believe that Proposition 2 is unsurprising. To see why, note that both results provide sufficient conditions for the Coase Outcome to occur. Importantly, when the Coase Outcome occurs, the monopolist has demonstrated essentially a nil level of bargaining power. That is, ideally he would set prices so that each customer would pay her reservation price for the product, and thus he would extract all of the possible rents from the customers. However, instead, he offers only one price. This price equals the *lowest* reservation price of the customers. Thus, his behavior is more like a seller in a perfectly competitive industry than a monopolist in a static, one-period market.

Next, consider the following general principle of bargaining games: If a player becomes more patient, then this usually aids his or her bargaining power. The GSW result analyzes the case where the monopolist has perfect patience—more specific, his discount rate approaches one. Despite this, (since the result is the Coase Outcome) his bargaining power is essentially nil.

Meanwhile, Proposition 2 considers a case where the monopolist's discount rate is fixed at a level below one. Thus, his patience is less than it is for the GSW case. Thus, compared to the GSW result, Proposition 2 *decreases* the monopolist's patience. Accordingly, we should expect his bargaining power to decrease. But since it is already essentially nil, we should expect it to remain essentially nil. Consequently, we should expect the Coase Outcome to occur in Proposition 2, since we already know that it occurs in the GSW result.

Given this, I believe that the main contribution of Proposition 2, if any, is the following: It presents a slightly less general, but more simple, version of GSW's result, while proving the result in a way that is more transparent and easier to understand.

Related, I believe that another contribution of Proposition 2 is that it helps to illustrate the brilliance of GSW's result. To see this, consider the following thought experiment. As I mention above, given GSW's result, I do not believe that Proposition 2 is very surprising. Further, given that I have followed many of GSW's steps, I do not believe my proof is very impressive. However, suppose I had derived Proposition 2 *before* GSW had derived their result. If so, then I think readers would have been very surprised and impressed by my result.

After all, consider what Proposition 2 proves. It examines a case in which a dynamic monopolist is given more options, relative to those given to a static monopolist. Specifically, rather than being allowed to choose only one price, he is allowed to choose many prices over different periods. Yet, strangely, the increase in options causes his profits to *drop*. Perhaps more surprising, even though he is a monopolist, his profits become essentially nil.

Moreover, Proposition 2 shows that this is true for a very general set of conditions. That is, for instance, the result assumes very little about the demand curve or the monopolist's discount rate. I believe that most readers, a priori, would have considered it extremely difficult to prove such a result, and I believe that some would have (mistakenly) considered it impossible.

But now suppose we make the task even more difficult. We not only raise the monopolist's level of patience, we make it perfect. After doing this, we ask a researcher to prove the same result—to show that even in this situation the monopolist still earns profits that are essentially nil. This is exactly what GSW do. It is a difficult task, one that I believe should be considered one of the all-time great achievements in economic theory.

5. Discussion II: Proposition 3

More surprising than Proposition 2, I believe, is my second result, Proposition 3. It addresses the following thought experiment. Suppose a researcher wanted to prove a slightly stronger version of Proposition 2. Specifically, suppose that he or she wanted to show that, instead of a perfect level of patience for the customers, merely a very high level of patience—say, one with a discount rate of .999—is also sufficient to generate the Coase Outcome. Proposition 3 shows that it is impossible to prove such a result.

Importantly, Proposition 3 makes very few assumptions about customers' preferences. It only assumes some weak technical conditions such as that the inverse demand curve is everywhere differentiable and decreasing. This means that, even if a researcher were to choose a very special demand curve—say, one that is extremely elastic or one that is extremely inelastic—then he or she still could not prove such a stronger version of the Coase Conjecture.

6. Final Word: Coase's Conjecture and His *Lack* of Discussion of Discount Rates

The following is the passage in which Coase explains his Conjecture. To make his point concrete, he constructs an example in which a monopolist owns all the land in the United States and all the land is of uniform quality.

Most curious, note that the passage does not even mention the patience of the monopolist or the customers, much less formally examine the extent to which discount rates of those actors affect outcomes.[10]

> We now have to determine the price which the monopolistic landowner would charge for a unit of land in the assumed conditions. [Figure 2] would seem to suggest (and has, I believe, suggested to some) that such a monopolistic landowner would charge the price OA, would sell the quantity of land OM, thus maximizing his receipts, and would hold off the market the quantity of land, MQ. But suppose that he did this. MQ land and money equal to OA × OM would be in the possession of the original landowner while OM land would be owned by the others. In these circumstances, why should the original landowner continue to hold MQ off the market? The original landowner could obviously improve his position by selling more land since he could by this means

10. The passage is the second and third paragraph of Coase's article (pp. 143-4). Only once in the entire article does Coase mention discount rates or any similar concept. This occurs in the second-to-last paragraph of the article. I refer to this paragraph earlier in the paper, where I note that Coase uses the phrase "*the* [my emphasis] rate of discount," thus suggesting that Coase assumes that the monopolist and customers share the same rate of discount. The paragraph is not part of Coase's central argument; rather, it is more like an afterthought. Indeed, it appears near the end of the article and its opening sentence is: "One other qualification should be mentioned."

> acquire more money. It is true that this would reduce the value of the land OM owned by those who had previously bought land from him—but the loss would fall on them, not on him. If the same assumption about his behavior was made as before, he would then sell part of MQ. But this is not the end of the story, since some of MQ would still remain unsold. The process would continue as long as the original landowner retained any land, that is, until OQ had been sold. And if there were no costs of disposing of the land, the whole process would take place in the twinkling of an eye.
>
> It might be objected to this supposed behavior under which land is sold in separate transactions involving blocks of land, probably of diminishing size, that it would be even better if the landowner sold the land by infinitesimal units, thus maximizing his total revenue. But this is neither her nor there. Whatever the intermediate steps are assumed to be, OQ land will be sold. And given that OQ is going to be sold, the value of a unit of land is going to be OB and given this, no buyer of land will pay more than OB for it. Although the demand schedule may be correctly drawn in that, if the quantity of land is OM, the price would be OA, the landowner would find himself in the position that, if he were charged more than OB, he would sell nothing.[11] The demand schedule facing the original landowner would be infinitely elastic at the competitive price and this even though he was the sole supplier. With complete durability, the price becomes independent of the number of suppliers and is thus always equal to the competitive price.

Coase's argument relies on two key points. First, the monopolist will eventually set price equal to marginal cost. Second, because of this, any

11. I believe Coase made a typo in this sentence. Where he writes "were charged" I think he meant "were to charge."

customer would be rational to forego buying until the monopolist offers such a price.

The second point, however, ignores the possibility that customers may be less than perfectly patient. If so, they may prefer to buy at a higher price now instead of a lower price later. Coase does not discuss this possibility.

For these reasons Proposition 3 may be especially important. As it shows formally, for some discount rates, the Coase Outcome is guaranteed *not* to occur. Consequently, Coase's Conjecture, strictly speaking, is not true unless one adds some caveats about the patience of the monopolist and customers.

7. Appendix

7.1. Preliminary Results

The following lemmas follow readily from the assumptions and definitions of the model. Lemmas 1 and 4 are nearly identical, respectively, to GSW's Lemmas 1 and 2. To prove these lemmas I borrow heavily from GSW's proofs. However, in at least two important respects my proofs differ from those of GSW. First, for every instance in which GSW list "δ" (the common discount factor of the monopolist and customers) I substitute δ_C, δ_M, or some function of the two. Second, GSW's proofs are generally more sparse than my proofs. In particular, for some parts of their proofs, GSW skip steps that are perhaps obvious to highly skilled theoreticians but not so obvious to the rest of us. For instance, my proof of Lemma 1 is approximately double the length of GSW's corresponding proof.

Lemma 1: In any subgame of any equilibrium, all customers (except for possibly customer $q = 1$) are willing to buy if the price is $f(1)$ or less, and therefore the monopolist in equilibrium never offers a price less than $f(1)$.

Proof: Suppose at some subgame a set of customers $Q \subseteq [0, 1]$ have not bought. Label the remaining periods $t = 1, 2, \ldots$ Define $c_t(q)$as the supremum price that customer q will accept in period t (in any subgame). First, it must be the case that $c_t(q) \geq 0$. (To see this, first note that if the monopolist is playing a subgame-perfect strategy, then he will never offer a price lower than his marginal costs, zero. Thus, if q rejects the monopolist's offer in period t, then the maximum she can receive in utility is if the the monopolist offers a price of zero and he does so in the very next period, period $t + 1$. This would give the customer a utility of $\delta_C^{t+1}[f(q) - 0]$. Meanwhile, if the monopolist offers a price of zero in period t, then q's utility is $\delta_C^t f(q)$. Since $\delta_C < 1$ and $f(q)$ is positive, q strictly prefers to accept the price of zero in period t, which contradicts the assertion that $c_t(q) < 0$.) Next define $\tilde{c}_t = \inf_{q \in Q} c_t(q)$. Define $c = \inf_{t \in \mathbb{N}} \tilde{c}_t$. It follows that, for all t and for all q, $c_t(q) \geq 0$. Consequently, it follows that $c \geq 0$. (Importantly, this means that c is not negative infinity, and thus we can treat c

as a finite number in various equations and inequalities.) We want to show that $c \geq f(1)$. To do this, suppose not. Then $c < f(1)$. Now consider an arbitrary period t, and suppose the producer offers the price

$$\tilde{p}_t = (\frac{1-\delta_C}{2})f(1) + (\frac{1+\delta_C}{2})c. \tag{2}$$

Note that this implies $c < \tilde{p}_t < f(1)$. If a customer, q, accepts this price, then her utility is $f(q) - \tilde{p}_t$. Meanwhile, if she rejects the price, then she can do no better than if, in the next period, the producer offers a price of c. Thus, if she rejects the producer's offer of $\tilde{p}_t$, then her utility will be no greater than $\delta_C[f(q) - c]$. Once we substitute for the definition of $\tilde{p}_t$, it is easily shown that, for all q, the former utility is greater than the latter utility. (To see this, first recall that $\tilde{p}_t > c$. Thus, $-\tilde{p}_t < -c$. Adding $2\tilde{p}_t$ to both sides gives $\tilde{p}_t < 2\tilde{p}_t - c$. For the $\tilde{p}_t$ on the right-hand side of the latter, let us substitute the definition of $\tilde{p}_t$ (from (2)). This implies $\tilde{p}_t < (1-\delta_C)f(1) + \delta_C c$. Since $f(q) \geq f(1)$, the latter implies $\tilde{p}_t < (1-\delta_C)f(q) + \delta_C c$, which implies $f(q) - \tilde{p}_t > \delta_C[f(q) - c]$.)

Thus, q prefers to accept the producer's offer of $\tilde{p}_t$. This means that the supremum price at which she is willing to buy is at least as large as $\tilde{p}_t$, which implies that $c_t(q) \geq \tilde{p}_t \equiv (\frac{1-\delta_C}{2})f(1) + (\frac{1+\delta_C}{2})c$. Since this is true for all q, it follows that $\tilde{c}_t \equiv \inf_{q \in Q} c_t(q) \geq (\frac{1-\delta_C}{2})f(1) + (\frac{1+\delta_C}{2})c$. Since we chose t arbitrarily, the latter relation holds for all t. This implies that the latter relation holds for the infimum of $\tilde{c}_t$, which implies $c \geq (\frac{1-\delta_C}{2})f(1) + (\frac{1+\delta_C}{2})c$. But since $f(1) > c$ and $\delta_C < 1$, the latter implies $c > c$, a contradiction. It follows that $c \geq f(1)$. ■

The next lemma, due to Fudenberg, Levine, and Tirole (1985)—see their Lemma 1—shows that if a customer prefers to buy at a certain price in a given round, then all customers with higher valuations must also prefer to buy at that price in the same round.

Lemma 2: Suppose that, in some equilibrium and in some period i, customer q buys. Then, any customer $q' < q$ must also buy in that round (if she has not already bought in a prior round).

Define $q_0 = 0$, and for any equilibrium define q_{i+1} as the supremum of the customers who have bought in period i or before. Note that the definition implies $q_0 \leq q_1 \leq q_2 \ldots$. The next lemma directly follows from this definition and Lemma 2.

> *Lemma 3:* For any equilibrium, at the end of period i, all customers in $[0, q_{i+1})$ have bought, and (assuming $q_{i+1} \neq 1$) all customers in $(q_{i+1}, 1]$ have not.

It is useful to call q_i the *state* of the game. Lemma 3 implies that, at the beginning of period i, just before the monopolist names price p_i, q_i is the (unique) point that separates all customers who have bought from all those who have not.

The two lemmas imply that there will be a "skimming" of customers, ordered by their levels of demand. That is, the highest-demanding customers will buy in the first round, the next highest will buy in the second round, and so on. It cannot happen, for example, that in some round a low-demanding customer buys while a higher-demanding customer chooses not to buy.

Although it may seem obvious that this behavior must occur in equilibrium, it is difficult to prove. In my judgment, Fudenberg, Levine, and Tirole's Lemma 1 is a very important contribution, and the authors exercised much skill in proving it.[12]

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12. Moreover, an argument can be made that it is *not* completely obvious that such skimming behavior must occur in equilibrium. Indeed, in a recent pioneering paper, Ali, Kartik, and Kleiner (2022) show that it need not occur in a political bargaining model, which otherwise has similar rules as the Coase Conjecture model. The following is an example of their model. In a uni-dimensional policy space the status quo is leftwing and the ideal point of the "setter" is rightwing. The setter proposes policies, which a (median) voter accepts or rejects. Like the Coase Conjecture case, only one player, the setter, proposes policies. He continues to do so, possibly indefinitely, until the voter eventually accepts one of his proposals. The setter does not know the voter's ideal point, only the pdf of the point. If the setter, like the monopolist in the Coase Conjecture, adopted a skimming strategy, then he would begin by proposing a fairly rightwing policy, and if that is rejected, he would

The following lemma shows that if the state, q_i, becomes large enough, then the state in the next round, q_{i+1}, becomes 1—that is, all remaining customers buy. The lemma is very similar to GSW's Lemma 2, and I borrow heavily from their proof.[13]

Lemma 4: Define $\tilde{q} \equiv 1 - \frac{(1-\delta_M)f(1)}{(1+\delta_M)k}$. Let $\{p_i\}$ and $\{q_i\}$ be the prices and states realized in an equilibrium (possibly the realizations of random variables from mixed strategies). Then, for any i, if $q_i > \tilde{q}$, then $q_{i+1} = 1$.

Proof: Suppose in some equilibrium, in some round i, $q_i > \tilde{q}$. Then, from the definition of $\tilde{q}$ it follows that

$$-(1-\delta_M)f(1) + (1-q_i)(1+\delta_M)k < 0. \tag{3}$$

Note that in period i the monopolist's equilibrium revenue is $(q_{i+1} - q_i)p_i$. In future rounds only customers $(q_{i+1}, 1)$ have not bought, and none of these customers will have a valuation for the good higher than $f(q_{i+1})$. Hence, the monopolist's revenue in future rounds will be no greater than $(1-q_{i+1})f(q_{i+1})$. Define DR_i as the monopolist's discounted revenue from round i onward. Then

$$DR_i \leq (q_{i+1} - q_i)p_i + \delta_M(1-q_{i+1})f(q_{i+1}).$$

Since (by Lemma 3) all $q \in (q_i, q_{i+1})$ buy at price p_i, it must be the case that $p_i \leq f(q_{i+1})$. This and the above inequality imply

$$DR_i \leq (q_{i+1} - q_i)f(q_{i+1}) + \delta_M(1-q_{i+1})f(q_{i+1}).$$

Recall our assumption that $\forall q, -f'(q) < k$. This implies $f(q_{i+1}) \leq f(1) + (1-q_{i+1})k$. Substituting this into the above gives

$$DR_i \leq (q_{i+1} - q_i)f(1) + (q_{i+1} - q_i)k(1-q_{i+1}) + \delta_M(1-q_{i+1})[f(1) + k(1-q_{i+1})].$$

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propose more and more leftwing policies until the voter eventually accepts his proposal. However, Ali, et. al. show that there can be an equilibrium in which, instead, the setter adopts a "leapfrog" strategy. That is, he begins by proposing a very leftwing policy–one just slightly to the right of the status quo. Then, if the voter rejects that, the setter reverts to a skimming strategy—i.e. he proposes a fairly rightwing policy, then proposes more and more leftwing policies until the voter accepts one.

13. The lemma is also similar to the first part of Fudenberg, Levine, and Tirole's (1985) proof of their Lemma 3.

Since $q_{i+1} \leq 1$, $q_{i+1} - q_i \leq 1 - q_i$. Substituting this fact into the second term of the right-hand side of the above inequality gives

$$DR_i \leq (q_{i+1} - q_i)f(1) + (1 - q_i)k(1 - q_{i+1}) + \delta_M(1 - q_{i+1})[f(1) + k(1 - q_{i+1})].$$

By Lemma 1, if the monopolist were to charge a price of $f(1)$ in round i, then all remaining customers (except possibly $q = 1$) would buy, and this would give him a discounted revenue of $(1 - q_i)f(1)$. His equilibrium discounted revenue must be at least that large. Hence, $DR_i \geq (1 - q_i)f(1)$. This and the above inequality imply

$$(1 - q_i)f(1) \leq (q_{i+1} - q_i)f(1) + (1 - q_i)k(1 - q_{i+1}) + \delta_M(1 - q_{i+1})[f(1) + k(1 - q_{i+1})],$$

which implies

$$0 \leq (1 - q_{i+1})\{-f(1) + (1 - q_i)k + \delta_M[f(1) + k(1 - q_{i+1})]\},$$

which implies

$$0 \leq (1 - q_{i+1})\{-(1 - \delta_M)f(1) + (1 - q_i)k + \delta_M k(1 - q_{i+1})\}.$$

Since (by the definition of q_i) $q_{i+1} \geq q_i$, $\delta_M k(1 - q_i) \geq \delta_M k(1 - q_{i+1})$. This and the above inequality imply

$$0 \leq (1 - q_{i+1})\{-(1 - \delta_M)f(1) + (1 - q_i)(1 + \delta_M)k\}.$$

(3) implies that the term in braces is negative, which implies that $1 - q_{i+1} = 0$, which implies $q_{i+1} = 1$. ∎

The next two results are due to Fudenberg, Levine, and Tirole (1985) (and are expressed in their Lemma 3). However, I modify their proposition and proof to fit my terminology and setup.

Lemma 5 shows that there exists a natural number K and a real number $r > 0$ such that, for any state q_i that is less than $\tilde{q}$ (as defined in Lemma 4), in K rounds the new state, q_{i+K} must increase by more than r units. A corollary of this, Lemma 6, is that the market must clear in a finite number of rounds—specifically, no more than $\tilde{q}K/r + 1$.

Lemma 5: As in Lemma 4, define $\tilde{q} = 1 - \frac{(1-\delta_M)f(1)}{(1+\delta_M)k}$. Define $r = (1 - \tilde{q})f(1)/[2f(0)])$ and $K = \text{INT}[ln(r)/ln(\delta_M)] + 1$. Let $\{q_j\}$ be the realized states of an equilibrium, and for a particular state q_i assume $q_i < \tilde{q}$. Then $q_{i+K} > q_i + r$.

Proof: Let r, K, and $\tilde{q}$ be defined as above. Then $K > ln(r)/ln(\delta_M)$. This implies $\delta_M^K < r$. This and the definition of r imply

$$\delta_M^K < (1-\tilde{q})f(1)/[2f(0)]. \tag{4}$$

Assume $q_i < \tilde{q}$. We want to show $q_{i+K} > q_i + r$. To do this, suppose not. That is, suppose

$$q_{i+K} \leq q_i + r. \tag{5}$$

The monopolist's revenue in the first K rounds beginning with round i is

$$(q_{i+1} - q_i)p_i + (q_{i+2} - q_{i+1})p_{i+1} + \ldots + (q_{i+K} - q_{i+K-1})p_{i+K-1}.$$

Since no customer has a valuation greater than $f(0)$, all of the prices in the above expression must be less than or equal to $f(0)$. Thus, the above expression is less than or equal to $(q_{i+K} - q_i)f(0)$. After round K, $1 - q_{i+K}$ customers remain. None of these customers will buy if the price is greater than $f(0)$. Since $1 - q_{i+K} \leq 1$, the discounted revenue after round $i + K$ (discounted from round i) is no greater than $\delta_M^K f(0)$. Define DR_i as the discounted revenue from round i onward. Thus,

$$DR_i \leq (q_{i+K} - q_i)f(0) + \delta_M^K f(0).$$

The above, (4) and (5) imply

$$DR_i \leq rf(0) + (1-\tilde{q})f(1)/2.$$

After substituting the definition of r and simplifying, the above implies

$$DR_i < (1-\tilde{q})f(1). \tag{6}$$

Now consider the following alternative strategy for the monopolist. Suppose in round i he sets price equal $f(1)$. Then, by Lemma 1, all remaining customers—i.e. all $q \in (q_i, 1)$—must buy. This gives a discounted revenue of $(1 - q_i)f(1)$. By our hypothesis, $q_i < \tilde{q}$. Hence, this strategy brings a discounted revenue greater than $(1-\tilde{q})f(1)$, which contradicts (6). It follows that $q_{i+K} > q_i + r$. ■

Lemma 6: In any equilibrium the market clears in a finite number of rounds. That is, there exists a round j such that $p_j = f(1)$ and $q_{j+1} = 1$.

Proof: By definition, $q_0 = 0$. By Lemma 5, in K rounds $q_K > r$. In K more rounds $q_{2K} > 2r$. And so on. Define $n = \text{INT}[\tilde{q}/r] + 1$, where $\tilde{q}$ is defined as in Lemma 5. Then in nK rounds $q_{nK} > \tilde{q}$. By Lemma 3, $q_{nK+1} = 1$. Since $nK + 1$ is finite, the market clears in a finite number of rounds. ■

I use the next two lemmas to prove Proposition 3, which shows that, as δ_M approaches one, the Coase Outcome is guaranteed *not* to occur. I do this in two parts. The first part, Lemma 7, shows that there is an upper bound on the discounted revenue of the monopolist and that this bound is lower when the monopolist sets a low initial price. This has important implications for what I call a Coase Strategy. By this I mean a strategy in which the monopolist chooses an initial price very near $f(1)$. The lemma shows that, if the monopolist adopts a Coase Strategy, then his discounted revenue cannot be too much greater than $f(1)$.

The next lemma, Lemma 8, considers the case when δ_M approaches one. It considers what might be called an intermediate-price strategy for the monopolist. Loosely speaking, it causes about half the customers to buy in the initial round. Lemma 8 shows that this strategy gives the monopolist a strictly higher discounted revenue than he would receive from a Coase Strategy. Thus, the two lemmas show that, when δ_M is close to one, the monopolist cannot be optimizing if he adopts a Coase Strategy.

Lemma 7: Suppose in the initial round the monopolist sets price equal to p_0, which might or might not be part of an equilibrium strategy. Suppose in the remaining subgame the customers and the monopolist play equilibrium strategies. Then the monopolist's total expected discounted revenue is less than or equal to $f(1) + [p_0 - f(1)]/(1 - \delta_C)$.

Proof: Let p_0 be the monopolist's initial price. By Lemma 3 there exists a q_1 such that all $q < q_1$ buy and all $q > q_1$ do not. Accordingly, the monopolist's revenue in the initial round is $(q_1 - q_0)p_0 = q_1 p_0$. Since $f()$ is decreasing, all remaining customers (i.e. all customers $q \in [q_1, 1]$) have valuations less than or equal to $f(q_1)$, and hence they will not buy if the price is above $f(q_1)$. Consequently, the monopolist's discounted revenue after the initial round is less than or equal to $\delta_M(1 - q_1)f(q_1)$. Define DR as the monopolist's total discounted revenue. Then

$$DR \leq q_1 p_0 + \delta_M(1 - q_1)f(q_1).$$

Define q_1' as follows. Suppose that customers assume that the monopolist, instead of playing an equilibrium strategy after the initial round, will set the next round's price equal to $f(1)$ and hence, by Lemma 1, would clear the market. Define q_1' as the infimum q who would prefer to wait instead of buying at p_0 in the initial round. This alternative strategy is maximally accommodating to the customers. That is, if he is optimizing, the monopolist would never set price below this value. This means that, under this alternative strategy, the number of customers who prefer to wait (instead of buying in the initial round) is at least as many as the customers who prefer to wait under the original strategy, which means that $q_1' \leq q_1$. Since $f()$ is decreasing, this means $f(q_1') \geq f(q_1)$. This and the above inequality imply

$$DR \leq q_1 p_0 + \delta_M (1 - q_1) f(q_1').$$

By the definition of q_1' one of two cases must be true: (i) $q_1' = 0$ and all q weakly prefer to wait (and buy at $f(1)$) instead of buying now at p_0, or (ii) $q_1' > 0$ and customer q_1' is indifferent between buying now at p_0 or waiting a round to buy at $f(1)$. If (i), then $f(0) - p_0 \leq \delta_C[f(0) - f(1)]$, which implies $f(0) \leq [p_0 - \delta_C f(1)]/(1 - \delta_C)$. Since in this case $q_1' = 0$, it follows that $f(q_1') \leq [p_0 - \delta_C f(1)]/(1 - \delta_C)$. If (ii) is true, then $f(q_1') - p_0 = \delta_C[f(q_1') - f(1)]$. Thus, either case implies $f(q_1') \leq [p_0 - \delta_C f(1)]/(1 - \delta_C)$. Substituting this into the above inequality gives

$$DR \leq q_1 p_0 + \delta_M (1 - q_1)[p_0 - \delta_C f(1)]/(1 - \delta_C).$$

Since $\delta_M < 1$,

$$DR \leq q_1 p_0 + (1 - q_1)[p_0 - \delta_C f(1)]/(1 - \delta_C).$$

Since $q_1 p_0 = q_1 f(1) + q_1[p_0 - f(1)]$, the above implies

$$DR \leq q_1 f(1) + q_1[p_0 - f(1)] + (1 - q_1)[p_0 - \delta_C f(1)]/(1 - \delta_C).$$

Since $-\delta_C f(1) = (1 - \delta_C) f(1) - f(1)$, the above implies

$$DR \leq q_1 f(1) + q_1[p_0 - f(1)] + (1 - q_1)[(1 - \delta_C) f(1) + p_0 - f(1)]/(1 - \delta_C),$$

which simplifies to

$$DR \leq q_1 f(1) + q_1[p_0 - f(1)] + (1 - q_1) f(1) + (1 - q_1)[p_0 - f(1)]/(1 - \delta_C),$$

which simplifies to

$$DR \leq f(1) + q_1[p_0 - f(1)] + (1 - q_1)[p_0 - f(1)]/(1 - \delta_C),$$

Since $\delta_C \geq 0$, $q_1[p_0 - f(1)] \leq q_1[p_0 - f(1)]/(1 - \delta_C)$. This and the above imply

$$DR \leq f(1) + [p_0 - f(1)]/(1 - \delta_C).$$

■

Lemma 8: Define $\varepsilon = \frac{1}{4}(1-\delta_C)^2[f(\frac{1}{2}) - f(1)]$ and $\bar{\delta}_M = \max\{0, 1 - \frac{2\varepsilon}{f(1)(1-\delta_C)}\}$. Assume $\delta_M \in (\bar{\delta}_M, 1)$. Suppose in the initial round the monopolist sets price equal to $p_0 = (1-\delta_C)f(\frac{1}{2}) + \delta_C f(1)$, and suppose that in the remaining subgame the monopolist and customers play equilibrium strategies. Then the monopolist's total expected discounted revenue is greater than $f(1) + \varepsilon/(1 - \delta_C)$.

Proof: By the definition of ε and $\bar{\delta}_M$,

$$\frac{\varepsilon}{1-\delta_C} = \frac{1}{4}[f(\frac{1}{2}) - f(1)](1-\delta_C) \tag{7}$$

$$\bar{\delta}_M \geq 1 - \frac{1}{2f(1)}[f(\frac{1}{2}) - f(1)](1-\delta_C) \tag{8}$$

Suppose, according to the Lemma, that the the monopolist sets the initial price, p_0, equal to $(1-\delta_C)f(\frac{1}{2}) + \delta_C f(1)$, and suppose that in the remaining subgame the monopolist and customers play equilibrium strategies. Then, by Lemma 3 there exists a q_1 such that all $q < q_1$ buy and all $q > q_1$ do not. In the next round, if the monopolist set price equal to $f(1)$, then, by Lemma 1, all customers would buy, and his discounted revenue from round 1 onward would be $(1-q_1)f(1)$. Thus, when he optimizes, the monopolist's discounted revenue must be at least this amount. Define DR as the monopolist's total discounted revenue. It thus follows that

$$DR \geq q_1 p_0 + \delta_M(1-q_1)f(1). \tag{9}$$

As in Lemma 7, define q_1' as follows. Suppose that customers assume that the monopolist, instead of playing an equilibrium strategy after round zero, will set the next round's price equal to $f(1)$ and hence, by Lemma 1, would clear the market. Define q_1' as the infimum customer who prefers to wait instead of buying at p_0 in the initial round. This alternative strategy is maximally accommodating to the customers. That is, if he is optimizing, the monopolist would never set price below this value. This means that, under this alternative strategy, the number of customers who prefer to wait (instead of buying in the initial round) is at least as many as the customers who prefer to wait under the original strategy, which means that

$$q_1' \leq q_1. \tag{10}$$

Since $\delta_C < 1$, $p_0 > f(1)$. This means that customer $q = 1$ strictly prefers to wait and buy at $f(1)$, instead of buying at p_0 in the initial round. Likewise, it can be shown that customer $q = 0$ strictly prefers to buy at p_0 in the initial round instead of waiting until the next round to buy at $f(1)$. (If not, then $f(0) - p_0 \leq \delta_C[f(0) - f(1)]$, which implies $p_0 \geq (1-\delta_C)f(0) + \delta_C f(1)$. Substituting for p_0 gives $(1-\delta_C)f(\frac{1}{2}) + \delta_C f(1) \geq (1-\delta_C)f(0) + \delta_C f(1)$, which implies $f(\frac{1}{2}) \geq f(0)$, which contradicts our assumption that $f()$ is strictly decreasing.). This means that there exists a customer $q_1' \in (0,1)$ who is indifferent between buying at p_0 and waiting until the next round to buy at $f(1)$. This means that $f(q_1') - p_0 = \delta_C[f(q_1') - f(1)]$, which implies

$$f(q_1') = \frac{p_0 - \delta_C f(1)}{1 - \delta_C}.$$

Substituting $p_0 = (1-\delta_C)f(\frac{1}{2}) + \delta_C f(1)$ into the above, we get $f(q_1') = f(\frac{1}{2})$. Since $f()$ is strictly decreasing, this implies $q_1' = \frac{1}{2}$. This and (10) imply $\frac{1}{2} \leq q_1$. Note that the right-hand side of (9) is a weighted average of p_0 and $\delta_M f(1)$, where the weights are q_1 and $1-q_1$. Since $p_0 > \delta_M f(1)$, if we decrease the weight on p_0 (from q_1 to $\frac{1}{2}$, then we make the right-hand side of (9) smaller. Accordingly,

$$DR \geq \frac{1}{2}p_0 + \frac{1}{2}\delta_M f(1).$$

Substituting $p_0 = (1-\delta_C)f(\frac{1}{2}) + \delta_C f(1)$ into the above gives

$$DR \geq \frac{1}{2}[(1-\delta_C)f(\frac{1}{2}) + \delta_C f(1)] + \frac{1}{2}\delta_M f(1).$$

Since $\bar{\delta}_M < \delta_M$, the above implies

$$DR > \frac{1}{2}[(1-\delta_C)f(\frac{1}{2}) + \delta_C f(1)] + \frac{1}{2}\bar{\delta}_M f(1).$$

The above and (8) imply

$$DR > \frac{1}{2}[(1-\delta_C)f(\frac{1}{2}) + \delta_C f(1)] + \frac{1}{2}f(1) - \frac{1}{4}[f(\frac{1}{2}) - f(1)](1-\delta_C).$$

Note that $(1-\delta_C)f(\frac{1}{2}) + \delta_C f(1) = (1-\delta_C)f(\frac{1}{2}) - (1-\delta_C)f(1) + f(1) = (1-\delta_C)[f(\frac{1}{2}) - f(1)] + f(1)$. This and the above inequality imply

$$DR > \frac{1}{2}\{(1-\delta_C)[f(\frac{1}{2}) - f(1)] + f(1)\} + \frac{1}{2}f(1) - \frac{1}{4}[f(\frac{1}{2}) - f(1)](1-\delta_C),$$

which implies

$$DR > \frac{1}{4}(1-\delta_C)[f(\frac{1}{2}) - f(1)] + f(1).$$

By (7), the above implies

$$DR > f(1) + \varepsilon/(1-\delta_C).$$

■

7.2. Main Results

Proposition 1 (GSW): Let $\delta_C = \delta_M \equiv \delta$. Assume $\delta \in (0,1)$. Then for all $\varepsilon > 0$, there exist $\bar{\delta}$ such that, if $\delta > \bar{\delta}$, then in any equilibrium $p_0 < f(1) + \varepsilon$.

Proposition 2: There exists $\bar{\delta}_C$ such that for all $\delta_C \in (\bar{\delta}_C, 1)$, in any equilibrium, p_0 equals $f(1)$.

Proof: Let

$$\bar{\delta}_C = \max\{0, 1 - (1 - \delta_M)f(1)/k\}. \tag{11}$$

Assume $\delta_C \in (\bar{\delta}_C, 1)$. Let $\{p_i\}$ and $\{q_i\}$ be the realized prices and states in some equilibrium. By Lemma 6 there exists j such that $p_j = f(1)$. Define T as the minimum j such that this is true. Hence, $p_T = f(1)$, and by Lemma 6, $q_{T+1} = 1$.

Assume $T > 0$. (I show that this leads to a contradiction, thus showing that $T = 0$, which implies that $p_0 = p_T = f(1)$.)

Given this, it must be the case that $q_T \in (q_{T-1}, q_{T+1})$. (To see this, first note that, by the definition of q_i, $q_T \in [q_{T-1}, q_{T+1}]$. Next, if $q_T = q_{T+1}$, then $q_T = 1$, which contradicts our definition of T — that it is the smallest i such that $q_{i+1} = 1$. If $q_T = q_{T-1}$, then this means that no customers buy in period $T-1$. This means the monopolist could strictly increase his discounted revenue by setting price in that period equal to $f(1)$ [thus keeping total revenue the same but transferring all revenue from period T to period $T-1$].)

This implies that in period $T-1$ the monopolist's price p_{T-1} causes some, but not all, customers to buy. This and Lemma 3 mean that there exists a unique customer, q_T, who is indifferent between buying at price p_{T-1} in period $T-1$ and buying at price $f(1)$ in period T. This implies $f(q_T) - p_{T-1} = \delta_C[f(q_T) - f(1)]$, which implies

$$p_{T-1} = (1 - \delta_C)f(q_T) + \delta_C f(1).$$

Define DR_{T-1} as the monopolist's discounted revenue from period $T-1$ onward. It follows that $DR_{T-1} = (q_T - q_{T-1})p_{T-1} + \delta_M(1 - q_T)f(1)$. Given the above relation for p_{T-1}, this can be rewritten as

$$DR_{T-1} = (q_T - q_{T-1})[(1 - \delta_C)f(q_T) + \delta_C f(1)] + \delta_M(1 - q_T)f(1). \tag{12}$$

Now suppose that, instead of playing his equilibrium strategy, the monopolist in period $T-1$ sets price equal to $f(1)$. Then, by Lemma 1, all remaining customers would buy, and his discounted revenue would be $(1 - q_{T-1})f(1)$. I now show that the latter term is greater than DR_{T-1} (which implies a contradiction, thus implying that $T = 0$ and $p_0 = f(1)$).

To do this, first note that, since $-f'(q) < k \ \forall q \in [0, 1]$, $f(q_T) < f(1) + (1 - q_T)k$. Substituting this into (12), gives

$$DR_{T-1} < (q_T - q_{T-1})\{(1 - \delta_C)[f(1) + (1 - q_T)k] + \delta_C f(1)\} + \delta_M(1 - q_T)f(1).$$

$$= (q_T - q_{T-1})\{f(1) + (1 - \delta_C)(1 - q_T)k\} + \delta_M(1 - q_T)f(1).$$

By (10) and our assumption that $\delta_C > \bar{\delta}_C$, $(1-\delta_C)k < (1-\delta_M)f(1)$. The above and the latter fact imply

$$DR_{T-1} < (q_T - q_{T-1})\{f(1) + (1-\delta_M)f(1)(1-q_T)\} + \delta_M(1-q_T)f(1)$$

$$= (q_T - q_{T-1})f(1) + (q_T - q_{T-1})(1-\delta_M)(1-q_T)f(1) + \delta_M(1-q_T)f(1).$$

Since $q_T - q_{T-1} \leq 1$, we can re-write the second term in the above expression. This gives

$$DR_{T-1} < (q_T - q_{T-1})f(1) + (1-\delta_M)(1-q_T)f(1) + \delta_M(1-q_T)f(1)$$

$$= (q_T - q_{T-1})f(1) + (1-q_T)f(1)$$

$$= (1-q_{T-1})f(1).$$

Thus, the above implies that the monopolist is not optimizing under his supposed equilibrium strategy, which is a contradiction. It follows that $T = 0$. Recall that T was defined as the smallest i such that $p_i = f(1)$. Thus, $p_0 = f(1)$. ■

Proposition 3: There exists an $\varepsilon > 0$ and a $\bar{\delta}_M < 1$ such that for all $\delta_M \in (\bar{\delta}_M, 1)$, in any equilibrium $p_0 > f(1) + \varepsilon$.

Proof: Let ε and $\bar{\delta}_M$ equal the following:

$$\varepsilon = \frac{1}{4}[f(\frac{1}{2}) - f(1)](1-\delta_C)^2$$

$$\bar{\delta}_M = \max\{0, 1 - \frac{2\varepsilon}{f(1)(1-\delta_C)}\}.$$

Assume $\delta_M > \bar{\delta}_M$. To prove the proposition, assume not—that is, in an equilibrium the monopolist sets initial price, p_0, less than or equal to $f(1) + \varepsilon$. Note this implies $p_0 - f(1) \leq \varepsilon$. Define DR as the monopolist's expected discounted revenue. By Lemma 7,

$$DR \leq f(1) + \frac{\varepsilon}{1-\delta_C}. \tag{13}$$

Now suppose the monopolist chooses an alternative strategy. Namely, suppose in the initial round he sets price equal to $p_0' = (1-\delta_C)f(\frac{1}{2}) + \delta_C f(1)$. Suppose that in the remaining subgame the monopolist and the customers play equilibrium strategies. Then, as Lemma 8 shows, this strategy brings a discounted revenue greater than $f(1) + \varepsilon/(1-\delta_C)$. This and (13) imply that the original strategy cannot be an equilibrium. It follows that in any equilibrium, $p_0 > f(1) + \varepsilon$. ■

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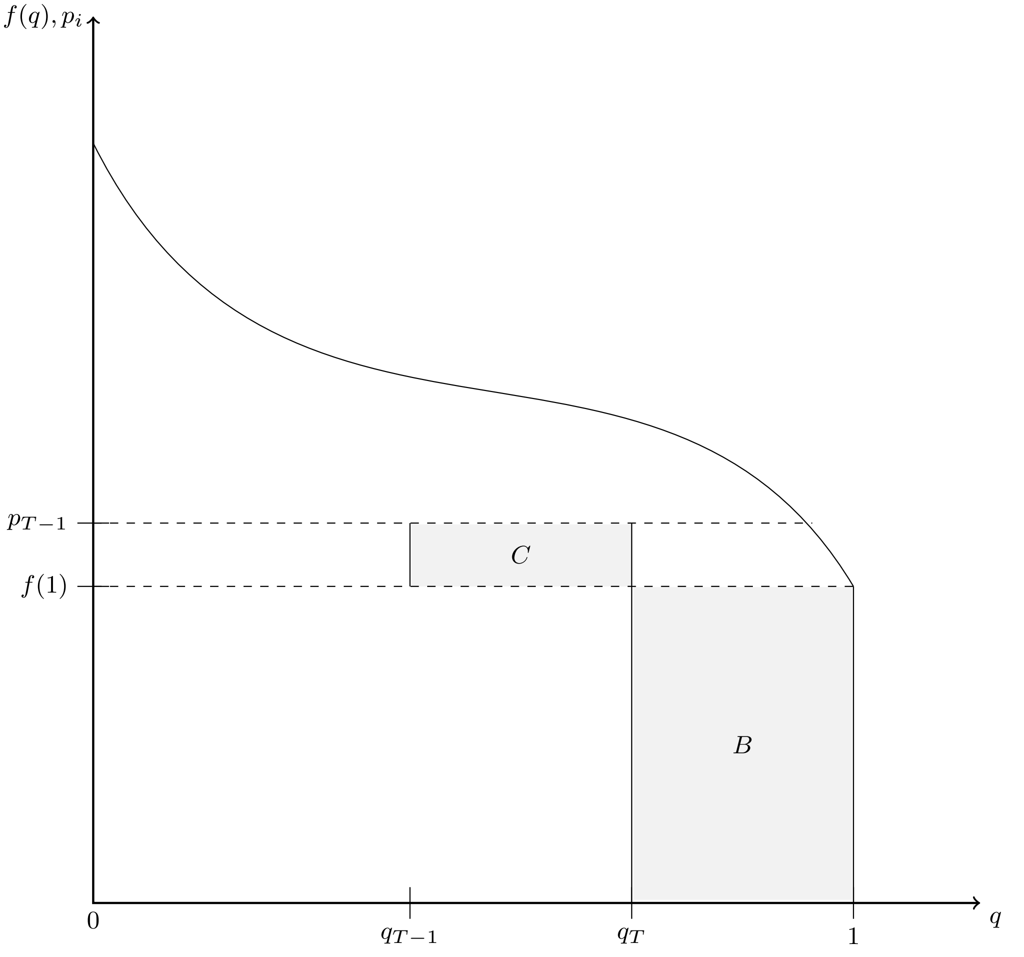
Figure 1. Illustration of Proposition 2



Key: In equilibrium the monopolist charges price p_{T-1} in period $T-1$ and price $f(1)$ in period T. Customers $q \in (q_{T-1}, q_T)$ buy in period $T-1$ and $q \in (q_T, 1)$ buy in period T. If the monopolist deviates by charging $f(1)$ in period $T-1$, then the loss in revenue from customers (q_{T-1}, q_T) is $[p_{T-1} - f(1)](q_T - q_{T-1})$ which is represented by area C. The benefits of the deviation are that the revenue he receives from customers $(q_T, 1)$ comes one period earlier. This revenue equals $(1 - q_T)f(1)$ and is represented by area B.

Figure 2. The Figure Coase Used to Explain his Conjecture