Vibration Control of Nonlinear Three-dimensional Marine Riser Model with Output Constraints

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Abstract: A typical three-dimensional flexible marine riser system described as a distributed parameter system with several partial differential equations and ordinary differential equations is taken into account in this article, we are aiming at limiting the displacement of riser within restricted range. Appropriate boundary control laws by integrating backstepping technique with barrier Lyapunov functions are put forwarded to suppress the vibration of flexible riser under the external disturbances. With the designed boundary control, Lyapunov synthesis method is used to prove the stability of the closed-loop system without any discretization or simplification of the dynamic in the time and space when the initial conditions meet the requirements. Furthermore, in order to illustrate the effectiveness of proposed control laws, numerical simulation studies are carried.

Keywords: Partial Differential Equations; Backstepping Technique; Flexible Marine Riser; Boundary Control; Three-dimensional Model

1. Introduction

The flexible structures are widely used in various fields such as manipulators [1-2], wings [3-4] and other engineering applications due to their light weight, low cost and high efficiency. Marine risers are modeled as typical flexible structures [5-7]. In the process of deep-water drilling operations, marine risers play an increasingly critical role, which is used to isolated the internal drilling fluid from the external seawater, establish drilling fluid circulation channels and transport oil and gas raw materials and other undersea economic resources to production lines. However, it is highly likely to cause vibration and deformation influenced by the surrounding disturbance, which can not only cause premature vulnerability problems, shorten its service life, but also require expensive maintenance [8-9].

Motivated by this, an increasingly number of scholars devoted themselves to the studies on vibration control of marine riser and many control techniques have been presented. Such as finite element method, modal analysis, boundary control and other control methods. The flexible marine riser system can be considered as an infinite dimension model, when the high-frequency parts of model are deleted, the system will appear "spillover" phenomenon, and boundary control can solve

the phenomenon well. Therefore boundary control is more suitable than other methods for infinite dimension model. In the existing corresponding works, the riser system is modeled as a Euler-Bernoulli beam because the ratio of its diameter to length is very small, and the dynamic behavior of it is described as a distributed parameter system composed of partial differential equations (PDEs) and ordinary differential equations (ODEs). For control problems of DPS [10]-[12], one of the inevitable challenges is how to use finite sensors to achieve the goal of controlling an infinite-dimensional model. This challenge has been overcome by boundary control, the designed boundary control can decrease the vibration of marine riser [13]-[15]. In [16], an adaptive feedback controller based on boundary control is proposed to stabilize the riser system, the high-gain observer and the robust strategy are used to handle the uncertainty of the system. Besides, combine boundary control with backstepping technique and adaptive observer to tackle with the uncertain parameter in system, the actuators can obtain better control performance which can be seen in [17]. Despite the researches on control of flexible riser have acquired desired results, they only focus on the transverse vibration in two-dimensional space. In fact, the marine riser produces more than just transverse vibration under the complex environmental interference. To reduce the lateral and longitudinal vibration of a coupled nonlinear flexible riser, two actuators are set on the top of the riser through designing proper boundary control in [18]. The barrier Lyapunov function (BLF) has been increasingly used in partial differential equations, although it was originally proposed to deal with ordinary differential equations and many remarkable results about BLF have been achieved. By combining backstepping and Lyapunov method, boundary control with observer and barrier Lyapunov function is introduced to solve the output constraint problem of flexible riser system [19]. In work [20], a controller with integral-barrier Lyapunov function and boundary control is designed to suppress the riser's vibration with a top tension constraint. To handle with top-level tension and input saturation constraint, boundary control with barrier term and auxiliary system is exploited in [21].

In order to make further study, the flexible riser system is studied in three-dimensional space. On the basis of integral-barrier Lyapunov function, the objective of reducing the vibration of riser under the external nonlinear disturbances has been accomplished, three actuators are equipped at the top of the riser and boundary control makes sure the joint angles in the limited range at [22]. Inspired by this work, the author proposed different control laws aimed at limiting the amplitude at the top of the riser within constrained range and suppressing the vibration of riser in three-dimensional space.

Boundary control integrated with backstepping technique and barrier Lyapunov function is employed. Compared with the existing works, the main contributions of this paper are summarized as follows:

(i) The flexible riser system under the external disturbance in three-dimensional space is described as a distributed parameters system which increases the accuracy of establishing model and the difficulties of the control designing greatly.

(ii) Based on the backstepping technique and barrier Lyapunov function, boundary control with barrier terms is proposed to guarantee the stability of the system and achieve the constraint of the top displacement of riser system.

(iii) By comparing the proposed control with the traditional PD control via numerical simulations, it can be concluded that regardless of whether there is distributed interference or not, the performance of proposed control is better than that of PD control.

2. Dynamics and preliminaries

A typical marine riser in three-dimensional is shown in **Figure 1**. The controllers are implemented from three actuators installed at the top boundary of the riser. We ignore the effect of gravity and make s, t be independent spatial and time variables. For clarity, the notations, $(\dot{*}) = \frac{\partial(*)}{\partial t}$,

$(\ddot{*}) = \frac{\partial^2(*)}{\partial t^2}$, $(*)' = \frac{\partial(*)}{\partial s}$, $(*)'' = \frac{\partial^2(*)}{\partial s^2}$, $(*)''' = \frac{\partial^3(*)}{\partial s^3}$, $(*)'''' = \frac{\partial^4(*)}{\partial s^4}$ are introduced throughout the whole paper.

Figure 1. A typical three-dimensional marine riser

2.1 Dynamic Analysis

The kinetic energy of the system E_k can be represented as:

$$\begin{aligned} E_k = & \frac{1}{2}M[(\dot{x}(L,t))^2 + \dot{y}(L,t)^2 + \dot{z}(L,t)^2] + \frac{1}{2}\rho \int_0^L \dot{x}(s,t)^2 ds \\ & + \frac{1}{2}\rho \int_0^L \dot{y}(s,t)^2 ds + \frac{1}{2}\rho \int_0^L \dot{z}(s,t)^2 ds + \rho \int_0^L \dot{z}(s,t)\dot{x}(s,t) ds \\ & + \rho \int_0^L \dot{z}(s,t)\dot{y}(L,t) ds + \rho \int_0^L \dot{x}(s,t)\dot{y}(L,t) ds \end{aligned} \quad (1)$$

Where the M denotes the mass of the vessel, $x(L,t), y(L,t), z(L,t)$ and $\dot{x}(L,t), \dot{y}(L,t), \dot{z}(L,t)$ are the position and velocity of the vessel. $x(s,t), y(s,t), z(s,t)$ are the displacement of the riser at position, s for time t . $\rho > 0$ is the uniform mass per unit length of the riser, and L is the length of the riser. μ is a positive constant.

The potential energy of the system E_p can be represented as:

$$\begin{aligned} E_p = & \frac{1}{2}T \int_0^L [x'(s,t)^2 + y'(s,t)^2] ds + \frac{1}{2}EI \int_0^L [x''(s,t)^2 + y''(s,t)^2] ds \\ & + \frac{1}{2}EA \int_0^L [z'(s,t) + \frac{1}{2}(y'(s,t))^2 + \frac{1}{2}(x'(s,t))^2] ds \end{aligned} \quad (2)$$

Where EI is the bending stiffness of the riser and T is the top tension of the riser. EA is the axial stiffness of the riser.

The virtual work done by distributed ocean current disturbance f_x, f_y, f_z on the riser and the boundary disturbance d_x, d_y, d_z on the tip payload is given as:

$$\delta W_f(t) = \int_0^L (f_x \delta x + f_y \delta y + f_z \delta z) ds + d_x \delta x(L,t) + d_y \delta y(L,t) + d_z \delta z(L,t) \quad (3)$$

The virtual work done by boundary control force U_x, U_y, U_z to suppress the vibration and prevent the top amplitude from the constraint can be represented as:

$$\delta W_m(t) = U_x \delta x(L,t) + U_y \delta y(L,t) + U_z \delta z(L,t) \quad (4)$$

Therefore, the total virtual work $\delta W(t)$ done on the system can be described as:

$$\delta W(t) = \delta W_f(t) + \delta W_m(t) \quad (5)$$

Based on the Hamilton's principle $\int_{t_1}^{t_2} \delta[E_k - E_p + W] dt = 0$, we can obtain the following governing equations:

$$\rho\ddot{x} = Tx'' + EA(z''x' + x''z') + \frac{3}{2}EA(x')^2x'' + \frac{1}{2}EA[x''(y')^2 + 2x'y'y''] - EIx''' + f_x \quad (6)$$

$$\rho\ddot{y} = Ty'' + EA(z''y' + y''z') + \frac{3}{2}EA(y')^2y'' + \frac{1}{2}EA[y''(x')^2 + 2y'x'x''] - EIy''' + f_y \quad (7)$$

$$\rho\ddot{z} = EA[z'' + x'x'' + y'y''] + f_z \quad (8)$$

$\forall (s, t) \in [0, L] \times [0, \infty)$, and the boundary conditions are as follows:

$$x(0, t) = y(0, t) = z(0, t) \quad (9)$$

$$x''(0, t) = y''(0, t) = z''(0, t) \quad (10)$$

$$x''(L, t) = y''(L, t) = z''(L, t) \quad (11)$$

$$\begin{aligned} M\ddot{x}(L, t) = & U_x + d_x - Tx'(L, t) - \frac{1}{2}EA(x'(L, t))^3 - EAx'(L, t)z'(L, t) \\ & - \frac{1}{2}EAx'(L, t)(y'(L, t))^2 + EIx'''(L, t) \end{aligned} \quad (12)$$

$$\begin{aligned} M\ddot{y}(L, t) = & U_y + d_y - Ty'(L, t) - \frac{1}{2}EA(y'(L, t))^3 - EAy'(L, t)z'(L, t) \\ & - \frac{1}{2}EAy'(L, t)(x'(L, t))^2 + EIy'''(L, t) \end{aligned} \quad (13)$$

$$\begin{aligned} M\ddot{z}(L, t) = & U_z + d_z - EAz'(L, t) - \frac{1}{2}EA(y'(L, t))^2 \\ & - \frac{1}{2}EA(x'(L, t))^2 \end{aligned} \quad (14)$$

2.2 Preliminaries

Definition 1 [25]: A barrier Lyapunov function (BLF) which is a scalar function $V_\varsigma(x)$ define with respect to the system $\dot{x} = f(x)$ on an open region D containing the origin, which is continuous , positive definite, has continuous first-order partial derivatives at every point of D ,has the property $V(x) \rightarrow \infty$ as x approaches the boundary of D and satisfies $V(x(t)) \leq b, \forall t \geq 0$ along the solutions of $\dot{x} = f(x)$ for $x(0) \in D$ and a positive constant b .

Lemma 1[26]: Let $\varphi_1(s, t), \varphi_2(s, t) \in R$,the following inequalities hold:

$$\varphi_1\varphi_2 \leq |\varphi_1\varphi_2| \leq \frac{1}{\sigma}\varphi_1^2 + \sigma\varphi_2^2, \quad \forall \varphi_1, \varphi_2 \in R, \quad \sigma > 0 \quad (15)$$

Lemma 2 [30]: Let $\varphi(s, t) \in R$ be a function defined on $x \in [0, L]$ and $t \in [0, \infty)$ that satisfies the boundary condition $\varphi(0, t) = 0, \forall t \in [0, \infty)$, then the following inequalities hold:

$$\varphi^2 \leq L \int_0^L [\varphi']^2 ds \quad (16)$$

Remark 1: The control goal is to suppress the vibration $x(L, t), y(L, t), z(L, t)$ in three-dimensional space and guarantee $|x(L, t)| < C_1, |y(L, t)| < C_2, |z(L, t)| < C_3$, under the environmental disturbances. Where C_1, C_2, C_3 represent the constraints of three directions.

Lemma 3 [25]: Let $Z_1 := \{x_1, y_1, z_1 \in R : |x_1| < C_1, |y_1| < C_2, |z_1| < C_3\} \subset R$ and $N := R^l \times Z_1 \subset R^{l+1}$ be open sets. Consider the system $\dot{\eta} = h(t, \eta)$, where $\eta = [w, x_1, y_1, z_1]^T \in N$ and $h : R_+ \times N \rightarrow R^{l+1}$ is piecewise continuous in t and locally Lipschitz in x_1, y_1, z_1 , uniformly in t , on $R^l \times N$. Suppose that there exist functions $H : R^l \rightarrow R_+$ and $V_c : Z_1 \rightarrow R_+$, continuously differentiable and positive definite in their respective domains, such that:

$$V_c(x_1, y_1, z_1) \rightarrow \infty \text{ as } |x_1| \rightarrow C_1 \text{ or } |y_1| \rightarrow C_2 \text{ or } |z_1| \rightarrow C_3 \quad (17)$$

$$\theta_1(\|w\|) \leq H(w) \leq \theta_2(\|w\|) \quad (18)$$

Where θ_1, θ_2 are class K_∞ functions. Let $V_D(\eta) = H(w) + V_c(w, x_1, y_1, z_1)$ and $x_1(0), y_1(0), z_1(0)$ belong to the set $|x_1| < C_1, |y_1| < C_2, |z_1| < C_3$. If the inequality holds:

$$\dot{V}_D = \frac{\partial V_D}{\partial \eta} h \leq 0 \quad (19)$$

Then x_1, y_1, z_1 remains in the open set $|x_1| < C_1, |y_1| < C_2, |z_1| < C_3$.

Remark 2: For conciseness, $x = x(s, t), y = y(s, t), z = z(s, t), f_i = f_i(s, t), i = x, y, z$, in the rest of paper.

Property 1 [27]: If kinetic energy given by equation (1) of the system (6)-(14) is bounded, then the functions $\dot{x}', \dot{y}', \dot{z}', \ddot{x}', \ddot{y}', \ddot{z}'$ are bounded. $\forall (s, t) \in [0, L] \times [0, \infty)$.

Property 2 [27]: If potential energy given by equation (2) of the system (6)-(14) is bounded, then the

functions $x'', y'', z'', x''', y''', z'''$ are bounded. $\forall (s, t) \in [0, L] \times [0, \infty)$.

Assumption 1 [22]: For the known distributed ocean disturbances f_x, f_y, f_z and the unknown boundary disturbances d_x, d_y, d_z , there exist constants $F_x, F_y, F_z, D_x, D_y, D_z$ such that $|f_x| \leq F_x, |f_y| \leq F_y, |f_z| \leq F_z, \forall (s, t) \in [0, L] \times [0, \infty)$ and $|d_x(t)| \leq D_x, |d_y(t)| \leq D_y, |d_z(t)| \leq D_z, \forall [s, t) \in [0, \infty)$, these are reasonable assumptions, because the disturbances have finite and bounded energy. Furthermore, the knowledge of exact values of the disturbances are not required.

3. Control design

3.1 Boundary Control Design

The control objective is to suppress the vibration of the riser and stabilize the riser system at constrained range in three-dimensional space, even though the presence of distributed disturbance and boundary disturbance. The control inputs U_x, U_y, U_z are proposed to stabilize the closed-loop system and make the $x(L, t), y(L, t), z(L, t)$ stay in the limited range.

For the given riser system, we propose the following control laws :

$$U_x = -d_x + \phi_x - \gamma_x e_1 - \frac{(\phi_x + \mu \rho L x_v + x_1)}{\Delta_1} - \frac{M e_1 x_1}{\Delta_1 (C_1^2 - x_1^2)} - \frac{r_1 e_1}{\Delta_1} - M \dot{x}_v \quad (20)$$

$$U_y = -d_y + \phi_y - \gamma_y e_2 - \frac{(\phi_y + \mu \rho L y_v + y_1)}{\Delta_2} - \frac{M e_2 y_1}{\Delta_2 (C_2^2 - y_1^2)} - \frac{r_2 e_2}{\Delta_2} - M \dot{y}_v \quad (21)$$

$$U_z = -d_z + \phi_z - \gamma_z e_3 - \frac{(\phi_z + \mu \rho L z_v + z_1)}{\Delta_3} - \frac{M e_3 z_1}{\Delta_3 (C_3^2 - z_1^2)} - \frac{r_3 e_3}{\Delta_3} - M \dot{z}_v \quad (22)$$

Where $\mu, \gamma_x, \gamma_y, \gamma_z, r_1, r_2, r_3$, are positive constants and ϕ_x, ϕ_y, ϕ_z are defined as:

$$\begin{aligned} \phi_x &= T x'(L, t) + \frac{1}{2} E A (x'(L, t))^3 + E A x'(L, t) z'(L, t) \\ &+ \frac{1}{2} E A x'(L, t) (y'(L, t))^2 - E I x'''(L, t) \end{aligned} \quad (23)$$

$$\begin{aligned} \phi_y &= T y'(L, t) + \frac{1}{2} E A (y'(L, t))^3 + E A y'(L, t) z'(L, t) \\ &+ \frac{1}{2} E A y'(L, t) (x'(L, t))^2 - E I y'''(L, t) \end{aligned} \quad (24)$$

$$\phi_z = EAz'(L,t) + \frac{1}{2}EA(x'(L,t))^2 + \frac{1}{2}EA(y'(L,t))^2 \quad (25)$$

To use the backstepping technique conveniently, the PDE dynamics (6)-(14) of considered riser system can be transformed as:

$$\left\{ \begin{array}{l} \rho \ddot{x} = Tx'' + EA(z''x' + x''z') + \frac{3}{2}EA(x')^2 x'' + \frac{1}{2}EA[x''(y')^2 + 2x'y'y''] - EIx'''' + f_x \\ \rho \ddot{y} = Ty'' + EA(z''y' + y''z') + \frac{3}{2}EA(y')^2 y'' + \frac{1}{2}EA[y''(x')^2 + 2y'x'x''] - EIy'''' + f_y \\ \rho \ddot{z} = EA[z'' + x'x'' + y'y''] + f_z \\ x(0,t) = y(0,t) = z(0,t) = 0 \\ x''(0,t) = y''(0,t) = z''(0,t) = 0 \\ x''(L,t) = y''(L,t) = z''(L,t) = 0 \\ x_1 = x(L,t), \dot{x}_1 = x_2 \\ y_1 = y(L,t), \dot{y}_1 = y_2 \\ z_1 = z(L,t), \dot{z}_1 = z_2 \\ \dot{x}_2 = \frac{1}{M}(U_x + d_x - \phi_x) \\ \dot{y}_2 = \frac{1}{M}(U_y + d_y - \phi_y) \\ \dot{z}_2 = \frac{1}{M}(U_z + d_z - \phi_z) \end{array} \right. \quad (26)$$

Step one :

First, we use x_v, y_v, z_v to express the virtual control of x_2, y_2, z_2 and define corresponding errors

e_1, e_2, e_3 as:

$$e_1 = x_2 - x_v \quad (27)$$

$$e_2 = y_2 - y_v \quad (28)$$

$$e_3 = z_2 - z_v \quad (29)$$

Consider the Lyapunov function candidate as:

$$V_a = V_1 + V_2 + V_3 \quad (30)$$

Where

$$V_1 = \frac{1}{2} \rho \int_0^L [(\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2] ds + \frac{1}{2} EI \int_0^L [(x'')^2 + (y'')^2] ds + \frac{1}{2} T \int_0^L [(x')^2 + (y')^2] ds + \frac{1}{2} EA \int_0^L [(z' + (\frac{1}{2} y')^2 + (\frac{1}{2} x')^2)]^2 ds \quad (31)$$

$$V_2 = \mu \rho \int_0^L s(\dot{x}x' + \dot{y}y' + \dot{z}z') ds \quad (32)$$

$$V_3 = \frac{1}{2} (x_1)^2 + \frac{1}{2} (y_1)^2 + \frac{1}{2} (z_1)^2 \quad (33)$$

Where V_1 is energy term, which is designed on account of the kinetic and potential energies of riser system. The small crossing term V_2 is designed to promote stability analysis. V_3 is associated with the state of the system.

Lemma 4: For the Lyapunov function candidate V_a , it is a positive definite function and there exist constants η_1 and $\eta_2 \in R^+$, such that the following inequality holds:

$$0 < \eta_1 (V_1 + V_3) \leq V_a \leq \eta_2 (V_1 + V_3) \quad (34)$$

Proof. See Appendix A.

Differentiating (30) and combining the riser model in three-dimension (26) with (27)-(29), (31)-(33), we derive:

$$\begin{aligned} \dot{V}_a = & \phi_x(e_1 + x_v) + \phi_y(e_2 + y_v) + \phi_z(e_3 + z_v) + \frac{\mu \rho L}{2} (e_1 + x_v)^2 \\ & + \frac{\mu \rho L}{2} (e_2 + y_v)^2 + \frac{\mu \rho L}{2} (e_3 + z_v)^2 + x_1(e_1 + x_v) + y_1(e_2 + y_v) \\ & + z_1(e_3 + z_v) + \mu L \phi_x x'(L, t) + \mu L \phi_y y'(L, t) + \mu L \phi_z z'(L, t) \\ & + \int_0^L f_x \dot{x} ds + \int_0^L f_y \dot{y} ds + \int_0^L f_z \dot{z} ds - \frac{\mu T L}{2} (x'(L, t))^2 - \frac{\mu T L}{2} (y'(L, t))^2 \\ & - \frac{\mu \rho}{2} \int_0^L (\dot{x})^2 ds - \frac{\mu \rho}{2} \int_0^L (\dot{y})^2 ds - \frac{\mu \rho}{2} \int_0^L (\dot{z})^2 ds \\ & - \frac{\mu E A L}{2} \left[\frac{1}{2} (x'(L, t))^2 + \frac{1}{2} (y'(L, t))^2 + z'(L, t) \right]^2 \\ & - \frac{\mu T}{2} \int_0^L (x')^2 ds - \frac{\mu T}{2} \int_0^L (y')^2 ds - \frac{\mu E A}{2} \int_0^L (z')^2 ds - \frac{3 \mu E A}{8} \int_0^L (x')^4 ds \\ & - \frac{3 \mu E A}{8} \int_0^L (y')^4 ds - \frac{3 \mu E A}{4} \int_0^L (x'y')^2 ds - \frac{\mu E I}{2} \int_0^L (x'')^2 ds - \frac{\mu E I}{2} \int_0^L (y'')^2 ds \\ & - \mu E A \int_0^L (x')^2 z' ds - \mu E A \int_0^L (y')^2 z' ds + \mu \int_0^L s(f_x x' + f_y y' + f_z z') ds \end{aligned} \quad (35)$$

From the (35), the virtual controls x_v, y_v, z_v can be designed as:

$$x_v = -\xi_1 \phi_x - k_1 x_1 \quad (36)$$

$$y_v = -\xi_2 \phi_y - k_2 y_1 \quad (37)$$

$$z_v = -\xi_3 \phi_z - k_3 z_1 \quad (38)$$

Where $\xi_i, i=1,2,3$ and $k_i, i=1,2,3$ are positive constants.

Take the inequality (15) and (36)-(38) into (35), we can derive:

$$\begin{aligned} \dot{V}_a \leq & e_1(\phi_x + \mu\rho Lx_v + x_1) + e_2(\phi_y + \mu\rho Ly_v + y_1) + e_3(\phi_z + \mu\rho Lz_v + z_1) \\ & + \frac{e_1^2}{2} + \frac{e_2^2}{2} + \frac{e_3^2}{2} - [\xi_1 - \frac{\mu\rho L\xi_1^2}{2} - \frac{(\mu\rho Lk_1\xi_1 - k_1 - \xi_1)}{\lambda_1} - \frac{1}{\delta_1}\mu L]\phi_x^2 \\ & - [\xi_2 - \frac{\mu\rho L\xi_2^2}{2} - \frac{(\mu\rho Lk_2\xi_2 - k_2 - \xi_2)}{\lambda_2} - \frac{1}{\delta_2}\mu L]\phi_y^2 - [\xi_3 - \frac{\mu\rho L\xi_3^2}{2} \\ & - \frac{(\mu\rho Lk_3\xi_3 - k_3 - \xi_3)}{\lambda_3} - \frac{1}{\delta_3}\mu L]\phi_z^2 - [k_1 - \frac{\mu\rho Lk_1^2}{2} - \lambda_1(\mu\rho Lk_1\xi_1 - k_1 - \xi_1)]x_1^2 \\ & - [k_2 - \frac{\mu\rho Lk_2^2}{2} - \lambda_2(\mu\rho Lk_2\xi_2 - k_2 - \xi_2)]y_1^2 - [k_3 - \frac{\mu\rho Lk_3^2}{2} \\ & - \lambda_3(\mu\rho Lk_3\xi_3 - k_3 - \xi_3)]z_1^2 - (\frac{\mu TL}{2} - \delta_1\mu L)[x'(L,t)]^2 \\ & - (\frac{\mu TL}{2} - \delta_2\mu L)[y'(L,t)]^2 - [\frac{\mu EAL}{2} - \delta_3\mu L - \alpha_1\mu EAL][z'(L,t)]^2 \\ & - (\frac{\mu EAL}{2} - \frac{\mu EAL}{\alpha_1})\left[\frac{1}{2}(x'(L,t))^2 + \frac{1}{2}(y'(L,t))^2\right]^2 - (\frac{\mu\rho}{2} - \frac{1}{v_1})\int_0^L (\dot{x})^2 ds \\ & - (\frac{\mu\rho}{2} - \frac{1}{v_2})\int_0^L (\dot{y})^2 ds - (\frac{\mu\rho}{2} - \frac{1}{v_3})\int_0^L (\dot{z})^2 ds - (\frac{\mu T}{2} - \frac{\mu L}{\beta_1})\int_0^L (x')^2 ds \\ & - (\frac{\mu T}{2} - \frac{\mu L}{\beta_2})\int_0^L (y')^2 ds - (\frac{\mu EA}{2} - \frac{\mu L}{\beta_3} - \frac{\mu EA}{\alpha_2} - \frac{\mu EA}{\alpha_3})\int_0^L (z')^2 ds \\ & - (\frac{3\mu EA}{8} - \alpha_2\mu EA)\int_0^L (x')^4 ds - (\frac{3\mu EA}{8} - \alpha_3\mu EA)\int_0^L (y')^4 ds \\ & - \frac{3\mu EA}{4}\int_0^L (x'y')^2 ds - \frac{\mu EI}{2}\int_0^L (x'')^2 ds - \frac{\mu EI}{2}\int_0^L (y'')^2 ds \\ & + \int_0^L (\mu L\beta_1 + v_1)(f_x)^2 ds + \int_0^L (\mu L\beta_2 + v_2)(f_y)^2 ds \\ & + \int_0^L (\mu L\beta_3 + v_3)(f_z)^2 ds \end{aligned} \quad (39)$$

Where $\alpha_i, \beta_i, \delta_i, v_i, i=1,2,3$ are the proper parameters and are positive constants.

Step two:

In this step, the controllers are designed to regulate the errors e_1, e_2, e_3 in a small neighborhood of

the origin. Combining (26) and (27)-(29) with (36)-(38), the time derivative of e_1, e_2, e_3 are:

$$\dot{e}_1 = \frac{1}{M}(U_x + d_x - \phi_x) - \dot{x}_v \quad (40)$$

$$\dot{e}_2 = \frac{1}{M}(U_y + d_y - \phi_y) - \dot{y}_v \quad (41)$$

$$\dot{e}_3 = \frac{1}{M}(U_z + d_z - \phi_z) - \dot{z}_v \quad (42)$$

Choose the Lyapunov function candidate V_b as:

$$V_b(t) = V_a + V_4 \quad (43)$$

Where V_4 represented:

$$V_4 = \frac{Me_1^2}{2} \ln \frac{2C_1^2}{C_1^2 - x_1^2} + \frac{Me_2^2}{2} \ln \frac{2C_2^2}{C_2^2 - y_1^2} + \frac{Me_3^2}{2} \ln \frac{2C_3^2}{C_3^2 - z_1^2} \quad (44)$$

Differentiating (43) and (44):

$$\dot{V}_b(t) = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 \quad (45)$$

$$\begin{aligned} \dot{V}_4 = & Me_1 \dot{e}_1 \Delta_1 + Me_1^2 \frac{x_1 \dot{x}_1}{C_1^2 - x_1^2} + Me_2 \dot{e}_2 \Delta_2 + Me_2^2 \frac{y_1 \dot{y}_1}{C_2^2 - y_1^2} \\ & + Me_3 \dot{e}_3 \Delta_3 + Me_3^2 \frac{z_1 \dot{z}_1}{C_3^2 - z_1^2} \end{aligned} \quad (46)$$

$$\text{Where } \Delta_1 = \ln \frac{2C_1^2}{C_1^2 - x_1^2}, \Delta_2 = \ln \frac{2C_2^2}{C_2^2 - y_1^2}, \Delta_3 = \ln \frac{2C_3^2}{C_3^2 - z_1^2}.$$

According (34) and (43), we can derive:

$$\mathcal{G}_1[V_1 + V_3 + V_4] \leq \dot{V}_b(t) \leq \mathcal{G}_2[V_1 + V_3 + V_4] \quad (47)$$

Where $\mathcal{G}_1 = \min(\eta_1, 1), \mathcal{G}_2 = \max(\eta_2, 1)$.

Lemma 5: The time derivative of function V_b is upper bounded with

$$\dot{V}_b \leq -\varpi V_b(t) + \tau \quad (48)$$

Where $\varpi, \tau > 0$.

proof: See Appendix B.

3. 2 Stability Analysis.

Theorem 1: For the considered riser system (6)-(14), under the proposed controls (20)-(22) with bounded

initial conditions and Assumption 1, when we choose the suitable control parameters $\gamma_x, \gamma_y, \gamma_z, r_1, r_2, r_3$, we can conclude that system is uniformly ultimately bounded. The states of the system $x(s, t), y(s, t), z(s, t)$ will remain in the compact set Ω_1 which is defined as:

$$\Omega_1 := \{x(s, t), y(s, t), z(s, t) \in R \mid |x(s, t)|, |y(s, t)|, |z(s, t)| \leq D_1 \forall (s, t) \in [0, L] \times [0, \infty)\} \quad (49)$$

Where the constant $D_1 = \sqrt{\frac{L}{\zeta} V_b(0) + \frac{\tau}{\varpi}}$. the states of the system will eventually converge to the

compact Ω_2 defined by:

$$\Omega_2 := \{x(s, t), y(s, t), z(s, t) \in R \mid |x(s, t)|, |y(s, t)|, |z(s, t)| \forall t \in [0, \infty)\} \quad (50)$$

Where the constant $D_2 = \sqrt{\frac{L\tau}{\zeta\varpi}}$.

Proof: Multiplying (48) by $e^{\varpi t}$ yields:

$$\frac{\partial(V_b e^{\varpi t})}{\partial t} \leq \tau e^{\varpi t} \quad (51)$$

Integration of the above inequality and multiply by $e^{-\varpi t}$, we obtain:

$$V_b(t) \leq V(0)e^{-\varpi t} + \frac{\tau}{\varpi} \in L_\infty \quad (52)$$

which implies $V_b(t)$ is bounded. Utilizing inequality (16), (34) and (47), we obtain

$$\frac{1}{L} [x(s, t)]^2 \leq \int_0^L [x'(s, t)]^2 ds \leq \eta_2 V_1 \leq \frac{1}{\zeta} V_b(t) \in L_\infty \quad (53)$$

$$\frac{1}{L} [y(s, t)]^2 \leq \int_0^L [y'(s, t)]^2 ds \leq \eta_2 V_1 \leq \frac{1}{\zeta} V_b(t) \in L_\infty \quad (54)$$

$$\frac{1}{L} [z(s, t)]^2 \leq \int_0^L [z'(s, t)]^2 ds \leq \eta_2 V_1 \leq \frac{1}{\zeta} V_b(t) \in L_\infty \quad (55)$$

Rearranging the terms of above three inequalities, we can obtain

$$\begin{aligned} |x(s, t)|, |y(s, t)|, |z(s, t)| &\leq \sqrt{\frac{L}{\zeta} [V_b(0)e^{-\varpi t} + \frac{\tau}{\varpi}]} \\ \forall (s, t) &\in [0, L] \times [0, \infty) \end{aligned} \quad (56)$$

Furthermore, from (56) we can obtain

$$\lim_{t \rightarrow \infty} |x(s, t)| \leq \sqrt{\frac{L\tau}{\zeta\varpi}}, \forall s \in [0, L] \quad (57)$$

$$\lim_{t \rightarrow \infty} |y(s, t)| \leq \sqrt{\frac{L\tau}{\zeta\varpi}}, \forall s \in [0, L] \quad (58)$$

$$\lim_{t \rightarrow \infty} |z(s, t)| \leq \sqrt{\frac{L\tau}{\zeta\varpi}}, \forall s \in [0, L] \quad (59)$$

Remark 3: From **Lemma 3** and (47). We know that V_a and V_b are bounded and positive. From the definition of barrier Lyapunov function, we know that $V_4(t) \rightarrow \infty$ as $|x_1| \rightarrow C_1, |y_1| \rightarrow C_2, |z_1| \rightarrow C_3$, Consequently, we know that $|x_1| \neq C_1, |y_1| \neq C_2, |z_1| \neq C_3$. Given that $x_1(L, 0) < C_1, y_1(L, 0) < C_2, z_1(L, 0) < C_3$, according to the properties of barrier Lyapunov function, we can further infer that x_1, y_1, z_1 remain in the sets $x_1 < C_1, y_1 < C_2, z_1 < C_3, \forall t \in [0, \infty)$.

Remark 4: According to the **property 1** and **property 2**, we can draw a conclusion that $\ddot{x}(s, t), \ddot{y}(s, t), \ddot{z}(s, t)$ are bounded, $\forall (s, t) \in [0, L] \times [0, \infty)$. And we can conclude that control inputs U_x, U_y, U_z are bounded $\forall (s, t) \in [0, L] \times [0, \infty)$.

4. Simulation

In this part, to verify the effectiveness and practicability of the proposed control laws (20)-(22), the finite difference method (FDM) [32]-[35] is chosen to simulate the performance of system. The corresponding initial conditions of the system are given as: $\dot{x}(s, 0) = \dot{y}(s, 0) = \dot{z}(s, 0) = 0, x(s, 0) = 0$, and $y(s, 0) = z(s, 0) = 0$. The length of riser L is $1000m$, the uniform mass per unit length of riser ρ is $108kg/m$, the mass of the vessel M is 9.6×10^6 , the bending stiffness of the riser EI is $1.22 \times 10^5 Nm^2$, the axial stiffness of the riser EA is $3.92 \times 10^8 N$, the top tension of the riser T is $1.11 \times 10^8 N$, the constraints C_1, C_2, C_3 , on x, y, z are $0.05m$.

The boundary disturbances are generated by the following equations:

$$d_x(t) = d_y(t) = 3 \times 10^5 + [0.8 \sin(0.7t) + 0.2 \sin(0.5t) + 0.2 \sin(0.9t)] \times 10^5 \quad (60)$$

$$d_z(t) = (3 + 0.2 \sin(0.5t)) \times 10^4 \quad (61)$$

According to (60), (61), we can know that the periods of boundary disturbance are $T_x = T_y = 20\pi$, $T_z = 4\pi$, and the distributed disturbance are as follows:

$$f(x, t) = \frac{1}{2} \rho_s C_D(x, t) U(x, t)^2 + A_D \cos(4\pi f_v t + \beta) \quad (62)$$

Reader can refer to [8] for the detailed parameters of distributed disturbance on the riser. In this paper, the distributed disturbance cause effects in all three directions, therefore, we assume that $f_x = f_y = f_z$, respectively, the length of simulation is 400s.

For validating the control performance of proposed control laws (20)-(22), we consider three cases:

(i): Without any control, the riser suffers boundary disturbances and distributed disturbance in the same time. the corresponding spatial time responses are depicted in **Figure2**. For convenience, (a) stands for $x(s, t)$, (b) stands for $y(s, t)$, (c) represents $z(s, t)$.

Figure 2. Displacement of the riser without any control.

(ii): When there exist no distributed disturbance, the responses of riser system with PD control (63)-(65) are shown in **Figure 3**. The top displacement of riser with PD control is shown in **figure 4**.

The performance of proposed control is shown in **Figure 5**. The top displacement of riser with proposed control is shown in **figure 6**. Where the PD control parameters are selected as:

$K_{p1} = K_{p2} = K_{p3} = 8 \times 10^4$, $K_{d1} = K_{d2} = K_{d3} = 10 \times 10^6$. By selecting the control parameters $\gamma_x = 0.01$, $\gamma_y = \gamma_z = 0.01$, $r_1 = r_2 = r_3 = 5.5 \times 10^5$, $k_1 = k_2 = k_3 = 0.05$, $\xi_1 = \xi_2 = \xi_3 = 1 \times 10^{-7}$

$$U_x = -K_{p1}x(L, t) - K_{d1}\dot{x}(L, t) \quad (63)$$

$$U_y = -K_{p2}y(L,t) - K_{d2}\dot{y}(L,t) \quad (64)$$

$$U_z = -K_{p3}z(L,t) - K_{d3}\dot{z}(L,t) \quad (65)$$

Figure 3. Displacement of the riser without distributed disturbance with PD control.

Figure 4. Top displacement of the riser without distributed disturbance with PD control.

Figure 5. Displacement of the riser without distributed disturbance with proposed control.

Figure 6. Top displacement of the riser without distributed disturbance with proposed control.

(iii): When there exist distributed disturbance, the responses of riser system with above PD control and parameters are shown in **Figure 7**. Top displacement of the riser with distributed disturbance under PD control is depicted in **Figure 8**. The performance of proposed control with above control parameters is depicted in **Figure 9**. Top displacement of the riser with distributed disturbance under proposed control is depicted in **Figure 10**.

Figure 7. Displacement of the riser with distributed disturbance under PD control.

(a) (b) (c)
Figure 8. Top displacement of the riser with distributed disturbance under PD control

(a) (b) (c)
Figure 9. Displacement of the riser with distributed disturbance under proposed control.

(a) (b) (c)
Figure 10. Top displacement of the riser with distributed disturbance under proposed control

Remark 5: Although distributed interference makes a certain negative effect on PD control and the proposed control, the proposed control can still control the top amplitudes of riser in smaller vibration range than PD control in all directions, which can be seen in **Figure 11**. In addition, we compare the performances of the proposed control with distributed interference and without distributed disturbance, the results are shown in **Figure 12**.

Figure 11. Comparison of PD control and the proposed control with distributed disturbance

Figure 12. Performance of the proposed control

In order to show the simulation results more intuitively, we list the following table:

Table 1. Comparison of simulation results

	Direction	Without control	With PD control	With proposed control
Without distributed disturbance	$x(L, t)_{\max}$	$5.14m$	$1.98m$	$5.80 \times 10^{-4}m$
	$y(L, t)_{\max}$	$5.14m$	$1.98m$	$5.80 \times 10^{-4}m$
	$z(L, t)_{\max}$	$0.19m$	$0.08m$	$5.76 \times 10^{-5}m$
With distributed disturbance	$x(L, t)_{\max}$	$5.22m$	$2.02m$	$4.65 \times 10^{-3}m$
	$y(L, t)_{\max}$	$5.22m$	$2.02m$	$4.65 \times 10^{-3}m$
	$z(L, t)_{\max}$	$0.31m$	$0.14m$	$7.75 \times 10^{-3}m$

Remark 6: From above table, we can find that it is easy to violate the requirements of the American Petroleum Institute which is that the average value of the upper and lower joint angle keep within 2° and the maximum non drilling angle should be limited to 4° [14] when there is no control. As we can know from above simulation results, both PD control and proposed control can reduce the amplitudes of vibration in three directions. Obviously, the performance of proposed control are far better than that of PD control. Whether there is distributed disturbance or not, the vibration of three directions will stay in a very small range with proposed control while PD control is only to reduce the amplitude of vibration a little. At the same time, we can know that the distributed disturbance decrease the control effects on riser.

5 .Conclusion

This work has discussed the problem of the vibration control of riser system in three-dimensional space. Based on the backstepping and barrier Lyapunov function, three control laws are proposed. With the proposed control, the vibration of riser system has been suppressed and achieved the constraint of the displacement without discretization and simplification of the dynamic model in time and space. The stability analysis of the closed-loop system has been proved by Lyapunov's synthetic method. While from the simulation results, we can know that the distributed disturbance can reduce the effects of vibration control. Therefore, how to overcome the distributed disturbance will be a meaningful study in the future.

Appendix A. proof of lemma 3

Proof. Applying the inequality (15) and equality (31) yields:

$$|V_2| \leq \mu \rho L \int_0^L [(\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2 + (x')^2 + (y')^2 + (z')^2] ds \leq o_1 V_1 \quad (\text{A.1})$$

Where $o_1 = \frac{2\mu\rho L}{\min(\rho, T, EA)}$. Then we can obtain:

$$-o_1 V_1 \leq V_2 \leq o_1 V_1 \quad (\text{A.2})$$

Consider μ is a small positive weighting constant satisfying $0 < \mu < \frac{\min(\rho, T, EA)}{2\rho L}$, we can obtain:

$$o_2 = 1 - o_1 = 1 - \frac{2\mu\rho L}{\min(\rho, T, EA)} > 0 \quad (\text{A.3})$$

$$o_3 = 1 + o_1 = 1 + \frac{2\mu\rho L}{\min(\rho, T, EA)} > 1 \quad (\text{A.4})$$

Then we further yield:

$$0 \leq o_2 V_1 \leq V_1 + V_2 \leq o_3 V_1 \quad (\text{A.5})$$

Combining with the Lyapunov function (30), we obtain:

$$0 < \eta_1 (V_1 + V_3) \leq V_a \leq \eta_2 (V_1 + V_3) \quad (\text{A.6})$$

Where $\eta_1 = \min(o_2, 1) = o_2$, $\eta_2 = \max(o_3, 1) = o_3$ are two positive constants.

Appendix B. proof of lemma 4

Proof. Differentiating the equality (43) and combining (40)-(42) leads to:

$$\begin{aligned}
\dot{V}_b = & \phi_x(e_1 + x_v) + \phi_y(e_2 + y_v) + \phi_z(e_3 + z_v) + \frac{\mu\rho L}{2}(e_1 + x_v)^2 + \frac{\mu\rho L}{2}(e_2 + y_v)^2 \\
& + \frac{\mu\rho L}{2}(e_3 + z_v)^2 + x_1(e_1 + x_v) + y_1(e_2 + y_v) + z_1(e_3 + z_v) + \mu L \phi_x x'(L, t) \\
& + \mu L \phi_y y'(L, t) + \mu L \phi_z z'(L, t) + M e_1 \dot{e}_1 \Delta_1 + M(e_1)^2 \frac{x_1(e_1 + x_v)}{C_1^2 - x_1^2} + M e_2 \dot{e}_2 \Delta_2 \\
& + M(e_2)^2 \frac{y_1(e_2 + y_v)}{C_2^2 - y_1^2} + M e_3 \dot{e}_3 \Delta_3 + M(e_3)^2 \frac{z_1(e_3 + z_v)}{C_3^2 - z_1^2} + \int_0^L f_x \dot{x} ds + \int_0^L f_y \dot{y} ds \\
& + \int_0^L f_z \dot{z} ds - \frac{\mu T L}{2} (x'(L, t))^2 - \frac{\mu T L}{2} (y'(L, t))^2 - \frac{\mu \rho}{2} \int_0^L (\dot{x})^2 ds - \frac{\mu \rho}{2} \int_0^L (\dot{y})^2 ds \quad (B1) \\
& - \frac{\mu \rho}{2} \int_0^L (\dot{z})^2 ds - \frac{\mu E A L}{2} \left[\frac{1}{2} (x'(L, t))^2 + \frac{1}{2} (y'(L, t))^2 + z'(L, t) \right]^2 - \frac{\mu T}{2} \int_0^L (x')^2 ds \\
& - \frac{\mu T}{2} \int_0^L (y')^2 ds - \frac{\mu E A}{2} \int_0^L (z')^2 ds - \frac{3\mu E A}{8} \int_0^L (x')^4 ds - \frac{3\mu E A}{8} \int_0^L (y')^4 ds \\
& - \frac{3\mu E A}{4} \int_0^L (x' y')^2 ds - \frac{\mu E I}{2} \int_0^L (x'')^2 ds - \frac{\mu E I}{2} \int_0^L (y'')^2 ds \\
& - \mu E A \int_0^L (x')^2 z' ds - \mu E A \int_0^L (y')^2 z' ds + \mu \int_0^L s (f_x x' + f_y y' + f_z z') ds
\end{aligned}$$

Combining (B.1), (40)-(42) and inequality (39) yields:

$$\begin{aligned}
\dot{V}_b \leq & e_1(\dot{\phi}_x + \mu\rho Lx_v + x_1) + e_2(\dot{\phi}_y + \mu\rho Ly_v + y_1) + e_3(\dot{\phi}_z + \mu\rho Lz_v + z_1) \\
& + \frac{e_1^2}{2} + \frac{e_2^2}{2} + \frac{e_3^2}{2} + e_1\Delta_1(U_x + d_x - \phi_x - M\dot{x}_v) + Me_1^2 \frac{x_1x_2}{C_1^2 - x_1^2} \\
& + e_2\Delta_2(U_y + d_y - \phi_y - M\dot{y}_v) + Me_2^2 \frac{y_1y_2}{C_2^2 - y_1^2} \\
& + e_3\Delta_3(U_z + d_z - \phi_z - M\dot{z}_v) + Me_3^2 \frac{z_1z_2}{C_3^2 - z_1^2} \\
& - [\xi_1 - \frac{\mu\rho L\xi_1^2}{2} - \frac{(\mu\rho Lk_1\xi_1 - k_1 - \xi_1)}{\lambda_1} - \frac{1}{\delta_1}\mu L]\phi_x^2 \\
& - [\xi_2 - \frac{\mu\rho L\xi_2^2}{2} - \frac{(\mu\rho Lk_2\xi_2 - k_2 - \xi_2)}{\lambda_2} - \frac{1}{\delta_2}\mu L]\phi_y^2 \\
& - [\xi_3 - \frac{\mu\rho L\xi_3^2}{2} - \frac{(\mu\rho Lk_3\xi_3 - k_3 - \xi_3)}{\lambda_3} - \frac{1}{\delta_3}\mu L]\phi_z^2 \\
& - [k_1 - \frac{\mu\rho Lk_1^2}{2} - \lambda_1(\mu\rho Lk_1\xi_1 - k_1 - \xi_1)]x_1^2 \\
& - [k_2 - \frac{\mu\rho Lk_2^2}{2} - \lambda_2(\mu\rho Lk_2\xi_2 - k_2 - \xi_2)]y_1^2 \\
& - [k_3 - \frac{\mu\rho Lk_3^2}{2} - \lambda_3(\mu\rho Lk_3\xi_3 - k_3 - \xi_3)]z_1^2 \\
& - (\frac{\mu TL}{2} - \delta_1\mu L)[x'(L, t)]^2 - (\frac{\mu TL}{2} - \delta_2\mu L)[y'(L, t)]^2 \\
& - [\frac{\mu EAL}{2} - \delta_3\mu L - \frac{\mu EAL}{\alpha_1}][z'(L, t)]^2 \\
& - (\frac{\mu EAL}{2} - \alpha_1\mu EAL)\left[\frac{1}{2}(x'(L, t))^2 + \frac{1}{2}(y'(L, t))^2\right]^2 \\
& - (\frac{\mu\rho}{2} - \frac{1}{\nu_1})\int_0^L (\dot{x})^2 ds - (\frac{\mu\rho}{2} - \frac{1}{\nu_2})\int_0^L (\dot{y})^2 ds - (\frac{\mu\rho}{2} - \frac{1}{\nu_3})\int_0^L (\dot{z})^2 ds \\
& - (\frac{\mu T}{2} - \frac{\mu L}{\beta_1})\int_0^L (x')^2 ds - (\frac{\mu T}{2} - \frac{\mu L}{\beta_2})\int_0^L (y')^2 ds \\
& - (\frac{\mu EA}{2} - \frac{\mu L}{\beta_3} - \frac{\mu EA}{\alpha_2} - \frac{\mu EA}{\alpha_3})\int_0^L (z')^2 ds \\
& - (\frac{3\mu EA}{8} - \alpha_2\mu EA)\int_0^L (x')^4 ds - (\frac{3\mu EA}{8} - \alpha_3\mu EA)\int_0^L (y')^4 ds \\
& - \frac{3\mu EA}{4}\int_0^L (x'y')^2 ds - \frac{\mu EI}{2}\int_0^L (x'')^2 ds - \frac{\mu EI}{2}\int_0^L (y'')^2 ds \\
& + \int_0^L (\mu L\beta_1 + \nu_1)(f_x)^2 ds + \int_0^L (\mu L\beta_2 + \nu_2)(f_y)^2 ds \\
& + \int_0^L (\mu L\beta_3 + \nu_3)(f_z)^2 ds
\end{aligned} \tag{B2}$$

Take (20)-(22) into (B2) yields:

$$\begin{aligned}
\dot{V}_b \leq & -\gamma_x e_1^2 \Delta_1 - \gamma_y e_2^2 \Delta_2 - \gamma_z e_3^2 \Delta_3 - (r_1 - \frac{1}{2}) e_1^2 - (r_2 - \frac{1}{2}) e_2^2 - (r_3 - \frac{1}{2}) e_3^2 \\
& - [\xi_1 - \frac{\mu \rho L \xi_1^2}{2} - \frac{(\mu \rho L k_1 \xi_1 - k_1 - \xi_1)}{\lambda_1} - \frac{1}{\delta_1} \mu L] \phi_x^2 \\
& - [\xi_2 - \frac{\mu \rho L \xi_2^2}{2} - \frac{(\mu \rho L k_2 \xi_2 - k_2 - \xi_2)}{\lambda_2} - \frac{1}{\delta_2} \mu L] \phi_y^2 \\
& - [\xi_3 - \frac{\mu \rho L \xi_3^2}{2} - \frac{(\mu \rho L k_3 \xi_3 - k_3 - \xi_3)}{\lambda_3} - \frac{1}{\delta_3} \mu L] \phi_z^2 \\
& - [k_1 - \frac{\mu \rho L k_1^2}{2} - \lambda_1 (\mu \rho L k_1 \xi_1 - k_1 - \xi_1)] x_1^2 \\
& - [k_2 - \frac{\mu \rho L k_2^2}{2} - \lambda_2 (\mu \rho L k_2 \xi_2 - k_2 - \xi_2)] y_1^2 \\
& - [k_3 - \frac{\mu \rho L k_3^2}{2} - \lambda_3 (\mu \rho L k_3 \xi_3 - k_3 - \xi_3)] z_1^2 \\
& - (\frac{\mu T L}{2} - \delta_1 \mu L) [x'(L, t)]^2 - (\frac{\mu T L}{2} - \delta_2 \mu L) [y'(L, t)]^2 \\
& - [\frac{\mu E A L}{2} - \delta_3 \mu L - \frac{\mu E A L}{\alpha_1}] [z'(L, t)]^2 \\
& - (\frac{\mu E A L}{2} - \alpha_1 \mu E A L) \left[\frac{1}{2} (x'(L, t))^2 + \frac{1}{2} (y'(L, t))^2 \right]^2 \\
& - (\frac{\mu \rho}{2} - \frac{1}{v_1}) \int_0^L (\dot{x})^2 ds - (\frac{\mu \rho}{2} - \frac{1}{v_2}) \int_0^L (\dot{y})^2 ds - (\frac{\mu \rho}{2} - \frac{1}{v_3}) \int_0^L (\dot{z})^2 ds \\
& - (\frac{\mu T}{2} - \frac{\mu L}{\beta_1}) \int_0^L (x')^2 ds - (\frac{\mu T}{2} - \frac{\mu L}{\beta_2}) \int_0^L (y')^2 ds \\
& - (\frac{\mu E A}{2} - \frac{\mu L}{\beta_3} - \frac{\mu E A}{\alpha_2} - \frac{\mu E A}{\alpha_3}) \int_0^L (z')^2 ds \\
& - (\frac{3 \mu E A}{8} - \alpha_2 \mu E A) \int_0^L (x')^4 ds - (\frac{3 \mu E A}{8} - \alpha_3 \mu E A) \int_0^L (y')^4 ds \\
& - \frac{3 \mu E A}{4} \int_0^L (x' y')^2 ds - \frac{\mu E I}{2} \int_0^L (x'')^2 ds - \frac{\mu E I}{2} \int_0^L (y'')^2 ds \\
& + \int_0^L (\mu L \beta_1 + v_1) (f_x)^2 ds + \int_0^L (\mu L \beta_2 + v_2) (f_y)^2 ds \\
& + \int_0^L (\mu L \beta_3 + v_3) (f_z)^2 ds \\
\leq & -\eta_3 [V_1 + V_3 + V_4] + \tau
\end{aligned} \tag{B3}$$

Where

$$\eta_3 = \min \left\{ \frac{2\gamma_x}{M}, \frac{2\gamma_y}{M}, \frac{2\gamma_z}{M}, \frac{\mu\rho}{2} - \frac{1}{\nu_1}, \frac{\mu\rho}{2} - \frac{1}{\nu_2}, \frac{\mu\rho}{2} - \frac{1}{\nu_3}, \frac{\mu T}{2} - \frac{\mu L}{\beta_1}, \frac{\mu T}{2} - \frac{\mu L}{\beta_2}, \right. \\ \left. \frac{\mu T}{2} - \frac{\mu L}{\beta_3} - \frac{\mu EA}{\alpha_2} - \frac{\mu EA}{\alpha_3}, \frac{\mu EI}{2} \right\} \quad (B4)$$

$$\tau = (\mu L \beta_1 + \nu_1) \int_0^L (f_x)^2 ds + (\mu L \beta_2 + \nu_2) \int_0^L (f_y)^2 ds \\ + (\mu L \beta_3 + \nu_3) \int_0^L (f_z)^2 ds \leq (\mu L \beta_1 + \nu_1) F_x + (\mu L \beta_2 + \nu_2) F_y \\ + (\mu L \beta_3 + \nu_3) F_z \quad (B5)$$

And the designed parameters are selected to satisfy the following conditions:

$$r_i - \frac{1}{2} > 0, i = 1, 2, 3 \quad (B6)$$

$$\xi_i - \frac{\mu\rho L \xi_i^2}{2} - \frac{(\mu\rho L k_i \xi_i - k_i - \xi_i)}{\lambda_i} - \frac{1}{\delta_i} > 0, i = 1, 2, 3 \quad (B7)$$

$$k_i - \frac{\mu\rho L k_i^2}{2} - \lambda_i (\mu\rho L k_i \xi_i - k_i - \xi_i) > 0, i = 1, 2, 3 \quad (B8)$$

$$\frac{\mu TL}{2} - \delta_i > 0, i = 1, 2 \quad (B9)$$

$$\frac{\mu EAL}{2} - \delta_3 \mu L - \frac{\mu EAL}{\alpha_1} > 0 \quad (B10)$$

$$\frac{\mu EAL}{2} - \alpha_1 \mu EAL > 0 \quad (B11)$$

Combining (B3) and (47), we have

$$\dot{V}_b(t) \leq -\varpi V_b(t) + \tau \quad (B12)$$

$$\text{Where } \varpi = \frac{\eta_3}{\mathcal{G}_2}.$$

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Figures

Figure 1

A typical three-dimensional marine riser

Figure 2

Displacement of the riser without any control.

Figure 3

Displacement of the riser without distributed disturbance with PD control.

Figure 4

Top displacement of the riser without distributed disturbance with PD control.

Figure 5

Displacement of the riser without distributed disturbance with proposed control.

Figure 6

Top displacement of the riser without distributed disturbance with proposed control.

Figure 7

Displacement of the riser with distributed disturbance under PD control.

Figure 8

Top displacement of the riser with distributed disturbance under PD control

Figure 9

Displacement of the riser with distributed disturbance under proposed control.

Figure 10

Top displacement of the riser with distributed disturbance under proposed control

Figure 11

Comparison of PD control and the proposed control with distributed disturbance

Figure 12

Performance of the proposed control