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The application of the strength reduction shortest path method to the stability analysis of shallow buried tunnel

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Abstract: In order to improve the theoretical analysis of surrounding rock stability of shallow buried tunnel. The strength reduction shortest path theory is applied to the stability analysis of shallow buried tunnel surrounding rock, combined with the ultimate equilibrium strength reduction theory. We discussed the influence of the depth and span ratio of tunnel, cohesion, and internal friction angle on the shortest path of the strength reduction, and studied the effects of various factors on shallow buried tunnel safety relationship by using strength reduction factor of safety and the shortest path of the shallow buried tunnel surrounding rock and the grey relational analysis theory. The results show that: In the analysis of shallow buried tunnel in strength reduction, the approximate distribution obeys parabolic between reduction path length and the reduction ratio. When the strength reduction of cohesion is the shortest path the reduction rate is greater than the internal friction angle. The internal friction angle and cohesive force have a great influence on the stability of shallow tunnel under the method of shortest path of strength reduction. Finally, the comprehensive safety factor of shallow buried tunnel calculated by the finite element strength reduction shortest path method is greater than that calculated by the limit equilibrium method.

Key words: shortest path of strength reduction; limit equilibrium method; shallow tunnel; safety factor

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0 Introduction

The stability analysis of a shallow buried tunnel is an important theoretical basis to guide practical engineering. According to relevant statistics, some analysis methods proposed at present mainly include the limit equilibrium method, slip line method, limit analysis method, and finite element method. The limit analysis and finite element analysis are the research hotspots at present [1-3].

These two methods are mainly based on the strength reduction theory [4-6], and the ultimate equilibrium state of the tunnel is achieved by reducing the shear strength parameters of rock-soil mass [7]. The traditional strength reduction method generally uses the same reduction coefficient for c and $\tan\varphi$ of the shear parameters of rock and soil mass [8-9]. However, Fen Tang and Yingren Zheng *et al* [10-11] believe that when the rock mass is damaged by sliding, the cohesion and internal friction angle of the soil mass play different roles, order, and degree.

On the other hand, the attenuation degree and speed of cohesive force c and internal friction angle φ are also different, and different reduction coefficients can accurately reflect the respective safety reserves of c and φ , and the concept of double reduction method is proposed accordingly. Yuan [12], Bai [13], Zhao [14], *etc.* also proposed their own strength double reduction methods in the slope stability analysis. Bai Bing *et al* [15] pointed out that there should be an infinite number of reduction paths for double strength reduction, but ISAKOV [16] believed that among all the reduction paths, there should be a path L_{min} to make the slope from the initial state to the limit state fastest, that is, the shortest path of strength reduction. Yuan Wei *et al* [17] also believed that in the slope stability analysis, cohesion and internal friction angle should be reduced along the path with the shortest distance from the c - $\tan\varphi$ critical curve in the process of attenuation failure.

At the same time, considering the complexity of soil parameters, the precision of the strength reduction method is affected by many factors. For this reason, Reginald *et al* [18] compared and analyzed the strength reduction method with the limit equilibrium method, and proved that *Poisson's ratio* and *Young's modulus* had little influence on slope stability. Jing Zhoubao [19] used the *ABAQUS* strength reduction method to study and showed that the cohesion and internal friction angle have significant effects on slope stability, the dilatancy angle has relatively little effect, and the elastic modulus and *Poisson's ratio* have almost no effect. Liu S Y *et al* [20] analyzed the influence of dilatancy angle and *Poisson's ratio* on the safety coefficient of the slope by strength reduction

method and found that the influence of dilatancy angle and *Poisson's ratio* on the safety coefficient of the slope was small. However, at present, the strength reduction shortest path method is mainly used in the slope ^[21-22], and there are little researches on the tunnel.

Based on this, the strength reduction shortest path method is applied to the finite element strength reduction and ultimate equilibrium strength reduction analysis of shallow buried tunnels. The relationship between the length of the strength reduction path and the reduction ratio and the reasonable strength reduction ratio between the cohesion and the angle of internal friction under different parameters are discussed. The influence laws of buried depth and hole span ratio H/D , cohesion force c , and internal friction angle φ on the shortest path of strength reduction were studied. Finally, the relationship between the reduction path, the safety coefficient of surrounding rock, and the grey correlation analysis theory are used to study the influence of various factors on the safety of the shallow buried tunnel. The research results enrich the analysis theory of surrounding rock stability of shallow-buried tunnel and provide some guidance for the construction of the shallow-buried tunnel.

1 Strength reduction shortest path theory

The strength reduction shortest path theory ^[23-24] assumes that the cohesion and internal friction angle of rock and soil mass is reduced by different proportions. The reduction coefficient of cohesion is F_c , and the reduction coefficient of internal friction angle is F_φ , the formula is as follows:

$$F_c = \frac{c_0}{c_{SRT}} \quad (1)$$

$$F_\varphi = \frac{\tan\varphi_0}{\tan\varphi_{SRT}} \quad (2)$$

Where, c_0 and φ_0 are the initial cohesion and internal friction angle, c_{SRT} and φ_{SRT} are the reduced cohesion and internal friction angle.

As shown in Figure 1, assuming that the abscissa is $1/F_c$ and the ordinate is $1/F_\varphi$, the initial state M_0 of the tunnel and the line connecting the limit state line are reduced paths. There are countless strength reduction paths, and each reduction path is $L_1 \cdots L_n$. There is a reduction path L_k , the intersection point M_k of which is the limit state line. Under this reduction path, the cohesion and the angle of internal friction adopt the same reduction coefficient. In this case, the reduction path is the traditional strength reduction path. Among all the reduced paths, there should be a path

L_{min} that causes the tunnel to reach the limit state at the earliest from the initial state. This path is the shortest reduced strength path.

The limit states under different reduction paths can be combined to obtain the limit state line shown in Figure 1, which can be expressed as a function of $F_\varphi = f(F_c)$. At the same time, considering the uncertainty of the reduction path, we cannot make a quantitative analysis of it. At this time, the reduction ratio $\lambda = \frac{F_c}{F_\varphi}$ is introduced, so that different strength reduction paths have a specific reduction ratio λ corresponding to it. At this time, we can artificially assume different reduction ratios λ to make the rock and soil mass be reduced according to different reduction paths, to quantitatively analyze the stability of the surrounding rock of the shallow buried tunnel.

Fig.1 Schematic diagram of the shortest path of strength reduction

From Figure 1, it is easy to get the reduced path length from the initial state to the limit state of the tunnel under different reduced paths, the expression can be expressed as:

$$L = \sqrt{\left(1 - \frac{1}{F_c}\right)^2 + \left(1 - \frac{1}{F_\varphi}\right)^2} \quad (3)$$

The area outside the limit state line near the origin is the instability zone, while the area inside the limit state line near the initial state of the tunnel is the safety zone. At this time, according to the path length distribution, a functional expression of the strength reduction path length and the tunnel comprehensive safety factor can be established, as in equation (4). When the strength reduction path takes the smallest value L_{min} , the minimum comprehensive safety factor can be obtained [8].

$$F_s = \frac{1}{1 - \frac{L}{\sqrt{2}}} \quad (4)$$

2 Finite element strength reduction shortest path analysis

According to the existing mechanism of strength reduction, its principle is to gradually reduce the c and φ of rock and soil mass [25-26]. Regarding the non-proportional reduction of the cohesive force c and the internal friction angle φ of the rock and soil, we can specify the magnitude of the *ABAQUS* midfield variable, and then determine the limit state of the tunnel surrounding rock according to the instability criterion. Finally, the safety coefficient of the surrounding rock of the tunnel is obtained by using the relation between the length of the strength reduction path and the comprehensive safety coefficient of the tunnel. As for the instability criterion of rock and soil mass, the commonly used methods at present mainly include non-convergence criterion, plastic zone penetration criterion, and displacement mutation criterion [27-28]. In this paper, the sudden change in the settlement of the tunnel vault is selected as the criterion for the failure of the tunnel, and the safety factor at this time is selected for calculation and analysis.

2.1 Finite element example analysis

For a shallow tunnel, the ratio of the initial buried depth H to the span D is 1, and both are $12m$, the tunnel height is $9.72m$, the surrounding rock mass density is $18kN/m^3$, the elastic modulus is $1.3Gpa$, and the *Poisson's ratio* is 0.35. In the initial state, the cohesive force $c_0 = 120kPa$, and the internal friction angle is $\varphi_0 = 27^\circ$. As shown in Figure. 2, the overall model length $\times$ depth $= 120m \times 80m$, and the tunnel height $\times$ width $= 9.72m \times 12m$. The Mohr-Coulomb constitutive model is used in the calculation. The nodes at the bottom of the model limit the displacement in the X and Y directions, and the nodes on both sides limit the displacement in the X -direction.

(a) Overall model size

(b) Tunnel size

Fig.2 Finite element calculation model

In order to make the distribution of strength reduction path more uniform as far as possible, this paper assumes that the value range of reduction ratio is 0.5~2.0, to realize the reduction of rock-soil mass parameters according to different reduction paths. Combining the results of finite element limit calculation and instability criterion, the strength reduction path length (expressed as the square of the length) under different reduction ratios can be obtained as shown in Figure 3. With the increase of the reduction ratio, the length of the reduction path first decreases and then increases, and the minimum value occurs when $\lambda = 1.5$. At this time, the corresponding reduction path L_{min} is the shortest path mentioned above, and the comprehensive safety factor of the tunnel is 1.5. By fitting the relevant data, the distribution between the reduction ratio and the path length is approximated by a parabola.

Figure 4 shows the distribution of the plastic zone obtained by reducing the strength of the rock mass extracted in the finite element calculation according to the reduction ratio $\lambda = 1.5$. By connecting the points with the largest plastic deformation of the surrounding rock into a line, the potential failure surface of the surrounding rock can be approximated.

(a) Shortest path diagram of strength reduction

(b) Path length change diagram

Fig.3 Path length under different reduction factor

Fig.4 Failure modes under different reduction factor

2.2 Effects of different parameters on tunnel strength reduction shortest path

Previous studies have shown that elastic modulus, dilation angle, and *Poisson's ratio* have little influence on rock mass stability [29-31]. This paper mainly studies the influence law of buried depth and hole span ratio H/D , cohesion c , and internal friction angle φ on the shortest path of finite element strength reduction.

Buried depth and hole span ratio (H/D) select five working conditions of 0.5, 0.75, 1.0, 1.25, and 1.5. Cohesion c_0 selects five working conditions of 80, 100, 120, 140, and 160 kPa. Five operating conditions of 17°, 19.5°, 22°, 24.5°, and 27° are selected for the initial internal friction angle φ_0 . The specific calculation conditions are shown in Table 1, and the values of other parameters of all operating conditions remain unchanged.

Table 1. Calculation condition

H/D	$C0$ (kPa)	φ_0 (°)
0.5、0.75、1.0、1.25、1.5	120	22
1.0	80、100、120、140、160	22
1.0	120	17、19.5、22、24.5、27

Figures. 5-7 show the relationship curves of the path length with the reduction ratio under different H/D , cohesion c_0 , and internal friction angle φ_0 . It can be seen from the figure that the square of the length L^2 of the strength reduction path and the reduction ratio λ approximately obey the parabolic distribution law.

It can be seen from Fig. 5 that as H/D increases, the reduced shortest path L_{min}^2 gradually decreases, and the corresponding reduction ratio λ gradually decreases.

Table 2. Variation of reduction ratio under different H/D

H/D	λ
0.5	1.7
0.75	1.6
1.0	1.5
1.25	1.4
1.5	1.3

When H/D is larger, the reduction ratio λ is closer to 1, that's to say, as the H/D increases, the shortest path of strength reduction is closer to the traditional strength reduction path.

It can be seen from Figure 6 that as the initial cohesion c_0 increases, the reduced shortest path L_{min}^2 gradually increases, and the corresponding reduction ratio λ becomes larger.

Table 3. Variation of reduction ratio under different cohesion c_0

c_0/kPa	λ
80	1.1
100	1.3
120	1.5
140	1.7
160	2.0

It can be seen from Figure 7 that when the initial internal friction angle increases, the shortest path length L_{min}^2 for strength reduction gradually increases, while the reduction ratio λ at this time remains unchanged.

Table 4. Variation of reduction ratio under different internal friction angle φ_0

φ_0	λ
17°	1.5
19.5°	1.5
22°	1.5
24.5°	1.5
27°	1.5

This means that the change of the internal friction angle will not affect the reduced path of the shortest path of strength reduction.

Fig.5 Reduction factor and path length under different H/D

Fig.6 Reduction factor and path length under different c_0

Fig.7 Reduction factor and path length under different φ_0

Through statistics of the reduction ratios under the shortest path of strength reduction, the reduction ratios under the shortest path of strength reduction are all greater than 1. This shows that when the rock and soil reach the limit state according to the shortest reduction path, the reduction of cohesion is greater than the reduction of the internal friction angle.

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Fig.8 Stress analysis diagram of limit equilibrium state

Substituting equation (7) into equation (6) can get:

$$N = \frac{chtan\alpha + T_x}{cos\alpha - sin\alpha tan\phi} \quad (8)$$

Considering that BD is not a sliding surface, therefore:

$$T \approx ca_1 \quad (9)$$

Suppose $p = \gamma H$, H is the buried depth of the tunnel, p is the surrounding rock pressure of a shallow tunnel, and $\alpha = 45^\circ - \varphi/2$.

From the above formula, the formula (4) is finally transformed into:

$$\frac{K}{c} = \frac{15.93+17.47\tan\varphi/(0.819-0.574\tan\varphi)}{183.6H+365.85} \quad (10)$$

When $K=1.0$, the shallow tunnel is in a state of limit equilibrium.

Suppose $\lambda = \frac{F_c}{F_\phi}$, $F_c = \frac{c_0}{c}$, $F_\phi = \frac{\tan\phi_0}{\tan\phi}$ the formula (10) can be obtained:

$$\lambda = \frac{17.47c_0 \tan \varphi_0 / (0.819F_\varphi^2 - 0.574F_\varphi \tan \varphi_0)}{183.6H + 365.85} + \frac{\frac{15.93c_0}{F_\varphi}}{183.6H + 365.85} \quad (11)$$

3.2 The influence of different parameters on the shortest path of strength reduction of shallow buried tunnel

The working conditions in Table 1 are still used for calculation, and the influence of the three major factors of buried depth and hole-span ratio, cohesion, and internal friction angle on the shortest path of ultimate equilibrium strength reduction is obtained.

Through comparison, it can be found that the calculation results of the limit equilibrium method are basically consistent with the finite element numerical calculation results:

(1) As H/D increases, the shortest path L_{min} of strength reduction gradually decreases, and the corresponding reduction ratio λ gradually decreases.

- (2) With the increase of initial cohesion force c_0 , the reduction shortest path L_{min} gradually increases, and the corresponding reduction ratio λ also gradually increases.
- (3) With the increase of the initial internal friction angle, the reduced shortest path L_{min} gradually increases, and the corresponding reduction ratio λ remains unchanged.

Fig.9 Reduction factor and path length under different H/D

Fig.10 Reduction factor and path length under different c_0

Fig.11 Reduction factor and path length under different φ_0

4 Comparison of safety factors and sensitivity analysis of influencing factors under different methods

4.1 Comparative analysis of comprehensive safety factors under different methods

It can be seen from Equation (4) that the length of the shortest path by strength reduction can indirectly represent the safety factor. To further reflect the difference and relation between the shortest path by strength reduction by finite element method and the shortest path by ultimate equilibrium strength reduction, the comprehensive safety factor of the two different methods under different parameters is obtained as shown in Figure 12~ 14.

It can be seen from Figures 12-14 that the greater the internal friction angle and cohesion of the tunnel in the initial state, the greater the overall safety factor, and the two are approximately in a linear relationship. When the buried depth and span ratio are small, the safety factors obtained under the finite element method and the limit equilibrium method are relatively close. However, the finite element method of safety coefficient calculated under different parameters is larger than the limit equilibrium method, which indicates that shallow-buried tunnels are more likely to be damaged under the limit equilibrium method. This is because in the calculation of the limit equilibrium method, the value of the surrounding rock pressure P is larger, and the actual surrounding rock pressure P is smaller than γh , but in general, the two laws are consistent.

Fig.12 The relationship curves between the safety factor and H/D under different method

Fig.13 The relationship curves between the safety factor and c_0 under different method

Fig.14 The relationship curves between the safety factor and ϕ_0 under different method

4.2 Sensitivity gray correlation analysis of influencing factors

The sensitivity analysis of factors affecting the safety of shallow tunnels belongs to the category of gray correlation analysis [32]. The main principle of grey correlation analysis is to take the minimum comprehensive safety factor as the reference sequence, take each influencing factor as the sub-sequence, and quantify their sensitivity. The main influencing factors are determined from the buried depth and span ratio, cohesion, and internal friction angle. In actual engineering, these factors should be paid attention to. Combined with the calculation of the comprehensive safety factor of the finite element method in the previous section, the specific working conditions are shown in Table 5 to Table 7.

Table 5. Safety factor under different H/D

H/D	0.5	0.75	1.0	1.25	1.5
F_s	1.632	1.574	1.500	1.434	1.344

Table 6. Safety factor under different c_0

c	80	100	120	140	160
F_s	1.179	1.351	1.500	1.615	1.740

Table 7. Safety factor under different φ_0

φ	17	19.5	22	24.5	27
F_s	1.422	1.454	1.500	1.537	1.588

According to different influencing factors and working conditions, the initial matrix of gray correlation calculation is obtained as follows:

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.75 & 1.0 & 1.25 & 1.5 \\ 80 & 100 & 120 & 140 & 160 \\ 17 & 19.5 & 22 & 24.5 & 27 \end{bmatrix}$$

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 1.632 & 1.574 & 1.500 & 1.434 & 1.344 \\ 1.179 & 1.351 & 1.500 & 1.615 & 1.740 \\ 1.422 & 1.454 & 1.500 & 1.537 & 1.588 \end{bmatrix}$$

1) Perform averaging on the above X and Y matrix sequence, let:

$$X'_i = \frac{\bar{X}_i}{\bar{x}_i} = (x_i(1), x_i(2), \dots, x_i(n)) \quad (12)$$

Where: $i=0,1,2,\dots,m$

2) Evaluation of sequence of difference

$$\Delta_i(k) = |y_i(k) - x_i(k)| \quad (13)$$

In the difference sequence matrix Δ , the elements need to be compared to extract the maximum and minimum values:

$$\Delta_{max} = \max(\Delta_{ij}) \Delta_{min} = \min(\Delta_{ij}) \quad (14)$$

3) Find the gray correlation coefficient matrix

Before the analysis, the distance between the comparison point and the reference point should be sorted out, and the overall analysis should be used to calculate the difference and correlation between the factors. Finally, the correlation coefficient between the comparison factor and the reference factor is obtained through formula (15), which reflects its correlation.

$$\gamma_{ij} = \frac{\Delta_{min} + \xi \Delta_{max}}{\Delta_{ij} + \xi \Delta_{max}} \quad (15)$$

In the formula: ξ is the resolution coefficient, which is generally 0.5.

4) Find the degree of grey correlation

As in formula (16), the degree of relevance is obtained by calculating the average value of the

relevant coefficient, and the relevance comparison of the impact size is made accordingly.

$$A_i = \frac{1}{n} \sum_{j=1}^n \gamma_{ij} \quad (16)$$

Using the above formula to calculate, the gray correlation degrees of H/D , cohesion c , and internal friction angle φ of the shallow tunnel are 0.5347, 0.7805, and 0.7661, respectively. Therefore, in the process of calculating the stability of shallow tunnels by the strength reduction shortest path method, the internal friction angle φ and cohesive force c have a greater impact on the safety of shallow tunnels.

5 Conclusion

(1) In shallow tunnels, the reduced path length (expressed by the square of the length) obtained by the finite element strength reduction method and the ultimate equilibrium strength reduction method and the reduction ratio approximately obey the parabolic distribution law.

(2) The strength reduction shortest path L_{min} and the corresponding reduction ratio λ gradually decrease with the increase of H/D , and gradually increase with the increase of the initial cohesive force c_0 ; with the increase of the initial internal friction angle, the reduction is the shortest. The path L_{min} gradually increases, and the corresponding reduction ratio λ remains unchanged.

(3) The comprehensive safety factor of shallow tunnel calculated by the finite element strength reduction shortest path method is greater than the comprehensive safety factor calculated by the ultimate equilibrium strength reduction shortest path method. The strength reduction shortest path method can be a new method to analyze the stability of shallow tunnels.

(4) Among the factors affecting the stability of shallow tunnels calculated by the strength reduction shortest path method, the internal friction angle φ , and cohesion c have a greater impact on the safety of shallow tunnels.

6 Conflict of interest statement

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled, "The application of the strength reduction shortest path method to the stability analysis of shallow buried tunnel".

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