On Computation of Some Open and Closed Neighbourhood Degree Sum-Based Topological Indices for Metal-Insulator Transition Super Lattice Network

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Abstract

The properties and activities of chemical compounds can be understood by computing topological descriptors of molecular compounds. We investigate the physical and topological aspects of crystal structure of metal-insulator transition superlattice (GST-SL) in this study. Recently, researchers have turned their attention to modifying this substance into a form that is useful for human life. Metal-insulator transition superlattices (GST-SL) are also useful as two-dimensional (2D) transition metal dichalcogenides (TMDs) in the form of thin films. For this Superlattice Network $\boldsymbol{SL_\eta}$, we calculate open and closed neighbourhood degree sum based on topological indices.

Keywords: Neighbourhood Degree Sum, Topological Indices, Super Lattice Network

1 Introduction

The topological index is a useful technique for describing the physical, chemical, and biological properties of chemical substances [1–3]. In Quantitative Structure Activity Relationship / Quantitative Structure Property Relationship (QSAR/QSPR), [4–6], topological indices play a crucial role in associating the diverse chemical compound structures with bioactivities and chemical properties.

Cheminformatics is an active area of research where the relationship between quantitative structural behaviour and structural properties helps predict the biological activity and properties [7–9]. The nodes in the chemical diagram represent atoms or molecules, and the links reflect the chemical bonds between them. Topological indexes are immutable graph attributes and numerical parameters associated with the graph that characterize the topology. The degree of a vertex is given by d_u or $d(u)$ [10] and reflects the number of edges at that vertex. The sum of the orders of all adjacent nodes of node u, expressed as $\delta(u)$ / δ_u, is the open neighborhood order of node $u \in V$.

Sourav Mondal et al. [11] defined the neighbourhood versions of Zagreb indices M_{1_N}, M_{2_N}, HM_N and Forgotten index F_N and the indices are

$$M_{1_N}(G) = \sum_{uv \in E} \Big[\delta(u) + \delta(v) \Big], \quad M_{2_N}(G) = \sum_{uv \in E} \Big[\delta(u) * \delta(v) \Big]$$

$$HM_N = \sum_{uv \in E} \Big[\delta(u) + \delta(v) \Big]^2, \quad F_N = \sum_{uv \in E} \Big[\delta(u)^2 + \delta(v)^2 \Big]$$

Further, Sourav Mondal et al. [12] defined the some more neighbourhood indices, the indices are

$$ND_1(G) = \sum_{uv \in E} \left[\sqrt{\delta(u) * \delta(v)} \right], \quad ND_2(G) = \sum_{uv \in E} \left[\frac{1}{\sqrt{\delta(u) + \delta(v)}} \right]$$

$$ND_3(G) = \sum_{uv \in E} \left[\Big(\delta(u)*\delta(v)\Big)\Big(\delta(u)+\delta(v)\Big) \right], \quad ND_4(G) = \sum_{uv \in E} \left[\frac{1}{\sqrt{\delta(u) * \delta(v)}} \right]$$

$$ND_5(G) = \sum_{uv \in E} \left[\frac{\delta(u)}{\delta(v)} + \frac{\delta(v)}{\delta(u)} \right]$$

The closed versions of the above defined neighbourhood indices are

$$M_{1_{N^c}}(G) = \sum_{uv \in E} \Big[\delta_c[u] + \delta_c[v] \Big], \quad M_{2_{N^c}}(G) = \sum_{uv \in E} \Big[\delta_c[u] * \delta_c[v] \Big]$$

$$HM_{N^c} = \sum_{uv \in E} \left[\delta_c[u] + \delta_c[v] \right]^2, \quad F_{N^c} = \sum_{uv \in E} \left[\delta_c[u]^2 + \delta_c[v]^2 \right]$$

$$N^cD_1(G) = \sum_{uv \in E} \left[\sqrt{\delta_c[u] * \delta(v)} \right], \quad N^cD_2(G) = \sum_{uv \in E} \left[\frac{1}{\sqrt{\delta_c[u] + \delta_c[v]}} \right]$$

$$N^cD_3(G) = \sum_{uv \in E} \left[\left(\delta_c[u]*\delta_c[v] \right) \left(\delta_c[u]+\delta_c[v] \right) \right], \quad N^cD_4(G) = \sum_{uv \in E} \left[\frac{1}{\sqrt{\delta_c[u] * \delta_c[v]}} \right]$$

$$N^cD_5(G) = \sum_{uv \in E} \left[\frac{\delta_c[u]}{\delta_c[v]} + \frac{\delta_c[v]}{\delta_c[u]} \right]$$

Vignesh and Kalyani Desikan defined some neighbourhood degree sum based topological indices along with their reciprocal versions [13]. These indices are

$$SK_N(G) = \sum_{uv \in E(G)} \left[\frac{\delta(u) + \delta(v)}{2} \right], \quad SK1_N(G) = \sum_{uv \in E(G)} \left[\frac{\delta(u) * \delta(v)}{2} \right]$$

$$SK2_N(G) = \sum_{uv \in E(G)} \left[\frac{\delta(u) + \delta(v)}{2} \right]^2, \quad mR_N(G) = \sum_{uv \in E(G)} \left[\frac{1}{max\{\delta(u), \delta(v)\}} \right]$$

Also Verma et al. [14] defined the neighborhood inverse sum index as

$$NI(G) = \sum_{uv \in E(G)} \left[\frac{\delta(u) * \delta(v)}{\delta(u) + \delta(v)} \right]$$

where $\delta(u) = \sum_{v \in N_G(u)} d(v)$, where $N_G(u)$ represents the open neighbourhood of vertex u.

In [15, 16], Vignesh and Kalyani Desikan introduced closed versions of the aforementioned topological descriptors.

$$SK_{N^c}(G) = \sum_{uv \in E(G)} \left[\frac{\delta_c[u] + \delta_c[v]}{2} \right], \quad SK1_{N^c}(G) = \sum_{uv \in E(G)} \left[\frac{\delta_c[u] * \delta_c[v]}{2} \right]$$

$$SK2_{N^c}(G) = \sum_{uv \in E(G)} \left[\frac{\delta_c[u] + \delta_c[v]}{2} \right]^2, \quad mR_{N^c}(G) = \sum_{uv \in E(G)} \left[\frac{1}{max\{\delta_c[u], \delta_c[v]\}} \right]$$

$$ISI_{N^c}(G) = \sum_{uv \in E(G)} \left[\frac{\delta_c[u] * \delta_c[v]}{\delta_c[u] + \delta_c[v]} \right]$$

where $\delta_c[u] = \left[\sum_{v \in N_G(u)} d(v)\right] + d(u)$, where $N_G(u)$ represents the open neighborhood of vertex u.

Metalloids are a varied group of elements with Physico-chemical properties that lie in between metals and nonmetals. They are found in the earth's crust along with a mixture of inorganic and organic compounds [17]. Elements such as germanium (Ge), antimony (Sb) and tellurium (Te) (GST) exist as metalloids. These metalloids generate thermal, electrical intermediates, and metals to form large-structure oxides.

Researchers are interested in modifying this substance into forms useful for human life in various areas such as: GST alloys in the form of thin films, followed by two-dimensional (2D) Transition Metal Dichalcogenides (TMDs). GST and TMDs are used in many fields [18].

The physico-chemical characteristics are helpful for sensing, in nonvolatile RAM, thermoelectric properties and face changing features to increase the bandgap energy of Ge-Sb-Te (GST) [19, 20]. Phase Change Material (PCM) properties of $Ge - Sb - Te$ (GST) complex with a group of chalcogenides are promising technology and well known for many years [21, 22]. Another type of PCM material, called the Ge-Sb-Te superlattice ($GST - SL$), has gained a lot of attention due to its very low power usage. This display has been attributed to a unique information storing technique called crystalline to crystalline stage transition [23].

Rongbing Huang et al. [24] recently worked on the computation of degree-based topological indices of Metal-Insulator Transition Superlattice Network. Inspired by that work, in this paper we compute open and closed neighbourhood degree sum based topological descriptors with the help of the corresponding open and closed neighbourhood degree edge partitions of Metal-Insulator Transition Superlattice Network.

2 Methodology

We use the method of edge partitions based on open and closed neighbourhood degree of a vertex to compute the topological indices. From the structure SLN_η, We observed that $V\big(SLN_\eta\big) = 9\eta + 3$ and $E\big(SLN_\eta\big) = 13\eta$, η is the number of levels.

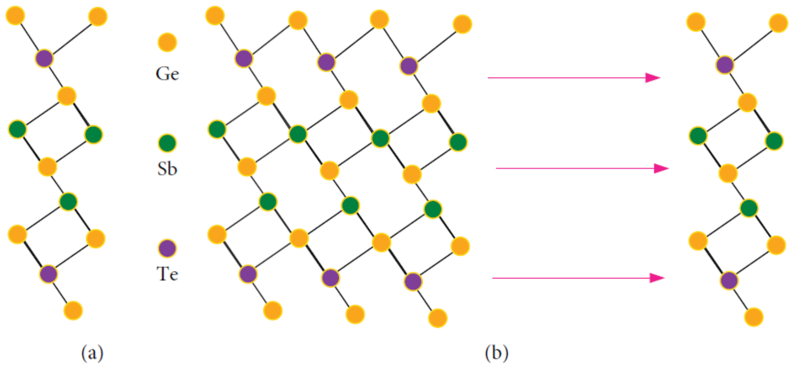

Fig. 1 (a) Unit cell. (b) General structure of $(GST - SL_\eta)$ [24].

Table 1 Open Neighbourhood Edge Partitions of Metal-Insulator Transition Super lattice Network when $\eta = 1$.

E_i	$(\delta(u), \delta(v))$	Count
E_1	(3, 5)	3
E_2	(5, 6)	2
E_3	(5, 7)	1
E_4	(6, 7)	6
E_5	(7, 7)	1

3 Results on Open Neighborhood Degree Based Descriptors of Metal-Insulator Transition Superlattice Network

In this section, we compute the open neighbourhood degree sum based topological indices of the Metal-Insulator Transition Superlattice Network.

3.1 Open Neighborhood Degree Based Descriptors of Metal-Insulator Transition Superlattice Network when $\eta \neq 1$

Let SLN_η denote the Metal-Insulator Transition Superlattice Network. Here, we compute the 14 open neighborhood descriptors, of SLN_η, when $\eta \neq 1$. Using the edge partitions of the network given in Table 2.

$$M_{1_N}\left(SLN_\eta\right) = \sum_{uv \in E_1} \left[3+6\right] + \sum_{uv \in E_2} \left[3+7\right] + \sum_{uv \in E_3} \left[3+9\right] + \sum_{uv \in E_4} \left[6+6\right]$$

Table 2 Open Neighbourhood Edge Partitions of Metal-Insulator Transition Super lattice Network when $\eta \neq 1$.

E_i	$(\delta(u), \delta(v))$	Count
E_1	(3, 6)	2
E_2	(3, 7)	2
E_3	(3, 9)	$\eta - 2$
E_4	(6, 6)	2
E_5	(6, 7)	$2\eta - 2$
E_6	(6, 9)	8
E_7	(7, 11)	$\eta - 2$
E_8	(7, 12)	2
E_9	(9, 9)	2
E_{10}	(9, 12)	$2\eta + 2$
E_{11}	(11, 11)	$\eta - 2$
E_{12}	(11, 12)	$6\eta - 12$

Table 3 Closed Neighbourhood Edge Partitions of Metal-Insulator Transition Super lattice Network when $\eta = 1$.

E_i	$(\delta_c[u], \delta_c[v])$	Count
E_1	(4, 8)	3
E_2	(8, 8)	2
E_3	(8, 10)	7
E_4	(10, 10)	1

$$
\begin{aligned}
&+ \sum_{uv \in E_5} \left[6 + 7\right] + \sum_{uv \in E_6} \left[6 + 9\right] + \sum_{uv \in E_7} \left[7 + 11\right] + \sum_{uv \in E_8} \left[7 + 12\right] \\
&+ \sum_{uv \in E_9} \left[9 + 9\right] + \sum_{uv \in E_{10}} \left[9 + 12\right] + \sum_{uv \in E_{11}} \left[11 + 11\right] + \sum_{uv \in E_{12}} \left[11 + 12\right] \\
&= 258\eta - 108.
\end{aligned}
$$

$$
\begin{aligned}
M_{2_N}\left(SLN_\eta\right) &= \sum_{uv \in E_1} \left[3 * 6\right] + \sum_{uv \in E_2} \left[3 * 7\right] + \sum_{uv \in E_3} \left[3 * 9\right] + \sum_{uv \in E_4} \left[6 * 6\right] \\
&+ \sum_{uv \in E_5} \left[6 * 7\right] + \sum_{uv \in E_6} \left[6 * 9\right] + \sum_{uv \in E_7} \left[7 * 11\right] + \sum_{uv \in E_8} \left[7 * 12\right] \\
&+ \sum_{uv \in E_9} \left[9 * 9\right] + \sum_{uv \in E_{10}} \left[9 * 12\right] + \sum_{uv \in E_{11}} \left[11 * 11\right] + \sum_{uv \in E_{12}} \left[11 * 12\right] \\
&= 1317\eta - 990.
\end{aligned}
$$

Table 4 Closed Neighbourhood Edge Partitions of Metal-Insulator Transition Super lattice Network when $\eta \neq 1$.

E_i	$(\delta_c[u], \delta_c[v])$	Count
E_1	(4, 9)	2
E_2	(4, 10)	2
E_3	(4, 12)	$\eta - 2$
E_4	(8, 9)	2
E_5	(8, 10)	$2\eta - 2$
E_6	(8, 12)	6
E_7	(9, 12)	2
E_8	(10, 14)	$\eta - 2$
E_9	(10, 16)	2
E_{10}	(12, 12)	2
E_{11}	(12, 16)	$2\eta + 2$
E_{12}	(14, 14)	$\eta - 2$
E_{13}	(14, 16)	$6\eta - 12$

$$\begin{aligned} HM_N\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\Big[3+6\Big]^2 + \sum_{uv\in E_2}\Big[3+7\Big]^2 + \sum_{uv\in E_3}\Big[3+9\Big]^2 + \sum_{uv\in E_4}\Big[6+6\Big]^2 \\ &+ \sum_{uv\in E_5}\Big[6+7\Big]^2 + \sum_{uv\in E_6}\Big[6+9\Big]^2 + \sum_{uv\in E_7}\Big[7+11\Big]^2 + \sum_{uv\in E_8}\Big[7+12\Big]^2 \\ &+ \sum_{uv\in E_9}\Big[9+9\Big]^2 + \sum_{uv\in E_{10}}\Big[9+12\Big]^2 \\ &+ \sum_{uv\in E_{11}}\Big[11+11\Big]^2 + \sum_{uv\in E_{12}}\Big[11+12\Big]^2 \\ &= 5346\eta - 3888. \end{aligned}$$

$$\begin{aligned} F_N\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\Big[3^2+6^2\Big] + \sum_{uv\in E_2}\Big[3^2+7^2\Big] + \sum_{uv\in E_3}\Big[3^2+9^2\Big] + \sum_{uv\in E_4}\Big[6^2+6^2\Big] \\ &+ \sum_{uv\in E_5}\Big[6^2+7^2\Big] + \sum_{uv\in E_6}\Big[6^2+9^2\Big] + \sum_{uv\in E_7}\Big[7^2+11^2\Big] + \sum_{uv\in E_8}\Big[7^2+12^2\Big] \\ &+ \sum_{uv\in E_9}\Big[9^2+9^2\Big] + \sum_{uv\in E_{10}}\Big[9^2+12^2\Big] + \sum_{uv\in E_{11}}\Big[11^2+11^2\Big] \\ &+ \sum_{uv\in E_{12}}\Big[11^2+12^2\Big] \end{aligned}$$

$$= 2712\eta - 1908.$$

$$\begin{aligned} ND_1\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\left[\sqrt{3*6}\right] + \sum_{uv\in E_2}\left[\sqrt{3*7}\right] + \sum_{uv\in E_3}\left[\sqrt{3*9}\right] + \sum_{uv\in E_4}\left[\sqrt{6*6}\right] \\ &+ \sum_{uv\in E_5}\left[\sqrt{6*7}\right] + \sum_{uv\in E_6}\left[\sqrt{6*9}\right] + \sum_{uv\in E_7}\left[\sqrt{7*11}\right] + \sum_{uv\in E_8}\left[\sqrt{7*12}\right] \\ &+ \sum_{uv\in E_9}\left[\sqrt{9*9}\right] + \sum_{uv\in E_{10}}\left[\sqrt{9*12}\right] + \sum_{uv\in E_{11}}\left[\sqrt{11*11}\right] \\ &+ \sum_{uv\in E_{12}}\left[\sqrt{11*12}\right] \\ &= 105.9156\eta - 20.5221. \end{aligned}$$

$$\begin{aligned} ND_2\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\left[\frac{1}{\sqrt{3+6}}\right] + \sum_{uv\in E_2}\left[\frac{1}{\sqrt{3+7}}\right] + \sum_{uv\in E_3}\left[\frac{1}{\sqrt{3+9}}\right] \\ &+ \sum_{uv\in E_4}\left[\frac{1}{\sqrt{6+6}}\right] + \sum_{uv\in E_5}\left[\frac{1}{\sqrt{6+7}}\right] + \sum_{uv\in E_6}\left[\frac{1}{\sqrt{6+9}}\right] \\ &+ \sum_{uv\in E_7}\left[\frac{1}{\sqrt{7+11}}\right] + \sum_{uv\in E_8}\left[\frac{1}{\sqrt{7+12}}\right] + \sum_{uv\in E_9}\left[\frac{1}{\sqrt{9+9}}\right] \\ &+ \sum_{uv\in E_{10}}\left[\frac{1}{\sqrt{9+12}}\right] + \sum_{uv\in E_{11}}\left[\frac{1}{\sqrt{11+11}}\right] + \sum_{uv\in E_{12}}\left[\frac{1}{\sqrt{11+12}}\right] \\ &= 2.9798\eta + 0.7767. \end{aligned}$$

$$\begin{aligned} ND_3\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\Big[(3*6)(3+6)\Big] + \sum_{uv\in E_2}\Big[(3*7)(3+7)\Big] \\ &+ \sum_{uv\in E_3}\Big[(3*9)(3+9)\Big] + \sum_{uv\in E_4}\Big[(6*6)(6+6)\Big] \\ &+ \sum_{uv\in E_5}\Big[(6*7)(6+7)\Big] + \sum_{uv\in E_6}\Big[(6*9)(6+9)\Big] \end{aligned}$$

$$+\sum_{uv\in E_7}\Big[(7*11)(7+11)\Big]+\sum_{uv\in E_8}\Big[(7*12)(7+12)\Big]$$
$$+\sum_{uv\in E_9}\Big[(9*9)(9+9)\Big]+\sum_{uv\in E_{10}}\Big[(9*12)(9+12)\Big]$$
$$+\sum_{uv\in E_{11}}\Big[(11*11)(11+11)\Big]+\sum_{uv\in E_{12}}\Big[(11*12)(11+12)\Big]$$
$$=28216\eta-27536.$$

$$ND_4\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{1}{\sqrt{3*6}}\right]+\sum_{uv\in E_2}\left[\frac{1}{\sqrt{3*7}}\right]+\sum_{uv\in E_3}\left[\frac{1}{\sqrt{3*9}}\right]$$
$$+\sum_{uv\in E_4}\left[\frac{1}{\sqrt{6*6}}\right]+\sum_{uv\in E_5}\left[\frac{1}{\sqrt{6*7}}\right]+\sum_{uv\in E_6}\left[\frac{1}{\sqrt{6*9}}\right]$$
$$+\sum_{uv\in E_7}\left[\frac{1}{\sqrt{7*11}}\right]+\sum_{uv\in E_8}\left[\frac{1}{\sqrt{7*12}}\right]+\sum_{uv\in E_9}\left[\frac{1}{\sqrt{9*9}}\right]$$
$$+\sum_{uv\in E_{10}}\left[\frac{1}{\sqrt{9*12}}\right]+\sum_{uv\in E_{11}}\left[\frac{1}{\sqrt{11*11}}\right]+\sum_{uv\in E_{12}}\left[\frac{1}{\sqrt{11*12}}\right]$$
$$=1.4206\eta+0.8150.$$

$$ND_5\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{3^2+6^2}{(3*6)}\right]+\sum_{uv\in E_2}\left[\frac{3^2+7^2}{(3*7)}\right]+\sum_{uv\in E_3}\left[\frac{3^2+9^2}{(3*9)}\right]$$
$$+\sum_{uv\in E_4}\left[\frac{6^2+6^2}{(6*6)}\right]+\sum_{uv\in E_5}\left[\frac{6^2+7^2}{(6*7)}\right]+\sum_{uv\in E_6}\left[\frac{6^2+9^2}{(6*9)}\right]$$
$$+\sum_{uv\in E_7}\left[\frac{7^2+11^2}{(7*11)}\right]+\sum_{uv\in E_8}\left[\frac{7^2+12^2}{(7*12)}\right]+\sum_{uv\in E_9}\left[\frac{9^2+9^2}{(9*9)}\right]$$
$$+\sum_{uv\in E_{10}}\left[\frac{9^2+12^2}{(9*12)}\right]+\sum_{uv\in E_{11}}\left[\frac{11^2+11^2}{(11*11)}\right]+\sum_{uv\in E_{12}}\left[\frac{11^2+12^2}{(11*12)}\right]$$
$$=27.8009\eta+1.3983.$$

$$SK_N\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{3+6}{2}\right]+\sum_{uv\in E_2}\left[\frac{3+7}{2}\right]+\sum_{uv\in E_3}\left[\frac{3+9}{2}\right]+\sum_{uv\in E_4}\left[\frac{6+6}{2}\right]$$

$$+\sum_{uv\in E_5}\left[\frac{6+7}{2}\right]+\sum_{uv\in E_6}\left[\frac{6+9}{2}\right]+\sum_{uv\in E_7}\left[\frac{7+11}{2}\right]+\sum_{uv\in E_8}\left[\frac{7+12}{2}\right]$$
$$+\sum_{uv\in E_9}\left[\frac{9+9}{2}\right]+\sum_{uv\in E_{10}}\left[\frac{9+12}{2}\right]+\sum_{uv\in E_{11}}\left[\frac{11+11}{2}\right]+\sum_{uv\in E_{12}}\left[\frac{11+12}{2}\right]$$
$$=129\eta-54.$$

$$SK1_N\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{3*6}{2}\right]+\sum_{uv\in E_2}\left[\frac{3*7}{2}\right]+\sum_{uv\in E_3}\left[\frac{3*9}{2}\right]+\sum_{uv\in E_4}\left[\frac{6*6}{2}\right]$$
$$+\sum_{uv\in E_5}\left[\frac{6*7}{2}\right]+\sum_{uv\in E_6}\left[\frac{6*9}{2}\right]+\sum_{uv\in E_7}\left[\frac{7*11}{2}\right]+\sum_{uv\in E_8}\left[\frac{7*12}{2}\right]$$
$$+\sum_{uv\in E_9}\left[\frac{9*9}{2}\right]+\sum_{uv\in E_{10}}\left[\frac{9*12}{2}\right]+\sum_{uv\in E_{11}}\left[\frac{11*11}{2}\right]+\sum_{uv\in E_{12}}\left[\frac{11*12}{2}\right]$$
$$=658.5\eta-495.$$

$$SK2_N\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{3+6}{2}\right]^2+\sum_{uv\in E_2}\left[\frac{3+7}{2}\right]^2+\sum_{uv\in E_3}\left[\frac{3+9}{2}\right]^2+\sum_{uv\in E_4}\left[\frac{6+6}{2}\right]^2$$
$$+\sum_{uv\in E_5}\left[\frac{6+7}{2}\right]^2+\sum_{uv\in E_6}\left[\frac{6+9}{2}\right]^2+\sum_{uv\in E_7}\left[\frac{7+11}{2}\right]^2+\sum_{uv\in E_8}\left[\frac{7+12}{2}\right]^2$$
$$+\sum_{uv\in E_9}\left[\frac{9+9}{2}\right]^2+\sum_{uv\in E_{10}}\left[\frac{9+12}{2}\right]^2+\sum_{uv\in E_{11}}\left[\frac{11+11}{2}\right]^2$$
$$+\sum_{uv\in E_{12}}\left[\frac{11+12}{2}\right]^2$$
$$=1336.5\eta-972.$$

$$mR_N\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{1}{max\{3,6\}}\right]+\sum_{uv\in E_2}\left[\frac{1}{max\{3,7\}}\right]+\sum_{uv\in E_3}\left[\frac{1}{max\{3,9\}}\right]$$
$$+\sum_{uv\in E_4}\left[\frac{1}{max\{6,6\}}\right]+\sum_{uv\in E_5}\left[\frac{1}{max\{6,7\}}\right]+\sum_{uv\in E_6}\left[\frac{1}{max\{6,9\}}\right]$$
$$+\sum_{uv\in E_7}\left[\frac{1}{max\{7,11\}}\right]+\sum_{uv\in E_8}\left[\frac{1}{max\{7,12\}}\right]+\sum_{uv\in E_9}\left[\frac{1}{max\{9,9\}}\right]$$

$$+ \sum_{uv \in E_{10}} \left[\frac{1}{max\{9,12\}} \right] + \sum_{uv \in E_{11}} \left[\frac{1}{max\{11,11\}} \right] + \sum_{uv \in E_{12}} \left[\frac{1}{max\{11,12\}} \right]$$
$$= 1.2453\eta + 0.5253.$$

$$NI\Big(SLN_\eta\Big) = \sum_{uv \in E_1} \left[\frac{(3*6)}{3+6)} \right] + \sum_{uv \in E_2} \left[\frac{(3*7)}{3+7)} \right] + \sum_{uv \in E_3} \left[\frac{(3*9)}{3+9)} \right]$$
$$+ \sum_{uv \in E_4} \left[\frac{(6*6)}{6+6)} \right] + \sum_{uv \in E_5} \left[\frac{(6*7)}{6+7)} \right] + \sum_{uv \in E_6} \left[\frac{(6*9)}{6+9)} \right]$$
$$+ \sum_{uv \in E_7} \left[\frac{(7*11)}{7+11)} \right] + \sum_{uv \in E_8} \left[\frac{(7*12)}{7+12)} \right] + \sum_{uv \in E_9} \left[\frac{(9*9)}{9+9)} \right]$$
$$+ \sum_{uv \in E_{10}} \left[\frac{(9*12)}{9+12)} \right] + \sum_{uv \in E_{11}} \left[\frac{(11*11)}{11+11)} \right] + \sum_{uv \in E_{12}} \left[\frac{(11*12)}{11+12)} \right]$$
$$= 63.2098\eta - 28.2588.$$

3.2 Numerical and Graphical Interpretations of Computed Results for Superlattice Network

Here, we performed computational techniques of superlattice network for all 14 open neighbourhood indices. The results are shown in Tables 5 and 6, and Figures 2 and 3 depict the graphical representation of computations based on the open neighbourhood indices.

Table 5 Computed Open Neighbourhood Topological Indices Values for $(SLN)_\eta$, $\eta > 1$

n	M_{1_N}	M_{2_N}	HM_N	F_N	ND_1	ND_2	ND_3
2	408	1644	6804	3516	191.3091	6.7363	28896
3	666	2961	12150	6228	297.2247	9.7161	57112
4	924	4278	17496	8940	403.1403	12.6959	85328
5	1182	5595	22842	11652	509.0559	15.6757	113544
6	1440	6912	28188	14364	614.9715	18.6555	141760
7	1698	8229	33534	17076	720.8871	21.6353	169976
8	1956	9546	38880	19788	826.8027	24.6151	198192
9	2214	10863	44226	22500	932.7183	27.5949	226408
10	2472	12180	49572	25212	1038.6339	30.5747	254624

Table 6 Computed Open Neighbourhood Topological Indices Values for $(SLN)_\eta$, $\eta > 1$

n	ND_4	ND_5	SK_N	$SK1_N$	$SK2_N$	mR_N	NI
2	3.6562	57.0001	204	822	1701	3.0159	98.1608
3	5.0768	84.801	333	1480.5	3037.5	4.2612	161.3706
4	6.4974	112.6019	462	2139	4374	5.5065	224.5804
5	7.918	140.4028	591	2797.5	5710.5	6.7518	287.7902
6	9.3386	168.2037	720	3456	7047	7.9971	351
7	10.7592	196.0046	849	4114.5	8383.5	9.2424	414.2098
8	12.1798	223.8055	978	4773	9720	10.4877	477.4196
9	13.6004	251.6064	1107	5431.5	11056.5	11.733	540.6294
10	15.021	279.4073	1236	6090	12393	12.9783	603.8392

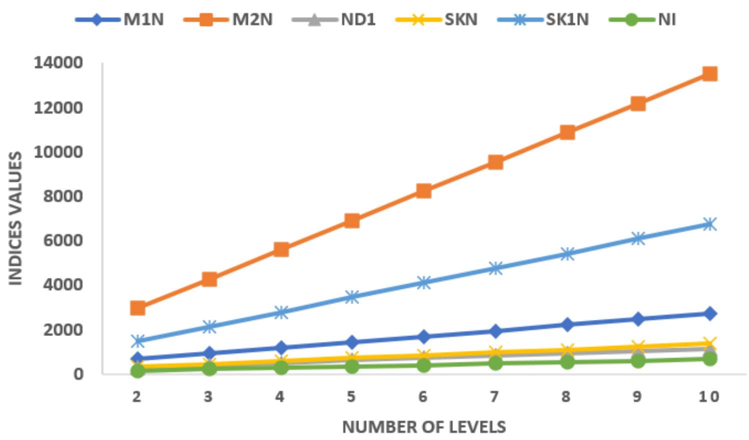

Fig. 2 Graphical Depiction of Numerical Computation based on the Open Neighbourhood Indices

3.3 Open Neighborhood Degree Based Descriptors of Metal-Insulator Transition Superlattice Network when $\eta = 1$

Let SLN_η denote the Metal-Insulator Transition Superlattice Network. Here, we compute the 14 open neighborhood descriptors, of SLN_η, when $\eta = 1$. Using the edge partitions of the network given in Table 1.

$$1.\ M_{1_N}(SLN_\eta) = 150, \quad 2.\ M_{2_N}(SLN_\eta) = 441$$
$$3.\ HM_N(SLN_\eta) = 1788, \quad 4.\ F_N(SLN_\eta) = 906$$
$$5.\ ND_1(SLN_\eta) = 74.3739, \quad 6.\ ND_2(SLN_\eta) = 3.8837$$
$$7.\ ND_3(SLN_\eta) = 5402, \quad 8.\ ND_4(SLN_\eta) = 2.3775$$
$$9.\ ND_5(SLN_\eta) = 27.1238, \quad 10.\ SK_{N^c}(SLN_\eta) = 75$$
$$11.\ SK1_N(SLN_\eta) = 220.5, \quad 12.\ SK2_N(SLN_\eta) = 447$$
$$13.\ mR_N(SLN_\eta) = 2.0762, \quad 14.\ NI(SLN_\eta) = 36.8808$$

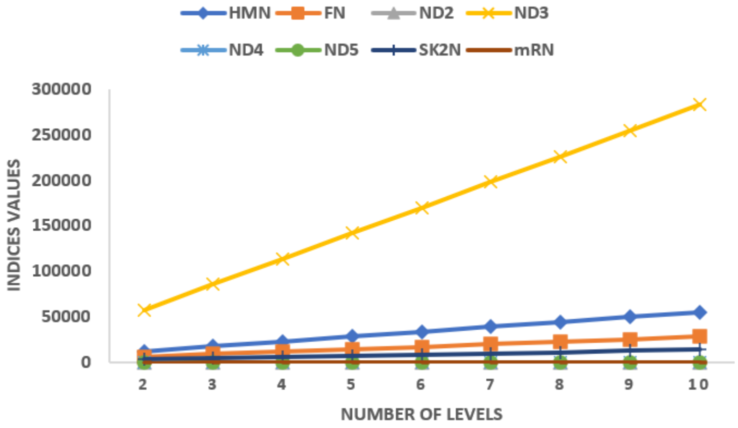

Fig. 3 Graphical Depiction of Numerical Computation based on the Open Neighbourhood Indices

4 Results on Closed Neighborhood Degree Based Descriptors of Metal-Insulator Transition Superlattice Network

In this section, we compute the closed neighbourhood degree sum based topological indices of the Metal-Insulator Transition Superlattice Network.

4.1 Closed Neighborhood Degree Based Descriptors of Metal-Insulator Transition Superlattice Network when $\eta \neq 1$

Let SLN_η denote the Metal-Insulator Transition Superlattice Network. Here, we compute the 14 closed neighborhood descriptors, of SLN_η, when $\eta \neq 1$. Using the edge partitions of the network given in Table 4.

$$
\begin{aligned}
M_{1_{N^c}}\left(SLN_\eta\right) = & \sum_{uv\in E_1}\left[4+9\right] + \sum_{uv\in E_2}\left[4+10\right] + \sum_{uv\in E_3}\left[4+12\right] + \sum_{uv\in E_4}\left[8+9\right] \\
& + \sum_{uv\in E_5}\left[8+10\right] + \sum_{uv\in E_6}\left[8+12\right] + \sum_{uv\in E_7}\left[9+12\right] + \sum_{uv\in E_8}\left[10+14\right] \\
& + \sum_{uv\in E_9}\left[10+16\right] + \sum_{uv\in E_{10}}\left[12+12\right] + \sum_{uv\in E_{11}}\left[12+16\right] \\
& + \sum_{uv\in E_{12}}\left[14+14\right] + \sum_{uv\in E_{13}}\left[14+16\right]
\end{aligned}
$$

$$= 340\eta - 126.$$

$$\begin{aligned}
M_{2_{N^c}}\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\Big[4*9\Big] + \sum_{uv\in E_2}\Big[4*10\Big] + \sum_{uv\in E_3}\Big[4*12\Big] + \sum_{uv\in E_4}\Big[8*9\Big] \\
&+ \sum_{uv\in E_5}\Big[8*10\Big] + \sum_{uv\in E_6}\Big[8*12\Big] + \sum_{uv\in E_7}\Big[9*12\Big] + \sum_{uv\in E_8}\Big[10*14\Big] \\
&+ \sum_{uv\in E_9}\Big[10*16\Big] + \sum_{uv\in E_{10}}\Big[12*12\Big] + \sum_{uv\in E_{11}}\Big[12*16\Big] \\
&+ \sum_{uv\in E_{12}}\Big[14*14\Big] + \sum_{uv\in E_{13}}\Big[14*16\Big] \\
&= 2272\eta - 1536.
\end{aligned}$$

$$\begin{aligned}
HM_{N^c}\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\Big[4+9\Big]^2 + \sum_{uv\in E_2}\Big[4+10\Big]^2 + \sum_{uv\in E_3}\Big[4+12\Big]^2 + \sum_{uv\in E_4}\Big[8+9\Big]^2 \\
&+ \sum_{uv\in E_5}\Big[8+10\Big]^2 + \sum_{uv\in E_6}\Big[8+12\Big]^2 + \sum_{uv\in E_7}\Big[9+12\Big]^2 \\
&+ \sum_{uv\in E_8}\Big[10+14\Big]^2 + \sum_{uv\in E_9}\Big[10+16\Big]^2 + \sum_{uv\in E_{10}}\Big[12+12\Big]^2 \\
&+ \sum_{uv\in E_{11}}\Big[12+16\Big]^2 + \sum_{uv\in E_{12}}\Big[14+14\Big]^2 + \sum_{uv\in E_{13}}\Big[14+16\Big]^2 \\
&= 9232\eta - 6018.
\end{aligned}$$

$$\begin{aligned}
F_{N^c}\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\Big[4^2+9^2\Big] + \sum_{uv\in E_2}\Big[4^2+10^2\Big] + \sum_{uv\in E_3}\Big[4^2+12^2\Big] \\
&+ \sum_{uv\in E_4}\Big[8^2+9^2\Big] + \sum_{uv\in E_5}\Big[8^2+10^2\Big] + \sum_{uv\in E_6}\Big[8^2+12^2\Big] \\
&+ \sum_{uv\in E_7}\Big[9^2+12^2\Big] + \sum_{uv\in E_8}\Big[10^2+14^2\Big] + \sum_{uv\in E_9}\Big[10^2+16^2\Big]
\end{aligned}$$

$$+\sum_{uv\in E_{10}}\left[12^2+12^2\right]+\sum_{uv\in E_{11}}\left[12^2+16^2\right]$$
$$+\sum_{uv\in E_{12}}\left[14^2+14^2\right]+\sum_{uv\in E_{13}}\left[14^2+16^2\right]$$
$$=4688\eta-2946.$$

$$N^cD_1\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\sqrt{4*9}\right]+\sum_{uv\in E_2}\left[\sqrt{4*10}\right]+\sum_{uv\in E_3}\left[\sqrt{4*12}\right]$$
$$+\sum_{uv\in E_4}\left[\sqrt{8*9}\right]+\sum_{uv\in E_5}\left[\sqrt{8*10}\right]+\sum_{uv\in E_6}\left[\sqrt{8*12}\right]$$
$$+\sum_{uv\in E_7}\left[\sqrt{9*12}\right]+\sum_{uv\in E_8}\left[\sqrt{10*14}\right]+\sum_{uv\in E_9}\left[\sqrt{10*16}\right]$$
$$+\sum_{uv\in E_{10}}\left[\sqrt{12*12}\right]+\sum_{uv\in E_{11}}\left[\sqrt{12*16}\right]$$
$$+\sum_{uv\in E_{12}}\left[\sqrt{14*14}\right]+\sum_{uv\in E_{13}}\left[\sqrt{14*16}\right]$$
$$=168.1615\eta-29.0287.$$

$$N^cD_2\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{1}{\sqrt{4+9}}\right]+\sum_{uv\in E_2}\left[\frac{1}{\sqrt{4+10}}\right]+\sum_{uv\in E_3}\left[\frac{1}{\sqrt{4+12}}\right]$$
$$+\sum_{uv\in E_4}\left[\frac{1}{\sqrt{8+9}}\right]+\sum_{uv\in E_5}\left[\frac{1}{\sqrt{8+10}}\right]+\sum_{uv\in E_6}\left[\frac{1}{\sqrt{8+12}}\right]$$
$$+\sum_{uv\in E_7}\left[\frac{1}{\sqrt{9+12}}\right]+\sum_{uv\in E_8}\left[\frac{1}{\sqrt{10+14}}\right]+\sum_{uv\in E_9}\left[\frac{1}{\sqrt{10+16}}\right]$$
$$+\sum_{uv\in E_{10}}\left[\frac{1}{\sqrt{12+12}}\right]+\sum_{uv\in E_{11}}\left[\frac{1}{\sqrt{12+16}}\right]+\sum_{uv\in E_{12}}\left[\frac{1}{\sqrt{14+14}}\right]$$
$$+\sum_{uv\in E_{13}}\left[\frac{1}{\sqrt{14+16}}\right]$$
$$=0.37513\eta-1.66723.$$

$$
\begin{aligned}
N^{c}D_{3}\Big(SLN_{\eta}\Big) &= \sum_{uv\in E_1}\Big[(4*9)(4+9)\Big] + \sum_{uv\in E_2}\Big[(4*10)(4+10)\Big] \\
&+ \sum_{uv\in E_3}\Big[(4*12)(4+12)\Big] + \sum_{uv\in E_4}\Big[(8*9)(8+9)\Big] \\
&+ \sum_{uv\in E_5}\Big[(8*10)(8+10)\Big] + \sum_{uv\in E_6}\Big[(8*12)(8+12)\Big] \\
&+ \sum_{uv\in E_7}\Big[(9*12)(9+12)\Big] + \sum_{uv\in E_8}\Big[(10*14)(10+14)\Big] \\
&+ \sum_{uv\in E_9}\Big[(10*16)(10+16)\Big] + \sum_{uv\in E_{10}}\Big[(12*12)(12+12)\Big] \\
&+ \sum_{uv\in E_{11}}\Big[(12*16)(12+16)\Big] + \sum_{uv\in E_{12}}\Big[(14*14)(14+14)\Big] \\
&+ \sum_{uv\in E_{13}}\Big[(14*16)(14+16)\Big] \\
&= 63568\eta - 56208.
\end{aligned}
$$

$$
\begin{aligned}
N^{c}D_{4}\Big(SLN_{\eta}\Big) &= \sum_{uv\in E_1}\Big[\frac{1}{\sqrt{4*9}}\Big] + \sum_{uv\in E_2}\Big[\frac{1}{\sqrt{4*10}}\Big] + \sum_{uv\in E_3}\Big[\frac{1}{\sqrt{4*12}}\Big] \\
&+ \sum_{uv\in E_4}\Big[\frac{1}{\sqrt{8*9}}\Big] + \sum_{uv\in E_5}\Big[\frac{1}{\sqrt{8*10}}\Big] + \sum_{uv\in E_6}\Big[\frac{1}{\sqrt{8*12}}\Big] \\
&+ \sum_{uv\in E_7}\Big[\frac{1}{\sqrt{9*12}}\Big] + \sum_{uv\in E_8}\Big[\frac{1}{\sqrt{10*14}}\Big] + \sum_{uv\in E_9}\Big[\frac{1}{\sqrt{10*16}}\Big] \\
&+ \sum_{uv\in E_{10}}\Big[\frac{1}{\sqrt{12*12}}\Big] + \sum_{uv\in E_{11}}\Big[\frac{1}{\sqrt{12*16}}\Big] + \sum_{uv\in E_{12}}\Big[\frac{1}{\sqrt{14*14}}\Big] \\
&+ \sum_{uv\in E_{13}}\Big[\frac{1}{\sqrt{14*16}}\Big] \\
&= 1.0691\eta + 0.5333.
\end{aligned}
$$

$$
\begin{aligned}
N^cD_5\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\left[\frac{4^2+9^2}{(4*9)}\right]+\sum_{uv\in E_2}\left[\frac{4^2+10^2}{(4*10)}\right]+\sum_{uv\in E_3}\left[\frac{4^2+12^2}{(4*12)}\right]\\
&+\sum_{uv\in E_4}\left[\frac{8^2+9^2}{(8*9)}\right]+\sum_{uv\in E_5}\left[\frac{8^2+10^2}{(8*10)}\right]+\sum_{uv\in E_6}\left[\frac{8^2+12^2}{(8*12)}\right]\\
&+\sum_{uv\in E_7}\left[\frac{9^2+12^2}{(9*12)}\right]+\sum_{uv\in E_8}\left[\frac{10^2+14^2}{(10*14)}\right]+\sum_{uv\in E_9}\left[\frac{10^2+16^2}{(10*16)}\right]\\
&+\sum_{uv\in E_{10}}\left[\frac{12^2+12^2}{(12*12)}\right]+\sum_{uv\in E_{11}}\left[\frac{12^2+16^2}{(12*16)}\right]+\sum_{uv\in E_{12}}\left[\frac{14^2+14^2}{(14*14)}\right]\\
&+\sum_{uv\in E_{13}}\left[\frac{14^2+16^2}{(14*16)}\right]\\
&= 27.8214\eta+1.7905
\end{aligned}
$$

$$
\begin{aligned}
SK_{N^c}\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\left[\frac{4+9}{2}\right]+\sum_{uv\in E_2}\left[\frac{4+10}{2}\right]+\sum_{uv\in E_3}\left[\frac{4+12}{2}\right]\\
&+\sum_{uv\in E_4}\left[\frac{8+9}{2}\right]+\sum_{uv\in E_5}\left[\frac{8+10}{2}\right]+\sum_{uv\in E_6}\left[\frac{8+12}{2}\right]\\
&+\sum_{uv\in E_7}\left[\frac{9+12}{2}\right]+\sum_{uv\in E_8}\left[\frac{10+14}{2}\right]+\sum_{uv\in E_9}\left[\frac{10+16}{2}\right]\\
&+\sum_{uv\in E_{10}}\left[\frac{12+12}{2}\right]+\sum_{uv\in E_{11}}\left[\frac{12+16}{2}\right]+\sum_{uv\in E_{12}}\left[\frac{14+14}{2}\right]\\
&+\sum_{uv\in E_{13}}\left[\frac{14+16}{2}\right]\\
&= 170\eta-63.
\end{aligned}
$$

$$
\begin{aligned}
SK1_{N^c}\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\left[\frac{4*9}{2}\right]+\sum_{uv\in E_2}\left[\frac{4*10}{2}\right]+\sum_{uv\in E_3}\left[\frac{4*12}{2}\right]\\
&+\sum_{uv\in E_4}\left[\frac{8*9}{2}\right]+\sum_{uv\in E_5}\left[\frac{8*10}{2}\right]+\sum_{uv\in E_6}\left[\frac{8*12}{2}\right]\\
&+\sum_{uv\in E_7}\left[\frac{9*12}{2}\right]+\sum_{uv\in E_8}\left[\frac{10*14}{2}\right]+\sum_{uv\in E_9}\left[\frac{10*16}{2}\right]
\end{aligned}
$$

$$+\sum_{uv\in E_{10}}\left[\frac{12*12}{2}\right]+\sum_{uv\in E_{11}}\left[\frac{12*16}{2}\right]+\sum_{uv\in E_{12}}\left[\frac{14*14}{2}\right]$$
$$+\sum_{uv\in E_{13}}\left[\frac{14*16}{2}\right]$$
$$=1136\eta-768.$$

$$SK2_{N^c}\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{4+9}{2}\right]^2+\sum_{uv\in E_2}\left[\frac{4+10}{2}\right]^2+\sum_{uv\in E_3}\left[\frac{4+12}{2}\right]^2$$
$$+\sum_{uv\in E_4}\left[\frac{8+9}{2}\right]^2+\sum_{uv\in E_5}\left[\frac{8+10}{2}\right]^2+\sum_{uv\in E_6}\left[\frac{8+12}{2}\right]^2$$
$$+\sum_{uv\in E_7}\left[\frac{9+12}{2}\right]^2+\sum_{uv\in E_8}\left[\frac{10+14}{2}\right]^2+\sum_{uv\in E_9}\left[\frac{10+16}{2}\right]^2$$
$$+\sum_{uv\in E_{10}}\left[\frac{12+12}{2}\right]^2+\sum_{uv\in E_{11}}\left[\frac{12+16}{2}\right]^2+\sum_{uv\in E_{12}}\left[\frac{14+14}{2}\right]^2$$
$$+\sum_{uv\in E_{13}}\left[\frac{14+16}{2}\right]^2$$
$$=2308\eta-1504.5.$$

$$mR_{N^c}\Big(SLN_\eta\Big)=\sum_{uv\in E_1}\left[\frac{1}{max\{4,9\}}\right]+\sum_{uv\in E_2}\left[\frac{1}{max\{4,10\}}\right]+\sum_{uv\in E_3}\left[\frac{1}{max\{4,12\}}\right]$$
$$+\sum_{uv\in E_4}\left[\frac{1}{max\{8,9\}}\right]+\sum_{uv\in E_5}\left[\frac{1}{max\{8,10\}}\right]+\sum_{uv\in E_6}\left[\frac{1}{max\{8,12\}}\right]$$
$$+\sum_{uv\in E_7}\left[\frac{1}{max\{9,12\}}\right]+\sum_{uv\in E_8}\left[\frac{1}{max\{10,14\}}\right]+\sum_{uv\in E_9}\left[\frac{1}{max\{10,16\}}\right]$$
$$+\sum_{uv\in E_{10}}\left[\frac{1}{max\{12,12\}}\right]+\sum_{uv\in E_{11}}\left[\frac{1}{max\{12,16\}}\right]$$
$$+\sum_{uv\in E_{12}}\left[\frac{1}{max\{14,14\}}\right]+\sum_{uv\in E_{13}}\left[\frac{1}{max\{14,16\}}\right]$$
$$=0.9262\eta+0.3254.$$

$$\begin{aligned}
ISI_{N^c}\Big(SLN_\eta\Big) &= \sum_{uv\in E_1}\left[\frac{(4*9)}{4+9)}\right] + \sum_{uv\in E_2}\left[\frac{(4*10)}{4+10)}\right] + \sum_{uv\in E_3}\left[\frac{(4*12)}{4+12)}\right] \\
&+ \sum_{uv\in E_4}\left[\frac{(8*9)}{8+9)}\right] + \sum_{uv\in E_5}\left[\frac{(8*10)}{8+10)}\right] + \sum_{uv\in E_6}\left[\frac{(8*12)}{8+12)}\right] \\
&+ \sum_{uv\in E_7}\left[\frac{(9*12)}{9+12)}\right] + \sum_{uv\in E_8}\left[\frac{(10*14)}{10+14)}\right] + \sum_{uv\in E_9}\left[\frac{(10*16)}{10+16)}\right] \\
&+ \sum_{uv\in E_{10}}\left[\frac{(12*12)}{12+12)}\right] + \sum_{uv\in E_{11}}\left[\frac{(12*16)}{12+16)}\right] + \sum_{uv\in E_{12}}\left[\frac{(14*14)}{14+14)}\right] \\
&+ \sum_{uv\in E_{13}}\left[\frac{(14*16)}{14+16)}\right] \\
&= 83.2365\eta - 33.3245.
\end{aligned}$$

4.2 Numerical and Graphical Interpretations of Computed Results for Superlattice Network

Here, we performed computational techniques of superlattice network for all 14 closed neighbourhood indices. The results are shown in Tables 7 and 8, and Figures 4 and 5 depict the graphical representation of computations based on the closed neighbourhood indices.

Table 7 Computed Closed Neighbourhood Topological Indices Values for $(SLN)_\eta$, $\eta > 1$

n	$M_{1_{N^c}}$	$M_{2_{N^c}}$	HM_{N^c}	F_{N^c}	N^cD_1	N^cD_2	N^cD_3
2	554	3008	12446	6430	307.2943	-0.91697	70928
3	894	5280	21678	11118	475.4558	-0.54184	134496
4	1234	7552	30910	15806	643.6173	-0.16671	198064
5	1574	9824	40142	20494	811.7788	0.20842	261632
6	1914	12096	49374	25182	979.9403	0.58355	325200
7	2254	14368	58606	29870	1148.102	0.95868	388768
8	2594	16640	67838	34558	1316.263	1.33381	452336
9	2934	18912	77070	39246	1484.425	1.70894	515904
10	3274	21184	86302	43934	1652.586	2.08407	579472

Table 8 Computed Closed Neighbourhood Topological Indices Values for $(SLN)_\eta$, $\eta > 1$

n	N^cD_4	N^cD_5	SK_{N^c}	$SK1_{N^c}$	$SK2_{N^c}$	mR_{N^c}	ISI_{N^c}
2	2.6715	57.4333	277	1504	3111.5	2.1778	133.1485
3	3.7406	85.2547	447	2640	5419.5	3.104	216.385
4	4.8097	113.0761	617	3776	7727.5	4.0302	299.6215
5	5.8788	140.8975	787	4912	10035.5	4.9564	382.858
6	6.9479	168.7189	957	6048	12343.5	5.8826	466.0945
7	8.017	196.5403	1127	7184	14651.5	6.8088	549.331
8	9.0861	224.3617	1297	8320	16959.5	7.735	632.5675
9	10.1552	252.1831	1467	9456	19267.5	8.6612	715.804
10	11.2243	280.0045	1637	10592	21575.5	9.5874	799.0405

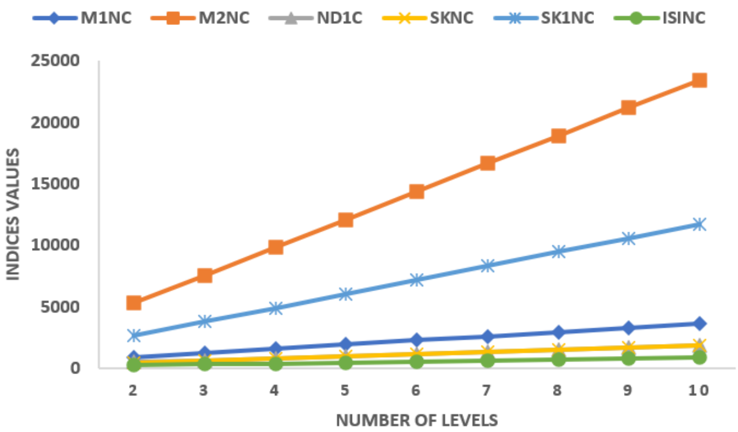

Fig. 4 Graphical Depiction of Numerical Computation based on the Closed Neighbourhood Indices

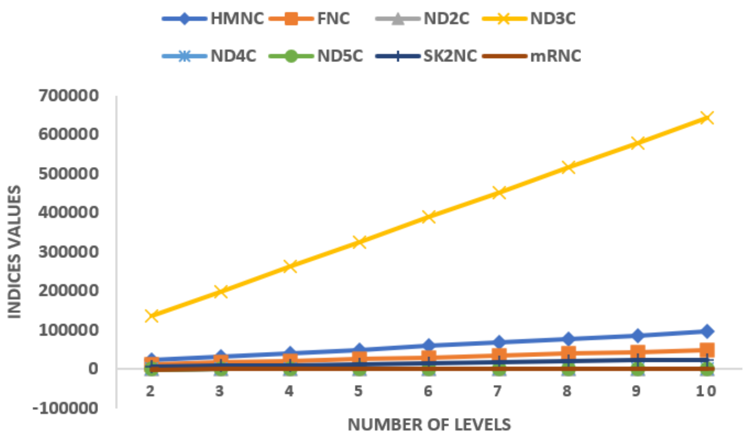

Fig. 5 Graphical Depiction of Numerical Computation based on the Closed Neighbourhood Indices

4.3 Closed Neighborhood Degree Based Descriptors of Metal-Insulator Transition Superlattice Network when $\eta = 1$

Let SLN_η denote the Metal-Insulator Transition Superlattice Network. Here, we compute the 14 closed neighborhood descriptors, of SLN_η, when $\eta = 1$. Using the edge partitions of the network given in Table 3.

$$
\begin{aligned}
&1.\ \ M_{1_{N^c}}(SLN_\eta) = 214, \quad 2.\ \ M_{2_{N^c}}(SLN_\eta) = 884 \\
&3.\ \ HM_{N^c}(SLN_\eta) = 3612, \quad 4.\ \ F_{N^c}(SLN_\eta) = 1844 \\
&5.\ \ N^cD_1(SLN_\eta) = 105.5805, \quad 6.\ \ N^cD_2(SLN_\eta) = 3.2395 \\
&7.\ \ N^cD_3(SLN_\eta) = 15280, \quad 8.\ \ N^cD_4(SLN_\eta) = 1.6630 \\
&9.\ \ N^cD_5(SLN_\eta) = 27.8500, \quad 10.\ \ SK_{N^c}(SLN_\eta) = 107 \\
&11.\ \ SK1_{N^c}(SLN_\eta) = 442, \quad 12.\ \ SK2_{N^c}(SLN_\eta) = 903 \\
&13.\ \ mR_{N^c}(SLN_\eta) = 1.4250, \quad 14.\ \ ISI_{N^c}(SLN_\eta) = 52.1111
\end{aligned}
$$

5 Conclusion

Topological descriptors serve an important role in chemistry by predicting the physical and chemical properties of molecules, as well as their bio-activities. In this paper, some open and closed neighbourhood degree sum based topological indices are computed for which can be used to predict different physico-chemical properties. As future work, we plan to apply these descriptors to various transformations of superlattice networks.

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Data Availability. Not Applicable

Declarations.

Conflict of Interest. The authors declare that they have no conflict of interest.

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