

# Study on the Influence Coefficient of Residual Stress used to Estimate the Stress Life Curve of Gears

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## Original Article

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## Title page

# Study on the Influence Coefficient of Residual Stress used to Estimate the Stress Life Curve of Gears

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## ORIGINAL ARTICLE

## Study on the Influence Coefficient of Residual Stress used to Estimate the Stress Life Curve of Gears

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**Abstract:** This study aims to estimate the gear S-N curve by the influence coefficient  $m$  of the residual stress at the root of the gear for providing theoretical basis and experimental support for the study of the bending fatigue performance of the gear. Based on the crack propagation theory and the linear damage accumulation theory, according to the Goodman relation, the residual stress is treated as the average stress. The spatial distribution of the residual stress is considered, and the residual stress is treated as a step function about depth, so the crack propagation process is divided into several stages. Through the fatigue test, the S-N curve of the gear is obtained. In order to avoid the influence of gear material and shape on the coefficient  $m$ , the S-N curve after heat treatment is used as the initial S-N curve, and the S-N curve after shot peening is used as the result curve. Through S-N curve, the influence coefficient  $m$  of tooth root residual stress is calculated, and the S-N curve of gears after shot peening is deduced by  $m$  value, which is compared with the S-N curve obtained by experiment. The influence coefficient  $m$  of residual stress calculated by S-N curve is 0.2132. The S-N curve of shot peening derived from  $m$  value is  $\ln S = 7.6963 - 0.0821 \ln N$ , which is consistent with the trend of S-N curve obtained by experiment, and the calculated data are more secure in the case of high cycle fatigue. The huge capital and labor cost of gear fatigue test can be saved by estimating gear S-N curve more accurately by  $m$  value. It provides a theoretical and experimental basis for the study of the influence coefficient of tooth root residual stress, and provides a solution for estimating the gear S-N curve.

**Key words :** Tooth root residual stress • The residual stress influence coefficient • S-N curve • Crack growth rate • Linear damage accumulation theory

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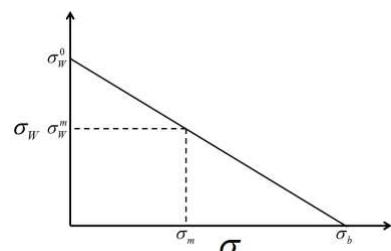
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## 1 Introduction

China's gear industry is developing towards surface integrity manufacturing and anti-fatigue manufacturing[1]. When studying the bending fatigue performance of gears, the residual stress is usually taken as the main basis and the fatigue limit is taken as the final criterion. The accurate acquisition of fatigue limit needs to be calculated by fatigue test, but the capital, time and labor cost of fatigue test are large[2]. Residual stress is an important reference in the study of fatigue performance, and the state of residual stress reflects the fatigue performance of materials: residual compressive stress can improve the fatigue performance of materials[3-4], and residual tensile stress will reduce the fatigue properties of materials[5-6]. Therefore, according to the characteristics of the influence of residual stress on fatigue performance, this paper proposes to use tooth root residual stress to estimate the fatigue limit.

At present, the residual stress is treated as the average stress when the residual stress is used to estimate the fatigue limit[7-9]. The effect of mean stress on fatigue limit is usually expressed in Goodman relationship[10-11], as shown in **Error! Reference source not found.**, where  $\sigma_m$  is the mean stress,  $\sigma_b$  is the tensile strength, and  $\sigma_w^0$  is the fatigue limit at  $\sigma_m = 0$ .



**Figure 1** Goodman Diagram

When the mean stress is  $\sigma_m$ , according to Goodman

relationship, the fatigue limit  $\sigma_w^m$  can be expressed as:

$$\sigma_w^m = \sigma_w^0 - (\sigma_w^0 / \sigma_b) \sigma_m = \sigma_w^0 - m \sigma_m, \quad (1)$$

where  $m = \sigma_w^0 / \sigma_b$  is the slope of the line called the mean stress sensitivity coefficient in **Error! Reference source not found.**[9].

When residual stress  $\sigma_r$  is present, Formula (1) is modified to

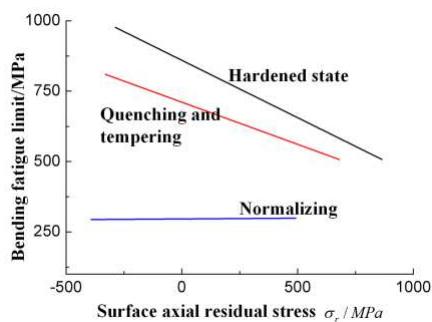
$$\sigma_w^{(m+r)} = \sigma_w^0 - m(\sigma_m + \sigma_r) = (\sigma_w^0 - m\sigma_m) - m\sigma_r, \quad (2)$$

According to Formulas (1) and (2), the fatigue limit changes due to residual stress are[12] :

$$\Delta\sigma_w^r = \sigma_w^{(m+r)} - \sigma_w^m = -m\sigma_r, \quad (3)$$

where  $m$  is also known as the residual stress influence coefficient[13] and residual stress action coefficient[9].

According to formulas (2) and (3), it can be seen that the fatigue limit of parts under residual stress can be estimated approximately as long as the  $m$  value is calculated. Scholars at home and abroad have done a lot of research on the calculation of  $m$  value. Syren B et al.[14] studied 45 steel under different heat treatment conditions. Figure 2 shows the relationship between surface residual stress and bending fatigue limit of parts under different conditions. We can obtain the  $m$  of normalizing state is 0, the  $m$  of quenching and tempering state is 0.27 and the  $m$  of hardening state is 0.4.



**Figure 2** Effect of processing residual stress on the bending fatigue limit of 45 steel planes in different states[14]

Hayama T et al.[15] obtained  $m$  of approximately 0.45 by conducting an experiment involving solid and hollow specimens after they subjected 45 steel notch specimens to induction quenching. Li Jinkui et al.[16] blasted 40Cr

steel-quenched and 550 tempered samples treated with shot peening and then performed a bending fatigue test to obtain  $m$  of 0.46; however,  $m$  of the sample treated with tempering plus shot peening at 200°C is 0.31. Zhang Dingxuan[9] comprehensively studied the surface deformation reinforcement of a low-temperature tempering state but with different carbon contents. The bending fatigue limit of the three chromium steel grinding states does not remarkably differ, but the fatigue limit after the surface deformation reinforcement changes greatly.  $m$  is as follows: 20Cr steel  $m = 0.18$ , 40Cr steel  $m = 0.15$ , and GCr15 steel  $m = 0.10$ .

The  $m$  values obtained by the abovementioned scholars differ greatly. In addition to different test conditions, the key lies in whether it separates the influence of various reinforcement (weakening) factors of the material. If no systematic analysis is carried out, the results of various effects are directly due to residual stress, which undoubtedly exaggerates its effect. As a result, a large error in  $m$  is obtained.

In a study on gear bending fatigue strength, material factors can affect  $m$ , and the shape of parts can cause errors in  $m$ . At the same time, because the distribution of residual stress in three-dimensional space is different, there will be a big error only using surface residual stress to calculate  $m$  value and estimate fatigue limit. The paper is based on crack propagation theory and linear damage accumulation theory. Considering the spatial distribution characteristics of residual stress, it is treated as a step function about depth. Then, through the  $S-N$  curve equation, analogizing formula (3), the  $m$  of tooth root is calculated. Because the  $S-N$  curve after heat treatment is used as the initial  $S-N$  curve and the  $S-N$  curve after shot peening is utilized as the result curve, the influence of a material and part shape on  $m$  can be avoided. Through that, an accurate  $m$  can be obtained, and the equation of  $S-N$  curve of gears can be derived. The present study provides the theoretical and experimental basis for predicting gear fatigue life.

## 2 Theoretical basis of solving the influence coefficient of residual stress by $S-N$ curve

In this paper, based on the linear damage accumulation theory of fatigue failure of parts, according to the characteristics of tooth root residual stress distribution along the depth, the crack propagation process is divided into several stages, and the proportion of each stage to the total life is calculated by the crack growth rate. Then the heat treatment  $S-N$  curve is used as the initial curve, the shot peening  $S-N$  curve as the result curve, and the  $S-N$  curve is used to solve the more accurate  $m$  value.

## 2.1 The theory of S-N curve needed to solve the value of $m$

In the case of a high-week fatigue, the fatigue performance of a material can be described as the relationship between the stress amplitude or the maximum stress which can characterize the cyclic load stress levels and the cycle time of material from initial to crack initiation (at this time, the given material is considered to be inoperative) which can characterize the fatigue life. The relationship is called the stress-life relationship or *S-N* curve[17].

The mathematical expressions of *S-N* curves are generally Wöhler formula, Basquin formula, Stromeyer formula, and Basquin's power function expression is the most commonly used[18]. In 1910, Basquin[19] studied the bending fatigue properties of materials and proposed a power function expression describing the *S-N* curve of materials:

$$S^n N = c, \quad (4)$$

Take the logarithm on both sides of Formula (4), and obtain:

$$\ln S = a \ln N + b, \quad (5)$$

where  $a = -1/n$ ,  $b = \ln c / n$ . Formula (5) indicates a log linear relationship between stress and life. By making an analogy of Formula (3), we find that  $S$  and  $N$  have a similar correspondence in the presence of residual stress:  $S = e^{(a \ln N + b)} - m\sigma_r$ , and  $\ln(S + m\sigma_r) = a \ln N + b$  is then obtained.

## 2.2 The theory of crack growth rate needed to solve the value of $m$

Fatigue crack growth rate  $dl/dN$  is the rate of change in crack length  $l$  with cycle time  $N$  under a given fatigue load, indicating the speed of crack growth[20-21].

In the calculation of the fatigue crack growth rate, the American Paris proposed the Paris formula for calculating the extended life of a crack under a constant load in 1961[22-23]:

$$\frac{dl}{dN} = C(\Delta K)^{m_k}, \quad (6)$$

where  $\Delta K$  is the range of stress intensity factors, and  $C$  and  $m_k$  are the material constants[24].

$$\Delta K = f \Delta \sigma \sqrt{\pi l}, \quad (7)$$

In Formula (7),  $f$  is a geometric correction factor. Using Formulas (6) and (7), we can obtain:

$$\frac{dl}{dN} = C(f \Delta \sigma \sqrt{\pi l})^{m_k} = Cf^{m_k} \pi^{\frac{m_k}{2}} \Delta \sigma^{m_k} l^{\frac{m_k}{2}}, \quad (8)$$

Given that  $C$ ,  $f$ , and  $\pi$  are constants,  $C_k = Cf^{m_k} \pi^{\frac{m_k}{2}}$  is constant, so we obtain:

$$\frac{dl}{dN} = C_k \Delta \sigma^{m_k} l^{\frac{m_k}{2}}, \quad (9)$$

Given that the Paris formula is highly accurate, simple, and convenient, it is widely used in fatigue design[25].

## 2.3 Linear damage accumulation theory needed to solve $m$ value

Palmgren (1924) and Miner (1945) independently proposed the theory of the linear injury accumulation of fatigue damage to quantitatively evaluate the contribution of different load levels to fatigue life.

Assuming that a part has a life  $N_i$  under the action of a constant cycle stress  $\sigma_i$ , we can define the damage it sustains after  $n_i$  times of cycles of that stress level as:

$$D_i = \frac{n_i}{N_i}, \quad (10)$$

For variable amplitude loads, if a part undergoes  $n_i$  times of cycles each under the action of  $k$  stress levels  $\sigma_i$ , the total damage to the part can be defined as:

$$D = \sum_{i=1}^k D_i = \sum_{i=1}^k \frac{n_i}{N_i}, \quad (11)$$

The total damage  $D=1$  corresponds to the complete damage of the part, that is fatigue failure[26]. The life  $N_i$  corresponding to different stress levels  $\sigma_i$  in Formula (11) must be determined on the basis of the *S-N* curve of the material.

## 2.4 Experimental determination of residual stress needed to solve $m$ value

Residual stress is the elastic stress that maintains its own balance caused by uneven plastic deformation inside a material[27]. Residual stress size varies with depth and is a function related to depth change:

$$\sigma_r = f(h), \quad (12)$$

where  $h$  is the depth, and this function can be obtained experimentally.

In this study, 20CrMnMo gear is used as an experimental material. The #1 gear is the gear after heat treatment, and the #2 gear is the gear after the shot peening reinforcement of the #1 gear. Tables 1 and 2 describe the specific parameters of the gear and the shot peening process parameters, respectively.

**Table 1** 20CrMnMo carburizing and quenching gear parameters

| Module                    | Number of teeth | Modification coefficient | Addendum coefficient |
|---------------------------|-----------------|--------------------------|----------------------|
| 5 mm                      | 32              | 0                        | 1                    |
| Top clearance coefficient | Pressure angle  | Tooth width              |                      |
| 0.25                      | 20°             | 20 mm                    |                      |

**Table 2** Shot peening process parameters

| Shot material               | Shot diameter | Shot peening pressure | Coverage rate |
|-----------------------------|---------------|-----------------------|---------------|
| pellets cut from steel wire | 0.6 mm        | 0.5 MPa               | 200%          |

Residual stress is detected through X-ray diffraction by using a Proto-LXRD high-power (1,200W) X-ray stress analyzer. Meanwhile, using an electrolytic polishing device, we can obtain the distribution of the residual stress at the root of the gear along the depth of the layer. Considering the gear material of 20CrMnMo, the Cr target is selected, the radiation type is Cr-K, the symmetrical face is BCC, the hkl plane is 211, and the Prague angle is 156.41°[28].

The data on the deep distribution of residual stress along the layer after the shot peening is obtained through an experiment (**Error! Reference source not found.**).

**Table 3** Data on the deep distribution situation of residual stress along the layer after shot peening

| Number of layers | Depth (μm) | Residual stress (MPa) |
|------------------|------------|-----------------------|
| 0                | 0          | -592.85               |
| 1                | 9.85       | -611.625              |
| 2                | 19.7       | -768.62               |

|   |        |           |
|---|--------|-----------|
| 3 | 29.55  | -1030.835 |
| 4 | 49.25  | -1078.2   |
| 5 | 59.1   | -1269.285 |
| 6 | 68.95  | -1172.88  |
| 7 | 88.65  | -985.555  |
| 8 | 108.35 | -837.09   |
| 9 | 147.75 | -553.59   |

Marines-Garcia I et al.[29] emphasized that fatigue life at the germination stage of fatigue cracks accounts for more than 90% of the total life span. Chen Q et al.[30] suggested that the germination life of cracks and the extended life of short cracks within 100 μm account for more than 80% of the total fatigue life when high-cycle life fatigue failure occurs. Thus, in this study, fatigue life is calculated on the basis of the crack expansion of 100 μm for 80% of the total life.

The value of residual stress is discrete in the experiment; thus, in this study,  $\sigma_r = f(h)$  function is considered a step function. According to **Error! Reference source not found.**, the residual stress function is expressed as follows:

$$\sigma_r = \begin{cases} -592.85, & 0 \leq h < 5 \\ -611.63, & 5 \leq h < 15 \\ -768.62, & 15 \leq h < 25 \\ -1030.84, & 25 \leq h < 45 \\ -1078.24, & 45 \leq h < 55 \\ -1269.29, & 55 \leq h < 65 \\ -1172.88, & 65 \leq h < 75 \\ -985.56, & 75 \leq h < 95 \\ -837.09, & 95 \leq h < 115 \end{cases}, \quad (13)$$

## 2.5 The method of solving $m$ value

To sum up, the residual stress obtained through experiments can be regarded as a ladder function of depth, and the crack propagation process can be divided into corresponding stages according to this function. When the  $S-N$  curve of the material is known, the material constant  $C_k$ ,  $m_k$  can be calculated according to the formula (9), thus the crack growth rate of each stage and the ratio of the life of each stage to the total life can be calculated. When the heat treatment  $S-N$  curve is used as the initial curve and the  $S-N$  curve after shot peening is derived from the root residual stress, assuming that there is no residual stress, then the two  $S-N$  curves are the same, and one stress  $S$  corresponds to one life  $N$ . Due to the existence of residual stress, the actual stress of the gear is  $S - m\sigma_r$ , and the life of the stress  $S$  of the gear after shot peening is the life corresponding to  $S - m\sigma_r$  of the heat treatment stress. Because the residual stress varies along the depth,

according to the linear damage accumulation theory, the proportion of the life of a stage to the total life corresponding to the actual stress in this stage is the same as that calculated by the crack growth rate. Therefore, a stress  $S$  is selected, and the life  $N_1$  derived from the heat treatment gear  $S-N$  curve and the tooth root residual stress after shot peening is equal to the life  $N_2$  corresponding to the stress  $S$  in the  $S-N$  curve of the gear after shot peening. As a result, a more accurate influence coefficient  $m$  of tooth root residual stress can be obtained, which provides a theoretical and experimental basis for later study of the effect of residual stress on fatigue performance.

### 3 The $S-N$ curve of gear is obtained by fatigue test

In this paper, the single tooth pulsating loading test of the test gear was carried out by using the national standard of gear bending fatigue test method (GB/T 14230) in order to obtain the  $S-N$  curve needed to calculate  $m$  value. The initial data of stress life are obtained, and then the  $S-N$  curve is fitted and solved[31].

#### 3.1 Determination of stress level in fatigue test

In this paper, five stress levels are determined to carry out the test, and the highest stress level should be as large as possible under the premise that the test material does not produce plastic deformation, so in order to determine the stress level reasonably, it is necessary to carry out the static load test of the gear. According to the conventional grouping method of GB/T 14230-93, we conduct a preparatory test, then, we get the stress levels of 20CrMnMo gears are 863.9MPa, 777.5MPa, 725.7MPa, 691.1MPa, 656.6MPa, respectively, and the stress levels of gears strengthened by shot peening are 950.3MPa, 863.9MPa, 777.5MPa, 742.9MPa, 725.7MPa[32].

#### 3.2 Determine the distribution function by fitting method

First, the life data obtained are arranged in an increasing order at the same stress level (**Error! Reference source not found. Error! Reference source not found.**, the relevant original data have been published in reference [32]). The failure probability of fatigue life is calculated on the basis of the empirical distribution function (14). The life data are then organized and calculated using the knowledge of probability theory. Lastly, whether the life data are consistent with the logarithm normal distribution or the two-parameter Weibull distribution is determined with the average value of the correlation coefficient and the average value of variance of the linear function fitted by the least square method.

$$F_{(N_i)} = \frac{i - 0.3}{z + 0.4}, \quad (14)$$

where  $i$  is the sequence number of each life value from small to large under the same stress level;  $z$  is the total number of samples at the same stress level, and  $F_{(N_i)}$  is the probability of failure of fatigue life  $N_i$  when the sequence number is  $i$ .

**Table 4** Bending fatigue life of ordinary gears[32]

| Sequence No. | Tooth root stress/MPa |       |        |        |          |
|--------------|-----------------------|-------|--------|--------|----------|
|              | 863.9                 | 777.5 | 725.7  | 691.1  | 656.6    |
| 1            | 14600                 | 20088 | 30700  | 52466  | 175676   |
| 2            | 20477                 | 20505 | 46612  | 52676  | 188648   |
| 3            | 26308                 | 37074 | 49087  | 58507  | 202856   |
| 4            | 27210                 | 39792 | 54451  | 87726  | 303060   |
| 5            | 34525                 | 51160 | 84020  | 94009  | 332132   |
| 6            |                       | 63416 | 96484  | 111292 | 419447   |
| 7            |                       |       | 112752 | 160327 | 504163   |
| 8            |                       |       |        | 169091 | 955716   |
| 9            |                       |       |        |        | 1047327  |
| 10           |                       |       |        |        | ≥3000000 |
| 11           |                       |       |        |        | ≥3000000 |
| 12           |                       |       |        |        | ≥3000000 |

**Table 5** Bending fatigue life of gears after shot peening[32]

| Sequence No. | Tooth root stress/MPa |       |        |        |        |
|--------------|-----------------------|-------|--------|--------|--------|
|              | 950.3                 | 863.9 | 777.5  | 742.9  | 725.7  |
| 1            | 9848                  | 26753 | 51762  | 52802  | 185353 |
| 2            | 11112                 | 34074 | 52281  | 74809  | 196583 |
| 3            | 14757                 | 41967 | 74526  | 76783  | 204638 |
| 4            | 23084                 | 51392 | 123374 | 112541 | 262523 |
| 5            | 27692                 | 56516 | 134523 | 153579 | 359025 |
| 6            |                       | 62262 | 193566 | 239174 | 578943 |
| 7            |                       |       | 279435 | 314675 | 641378 |

|    |        |          |
|----|--------|----------|
| 8  | 545325 | 835746   |
| 9  |        | 1314738  |
| 10 |        | ≥3000000 |
| 11 |        | ≥3000000 |

According to the above calculation and fitting formula, the data are processed. The related data of ordinary gear showed in **Error! Reference source not found.**; the relevant data of shot peening gear are presented in **Error! Reference source not found.**

**Table 6** The related data of ordinary gear

| Ordinary gear           | Tooth root stress/MPa           |     |     |     |     |       | Mean Variance | $m$   |
|-------------------------|---------------------------------|-----|-----|-----|-----|-------|---------------|-------|
|                         | 86                              | 77  | 72  | 69  | 65  | value |               |       |
|                         | 3.9                             | 7.5 | 5.7 | 1.1 | 6.6 |       |               |       |
| Correlation coefficient | Logarithmic normal distribution | 0.9 | 0.9 | 0.9 | 0.9 | 0.9   | 0.964         | 0.000 |
| n                       |                                 | 76  | 62  | 78  | 63  | 42    | 7             | 21184 |
|                         | Weibull distribution            | 7   | 8   | 4   | 8   | 0     | 9             |       |
|                         |                                 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9   | 0.951         | 0.001 |
|                         |                                 | 90  | 55  | 73  | 37  | 00    | 2             | 19674 |
|                         |                                 | 2   | 2   | 0   | 3   | 3     | 4             |       |

**Table 7** The relevant data of shot peening gear

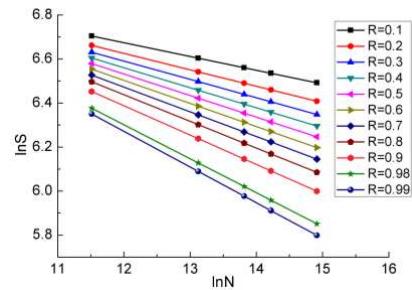
| Shot peening gear       | Tooth root stress/MPa           |     |     |     |     |       | Mean Variance | $m$   |
|-------------------------|---------------------------------|-----|-----|-----|-----|-------|---------------|-------|
|                         | 95                              | 86  | 77  | 74  | 72  | value |               |       |
|                         | 0.3                             | 3.9 | 7.5 | 2.9 | 5.7 |       |               |       |
| Correlation coefficient | Logarithmic normal distribution | 0.9 | 0.8 | 0.9 | 0.9 | 0.9   | 0.951         | 0.002 |
| n                       |                                 | 54  | 87  | 28  | 96  | 90    | 6             | 03082 |
|                         |                                 | 2   | 8   | 9   | 6   | 5     | 4             |       |
|                         | Weibull distribution            | 0.9 | 0.9 | 0.9 | 0.9 | 0.9   | 0.964         | 0.000 |
|                         |                                 | 33  | 39  | 68  | 95  | 85    | 6             | 75501 |
|                         |                                 | 7   | 4   | 2   | 8   | 8     | 1             |       |

The analysis of **Error! Reference source not found.** and **Error! Reference source not found.** indicates that the correlation coefficient of the logarithmic normal distribution function for ordinary gears is larger, and the variance is smaller. Thus, the logarithmic normal distribution is selected for the conventional gear. Similarly, the data of the shot peening gear conform to the two-parameter Weibull distribution.

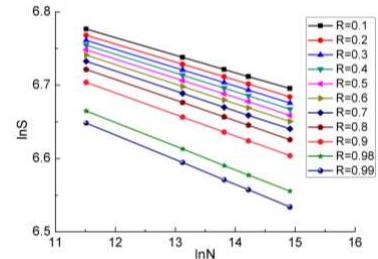
### 3.3 Solving the R-S-N curve of gear by fitting

R-S-N curve fitting is carried out in accordance with GB/T 14230-93. When encountering distortion (that the correlation coefficient of the fitted line is less than the

lowest critical value is called distortion), in order to obtain a more accurate S-N linear equation, it is necessary to analyze the undistorted part by using the fixed life method. Under the condition of determined life, the fatigue strength under different reliability is studied. Combined with the life data measured by the test, the determined life values selected  $10^5, 5 \times 10^5, 10^6, 1.5 \times 10^6, 3 \times 10^6$ , respectively. The final R-S-N curve is shown in **Error! Reference source not found.** and **Error! Reference source not found..**



**Figure 3** Distribution of fatigue strength at different reliabilities of ordinary gear life



**Figure 4** Distribution of fatigue strength at different reliabilities of shot peening gear life

When solving the  $m$  value, only one S-N curve of reliability is needed, so the S-N curve of reliability  $R=0.5$  is selected for calculation. When  $R=0.5$ , the S-N curve equation of the ordinary gear is  $\ln S = 7.7052 - 0.0978 \ln N$ , and the S-N curve equation of the shot peening gear is:  $\ln S = 7.0482 - 0.0261 \ln N$ .

### 4 Solve $m$ value and estimate S-N curve of shot peening gear

#### 4.1 Solving the material constant of crack growth rate formula by S-N curve

As shown in Formula (9),  $\frac{dl}{dN} = C_k \Delta \sigma^{m_k} l^{\frac{m_k}{2}}$ , the fatigue crack growth rate  $v$  is only related to  $\Delta \sigma$ , and  $C_k$  and  $m_k$  are the material constants. Thus,  $C_k$  and

$m_k$  can be obtained on the basis of the  $S-N$  curve of an ordinary gear.

In the case of taking the time consumed by crack propagation for 100 $\mu\text{m}$  as 80% of the total life, the fatigue crack growth rate is conservatively calculated. As indicated in Formula (9), the longer the crack, the faster the growth rate. The maximum rate is regarded as the average crack growth rate, then,  $C_k$  and  $m_k$  can be calculated.

The  $S-N$  equation is  $\ln S = 7.7052 - 0.0978 \ln N$ , and  $S=574.812$  and 516.254 MPa can be obtained when  $N=10^6, 3 \times 10^6$  is substituted into the calculation.

List the following equation:

$$\begin{cases} \frac{100}{C_k(\sqrt{574.812^2 \times 100})^{m_k}} = 10^6 \times 0.8 \\ \frac{100}{C_k(\sqrt{516.254^2 \times 100})^{m_k}} = 3 \times 10^6 \times 0.8 \end{cases}$$

Results:  $C_k = 3.192 \times 10^{-43}$ ,  $m_k = 10.225$ .

## 4.2 The proportion of each stage calculated by the crack growth rate

In general, cracks germinate from the subsurface. For the convenience of calculation, crack expansion is divided into several stages based on each step of the residual stress ladder function. The stage of the surface residual stress is removed. Therefore, the expansion is divided into eight stages in accordance with Formula (13). Conservatively, the maximum rate at each stage is calculated as the average rate.

Selecting 852.107MPa for the stress, and the life of each stage is calculated using  $n_i = \frac{l_i}{v_i}$ :

$$\begin{aligned} n_1 &= 2.63 \times 10^8, n_2 = 7.59 \times 10^6, n_3 = 4.39 \times 10^5, \\ n_4 &= 7.01 \times 10^4, n_5 = 2.76 \times 10^4, n_6 = 1.25 \times 10^4, \\ n_7 &= 6.94 \times 10^3, n_8 = 2.49 \times 10^3. \end{aligned}$$

Then,  $\sum n = \sum_{i=1}^8 n_i$ .  $\sum n$  is only 80% of the total

life, so the total life is  $\frac{\sum n}{0.8}$ . Thus, the proportion of each stage is respectively:

$$\begin{aligned} \frac{n_1}{n_t} &= 7.76 \times 10^{-1}, & \frac{n_2}{n_t} &= 2.24 \times 10^{-2}, \\ \frac{n_3}{n_t} &= 1.30 \times 10^{-3}, & \frac{n_4}{n_t} &= 2.07 \times 10^{-4}, \end{aligned}$$

$$\begin{aligned} \frac{n_5}{n_t} &= 8.16 \times 10^{-5}, & \frac{n_6}{n_t} &= 3.71 \times 10^{-5}, \\ \frac{n_7}{n_t} &= 2.05 \times 10^{-5}, & \frac{n_8}{n_t} &= 7.36 \times 10^{-6}. \end{aligned}$$

## 4.3 Calculating the influence coefficient $m$ of Tooth Root residual stress by $S-N$ Curve

The equation of the  $S-N$  curve of an ordinary gear is  $\ln S = 7.7052 - 0.0978 \ln N$ , and the  $S-N$  curve equation of the shot peening gear is  $\ln S = 7.0482 - 0.0261 \ln N$ .

In combination with the above and formulas (3), (5), (11), the following can be deduced:

$$\sum_{i=1}^8 \frac{n_i}{n_t} e^{\frac{\ln(S+m\sigma_{ri})-b}{a}} = 0.8 e^{\frac{\ln S - b_{peening}}{a_{peening}}} \quad (15)$$

Combined with the above paper and formula (15), we can calculate  $m=0.2132$ .

## 4.4 Estimation of $S-N$ curve of shot peening gear by $m$ value

The stress values of 700 and 1,000 MPa are selected for calculating the  $S-N$  curve equation of the shot peening gear in accordance with the abovementioned steps. Then, the result is:

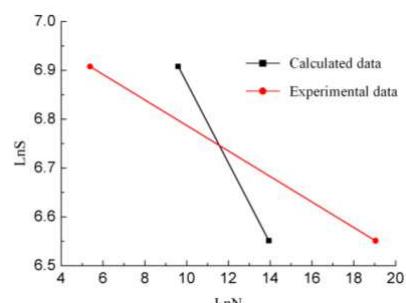
$$\ln S = 7.6963 - 0.0821 \ln N$$

Comparing with the experimental  $S-N$  curve, as shown in Figure 5, we can obtain:

(1) The trend of the  $S-N$  curve obtained by  $m$  is the same as that obtained by experiment;

(2) The calculated data is more safe in the case of high cycle fatigue, because the conservative calculation of  $m$  is adopted;

(3) When discussing the effect of residual stress on metal fatigue strength, it is only meaningful under high cycle fatigue[9].



**Figure 5** Comparison of the calculated *S-N* curve and the experimental *S-N* curve

## 5 Conclusion

The paper is based on crack propagation theory and linear damage accumulation theory. Considering the spatial distribution characteristics of residual stress, it is treated as a step function about depth. Then, through the *S-N* curve equation, the *m* of tooth root is calculated. Because the *S-N* curve after heat treatment is used as the initial *S-N* curve and the *S-N* curve after shot peening is utilized as the result curve, the influence of a material and part shape on *m* can be avoided. Through that, an accurate *m* can be obtained, and the equation of *S-N* curve of gears can be derived. The final calculated *m* of tooth root is 0.2132, and the *S-N* equation of shot peening gear obtained by calculation is  $\ln S = 7.6963 - 0.0821 \ln N$ . In the future, gears of similar heat treatment processes and materials can be calculated directly with *m*. The paper provides a theoretical and experimental basis for studying *m* and provides a feasible solution for *S-N* curve prediction.

In this paper, the initial data of gear fatigue test are fitted and solved according to the national standard (GB/T 14230-93), and the *R-S-N* curve clusters of heat treated gears and shot peening gears are obtained. When *R*=0.5, the *S-N* curve equation of the ordinary gear is  $\ln S = 7.7052 - 0.0978 \ln N$ , and the *S-N* curve equation of the shot peening gear is:  $\ln S = 7.0482 - 0.0261 \ln N$ .

The trend of the *S-N* curve obtained by *m* is the same as that obtained by experiment. The calculated data is more safe in the case of high cycle fatigue, because the conservative calculation of *m* is adopted. When discussing the effect of residual stress on metal fatigue strength, it is only meaningful under high cycle fatigue. The safe and effective estimation of gear *S-N* curve saves the cost of manpower and time, and provides a theoretical basis for anti-fatigue manufacturing of gears.

There are generally two methods to determine the gear fatigue stress life curve: the gear bench running test and the gear tooth pulsation loading test. The research results of this paper can be applied to the simplification of gear bench running test and further improve the accuracy of gear fatigue life prediction.

## 6 Declaration

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### Availability of data and materials

The data sets supporting the conclusions of this article are included within the article.

### Authors' contributions

The author' contributions are as follows: Zhen Wang and Zhong-Ming Liu were in charge of the whole trial; Zhen Wang and Long Chen wrote the manuscript; Jun-Wei Cheng and Long Chen were in charge of experiment.

### Competing interests

The authors declare no competing financial interests.

### Consent for publication

All the authors have agreed to publish.

### Ethics approval and consent to participate

Not applicable

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### Appendix

(1) The caption of the picture is as follows:

**Figure 2** Goodman Diagram

**Figure 3** Effect of processing residual stress on the bending fatigue limit of 45 steel planes in different states

**Figure 4** Distribution of fatigue strength at different reliabilities of ordinary gear life

**Figure 5** Distribution of fatigue strength at different reliabilities of shot peening gear life

**Figure 6** Comparison of the calculated S-N curve and the experimental S-N curve

(2) The caption of the table is as follows:

**Table 1** 20CrMnMo carburizing and quenching gear

parameters

**Table 2** Shot peening process parameters

**Table 3** Data on the deep distribution situation of residual stress along the layer after shot peening

**Table 4** Bending fatigue life of ordinary gears

**Table 5** Bending fatigue life of gears after shot peening

**Table 6** The related data of ordinary gear

**Table 7** The relevant data of shot peening gear

## Figures

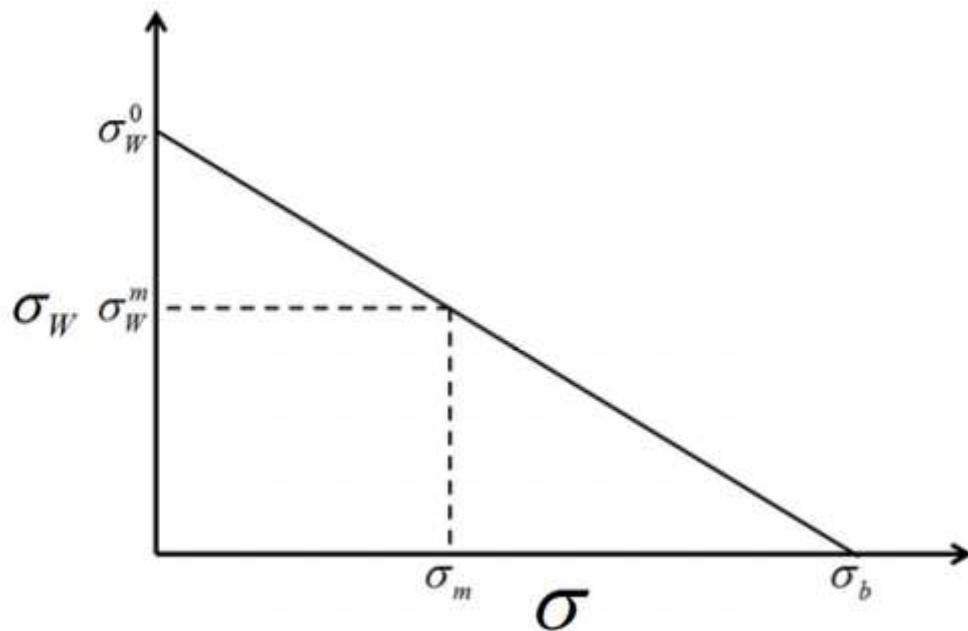


Figure 1

Goodman Diagram

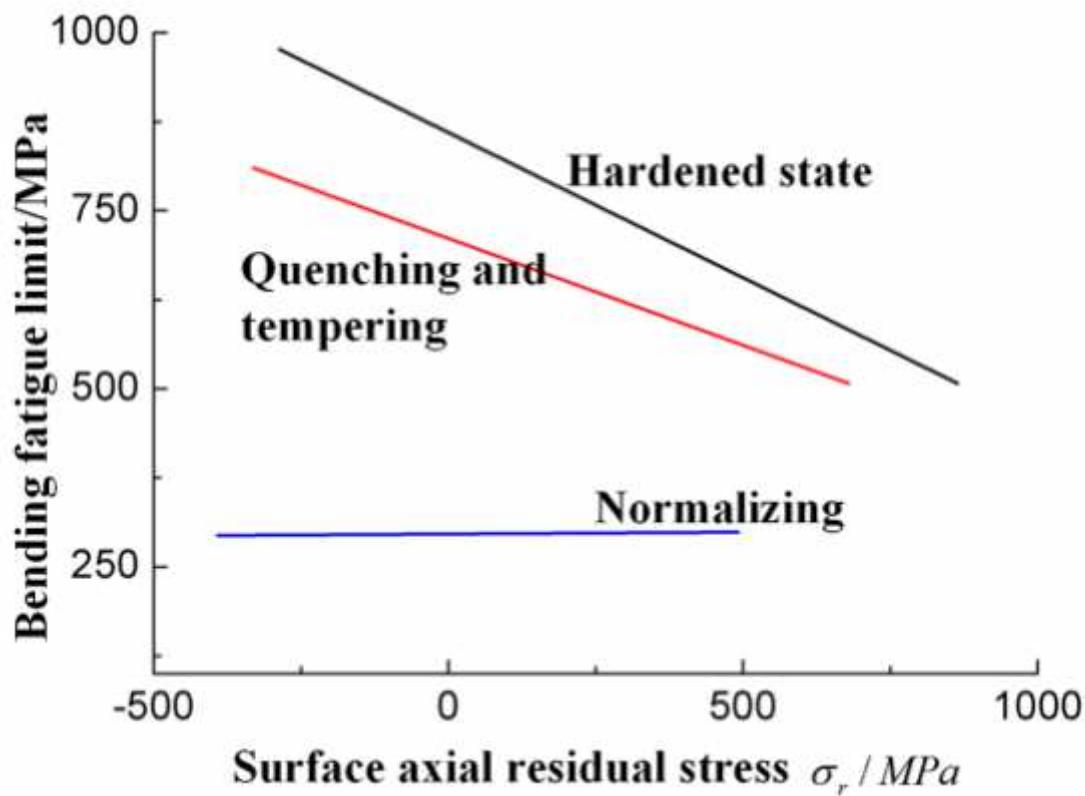


Figure 2

Effect of processing residual stress on the bending fatigue limit of 45 steel planes in different states[14]

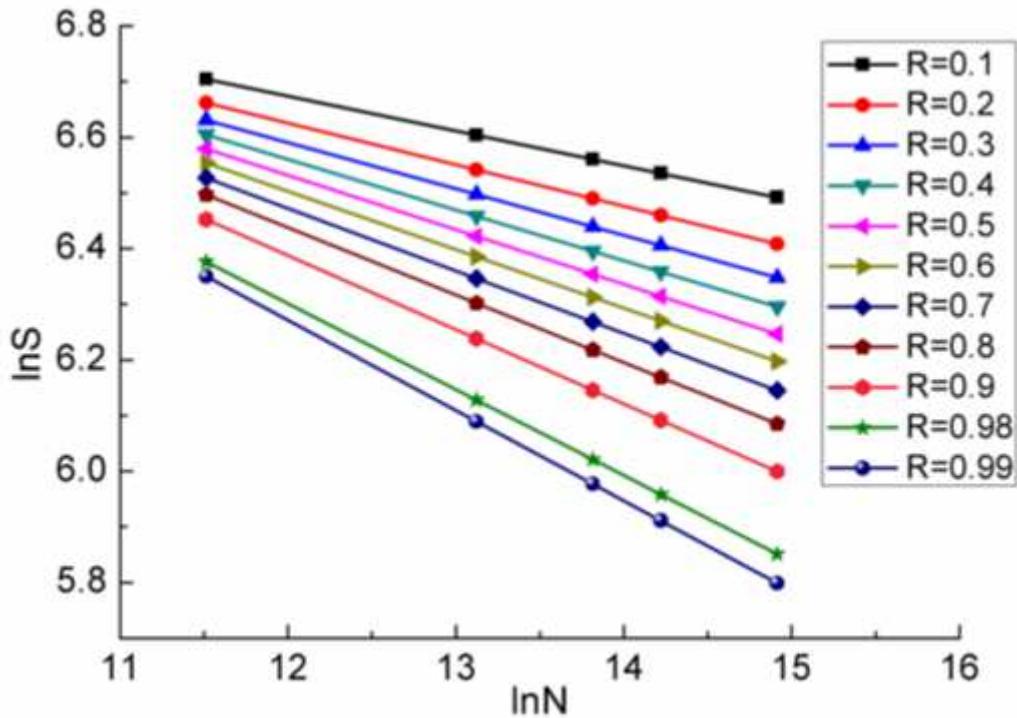


Figure 3

Distribution of fatigue strength at different reliabilities of ordinary gear life

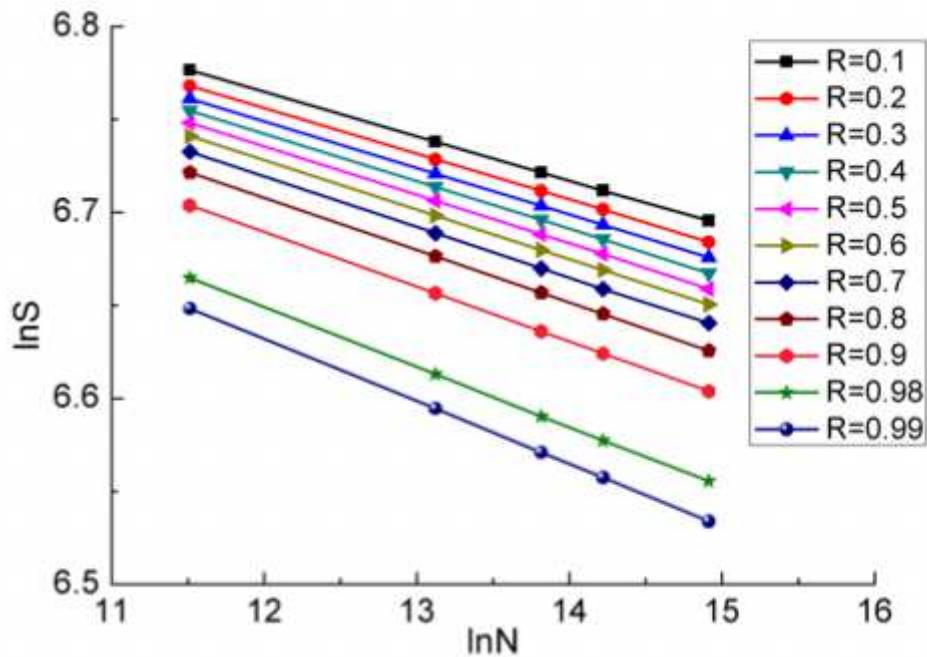
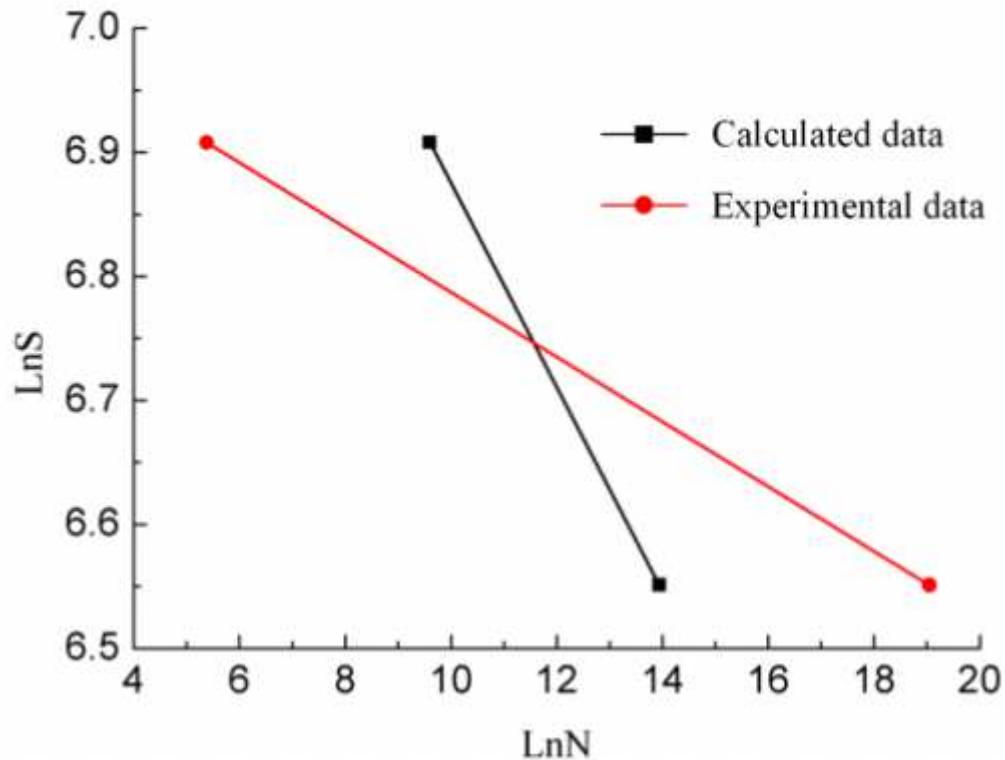


Figure 4

Distribution of fatigue strength at different reliabilities of shot peening gear life



**Figure 5**

Comparison of the calculated S-N curve and the experimental S-N curve