

# Comparison of representative heuristic algorithms for integrated reservoir optimal operation

**Wenzhuo Wang**

Hohai University

**Benyou Jia** (✉ [byjia@nhri.cn](mailto:byjia@nhri.cn))

Nanjing Hydraulic Research Institute <https://orcid.org/0000-0001-8363-0865>

**Slobodan P. Simonovic**

Western University

**Shiqiang Wu**

Nanjing Hydraulic Research Institute

**Ziwu Fan**

Nanjing Hydraulic Research Institute

**Li Ren**

Hohai University

---

## Research Article

**Keywords:** Differential evolution, Particle swarm optimization, Artificial physics optimization, Comparison of representative heuristic algorithms in integrated reservoir optimal operation, General process for solving integrated reservoir optimal operation.

**Posted Date:** March 30th, 2021

**DOI:** <https://doi.org/10.21203/rs.3.rs-201926/v1>

**License:**  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

---

# **Comparison of representative heuristic algorithms for integrated reservoir optimal operation**

**Wenzhuo Wang<sup>1</sup>, Benyou Jia<sup>2</sup>, Slobodan P. Simonovic<sup>3</sup>, Shiqiang Wu<sup>2</sup>,**

**Ziwu Fan<sup>2</sup>, Li Ren<sup>1</sup>**

<sup>1</sup> College of Hydrology and Water Resources, Hohai University, Nanjing, Jiangsu, 210098 China

<sup>2</sup> State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Nanjing Hydraulic Research Institute, Nanjing, Jiangsu, 210029 China

<sup>3</sup> Department of Civil and Environmental Engineering, Western University, London, Ontario, N6A 5B9 Canada

Corresponding author: Benyou Jia, Nanjing Hydraulic Research Institute, Nanjing, Jiangsu, 210029 China (byjia@nhri.cn)

## **Highlights:**

- The differential evolution (DE), particle swarm optimization (PSO), and artificial physics optimization (APO) were reviewed and compared using a designed experiment.

- The general model with constraints and fitness function, and the solving process using a hybrid feasible domain restoration method and penalty function method were presented for implementing HAs to reservoir.
- The computational time of DE and PSO is not significantly affected by the population size, while APO shows a geometric growth as the size increasing. DE maintains the population diversity and stability well compared with PSO and APO.
- The DE algorithm could be more appropriate for reservoir in terms of mean of optimal objective function values, the standard deviation of optimal objective function values, mean of computational time and population diversity.

## **Abstract**

Heuristic algorithms (HAs) are widely used in integrated reservoir optimal operation due to the fast calculation and simple design. Existing literature usually focuses on one or two categories of HAs and simply reviews the state of the art. To provide an overall understanding and specific comparison of HAs in integrated reservoir optimal operation, differential evolution (DE), particle swarm optimization (PSO), and artificial physics optimization (APO), which serves as typical examples of the three categories of HAs, were compared in terms of the development and applications using a designed experiment. Besides, the general model with constraints and fitness function, and the solving process using a hybrid feasible domain restoration method and penalty function method were also presented. Taking a designed experiment with multiple scenarios, four evaluation criteria are used for the comprehensive comparison. Results of the comparison show that (a) the problem of integrated reservoir optimal operation is a mathematic programming problem with the narrow feasible region and monotonic objective function; (b) it is easy to obtain the same optimal objective function value but different optimal solutions when using heuristic algorithms; and (c) DE, PSO and APO does not result in a clear winner, but DE could be more appropriate for integrated reservoir operation optimization according to the four evaluation criteria used in this study.

**Key Points:** Differential evolution; Particle swarm optimization; Artificial physics optimization; Comparison of representative heuristic algorithms in integrated reservoir optimal operation; General process for solving integrated reservoir optimal operation.

# 1 Introduction

The reservoir is the most effective engineering infrastructure that provides water storage and redistributes water inflow in time and space. Most reservoirs, especially large-scale water management projects, are designed as integrated projects that may provide for flood control, municipal and industrial water supply, hydroelectric power, agricultural irrigation, navigation, and wetland protection (Labadie 2004). Generally, reservoir operation is trying to answer the following question: *when to release the water from the storage and how much water should be released at that time?* This is a typical sequential decision-making problem (Oliveira and Loucks 1997). However, various factors and restrictions involved in the problem, such as politics, emergency, precipitation monitoring, flood forecasting, state of the equipment, water demand, decision-makers' preferences, uncertainty make the real-time reservoir operation quite complicated (Rani and Moreira 2010). Optimization techniques of integrated reservoir operation remain one of the hot topics in water resources management.

Optimization techniques have been gradually introduced to reservoir management and operations since the 1960s. Then a large number of publications and achievements have emerged with the classical optimization techniques, which is analytical. Yeh (1985) reviewed the reservoir management and operations models, including linear programming (LP), dynamic programming (DP), and non-linear programming (NLP). Generally, the traditional techniques contribute to the optimization with clear mathematical formulation, objective function, and constraints, but also exhibit limitations on non-convex optimization problems and real-time reservoir operation implementation (Ateş and Yeroğlu 2018). Besides, the computational cost of classical techniques grows considerably as the reservoir number increases (Sharif and Swamy 2014).

As computational intelligence rapidly develops in the past two decades, heuristic algorithms based on computational intelligence are becoming significant approaches for solving optimization problems and widely explored in solving reservoir operation problems (Al-Jawad and Tanyimboh 2017; Jia et al. 2015; Yang et al. 2020). These algorithms provide fast computation, simple design, global optimal solutions, and superior generality. They can easily overcome lots of limitations of classical reservoir optimization approaches, such as equality constraints, high-dimensional computation, nonlinear relationships, calculation efficiency, multiple objectives (Dahmani and Yebdri 2020).

According to the difference in simulation approach, heuristic algorithms are divided into two categories: algorithms based on simulation of biological intelligent behavior, which can be further divided into (a) evolutionary computation algorithms and (b) swarm intelligence algorithms, and (c) algorithms based on simulation of physical and chemical laws (L 2008). The evolutionary computation algorithms mainly include genetic algorithm (GA) (Srinivas and Deb 1994), differential evolution (DE) (Storn and Price 1997), evolutionary programming (EP) (David 1998). The swarm intelligence algorithms mainly include particle swarm optimization (PSO) (Kennedy and Eberhart 1995), artificial bee colony (ABC) (Karaboga 2005), ant colony optimization (ACO) (M 1996), cuckoo search (CS) (Yang and Suash 2009), tabu search (TS) (Glover 1986). The physical and chemical law-based algorithms mainly include central force optimization (CFO) (Formato 2007), gravitational search algorithm (GSA) (Esmat et al. 2009), artificial physics optimization (APO) (Spears and Gordon 1999), simulated annealing algorithm (SAA) (Kirkpatrick et al. 1983).

There are no free lunch theorems for optimization (Wolpert and Macready 1997). Every heuristic algorithm has advantages but also disadvantages and different heuristic

algorithms exhibit different performance on different problem instances (Chen et al. 1999). Much literature pays effort into the review of the applications of different algorithms. Labadie (2004) assessed the state-of-the-art in the optimization of reservoir system management using evolutionary algorithms, along with the application of neural networks and fuzzy rule-based systems for inferring reservoir system operating rules. Hossain and El-shafie (2013) discuss the performance of the evolutionary algorithms and the ability to integrate with other techniques on the ground of the benchmark problem. The results verify that the evolutionary algorithms overcome the drawbacks of traditional algorithms and the swarm intelligence is less comparatively than GA but provides a great scope for further development. Maier et al. (2015) reviewed the potential applications of different evolutionary algorithms in various application areas and paid significant research effort on the better solutions of reduced computational effort.

However, existing reviews usually focus on one or two types of heuristic algorithms comparing the different variants, which can not give an overall understanding of the advantages and disadvantages among the three categories. Besides, little literature gives a particular comparison of their performance of multi-objective reservoir optimal operation. To bridge these gaps, this paper presented specific comparison research with a designed experiment of differential evolution (DE), particle swarm optimization (PSO), and artificial physics optimization (APO) algorithms, which respectively represent evolutionary computation algorithms, swarm intelligence algorithms, and physical and chemical law-based algorithms. The goals include (a) characterizing multi-objective reservoir management and operation optimization problem; (b) identifying the difference between three typical heuristic algorithms on multi-objective reservoir optimal operation; (c) explaining why the three selected

heuristic algorithms result in different performance.

## 2 Review of DE, PSO, and APO

### 2.1 Differential evolution

Differential evolution (DE) was originally designed to solve the Chebyshev Polynomials and then widely used in reservoir operation. The core components of the DE algorithm include mutation operator, crossover operator, and selection operator.

#### (1) Mutation operator

DE can be divided into different modes based on the numbers and types of the selected parent population. The general convention used is DE/ $\alpha$ / $\beta$ / $\gamma$ , where  $\alpha$  is the vector to be perturbed (can be *Rand*: randomly selected or *Best*: best population),  $\beta$  the number of vectors considered for the mutation and  $\gamma$  the type of crossover used (can be *Exp*: exponential or *Bin*: binomial). The most commonly used two types of mutation operator are as follows:

DE/*Rand*/1/*Bin*

$$X(t)_{g+1}^i = X(t)_g^{r3} + P_S \cdot (X(t)_g^{r1} - X(t)_g^{r2}) , \quad \forall i \in [1, N_p] \quad (1)$$

DE/*Best*/1/*Bin*

$$X(t)_{g+1}^i = X(t)_g^{best} + P_S \cdot (X(t)_g^{r1} - X(t)_g^{r2}) , \quad \forall i \in [1, N_p] \quad (2)$$

where  $X(t)$  is an individual;  $P_S$  is the parameter scaling factor that controls the scaling weighting in mutation operator;  $i$  is an identifier for one individual;  $N_p$  is the total number of individuals in a population;  $g$  is an identifier for evolutionary generations; *best* is an identifier for the individual that has the best fitness;  $r1$ ,  $r2$ , and  $r3$  are random integers located in the interval  $[1, N_p]$ , they are not equal to each other as well as  $i$ , and also they are used as identifiers for individuals.

#### (2) Crossover operator

Test population will arise from the use of the crossover operator. The test population is generated between the parent population and mutation population according to the following (for minimization optimization problem):

$$Y(t)_{g+1}^i = \begin{cases} X(t)_g^i & , \text{ if } Rand \leq P_C \\ X(t)_{g+1}^i & , \text{ otherwise} \end{cases} , \quad \forall i \in [1, N_p] \quad (3)$$

where  $Y(t)$  is a test individual;  $P_C$  is the crossover rate parameter that controls the probability of mutation-to-test individual in crossover operator;  $Rand$  is a random value within uniform distribution  $[0, 1]$ ; other variables as shown above.

### (3) Selection operator

DE algorithm is a “greedy” selection, namely: comparison the fitness function value of test population with parent population, then the outstanding population is preserved and into the next evolution of the offspring population. The selection operator can be expressed as for minimization optimization problem):

$$X(t)_{g+1}^i = \begin{cases} Y(t)_{g+1}^i & , \text{ if } F[Y(t)_{g+1}^i] \leq F[X(t)_g^i] \\ X(t)_g^i & , \text{ otherwise} \end{cases} , \quad \forall i \in [1, N_p] \quad (4)$$

where  $F(\cdot)$  is the fitness function which is used for assessing the merit of the individual; other variables as shown above.

Since DE was proposed, many applications were documented in the literature as Table 1 shows. DE is computationally very efficient and offers a high level of robustness. Besides, DE is one of the best evolutionary algorithms for solving optimization problems with real-coded variables, which makes it more attractive for implementation in practice.

**Table 1.** Review of reservoir operation using differential evolution

Paper	Research
-------	----------

---

Reddy and Kumar (2008)	adopted multi-objective DE for the simultaneous evolution of optimal cropping pattern and operation policies for a multi-crop irrigation reservoir system with the objectives that maximize total net benefits and the total irrigated area.
Regulwar et al. (2010)	presented DE for the optimal operation of the multi-objective reservoir, in which the objective was the maximization of the hydropower production, while the irrigation supply was considered as a constraint. They showed that DE/ <i>best/1/bin</i> was the best strategy to obtain the optimal solution.
Qin et al. (2010)	introduced DE for multi-objective reservoir flood control operation optimization. This work tried to integrate the faith space of Cultural Algorithm into DE to form a mixed multi-objective algorithm for generating optimal trade-offs.
Basu (2014)	presented an improved DE for short-term hydrothermal operation, and an objective that minimizes the fuel cost of thermal plants while making use of the available hydropower as much as possible.
Schardong and Simonovic (2015)	came up with a novel fuzzy DE algorithm, as they adopted fuzzy numbers to represent vague knowledge and random alpha-cut levels for the search initialization in DE. Results from a reservoir operation case study problem showed fast convergence
Thang et al. (2018)	Proposed a self-tuned mutation operation to improve conventional DE, as it can open the local search zone in the quality of the solution, applied in solving two short-term hydrothermal scheduling problems showed well effectiveness.

---

## 2.2 Particle swarm optimization

Particle swarm optimization (PSO) is based on the foraging behavior of birds and takes advantage of sharing information (velocity and position) among particles

(vivid birds) in the whole swarm (vivid bird group). The movement of the whole swarm is generated from disorder to order evolutionary process in a search space of the optimization problem. The optimal solution is obtained within a finite number of evolutions.

The basic components of PSO are velocity and position operator. Each individual can be seen as a particle, which has no weight and volume, while a particle flies at a certain velocity in search space. The flight velocity is dynamically adjusted by the flight experience of the individual and the flight experience of the group. The equations of velocity and position operators for updating particles are as follows:

$$\begin{cases} V(t)_{g+1}^i = w \cdot V(t)_g^i + c_1 \cdot r_1 \cdot [X(t)_g^{gbest} - X(t)_g^i] + c_2 \cdot r_2 \cdot [X(t)_g^{pbest} - X(t)_g^i] \\ X(t)_{g+1}^i = X(t)_g^i + V(t)_{g+1}^i \end{cases}, \forall i \in [1, N_p] \quad (5)$$

where  $X(t)$  is a position vector, which represents a particle (an individual);  $V(t)$  is a velocity vector, which represents an attribute in particle;  $w$  is the parameter of inertia weighting;  $c_1$  is the parameter of social factor;  $c_2$  is the parameter of cognitive factor;  $r_1$  and  $r_2$  are two random values within uniform distribution  $[0,1]$ ;  $gbest$  is an identifier for the particle that has the best fitness in all past  $g$  populations;  $pbest$  is an identifier for the particle that has the best fitness in  $g$ -th population; other variables ( $i, g, N_p$ ) are as shown above in the discussion of DE.

Different variants of PSO were proposed as Table 2 shows. The PSO is characterized by the easy implementation, high accuracy, and rapid convergence. Therefore, it attracted the attention of water resources specialists and demonstrated its superiority in solving water resources management problems.

**Table 2.** Review of reservoir operation using particle swarm optimization

Paper	Research
-------	----------

---

Clerc and Kennedy (2002)	presented a constriction factor $\chi$ to replace the parameter of inertia weight $w$ to ensure the PSO convergence.
Ratnaweera et al. (2004)	presented a modified PSO by the introduction of time-varying acceleration coefficients: TVAC.
Kumar and Reddy (2007)	adopted PSO to derive reservoir operation policies for multipurpose reservoir systems. The PSO algorithm was further improved by incorporating a new strategic mechanism called elitist-mutation to improve its performance in their study.
Mandal and Chakraborty (2011)	optimized the short-term combined economic emission scheduling of hydrothermal system with four reservoirs. This problem considered both, cost and emission as competing objectives. They used the PSO algorithm for solving the problem and their results show a considerable reduction of the computational time.
Afshar (2012)	came up with a constrained PSO for large scale reservoir operation with simultaneous consideration of reservoir water supply operation and hydropower generation. In this work, a new set of bounds is defined for each reservoir storage volume to control the quality and quantity of each feasible solution in PSO.
Zhang et al. (2014)	proposed an improved adaptive PSO for reservoir hydropower operation optimization. They introduced a dynamic parameter control mechanism in parameter settings in PSO and a new strategy for reservoir constraints handling. Thus, greater effectiveness and robustness were demonstrated in the results. In the problem of reservoir flood control operation optimization
Luo et al. (2015)	proposed a hybrid multi-objective PSO–EDA algorithm, in which divided the particle population into several sub-populations and builds probability models for each process, for solving single reservoir multi-objective flood control problem.

came up with a parallel multi-objective particle swarm optimization algorithm, in which the swarm with large population size was divided into several smaller subswarms to be simultaneously optimized by different worker threads, for solving cascade hydropower reservoir operation balancing benefit and firm output problem.

Niu et al. (2018)

presented a hybrid algorithm of particle swarm optimization and grey wolf optimizer for solving a single reservoir long-term operation problem with the objectives to minimize water supply deficits, results showed better performance in overcoming trapping in the local minima.

Dahmani and Yebdri (2020)

---

### 2.3 Artificial physics optimization

Artificial physics optimization simulates Newton's Second Law of Motion for global optimization problems. Each individual in the algorithm has attributes of mass, position, and velocity and the movement of the individual is affected by the virtual force in a search space. The main components of the APO algorithm include mass operator, force operator, velocity, and position operators.

#### (1) Mass operator

The mass of an individual is not a constant, but a function related to the definition of fitness value of an individual, and this mass function must be non-negative, bounded, and monotonic. The value of the mass function should be restricted between 0 and 1, and can be used as the measure of fitness value. The bigger the value of mass of the individual is, the better the fitness of the individual is. Therefore, the equation of the mass operator derived from the concave function is:

$$M_g^i = \exp \left\{ \frac{F[X(t)^{best}] - F[X(t)_g^i]}{F[X(t)^{worst}] - F[X(t)^{best}]} \right\}, \quad \forall i \in [1, N_p] \quad (6)$$

where  $M$  is the mass of an individual;  $F(\cdot)$  is the fitness function which is used for assessing the quality of an individual;  $X(t)$  is a position vector, which represents an

individual;  $\exp$  is natural exponential;  $best$  and  $worst$  are identifiers for the individuals that have the best and worst fitness values, respectively; other variables ( $i, g, NP$ ) are as presented above in the description of DE.

## (2) Force operator

The rule of gravitational force is: an individual with better fitness attracts the individual with poor fitness; the individual with the optimum fitness level has an absolute attraction, and attracts all the other individuals not affected by any force. Taking the minimization problem as an example, the force operator among individuals is:

$$CF(t)_g^{i,j} = \begin{cases} G \cdot M_g^i \cdot M_g^j \cdot R(t)_g^{i,j} , & F[X(t)_g^i] < F[X(t)_g^j] \\ -G \cdot M_g^i \cdot M_g^j \cdot R(t)_g^{i,j} , & F[X(t)_g^i] \geq F[X(t)_g^j] \end{cases} , \forall i \neq j; i \neq best \quad (7)$$

where  $CF(t)$  is the virtual component force vector between  $i$ -th and  $j$ -th individual;  $G$  is the gravitational constant;  $R(t)$  is the distance vector, which can be calculated as  $R(t)_g^{i,j} = \|X(t)_g^i - X(t)_g^j\|$ ; other variables as shown above.

The virtual resultant force of an individual is the summation of the virtual component forces among all other individuals as follows:

$$RF(t)_g^i = \sum_{k=1, k \neq best}^{NP} CF(t)_g^{i,k} , \forall i \neq best \quad (8)$$

where  $RF(t)$  is the virtual resultant force vector; other variables as shown above.

## (3) Velocity and position operator

Each individual is going to move in the direction of the virtual resultant force according to Newton's Second Law of Motion. Besides, a random variable is added to the velocity equation to guarantee that individuals without 0 probability have access to all points in the search space. The velocity and position operator are calculated as

follows:

$$\begin{cases} V(t)_{g+1}^i = w \cdot V(t)_g^i + r \cdot RF(t)_g^i / M_g^i \\ X(t)_{g+1}^i = X(t)_g^i + V(t)_{g+1}^i \end{cases}, \forall i \neq best \quad (9)$$

where  $w$  is the weighting parameter of inertia;  $r$  is a random variable, which obeys the uniform distribution  $[0,1]$  or the normal distribution  $(0,1)$ ; and other variables as shown above.

The review of APO is given in Table 3. As Table 3 shows, APO is seldom used in reservoir operation.

**Table 3.** Review of artificial physics optimization

Paper	Research
Xie et al. (2011)	proved that the global convergence of APO is guaranteed 100% in terms of dissipative system theory.
Wang and Zeng (2013)	used APO as a multi-objective optimization algorithm with non-dominated sorting method. Currently, the research on APO is still in its infancy, and further studies are needed.
(Zhan et al. 2014)	adopted APO for minimizing the vulnerability of microgrid system, and proposed improvement of APO search vector. The results obtained were encouraging when compared with PSO

### 3 Integrated reservoir optimal operation model

The main purpose of the multi-objective reservoir optimal operation is finding the best way of meeting the water demand of multiple water users. The water users may include flood control, municipal and industrial water supply, hydroelectric power, agricultural irrigation, navigation, etc. When developing the long-term optimization operation model for a multipurpose reservoir, important purposes such as hydroelectric power generation and water supply are given priority (incorporated as the objective functions), and other purposes are considered as constraints.

### 3.1 Objective functions

For multi-objective reservoirs, with hydropower generation and water supply as the main users, maximization of hydropower generation and minimization of water supply deficit are the two most common goals considered in reservoir optimal operation. In this paper, the supplied water refers to the water demand taken directly from the reservoir by the diversion canal which does not participate in hydropower generation. The two goals can be expressed as the following objective functions:

$$\text{maximize } B = \sum_{t=1}^T (\eta \cdot H_t \cdot HR_t) \quad (10)$$

$$\text{minimize } C = \sum_{t=1}^T (ID_t - IR_t) \quad (11)$$

where B is the benefit function; C is the cost function; T is the total period of operation;  $\eta$  is the hydropower efficiency coefficient;  $H_t$  is the hydraulic head during the period t;  $HR_t$  is the power release during the period t;  $ID_t$  is the water supply-demand (such as agricultural irrigation and domestic water) during the period t;  $IR_t$  is the water supply from the reservoir during the period t.

The two objectives conflict with each other. Maximization of hydropower production requires a higher volume of water stored in the reservoir to produce more electricity, while minimization of water supply deficit tries to minimize the water supply deficit requiring more water from the reservoir to meet the demand of water supply. To handle this multi-objective competition problem, a standardization approach was applied. It is able to express two objectives in the same units by treating the water supply deficit ( $ID_t - IR_t$ ) the same as the water  $HR_t$  for hydropower during each period t, and taking hydroelectric power quantity as an indicator. Then, the final single objective function for the model can be written as:

$$\text{maximize } \Phi = \sum_{t=1}^T \eta \cdot H_t \cdot (HR_t - ID_t + IR_t) \quad (12)$$

where  $\Phi$  is the objective function of the reservoir optimal operation model.

### 3.2 Constraints

In the multi-objective reservoir optimal operation model, the following constraints are considered.

(1) Water balance:

$$V_t = V_{t-1} + (IN_t - HR_t - IR_t - S_t - L_t) \quad (13)$$

where  $V_{t-1}$  and  $V_t$  are the water storage at the beginning and end of the period  $t$ , respectively;  $IN_t$  is the water inflow during the period  $t$ ;  $S_t$  is the surplus water (such as spill) during the period  $t$ ;  $L_t$  is the total water loss (such as evaporation and leakage) during the period  $t$ ; and other variables as discussed above.

(2) Water storage limits:

$$V_{t,\min} \leq V_t \leq V_{t,\max} \quad (14)$$

where  $V_{t,\min}$  and  $V_{t,\max}$  are the lower and upper bounds, respectively, of water storage during the period  $t$ .

(3) Water for power generation limits:

$$HR_{t,\min} \leq HR_t \leq HR_{t,\max} \quad (15)$$

where  $HR_{t,\min}$  and  $HR_{t,\max}$  respectively represent the lower and upper bounds of water for power generation during the period  $t$ .

(4) Water supply limits:

$$IR_{t,\min} \leq IR_t \leq \min[IR_{\max}, ID_t] \quad (16)$$

where  $IR_{t,\min}$  is the minimum water supply requirement during the period  $t$ ;

$\min[IR_{\max}, ID_t]$  is the maximum water supply capacity.

(5) Minimum hydraulic head limit:

$$H_{\min} \leq H_t \quad (17)$$

where  $H_{\min}$  is the minimum hydraulic head.

(6) Initial and terminal conditions:

$$V_0 = V_{start} \quad (18)$$

$$V_T = V_{end} \quad (19)$$

where  $V_{start}$  and  $V_{end}$  respectively represent the initial and terminal conditions of water storage during the whole operation period.

## 4 Reservoir optimization with heuristic algorithms

The common heuristic strategy is usually adopted when heuristic algorithms are used to solve an optimization problem. Namely, algorithms use the random number for initiating the search and stop after a finite number of iterative operations resulting in the solution that is close to, or equal to, the optimal solution during the acceptable computational time. Reservoir optimal operation model belongs to the multi-constraint and nonlinear optimization problem. The process of solving this problem by heuristic algorithm requires (a) mapping relation between algorithm search space and reservoir feasible domain that is formed by the various constraints in the reservoir, and (b) mapping relation between algorithm fitness space and reservoir objective function domain (represented by the fitness function of the algorithm).

In this paper, a general process of implementing a heuristic algorithm to solve the multi-objective reservoir optimal operation model is put forward by using a hybrid feasible domain restoration method and penalty function method. The comparison of

the three selected heuristic algorithms is possible within the general process presented in the following section.

## 4.1 Process of computation

### Step 1. Individual initialization

A set of randomly generated reservoir outflows or reservoir water levels is regarded as an individual (for example, the individual will be represented by the reservoir outflow sequence in this paper), and the reservoir outflow sequence is encoded real numbers corresponding to each attribute. In this way, the search space of the heuristic algorithm is covered by randomly distributed multiple individuals that form a search swarm.

### Step 2. Search space definition

It is necessary to make clear the relationship between the feasible domain and the infeasible domain due to the multiple constraints in the reservoir optimal operation model. In the model, an individual of the heuristic algorithm is selected to be water release for hydropower generation ( $HR_t$ ) and water release for water supply ( $IR_t$ ). Therefore, the water outflow and water level can be calculated using the water balance constraint (Eq. 13) together with the initial conditions (Eq. 18). The feasible domain restoration method is adopted for the power generation limits (Eq. 15) and the water supply limits (Eq. 16). This process needs a restoration operator (see Section 4.2). The violations between the remaining unrestored constraints (Eq.14, Eq.17, and Eq.19) and individual attributes are calculated separately. The results are collectively calculated using the violation function of the other constraints ( $Viol(\cdot)$ ).

### Step 3. Fitness space definition

A clear relationship between the objective function domain and the non-

objective function domain in the multi-objective reservoir optimal operation model is necessary. The fitness space is formed when the penalty function method is implemented to establish a penalty function to be added to the objective function (Eq. 12). This process is called fitness function design (see Section 4.3). Then, following the principle of the “bigger the better” fitness function value, the individual carrying the maximum fitness function value is identified.

#### Step 4. Generate new individuals

Various operators, such as mutation, selection, position, mass, etc., are used to update the parental individuals, and to generate new offspring individuals for the next assessment and iteration. (Different operators may carry different names in different heuristic algorithms, however, they have the same meaning).

#### Step 5. Termination condition

In this step, the check is performed to see whether the iteration number reached the preselected maximum number of iterations  $g_{max}$ . If this criterion is not met, the process returns to Step 2 to continue the iterative calculations; if it is met, the iteration calculation ends with the optimal individual values and their fitness.

## 4.2 Constraints handling process

Water release for hydropower generation ( $HR_t$ ) and water supply ( $IR_t$ ) are encoded as the individuals of the heuristic algorithm. Constraints associated with these two variables directly (Eq. 15 and Eq. 16) can be restored by modifying the attributes of each individual, while the other constraints are harder to restore. Therefore, for reducing the difficulty of algorithm design, it is better to handle some of the constraints first. The restoration operators are as follows.

When the release  $HR_t$  encoded in the attribute of an individual violates the

power generation limits (Eq. 15), the repair of the value of the attribute is made by making the attribute of the individual equal to the value of the limit. This restoration operator is expressed as:

$$\text{If } HR_t < HR_{t,\min} \text{ Then } HR_t = HR_{t,\min} \quad (20)$$

$$\text{If } HR_t > HR_{t,\max} \text{ Then } HR_t = HR_{t,\max} \quad (21)$$

when the  $IR_t$  encoded attribute of an individual violates the water supply limits (Eq. 16), the repair of the value of the attribute is made by making the attribute value equal to the value of the limit. This restoration operator is expressed as:

$$\text{If } IR_t < IR_{t,\min} \text{ Then } IR_t = IR_{t,\min} \quad (22)$$

$$\text{If } IR_t > \min[IR_{\max}, ID_t] \text{ Then } IR_t = \min[IR_{\max}, ID_t] \quad (23)$$

### 4.3 Fitness function design in the heuristic algorithm

The two constraints are restored before, while the other constraints (Eq. 14, Eq. 17, and Eq. 19) are still in the unknown state. Fitness function design, which consists of the objective function and penalty function, will help in dealing with this unknown state. The fitness function is expressed in the heuristic algorithm as:

$$Fit(\cdot) = \Phi - \mu_g \cdot \sum_{t=1}^T Viol(V_t, H_t, HR_t, IR_t) \quad (24)$$

where  $u_g$  is a dynamic penalty function, and it can be calculated by  $\mu_g = \beta \cdot g / g_{\max}$ , and  $\beta$  is the maximum penalty value;  $g$  is the iteration number in the heuristic algorithm;  $g_{\max}$  is the maximum number of iterations; and  $Viol(V_t, H_t, HR_t, IR_t)$  is the violation function of the other constraints (Eq. 14, Eq. 17, and Eq. 19).

## 5 Experiment design

### 5.1 Case study reservoir

The multipurpose reservoir located in the south-west coastal region of India, where the rainfall is abundant with a tropical monsoon climate, is selected as a case

study reservoir. The rainy season of this region usually covers June, July, August, and September. The catchment of this reservoir is elongated, hilly with steep slopes with an area of about 892 km<sup>2</sup>. The reservoir is designed for purposes of hydropower generation and agricultural irrigation water supply, and the total storage is 2797×10<sup>6</sup> m<sup>3</sup>, corresponding to the water level of 659.9 m. The dead storage is 145×10<sup>6</sup> m<sup>3</sup>, corresponding to the water level of 609.6 m. The annual water release from the reservoir for hydropower generation is expected to be 1912×10<sup>6</sup> m<sup>3</sup>, while agricultural irrigation demand is 850×10<sup>6</sup> m<sup>3</sup>. The normal reservoir water level is 657.9 m. The comprehensive efficiency of the hydropower station is  $\eta=7.61$ .

## 5.2 Experiment scenario

To investigate the difference among the three selected algorithms, we have designed 3 experiment scenarios of the reservoir operation problem. The algorithms are set with the same common parameters, the population number, and the maximum number of iterations. The algorithm-specific parameters are set as good as possible, to provide consistent conditions for achieving the superior performance of each algorithm.

Three particular inflow years are selected, dry year (2871×10<sup>6</sup> m<sup>3</sup> total annual volume), normal year (4203×10<sup>6</sup> m<sup>3</sup> total annual volume), and wet year (6556×10<sup>6</sup> m<sup>3</sup> total annual volume). For the three heuristic algorithms, the population number is set to be 60, 100, and 150, and the maximum number of iterations is limited to 500, 800, and 1200, respectively.

The proposed procedure of solving the operation model with the heuristic algorithm is implemented using the computer programming language (VB.NET) and database technology (SQL 2012). The data and parameters are stored in the database to facilitate unified input, and each solution of the algorithm is recorded into an independent calculation module. Each algorithm is executed 10 times independently.

### 5.3 Evaluation criteria

Most applications of the optimization approach use the optimal objective function value as the only criterion for evaluation. Mean of optimal objective function values, the standard deviation of optimal objective function values, mean of computational time and population diversity are chosen to evaluate the specific performance of the three representative heuristic algorithms.

(1) Mean of optimal objective function values

$$Ave = \sum_{k=1}^n Fit(\cdot)_k / n \quad (25)$$

where  $Ave$  is the mean of optimal objective function values and obtained from  $n$  repetitions of independent calculations.

(2) Standard deviation of optimal objective function values

$$Dev = \sqrt{\frac{1}{n} \cdot \sum_{k=1}^n [Fit(\cdot)_k - Ave]^2} \quad (26)$$

where  $Dev$  is the deviation of optimal objective function values, and this value is obtained from  $n$  repetitions of independent calculations.

(3) Mean of computational time

$$Tim = \sum_{k=1}^n TCC(\cdot)_k \quad (27)$$

where  $Tim$  is the mean of computational time;  $TCC(\cdot)$  is the optimization computational time of (closely related to the computer's  $CPU$ , the optimization dimension  $T$ , the maximum number of iterations  $g_{max}$ , and the number of individuals  $N_P$ ).

(4) Population diversity

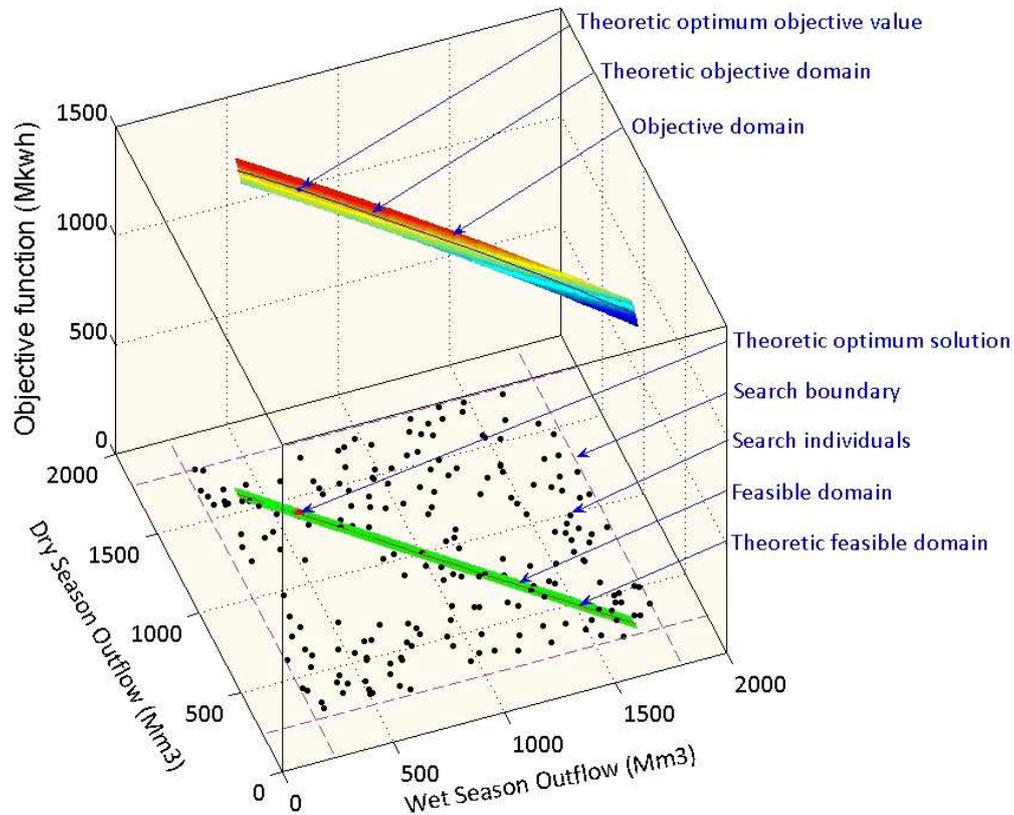
$$Div_g = \frac{\sum_{i=1}^{N_p} \sqrt{\sum_{t=1}^T \left[ X(t)_g^i - \sum_{i=1}^{N_p} X(t)_g^i / N_p \right]^2}}{N_p \times T} \quad (28)$$

where  $Div_g$  is the population diversity at  $g$ -th iteration in the calculation procedure. This criterion is derived from biology. In this paper, each heuristic algorithm has its search individual, and the individuals form a population, the proportion of the population in the whole search space can reflect the evolution degree of the heuristic algorithm.

## 6 Results and discussion

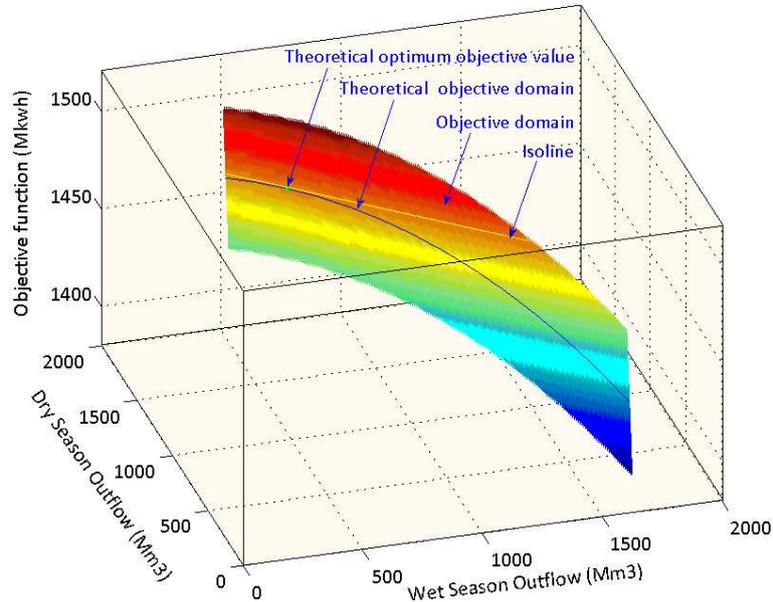
### 6.1 Mathematical model of optimal reservoir long-term operation

Three-dimensional (3D) representation is used in this paper, to better illustrate the multipurpose reservoir long-term optimal operation model solution obtained using heuristic algorithms. The year is divided into only two seasons (wet and dry), namely  $T=2$ , and then the optimization is performed under various control conditions. 3D schematic diagrams are shown in Figure 1 and Figure 2.



**Figure 1.** 3D schematic diagram of optimization results, feasible region and objective function

value



**Figure 2.** Enlarged view of the objective function domain

Figures 1 and 2 illustrate that:

(1) The dimensions of the theoretically feasible region with one dimension less than the number of decision variables (see the theoretical feasible solid line in Figure 1), due to the presence of initial water level and terminal water level equality constraints. However, because of pre-assigned optimal solution accuracy in actual reservoir operation, the feasible domain is narrow (see the green area in Figure 1).

(2) That the theoretical objective function could be monotonic (see the blue solid line in Figure 2), and the theoretical optimum objective value could be unique (see the green point in Figure 2). However, because of the optimal solution accuracy in actual reservoir operation, the objective function domain is distributed in the tangential direction of the theoretical optimum objective function value (see the yellow solid line

in Figure 2).

(3) That when heuristic algorithms are used to solve the problem (see the black dot in Figure 1), it is easy to obtain the same optimum objective function values, but different optimal solutions.

## 6.2 Comparison of heuristic algorithms

DE, PSO, and APO are used to solve the case study reservoir operation model, and all the evaluation criteria values are shown in Table 4. To more intuitively capture the different performance characteristics of compared algorithms, the Normal year inflow scenario is selected for the graphical presentation of results in Figures 3 to 6.

**Table 4.** The comparison results

Reservoir	Heuristic Algorithm	Population Size	Number of Iterations	Evaluation criterion			
				<i>Ave</i> (Mkwh)	<i>Dev</i>	<i>Tim</i> (s)	<i>Div<sub>gmax</sub></i>
Inflow	m	$N_p$	$g_{max}$				
$IN_t$	Name						
Dry	DE	60	500	1777.69	3.37	3.0	0.0668
		100	500	1778.92	3.49	3.2	0.0618
		150	500	1778.95	2.92	3.5	0.0528
	PSO	60	500	1772.30	5.93	3.0	0.1269
		100	500	1776.33	4.79	3.5	0.0528
		150	500	1778.92	4.64	3.8	0.0283
APO	60	500	1780.00	4.72	6.8	0.0100	
	100	500	1779.94	4.11	14.0	0.0046	
	150	500	1778.73	3.20	27.9	0.0060	
Normal	DE	60	800	3379.52	12.53	5.8	0.0830

		100	800	3376.74	5.64	6.2	0.0712
		150	800	3385.05	9.51	6.8	0.0707
		60	800	3382.74	10.16	6.3	0.0016
	PSO	100	800	3388.60	4.81	7.0	0.0160
		150	800	3387.25	5.51	7.8	0.0039
		60	800	3382.18	8.20	12.4	0.0077
	APO	100	800	3384.21	10.42	24.5	0.0046
		150	800	3388.69	6.55	47.3	0.0093
		60	1200	4969.27	12.84	9.1	0.1106
	DE	100	1200	4977.22	8.80	9.8	0.2060
		150	1200	4980.62	5.13	10.4	0.1608
		60	1200	4912.98	45.38	10.3	0.0000
Wet	PSO	100	1200	4892.50	12.74	11.8	0.0000
		150	1200	4916.17	49.36	13.0	0.0000
		60	1200	4987.55	2.34	19.1	0.0000
	APO	100	1200	4985.10	4.89	36.5	0.0000
		150	1200	4985.26	2.95	68.2	0.0000

The following conclusions are made from Table 4 and Figure 3 to Figure 6.

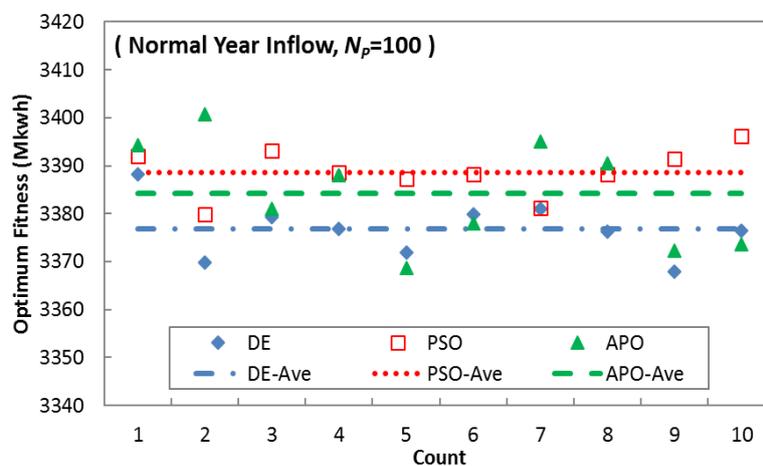
(1) For all three algorithms, the optimum objective values are improving with the increase of the population size. This phenomenon is reflected by the mean criterion of the optimal objective function values (see column 5 in Table 1). However, as the population size increases, the computational time for locating the optimal solution increases (see column 7 in Table 1).

(2) The mean of optimal objective function values is not a sufficient criterion to determine the best algorithm (see column 5 in Table 1 and Figure 3), because the algorithm search is random and the parameters of the algorithm have an important effect. However, from the deviation of optimal objective function value, DE shows better

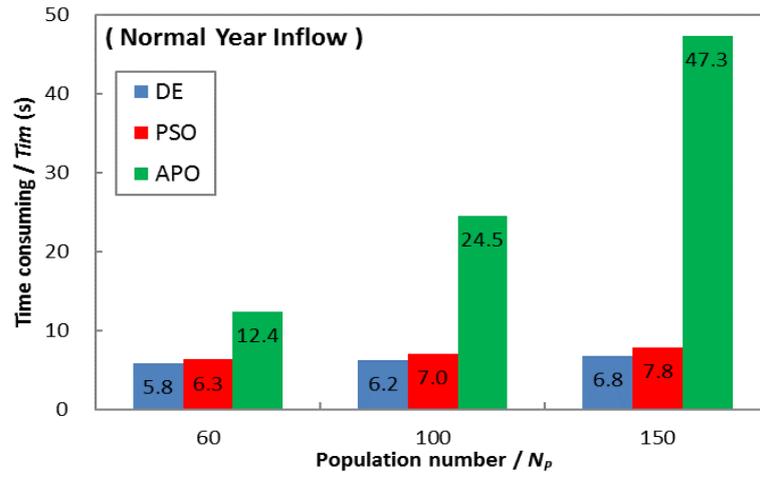
stability when compared to PSO and APO (see column 6 in Table 1).

(3) Under the same conditions, the computational time of DE and PSO is almost the same. DE's time is slightly shorter than PSO's, and the computational time of both algorithms increases with the increase in population size. The computational time of the APO is particularly affected by the population size showing a geometric increase with the increase of population size. This can be explained by the fact that APO involves the information exchange during the interaction between each pair of individuals (see column 7 in Table 1 and Figure 4).

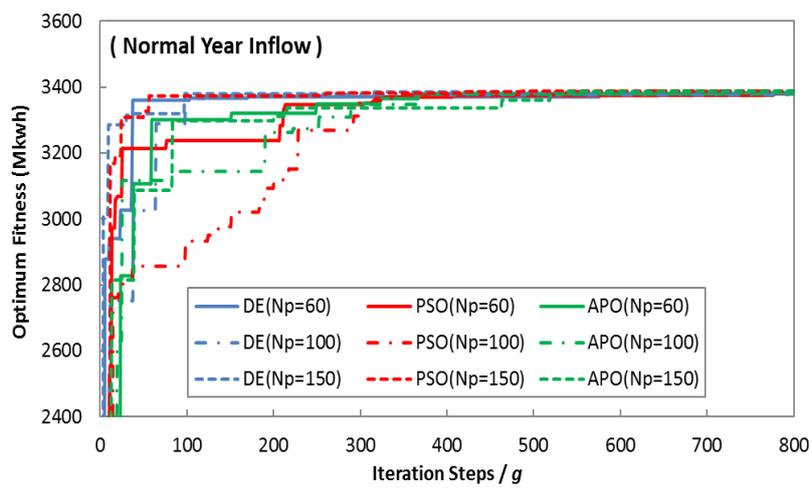
(4) The optimal objective function values obtained by the three algorithms are similar, but the search process is very different (see Figure 5, basically the first 200 steps show a fast convergence, and after 200 steps, the search becomes local). The population diversity criterion for DE still showing some numerical value at the maximum iteration step, but it has almost zero value for PSO and APO (see column 8 in Table 1). So the DE algorithm maintains a certain population diversity value unchanged, but PSO and APO show a decrease in value and fluctuation. Again, the APO performance is inferior to the other two algorithms (see Figure 6). Therefore, according to these analyses, DE shows a much higher potential of not falling into the trap of local optimal solution, while PSO and APO show the opposite.



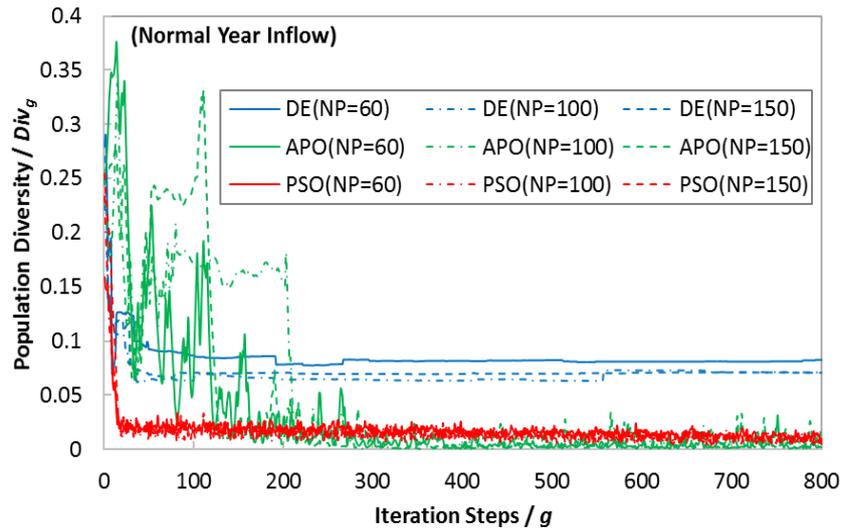
**Figure 3.** The optimum fitness value of 10 independent computations



**Figure 4.** The computational time of three heuristic algorithms



**Figure 5.** The optimum fitness value



**Figure 6.** The population diversity

### 6.3 Reservoir operation comparison

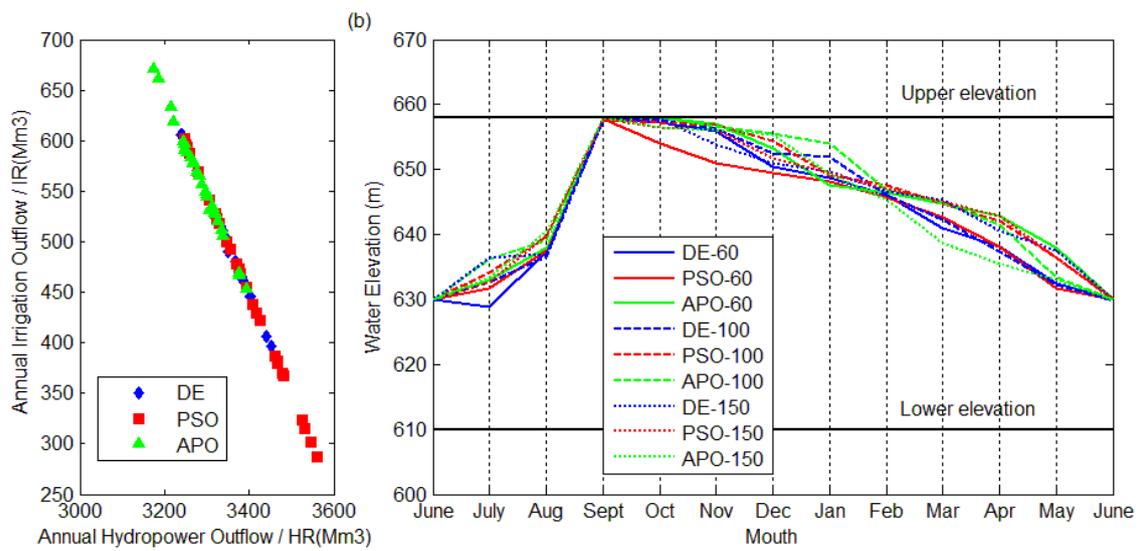
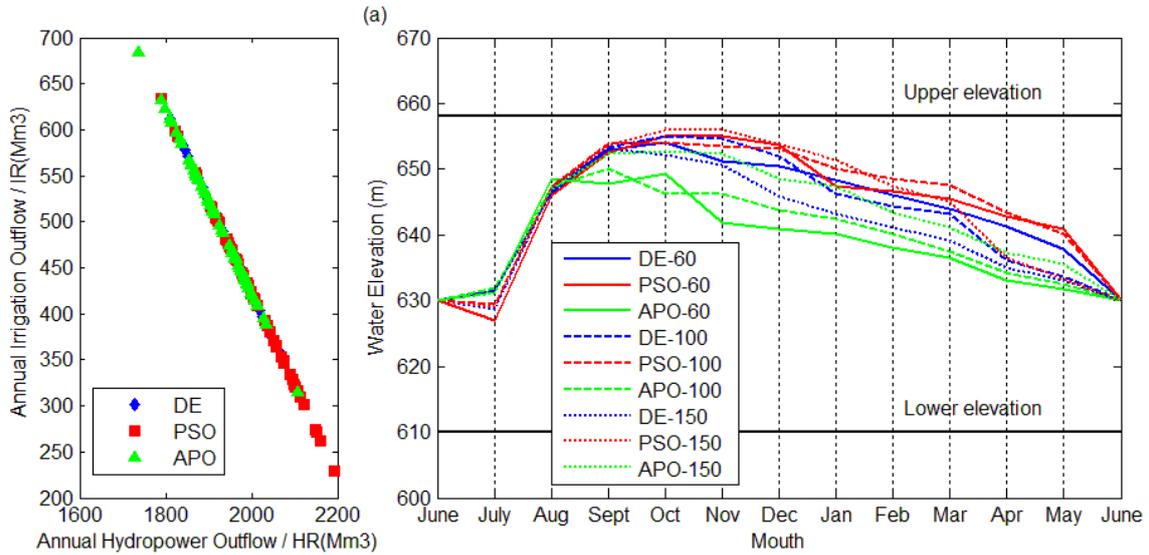
The optimal solutions of reservoir operations obtained using three heuristic algorithms are shown in Figure 7. The following observations are obtained from the analysis of results in Figure 7. The results show that:

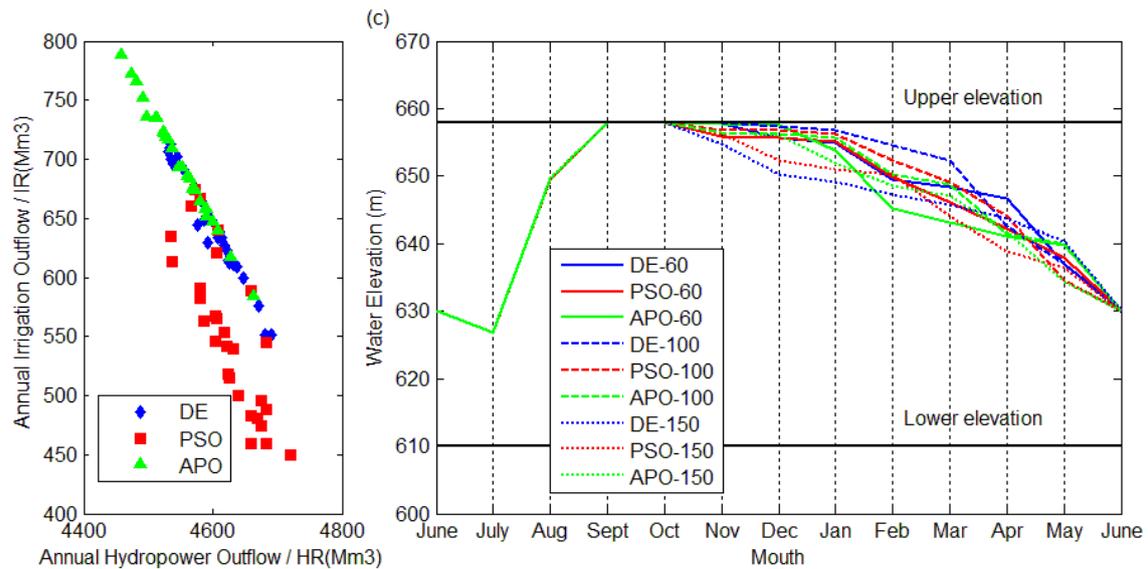
(1) The optimal solution obtained by APO tends to take provide more reservoir release for irrigation (the green triangles in Figure 7). The optimum solution obtained by PSO tends to release more water for hydropower generation (the red squares in Figure 7). The DE (blue diamonds in Figure 7) performance is superior to the other two algorithms.

(2) The PSO algorithm shows higher instability and may end up with the local non-convergence solution (in Figure 7, many red squares are out of a linear relation track).

(3) The reservoir water elevation, namely the optimum solution, is significantly different from month to month. This is also indicative of the same optimum objective value and different optimal solutions.

(4) The DE algorithm could be more appropriate for reservoir operation considering all of the four evaluation criteria.





**Figure 7.** The reservoir operation schedule results calculated by heuristic algorithms: (a) Dry year inflow, (b) Normal year inflow, (c) Wet year inflow.

## 7 Conclusions

This paper reviewed the application of the representative heuristic algorithm of the three categories, including DE, PSO, and APO, to reservoir optimal operation and compared the specific performance with a designed experiment. A general solution process with the implementation of heuristic algorithms is developed for the problem of integrated reservoir optimal operation together with the necessary techniques for dealing with constraints and fitness function design. Four evaluation criteria are used for the comparison of three algorithms of the designed experiment. The main conclusions obtained from the comparative analyses are as follows.

(1) The problem of integrated reservoir optimal operation is a mathematical programming problem characterized by the narrow feasible region and monotonic objective function. When this problem is solved using the heuristic algorithm based on the random search method, it is easy to obtain the same optimum objective function values but very different optimal solutions.

(b) the comparative analysis of DE, PSO, and APO algorithms based on the designed experiment shows similar performance in terms of the optimum objective value criterion. The computational time of DE and PSO is not significantly affected by the population size, while the computational time of APO shows a geometric growth as the size increases. DE maintains the population diversity and stability well compared with PSO and APO. When all four evaluation criteria are considered and resulting reservoir operations are taken into consideration, the DE seems to be more appropriate for integrated reservoir optimal operation than PSO and APO.

### **Acknowledgments, Samples, and Data**

This study has been funded by the National Key R & D Programme of China (2017YFC0404605), the National Natural Science Foundation of China (51709178) and Science and Technology Innovation Project of Shanghai Chengtou Group Corporation (CTKY-ZDXM-2020-005). The help provided by Dr. R. Arunkumar in furnishing the case study reservoir data is appreciated.

### **Declarations**

**Ethics Approval** Compliance with Ethical Standards

**Conflicts of Interest/Competing Interests** The authors declared that they have no conflicts of interest to this work..

**Consent to Participate** Informed consen.

**Consent to Publish** Informed consen.

### **References**

- Afshar MH (2012) Large scale reservoir operation by Constrained Particle Swarm Optimization algorithms Journal of Hydro-environment Research 6:75-87
- Al-Jawad JY, Tanyimboh TT (2017) Reservoir operation using a robust evolutionary

- optimization algorithm *Journal of Environmental Management* 197:275-286  
doi:<https://doi.org/10.1016/j.jenvman.2017.03.081>
- Ateş A, Yeroğlu C (2018) Modified Artificial Physics Optimization for Multi-parameter Functions *Iranian Journal of Science and Technology, Transactions of Electrical Engineering* 42:465-478 doi:10.1007/s40998-018-0082-4
- Basu M (2014) Improved differential evolution for short-term hydrothermal scheduling *International Journal of Electrical Power & Energy Systems* 58:91-100  
doi:<https://doi.org/10.1016/j.ijepes.2013.12.016>
- Chen CH, Wu SD, Dai L (1999) Ordinal comparison of heuristic algorithms using stochastic optimization *IEEE Transactions on Robotics & Automation* 15:44-56
- Clerc M, Kennedy J (2002) The particle swarm - Explosion, stability, and convergence in a multidimensional complex space *Ieee Transactions on Evolutionary Computation* 6:58-73 doi:10.1109/4235.985692
- Dahmani S, Yebdri D (2020) Hybrid Algorithm of Particle Swarm Optimization and Grey Wolf Optimizer for Reservoir Operation Management *Water Resources Management* 34:4545-4560 doi:10.1007/s11269-020-02656-8
- David BF (1998) Artificial Intelligence through Simulated Evolution. In: *Evolutionary Computation: The Fossil Record*. IEEE, pp 227-296.  
doi:10.1109/9780470544600.ch7
- Esmat et al. (2009) GSA: A Gravitational Search Algorithm *Information Sciences*
- Formato RA (2007) CENTRAL FORCE OPTIMIZATION: A NEW METAHEURISTIC WITH APPLICATIONS IN APPLIED ELECTROMAGNETICS *Progress in Electromagnetics Research* 77:425-491
- Glover F (1986) Future paths for integer programming and links to artificial intelligence *Computers & Operations Research* 13:533-549

doi:[https://doi.org/10.1016/0305-0548\(86\)90048-1](https://doi.org/10.1016/0305-0548(86)90048-1)

Hossain MS, El-shafie A (2013) Intelligent Systems in Optimizing Reservoir Operation

Policy: A Review Water Resources Management 27:3387-3407

doi:10.1007/s11269-013-0353-9

Jia B, Zhong PA, Wan X, Xu B, Chen J (2015) Decomposition–coordination model of

reservoir group and flood storage basin for real-time flood control operation

Hydrology Research 46:11

Karaboga D (2005) An idea based on honey bee swarm for numerical optimization

Kennedy J, Eberhart R Particle swarm optimization. In: Proceedings of ICNN'95 -

International Conference on Neural Networks, 27 Nov.-1 Dec. 1995 1995. pp

1942-1948 vol.1944. doi:10.1109/ICNN.1995.488968

Kirkpatrick S, Gelatt CD, Vecchi MP (1983) Optimization by Simulated Annealing

Science 220:671-680 doi:10.1126/science.220.4598.671

Kumar DN, Reddy MJ (2007) Multipurpose Reservoir Operation Using Particle Swarm

Optimization Journal of Water Resources Planning & Management 2006

L R (2008) Computational intelligence: Methods and techniques. Springer Berlin

Heidelberg. doi:DOI: 10.1007/978-3-540-76288-1

Labadie JW (2004) Optimal Operation of Multireservoir Systems: State-of-the-Art

Review Journal of Water Resources Planning & Management 130:93-111

Luo J, Qi Y, Xie J, Zhang X (2015) A hybrid multi-objective PSO–EDA algorithm for

reservoir flood control operation Applied Soft Computing 34:526-538

doi:<https://doi.org/10.1016/j.asoc.2015.05.036>

M D (1996) Ant system: optimization by a colony of cooperating agents IEEE

transactions on systems, man, and cybernetics Part B, Cybernetics : a

publication of the IEEE Systems, Man, and Cybernetics Society 1

- Maier HR, Kapelan Z, Kasprzyk J, Matott LS (2015) Thematic issue on Evolutionary Algorithms in Water Resources Environmental Modelling & Software 69:222-225 doi:<https://doi.org/10.1016/j.envsoft.2015.05.003>
- Mandal KK, Chakraborty N (2011) Short-term combined economic emission scheduling of hydrothermal systems with cascaded reservoirs using particle swarm optimization technique Applied Soft Computing 11:1295-1302 doi:<https://doi.org/10.1016/j.asoc.2010.03.006>
- Niu W-j, Feng Z-k, Cheng C-t, Wu X-y (2018) A parallel multi-objective particle swarm optimization for cascade hydropower reservoir operation in southwest China Applied Soft Computing 70:562-575 doi:<https://doi.org/10.1016/j.asoc.2018.06.011>
- Oliveira R, Loucks DP (1997) Operating rules for multireservoir systems Water Resources Research 33:839-852
- Qin H, Zhou J, Lu Y, Li Y, Zhang Y (2010) Multi-objective Cultured Differential Evolution for Generating Optimal Trade-offs in Reservoir Flood Control Operation Water Resources Management 24:2611-2632 doi:10.1007/s11269-009-9570-7
- Rani D, Moreira MM (2010) Simulation–Optimization Modeling: A Survey and Potential Application in Reservoir Systems Operation Water Resources Management 24:1107-1138 doi:10.1007/s11269-009-9488-0
- Ratnaweera A, Halgamuge SK, Watson HC (2004) Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients IEEE Transactions on Evolutionary Computation 8:240-255 doi:10.1109/TEVC.2004.826071
- Reddy MJ, Kumar DN (2008) Evolving strategies for crop planning and operation of irrigation reservoir system using multi-objective differential evolution Irrigation

Science 26:177-190 doi:10.1007/s00271-007-0084-x

Regulwar DG, Choudhari SA, Raj PA (2010) Differential Evolution Algorithm with Application to Optimal Operation of Multipurpose Reservoir Journal of Water Resource and Protection 2:560-568

Schardong A, Simonovic SP (2015) Coupled Self-Adaptive Multiobjective Differential Evolution and Network Flow Algorithm Approach for Optimal Reservoir Operation Journal of Water Resources Planning and Management 141:04015015 doi:doi:10.1061/(ASCE)WR.1943-5452.0000525

Sharif M, Swamy VSV (2014) Development of LINGO-based optimisation model for multi-reservoir systems operation International Journal of Hydrology ence & Technology 4:126-138

Spears WM, Gordon DF Using Artificial Physics to Control Agents. In: International Conference on Information Intelligence & Systems, 1999.

Srinivas N, Deb K (1994) Muultiobjective Optimization Using Nondominated Sorting in Genetic Algorithms Evolutionary Computation 2:221-248

Storn R, Price K (1997) Differential Evolution – A Simple and Efficient Heuristic for global Optimization over Continuous Spaces Journal of Global Optimization 11:341-359 doi:10.1023/A:1008202821328

Thang N, Nguyen VQ, Minh D, Le VD (2018) Modified Differential Evolution Algorithm: A Novel Approach to Optimize the Operation of Hydrothermal Power Systems while Considering the Different Constraints and Valve Point Loading Effects Energies 11:540

Wang Y, Zeng J-c (2013) A multi-objective artificial physics optimization algorithm based on ranks of individuals Soft Computing 17:939-952 doi:10.1007/s00500-012-0969-3

- Wolpert DH, Macready WG (1997) No free lunch theorems for optimization IEEE Transactions on Evolutionary Computation 1:67-82 doi:10.1109/4235.585893
- Xie L, Zeng J, Formato RA (2011) Convergence analysis and performance of the extended artificial physics optimization algorithm Applied Mathematics and Computation 218:4000-4011 doi:https://doi.org/10.1016/j.amc.2011.02.062
- Yang G, Guo S, Liu P, Liu X, Yin J (2020) Heuristic Input Variable Selection in Multi-Objective Reservoir Operation Water Resources Management: An International Journal, Published for the European Water Resources Association (EWRA) 34
- Yang X, Suash D Cuckoo Search via Lévy flights. In: 2009 World Congress on Nature & Biologically Inspired Computing (NaBIC), 9-11 Dec. 2009 2009. pp 210-214. doi:10.1109/NABIC.2009.5393690
- Yeh WW-G (1985) Reservoir Management and Operations Models: A State-of-the-Art Review Water Resources Research 21:1797-1818 doi:https://doi.org/10.1029/WR021i012p01797
- Zhan X, Xiang T, Chen H, Zhou B, Yang Z (2014) Vulnerability assessment and reconfiguration of microgrid through search vector artificial physics optimization algorithm International Journal of Electrical Power & Energy Systems 62:679-688 doi:https://doi.org/10.1016/j.ijepes.2014.05.024
- Zhang Z, Jiang Y, Zhang S, Geng S, Wang H, Sang G (2014) An adaptive particle swarm optimization algorithm for reservoir operation optimization Applied Soft Computing 18:167-177 doi:https://doi.org/10.1016/j.asoc.2014.01.034

# Figures

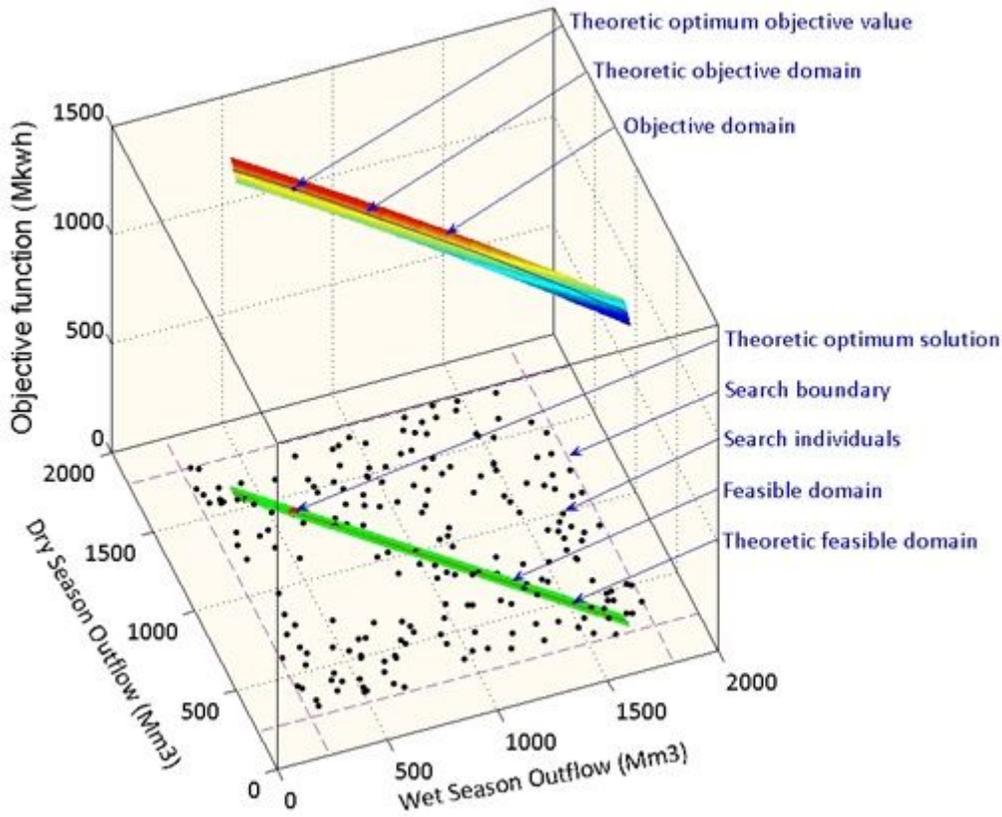


Figure 1

3D schematic diagram of optimization results, feasible region and objective function value

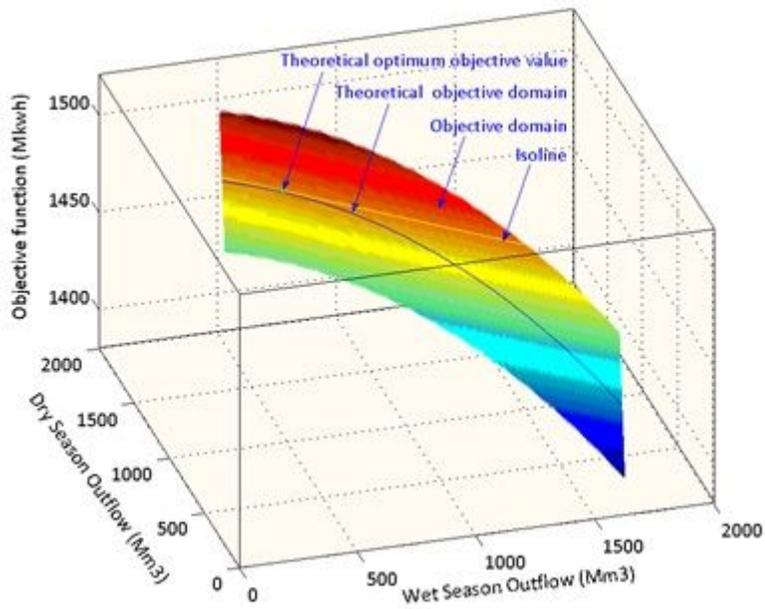


Figure 2

Enlarged view of the objective function domain

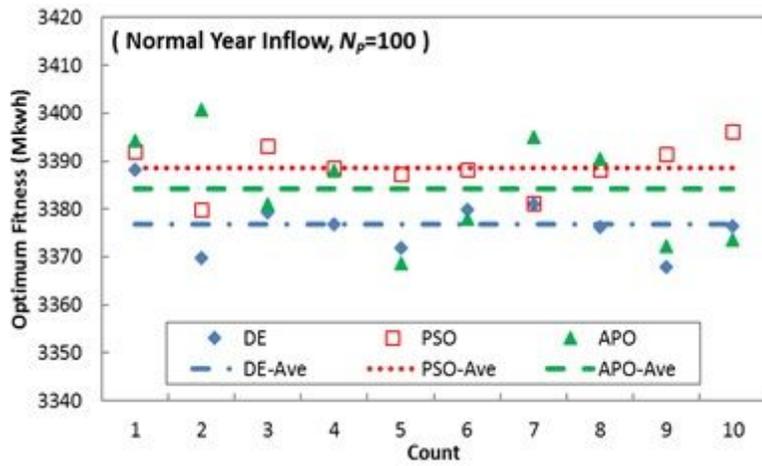


Figure 3

The optimum fitness value of 10 independent computations

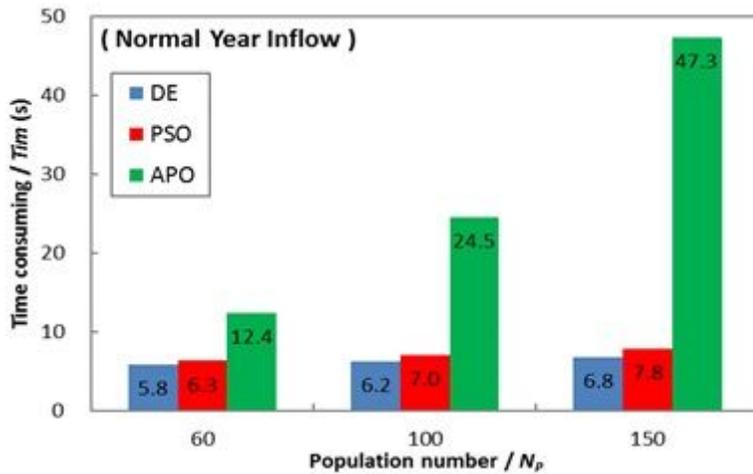


Figure 4

The computational time of three heuristic algorithms

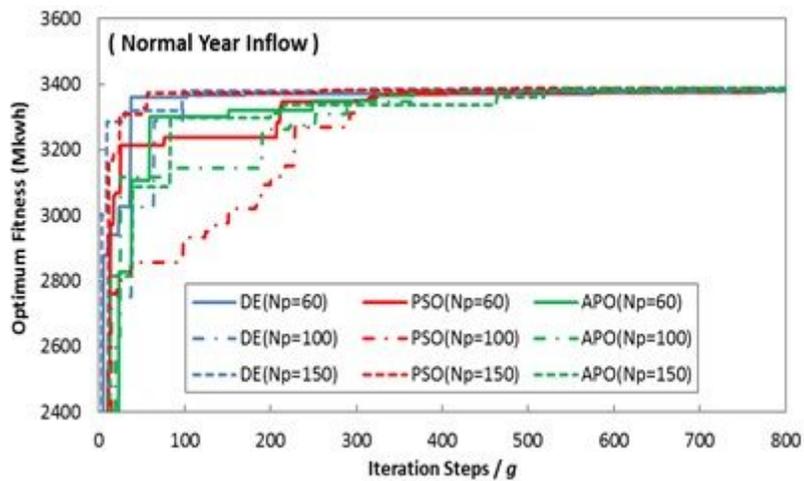


Figure 5

The optimum fitness value

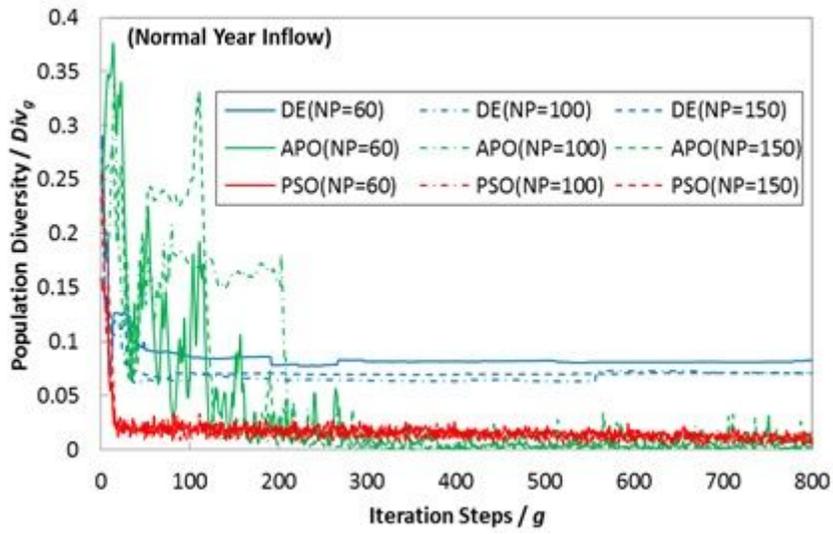
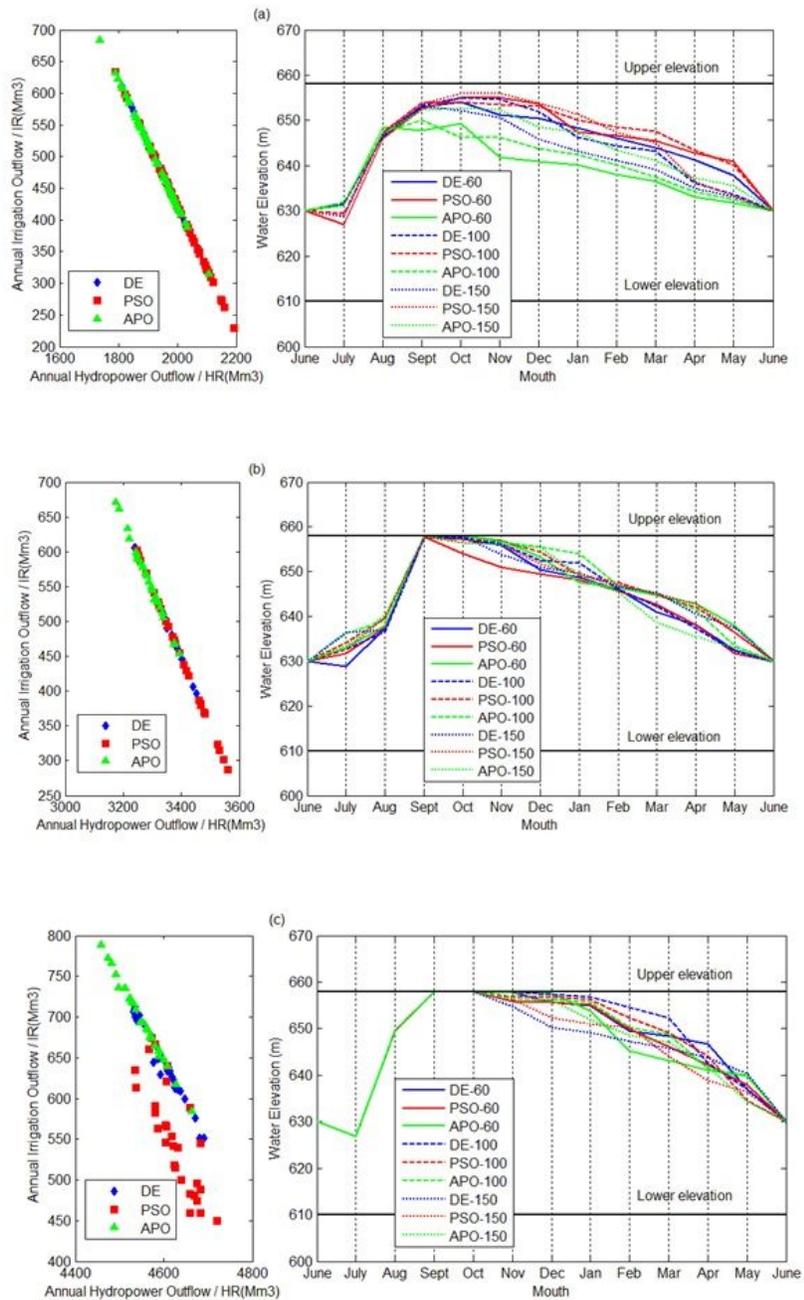


Figure 6

The population diversity



**Figure 7**

The reservoir operation schedule results calculated by heuristic algorithms: (a) Dry year inflow, (b) Normal year inflow, (c) Wet year inflow.