Finite-Time Consensus of Nonlinear Multi-Agent Systems via Impulsive Time Window Theory: A Two-Stage Control Strategy

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Finite-time consensus of nonlinear multi-agent systems via impulsive time window theory: a two-stage control strategy

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Abstract This paper concentrates on the problem of finite-time consensus of nonlinear multi-agent systems (MASs) via a impulsive time window theory: a two-stage control (TSC) strategy. The TSC divide the whole control period into two parts, separately variable impulsive control stage and finite-time consensus control stage. Different from the general single-stage control, TSC can adjust the time period of impulsive control and finite-time control dynamically according to the practical application requirements. The variable impulsive control is also discussed in this paper. Comparing with the traditional fixed impulsive theory, the impulsive sampling time occurs randomly within the impulsive time window, providing much more flexibility to the system. In addition, the switching topology scheme is introduced in this paper to strengthen the stability of the MASs. Finally, two numerical simulation examples on the leaderless case and leader-following case are given to demonstrate the theoretical analysis.

Keywords Two-stage control (TSC) · Finite-time consensus · Impulsive time window theory · Multi-agent systems (MASs)

1 Introduction

Recent decades, there has been a tremendous amount of research focused on the distributed collaborative evolution of MASs, such as unmanned aerial vehicles (UAVs) [1], sensor network [2], flocking [3] and so on [4, 5]. Currently, with the intensive study in this field, the topic of consensus problem has gradually attracted the attention of researchers [6–9].

However, the high communication cost produced by continuous control is not taken into account in [6–9]. Therefore, scholars pay attention to impulsive control strategy [10] and adopt appropriate control protocol to overcome this shortcoming. As a crucial and efficient control theory technique, the impulsive control scheme is studied in the MASs. This strategy only allows each node to transmit information in the impulsive instants, which dramatically reduces the network's communication load. Recently, numerous meaningful results of MASs consensus via impulsive control have been extensively investigated in [11–14]. In [13] proposed three kinds of impulsive protocols with fixed topology, switching topology and external disturbance to reach consensus on continuous-time MASs network. In [14] presented the distributed impulsive consensus protocol with input saturation to guarantee the dynamic consensus asymptotically.

Note that, the impulsive control theory proposed above [10–14] are performed under fixed impulsive interval, which means the impulsive instants and impulsive interval needs to be defined in advance. Nevertheless, the information exchange between agents cannot occur precisely at the fixed impulsive sampling time because of the practical system constraints. For example, the impulsive should occur at a fixed time of t_n . Still, the practical impulsive may occur during an

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impulsive time window $(t_n - R, t_n + R)$ of radius R . In other words, the conditions mentioned above may lead to incorrect impulsive consensus in the system. In order to obtain more flexible impulsive instants, some valuable methods of variable impulsive control strategy with impulse input constrained in impulsive time window are proposed in [15, 16]. In recent years, the application of variable impulsive control in the MASs is discussed in [17–19]. In [17, 18], the variable impulsive control with leader-following consensus for MASs in directed network topology is discussed. In [19] proposed the variable impulsive technique in an undirected interaction topology for the consensus of MASs.

Much study on impulsive control has been investigated, these research mainly concentrate on the final state of the system, ignoring the time it needs to reach consensus. In other words, the MASs can get into an equilibrium state when $t \rightarrow \infty$. Comparing with asymptotical convergence, the finite-time consensus improves the convergence rate of the system. Therefore, the combination of impulsive control and finite-time consensus theory takes both consideration of communication cost and convergence rate [20–23]. In [22] presented the consensus of MASs within finite time via impulsive control under disturbance. In [23] discussed finite-time consensus of second order MASs with impulsive effects. However, the previous study only considered the fixed impulsive control. Therefore, the combination of variable impulsive control and finite-time not only relaxes the restriction of the consensus condition, but also improves the flexibility of the system.

Last but not least, much existing study focus on fixed topology networks [24–26]. However, the communication topology between agents will change owing to external disturbances or link failures. Therefore, the switching topology is also discussed in this paper. Inspired by the related works, our paper concentrates on the problem of finite-time consensus of nonlinear MASs via impulsive time window theory: a TSC strategy. The knowledge of finite-time stability theory, algebraic graph theory, impulsive differential equation is adopted to achieve consensus of the MASs within finite time. The primary contributions of this paper can be epitomized as:

(i) Different from the previous research, the TSC is adopted to the nonlinear MASs. The TSC divide the whole control period into two parts, separately variable impulsive control stage and finite-time consensus control stage. This strategy can adjust the convergence period dynamically based on different practical application scenarios. Concretely speaking, to set larger number of impulsive samples to cope with the scarcity of communication resources. On the other hand, the system will step into the finite-time control stage earlier if we reduce the number of impulsive samples, improving the convergence speed of the system.

(ii) The variable impulsive control technology is introduced in this paper. Comparing with the existing studies of fixed impulsive theory, variable impulsive strategy does not need to preset the impulsive sampling interval. Furthermore, the impulsive instants can occur any time within the impulsive time window to increase system flexibility. This method not only improves the anti-interference performance of the MASs, but also relaxes the restriction of the consensus condition.

(iii) Due to the unreliability of the real communication network, the communication link between agents may be interrupted. Therefore, this paper also considers the switching topology criteria to improve the stability of the MASs networks.

This paper proceeds as follows. The algebraic graph theory is introduced in Sect. 1. We have formulated some related lemmas, definitions and assumptions in Sect. 2. In Sect. 3, a rigorous mathematical proof was made for the MASs of leaderless case and leader-following case with switching topology to reach a consensus state. Some numerical simulations will be shown in Sect. 4. Lastly, the conclusion is considered.

Notations $\mathbb{R}$ is the set of real numbers. $\mathbb{R}^m$ is an Euclidean space. $\mathbb{R}^{m \times k}$ is the real matrix set of $m \times k$. I_m is expressed as a matrix in which the whole elements are 1. $\mathbb{N}^+$ is defined as a positive integer. The symbol $\|\cdot\|$ represents Euclidean norm. $\otimes$ denotes the Kronecker product. $\lambda_{\max}(\mathcal{K})$, $\lambda_{\min}(\mathcal{K})$ and $\mathcal{K}^T$ define the biggest and smallest eigenvalue and the transpose of any matrix $\mathcal{K}$.

2 Preliminaries

2.1 Algebraic graph theory

In this section, based on knowledge of algebraic graph theory is given. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ describes the network structure of the MASs, where $\mathcal{V} = (v_1, v_2, \dots, v_n)$ represents the set of agents in the network, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ to describe a link set. Assume $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ presented the relationship between each agent. If agent i can communicate with agent j , then $a_{ij} > 0$, otherwise $a_{ij} = 0$, let $a_{ii} = 0$. Moreover, $\mathcal{G}$ is a directed balanced graph if $a_{ij} = a_{ji} > 0$. $\mathcal{N}_i = \{j | (v_i, v_j) \in \mathcal{E}\}$ depicts all neighbor agents of i agent. The number of edges connected to each node is represented by the degree matrix $\mathcal{D} = \text{diag}(d_1, d_2, \dots, d_n)$. Furthermore, the Laplacian matrix is expressed as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{L} = [l_{ij}]_{N \times N}$.

Since the topology graph describing agents varies over time, the directed switching topology is considered as follows. Here we define graphs $\mathcal{G}_g$, $g = \{1, 2, 3, \dots, g\}$, $g \in \mathbb{N}^+$, where g represents all possible topologies. The switch signal is denoted as $\sigma(t) : [t_0, +\infty) \rightarrow g$, which is piecewise constant. Obviously, $\mathcal{G}_{\sigma(t)} \in \mathcal{G}_g = \{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, \dots, \mathcal{G}_g\}$, $g \geq 1$

and $\mathcal{G}_{\sigma(t)}$ is fixed for $t \in [t_n, t_{n+1})$. Therefore, for each possible graph $\mathcal{G}_{\sigma(t)}$, the matrix mentioned above is redefined as follows $\mathcal{A}_{\sigma(t)} = [a_{ij}^{\sigma(t)}]$, $\mathcal{D}_{\sigma(t)} = [d_i^{\sigma(t)}]$ and $\mathcal{L}_{\sigma(t)} = \mathcal{D}_{\sigma(t)} - \mathcal{A}_{\sigma(t)}$, respectively.

2.2 Problem Statement

The classical model of nonlinear MASs is considered in this section. We propose a dynamic equation of N agents, which is expressed by

$$\dot{x}_i(t) = \Xi x_i(t) + \varphi(x_i(t)) + u_i(t), \quad i = 1, 2, \dots, N \quad (1)$$

where $x_i(t) \in \mathbb{R}^m$ denotes the state vector, and $x_i(t) = [x_{i1}, x_{i2}, \dots, x_{im}]^T$ is defined the state of MASs network. Ξ represents a constant matrix. A continuous nonlinear differentiable function is described by $\varphi(x_i(t))$, and $u_i(t)$ is the control protocol of the network.

Assumption 1 There is a directed spanning tree in $\mathcal{G}_{\sigma(t)}$ and a leader as the root node. Moreover, for all possible topologies $\mathcal{G}_g$, the directed graph $\mathcal{G}_{\sigma(t)} \in \mathcal{G}_g$ is strongly connected.

Assumption 2 If there exist Γ satisfying the following

$$\|\Phi(c) - \Phi(d)\| \leq \Gamma \|c - d\|, \quad \forall c, d \in \mathbb{R}, \quad (2)$$

then $\Phi(\cdot)$ satisfies the Lipschitz condition, where Γ is a known parameter.

Lemma 1 [25] Suppose Assumption 1 holds, let the directed digraph $\mathcal{G}_{\sigma(t)}$ can be associated by $\mathcal{L}_{\sigma(t)}$. Then all eigenvalues of $\mathcal{L}_{\sigma(t)}$ satisfy $0 = \lambda_1(\mathcal{L}_{\sigma(t)}) < \lambda_2(\mathcal{L}_{\sigma(t)}) \leq \dots \leq \lambda_N(\mathcal{L}_{\sigma(t)})$, $\mathcal{L}_{\sigma(t)} \mathbf{1}_m = 0$ and $\text{Re}(\lambda_i(\mathcal{L}_{\sigma(t)})) > 0, i = 2, 3, \dots$,

N if there is a directed spanning tree contained in $\mathcal{G}_{\sigma(t)}$.

Lemma 2 [27] If Assumption 1 holds, for any $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_m]^T$, there is the following rule

$$\gamma^T \mathcal{L}_{\sigma(t)} \gamma = \frac{1}{2} \sum_{i,j=1}^N a_{ij} (\gamma_i - \gamma_j)^2$$

and $\mathcal{L}_{\sigma(t)}$ is positive semi-definite.

Lemma 3 [28] Let $\gamma_1, \gamma_2, \dots, \gamma_m \geq 0$, $0 < z \leq 1$ and $u > 1$, then

$$\sum_{i=1}^m \gamma_i^z \geq \left(\sum_{i=1}^m \gamma_i \right)^z, \quad \sum_{i=1}^m \gamma_i^u \geq m^{1-u} \left(\sum_{i=1}^m \gamma_i \right)^u.$$

Lemma 4 [29] Assume there exist two constants $\varsigma \in (0, 1)$ and $\alpha > 0$, respectively, and the continuous function $V : U \rightarrow \mathbb{R}$ satisfying

$$\dot{V}(x) \leq -\alpha V^\varsigma(x), \quad x \in \mathfrak{V} \setminus \{0\},$$

the settling-time dependent on the initial condition $x_0 \in U$ is expressed as the following inequality

$$T(x_0) \leq \frac{V^{1-\varsigma}(x_0)}{\alpha(1-\varsigma)},$$

supposing there is an open neighborhood $\mathfrak{V} \subseteq U = \mathbb{R}$ and $V(x)$ is positive definite, then the MASs is global finite-time stable.

3 Main results

3.1 Leaderless consensus

A effective TSC protocol without leader is considered in this subsection as follows

$$u_i(t) = \begin{cases} b_n \sum_{n=1}^{\infty} \delta(t - t_n) \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} (x_i(t) - x_j(t)) & , t \in [t_0, T_1) \\ -\varpi x_i(t) - \nu \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} \text{sign}|x_i(t) - x_j(t)|^\varsigma & , t \in [T_1, T_2) \end{cases} \quad (3)$$

where $\delta(t - t_n)$ represents a impulse function, and b_n is a impulsive control gain. For simplicity, function $\text{sign}(y)|y|^\eta = \text{sign}|y|^\eta$. The impulsive sequence satisfies $0 < t_0 < t_1 < \dots < t_{n-1} < t_n < \dots$ and $\lim_{n \rightarrow \infty} t_n = +\infty$, and $\Pi = \tau_{n+1}^l - \tau_n^l$ denote the maximum upper bound of impulsive time window. We define $\Delta x_i(t_n) = x_i(t_n^+) - x_i(t_n^-)$, and let $x_i(t)$ is right-continuous, therefore, $x_i(t_n^+) = x_i(t)$ and $x_i(t_n^-) = \lim_{t \rightarrow t_n^-} x_i(t)$.

The control parameter of the finite-time protocol is defined as ϖ , playing a essential role on the smoothness of the consensus curve and convergence rate. $\nu > 0$ and $0 < \varsigma < 1$ are the positive constants. A sign function is presented as

$$\text{sign}(y) = \begin{cases} -1, & \text{if } y < 0 \\ 0, & \text{if } y = 0. \\ 1, & \text{if } y > 0 \end{cases}$$

For the first control stage, combine with controller (3) and system (1), the nonlinear impulsive control protocol can defined as

$$\begin{cases} \dot{x}_i(t) = \Xi x_i(t) + \varphi(x_i(t)), & t \neq t_n \\ \Delta x_i(t_n) = b_n \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} (x_i(t_n^-) - x_j(t_n^-)), & t = t_n \\ x_i(t_0^+) = x_i(t_0), & n \in \mathbb{N}^+. \end{cases} \quad (4)$$

Definition 1 The consensus of MASs (1) can reached within finite time under condition (3) for any initial conditions if

$$\lim_{t \rightarrow T_2} \|x_i(t) - \frac{1}{N} \sum_{i=1}^N x_i(t)\| = 0, \quad i = 1, 2, \dots, N. \quad (5)$$

Definition 2 The upper right-hand Dini derivative can described as

$$D^+ \varphi(x(t)) = \lim_{h \rightarrow +0} \sup \frac{1}{h} [\varphi(x(t+h)) - \varphi(x(t))].$$

Assumption 3 The first stage has the following rule

$$\tau_{n-1}^l < t_{n-1} < \tau_{n-1}^r < \tau_n^l < t_n < \tau_n^r < \tau_{n+1}^l < t_{n+1} < \tau_{n+1}^r.$$

Fig. 1 The TSC schematic diagram

Remark 1 As shown in Fig.1, the two endpoints of the n th variable impulsive time window are composed of τ_n^l and τ_n^r . t_n represents the sampling time of the n th impulsive and follows the criterion of Assumption 3. T_1 is the dividing point between the first controlling stage and the second controlling stage, namely, T_1 serves as the endpoint of the first controlling part and at the same time the start point of the second controlling part. Moreover, T_1 is independent of the agents' initial states, and it is determined by the number of impulsive intervals set in advance.

Remark 2 Different from other studies, this paper divides the control protocol into two stages. It is more efficient in practical applications. If the communication resources are limited, we can preset more impulsive samples in the first stage to reduce the communication cost. If the communication resources are relatively abundant, a smaller number of impulsive allows the MASs finishing the first control stage earlier and switching to the second control stage to obtain more faster convergence speed. Therefore, appropriate control scheme can be designed to fulfil the consensus of the system according to different scenarios.

Assumption 4 Supposing the switching graph $\mathcal{G}_{\sigma(t)}$ is connected and balanced, let $\hat{x}(t) = \frac{1}{N} \sum_{i=1}^N x_i(t) = \frac{1}{N} (1^T \otimes I_m) x(t)$ is the average state of all agents. Then, one get

$$\begin{aligned} \hat{x}(t_n^+) &= \frac{1}{N} (1^T \otimes I_m) x(t_n^+) \\ &= \frac{1}{N} (1^T \otimes I_m) x(t_n^-) \\ &\quad - \frac{1}{N} (1^T \otimes I_m) (b_n \mathcal{L}_{\sigma(t)} \otimes I_m) x(t_n^-) \\ &= \hat{x}(t_n^-). \end{aligned} \quad (6)$$

Since, $(1^T \otimes I_m)(b_n \mathcal{L}_{\sigma(t)} \otimes I_m) = b_n 1^T \mathcal{L}_{\sigma(t)}$ is a zero vector. Therefore, we denote $\varepsilon_i(t) = x_i(t) - \hat{x}(t)$ is an error vector, where $\varepsilon(t) = [\varepsilon_{i1}^T(t), \varepsilon_{i2}^T(t), \dots, \varepsilon_{im}^T(t)]^T$.

Though equation (6), the Kronecker product was adopted to rewrite equation (4) into matrix form

$$\begin{cases} \dot{\varepsilon}(t) = (I_m \otimes \Xi) \varepsilon(t) + \varphi(x(t)) - \frac{1}{N} (1^T \otimes I_m) \varphi(x(t)) \\ \quad, t \neq t_n \\ \Delta \varepsilon(t_n) = b_n (I_m \otimes \mathcal{L}_{\sigma(t)}) \varepsilon(t_n^-) \\ \quad, t = t_n. \end{cases}$$

Theorem 1 If Assumption 1-4 hold. Let $\lambda_2(\mathcal{L}_{\sigma(t)})$ represents the algebraic connectivity of the Laplace matrix $\mathcal{L}_{\sigma(t)}$. Supposing that λ and $0 < \rho_n < 1$ describe the maximum eigenvalue of $(\Xi^T + \Xi)$ and $((b_n(I_m \otimes \mathcal{L}_{\sigma(t)}) + I_{mN}))^T ((b_n(I_m \otimes \mathcal{L}_{\sigma(t)}) + I_{mN}))$, separately. If there exists a constant $\zeta > 1$ such that $(\tau_{n+1}^l - \tau_n^l)(\lambda + 2\Gamma) + \ln(\rho_n \zeta) \leq 0$ and $\lambda + 2\Gamma \leq 2\varpi$, then the leaderless case of the MASs can reach mean square consensus.

Proof In the first stage $t \in [t_0, T_1)$, the Lyapunov function is defined that

$$V(t) = \frac{1}{2} \varepsilon^T(t) \varepsilon(t). \quad (8)$$

When $t \neq t_n$, we can obtain the derivation of Lyapunov function as

$$\begin{aligned} D^+ V(t) &= \varepsilon^T(t) \dot{\varepsilon}(t) \\ &= \varepsilon^T(t) \left((I_m \otimes \Xi) \varepsilon(t) + \varphi(x(t)) \right. \\ &\quad \left. - \frac{1}{N} (1^T \otimes I_m) \varphi(x(t)) \right) \\ &= \frac{1}{2} \varepsilon^T(t) \varepsilon(t) ((\Xi^T + \Xi) \otimes I_m) \\ &\quad + \varepsilon^T(t) \left(\varphi(x(t)) - \frac{1}{N} (1^T \otimes I_m) \varphi(x(t)) \right). \end{aligned} \quad (9)$$

By Assumption 2, we can easy to obtain that

$$\|\varphi(x(t)) - \varphi(\hat{x}(t))\| \leq \Gamma \|x(t) - \hat{x}(t)\|. \quad (10)$$

Then,

$$\begin{aligned} D^+ V(t) &\leq (\lambda + 2\Gamma) \frac{1}{2} \varepsilon^T(t) \varepsilon(t) \\ &= (\lambda + 2\Gamma) V(t). \end{aligned} \quad (11)$$

For $t \in [\tau_n^l, \tau_{n+1}^l)$, this lead to

$$V(\tau_{n+1}^l) \leq V(\tau_n^l) \exp(\lambda + 2\Gamma)(\tau_{n+1}^l - \tau_n^l). \quad (12)$$

When $t = t_n$,

$$\begin{aligned} V(t_n^+) &= \frac{1}{2} \varepsilon^T(t_n^+) \varepsilon(t_n^+) \\ &= \frac{1}{2} (b_n ((I_m \otimes \mathcal{L}_{\sigma(t)}) + I_{mN}))^T \\ &\quad \times \varepsilon^T(t_n^-) (b_n ((I_m \otimes \mathcal{L}_{\sigma(t)}) + I_{mN})) \varepsilon(t_n^-) \\ &\leq \rho_n V(t_n^-). \end{aligned} \quad (13)$$

For $t \in [(n-1)\Pi, \tau_n^l)$, $t \in [\tau_n^l, t_n)$, $t \in [\tau_n^r, n\Pi)$, from (12) can get the following

$$V(t) \leq V((n-1)\Pi) \exp(\lambda + 2\Gamma)(t - (n-1)\Pi) \quad (14)$$

$$V(t) \leq V(\tau_n^l) \exp(\lambda + 2\Gamma)(t - \tau_n^l) \quad (15)$$

$$V(t) \leq V(\tau_n^r) \exp(\lambda + 2\Gamma)(t - \tau_n^r). \quad (16)$$

From (14) and (16), we obtain

$$\begin{aligned} V(t_n^-) &\leq V(\tau_n^l) \exp(\lambda + 2\Gamma)(t_n - \tau_n^l) \\ &\leq V((n-1)\Pi) \exp(\lambda + 2\Gamma)(t_n - (n-1)\Pi). \end{aligned} \quad (17)$$

When $t \in [t_0, \tau_1^l]$, by (12), we have

$$V(\tau_1^l) \leq V(t_0) \exp(\lambda + 2\Gamma)(\tau_1^l - t_0). \quad (18)$$

For $t \in [\tau_1^l, t_1]$, it follows from (18), then

$$V(t_1^-) \leq V(t_0) \exp(\lambda + 2\Gamma)(t_1 - t_0). \quad (19)$$

When $t = t_1$, by inequality (13), one has

$$V(t_1^+) \leq \rho_1 V(t_1^-). \quad (20)$$

Therefore, when $t \in [t_1, \tau_2^l]$, it can be derived from equations (12), (19) and (20) as follows

$$\begin{aligned} V(\tau_2^l) &\leq V(t_1^+) \exp(\lambda + 2\Gamma)(\tau_2^l - t_1) \\ &\leq \rho_1 V(t_1^-) \exp(\lambda + 2\Gamma)(\tau_2^l - t_1) \\ &\leq \rho_1 V(t_0) \exp(\lambda + 2\Gamma)(\tau_2^l - t_0). \end{aligned} \quad (21)$$

For $t \in [\tau_2^l, \tau_3^l]$, then one has

$$V(\tau_3^l) \leq \rho_1 V(t_0) \exp(\lambda + 2\Gamma)(\tau_3^l - t_0). \quad (22)$$

Similar to the above derivation process, for $t \in [\tau_2^l, \tau_3^l]$

$$V(\tau_3^l) \leq \rho_1 \rho_2 V(t_0) \exp(\lambda + 2\Gamma)(\tau_3^l - t_0). \quad (23)$$

In general, for $t \in [\tau_n^l, \tau_{n+1}^l]$, from inequality $\rho_n \exp(\lambda + 2\Gamma)(\tau_{n+1}^l - \tau_n^l) \leq \frac{1}{\zeta}$ one can get

$$\begin{aligned} V(t) &\leq \rho_1 \rho_2 \times \dots \times \rho_n V(t_0) \exp(\lambda + 2\Gamma)(t - t_0) \\ &\leq V(t_0) \exp(\lambda + 2\Gamma)(\tau_1^l - t_0) \rho_1 \exp(\lambda + 2\Gamma)(\tau_2^l - \tau_1^l) \\ &\quad \times \dots \times \rho_{n-1} \exp(\lambda + 2\Gamma)(\tau_n^l - \tau_{n-1}^l) \\ &\quad \times \rho_n^c \exp(\lambda + 2\Gamma)(t - \tau_n^l) \\ &\leq \frac{1}{\zeta^{n-1}} \rho_n^c V(t_0) \exp(\lambda + 2\Gamma)(t - \tau_n^l + \tau_1^l - t_0) \end{aligned} \quad (24)$$

$$\text{let } c = \begin{cases} 1, & t \geq t_n \\ 0, & t < t_n. \end{cases}$$

When $t_\omega = T_1, \omega \in \mathbb{N}^+$, the inequality (24) can further get

$$V(T_1) \leq \frac{1}{\zeta^\omega} V(t_0) \exp(\lambda + 2\Gamma)(t_\omega - \tau_\omega^l + \tau_1^l - t_0). \quad (25)$$

The error system of the finite-time control stage can define as follows

$$\begin{aligned} \dot{\varepsilon}_i(t) &= \Xi \varepsilon_i(t) + \varphi(\tilde{x}_i(t)) - \varpi \varepsilon_i(t) \\ &\quad - v \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} \text{sign}|\varepsilon_i(t) - \varepsilon_j(t)|^\varsigma \end{aligned} \quad (26)$$

let $\varphi(\tilde{x}_i(t)) = \varphi(x_i(t)) - \varphi(\hat{x}(t))$.

According to Theorem 1, for the second stage $t \in [T_1, T_2]$, the Lyapunov function expressed that $V(t) = \frac{1}{2} \varepsilon^T(t) \varepsilon(t)$, then one gets

$$\begin{aligned} D^+ V(t) &= \sum_{i=1}^N \varepsilon_i(t) \dot{\varepsilon}_i(t) \\ &= \sum_{i=1}^N \varepsilon_i(t) \left(\Xi \varepsilon_i(t) + \varphi(\tilde{x}_i(t)) - \varpi \varepsilon_i(t) \right. \\ &\quad \left. - v \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} \text{sign}|\varepsilon_i(t) - \varepsilon_j(t)|^\varsigma \right) \\ &\leq -v \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \varepsilon_i(t) a_{ij}^{\sigma(t)} \text{sign}|\varepsilon_i(t) - \varepsilon_j(t)|^\varsigma \\ &\leq -\frac{1}{2} v \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} (\varepsilon_i(t) - \varepsilon_j(t)) \\ &\quad \times \text{sign}|\varepsilon_i(t) - \varepsilon_j(t)|^\varsigma \\ &\leq -\frac{1}{2} v \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} |\varepsilon_i(t) - \varepsilon_j(t)|^{\varsigma+1}. \end{aligned} \quad (27)$$

By Lemma 1, 2 and 3, one obtains

$$\begin{aligned} D^+ V(t) &\leq -\frac{1}{2} v \left(\sum_{i=1}^N \sum_{j \in \mathcal{N}_i} (a_{ij}^{\sigma(t)})^{\frac{2}{1+\varsigma}} (\varepsilon_i(t) - \varepsilon_j(t))^2 \right)^{\frac{1+\varsigma}{2}} \\ &\leq -\frac{1}{2} v \left(\frac{2\varepsilon^T(t) \mathcal{L}_{\sigma(t)} \varepsilon(t)}{\frac{1}{2} \varepsilon^T(t) \varepsilon(t)} \times \frac{1}{2} \varepsilon^T(t) \varepsilon(t) \right)^{\frac{1+\varsigma}{2}} \\ &\leq -\frac{1}{2} v (4\lambda_2(\mathcal{L}_{\sigma(t)}))^{\frac{1+\varsigma}{2}} V^{\frac{1+\varsigma}{2}}(t). \end{aligned} \quad (28)$$

Thus, from Lemma 4, the settling-time of the MASs (1) can estimate that

$$T_2 \leq T_1 + \frac{4}{v (4\lambda_2(\mathcal{L}_{\sigma(t)}))^{\frac{1+\varsigma}{2}} (1-\varsigma)} V^{\frac{1-\varsigma}{2}}(T_1). \quad (29)$$

It can be derived from the above inequality that $V(t) \equiv 0$ for all $t \geq T_2$. Thus, the leaderless case of the MASs is global finite-time stability. The proof is completed. $\square$

3.2 Leader-following consensus

A dynamic model of the leader node 0 is considered in this subsection

$$\dot{x}_0(t) = \Xi x_0(t) + \varphi(x_0(t)). \quad (30)$$

Supposing $x_0(t) \in \mathbb{R}^n$ represented the state of the node 0, the switching interaction topology with a leader is defined by $\mathcal{G}_{\sigma(t)}$.

Remark 3 In the leaderless case, each node in the network can communicate with each other, and the state values of all agents can eventually converge to an average value. While, the leader-following case consensus can ignore information of follower nodes. Namely, all follower nodes will directly or indirectly communicate with the leader node, finally, all agents will be in the same state with the leader node. Furthermore, we can also choose different control schemes according to the requirements of practical applications. Therefore, for more careful consideration, the consensus of MASs with a leader is discussed in this subsection.

Consider a effective TSC protocol with leader as follow

$$u_i(t) = \begin{cases} b_n \sum_{n=1}^{\infty} \delta(t-t_n) \left(\sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} (x_i(t) - x_j(t)) \right. \\ \quad \left. + q_i (x_i(t) - x_0(t)) \right), t \in [t_0, T_1] \\ -\varpi (x_i(t) - x_0(t)) - v \left(\sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} \text{sign}|x_i(t) - x_j(t)|^\varsigma \right. \\ \quad \left. + \frac{1}{2} q_i \text{sign}|x_i(t) - x_0(t)|^\varsigma \right), t \in [T_1, T_2]. \end{cases} \quad (31)$$

Let $q_i > 0$ represents the relationship between the leader node 0 and the follower node, and the rest of the other parameters are the same as leaderless case.

Definition 3 For any initial state, the consensus of MASs with a leader can realized within finite time if satisfied

$$\lim_{t \rightarrow T_2} \|x_i(t) - x_0(t)\| = 0, \quad i = 1, 2, \dots, N \quad (32)$$

let $\delta_i(t) = x_i(t) - x_0(t)$ and $\delta(t) = [\delta_{i1}^T(t), \delta_{i2}^T(t), \dots, \delta_{im}^T(t)]^T$.

Similar to (7), for the first control stage, we can rewrite the error system of condition (31) into matrix from that

$$\begin{cases} \dot{\delta}(t) = (I_m \otimes \Xi) \delta(t) + \varphi(x(t)) - \frac{1}{N} (1^T \otimes I_m) \varphi(x(t)) \\ \quad, t \neq t_n \\ \Delta \delta(t_n) = b_n (I_m \otimes \mathcal{H}_{\sigma(t)}) \delta(t_n^-) \\ \quad, t = t_n \end{cases} \quad (33)$$

where $\mathcal{H}_{\sigma(t)} = \mathcal{L}_{\sigma(t)} + \mathcal{Q}$, which describe the structure of $\mathcal{G}_{\sigma(t)}$. We denote $\mathcal{Q} = \text{diag}[q_1, q_2, \dots, q_N]$ is a diag matrix.

Theorem 2 If Assumption 1-4 hold. Let $\lambda_{\min}(\mathcal{H}_{\sigma(t)})$ represents the smallest eigenvalue of matrix $\mathcal{H}_{\sigma(t)}$. Let λ and $0 < \theta_n < 1$ denote the maximum eigenvalue of $(\Xi^T + \Xi)$ and $[(b_n(I_m \otimes \mathcal{H}_{\sigma(t)}) + I_{mN})]^T [(b_n(I_m \otimes \mathcal{H}_{\sigma(t)}) + I_{mN})]$. Assume there is a known parameter $\zeta > 1$ such that $(\tau_{n+1}^l - \tau_n^l)(\lambda + 2\Gamma) + \ln(\theta_n \zeta) \leq 0$ and $\lambda + 2\Gamma \leq 2\varpi$, then the consensus of MASs with a leader can achieved within finite time.

Proof The Lyapunov function is chosen by

$$V(t) = \frac{1}{2} \delta^T(t) \delta(t). \quad (34)$$

The proof process for the first stage is similar to the leaderless case, and then for $t \in [\tau_n^l, \tau_{n+1}^l)$, we can get

$$V(t) \leq \frac{1}{\zeta^{n-1}} \theta_n^c V(t_0) \exp(\lambda + 2\Gamma)(t - \tau_n^l + \tau_1^l - t_0). \quad (35)$$

$$\text{where } c = \begin{cases} 1, & t \geq t_n \\ 0, & t < t_n. \end{cases}$$

When $t_\omega = T_1, \omega \in \mathbb{N}^+$, the inequality (35) can further get

$$V(T_1) \leq \frac{1}{\zeta^\omega} V(t_0) \exp(\lambda + 2\Gamma)(t_\omega - \tau_\omega^l + \tau_1^l - t_0). \quad (36)$$

Comparing with (26), the error equation of the second stage can be rewritten that

$$\begin{aligned} \dot{\delta}_i(t) &= \Xi \delta_i(t) + \varphi(\tilde{\delta}_i(t)) - \varpi \delta_i(t) \\ &\quad - v \left(\sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} \text{sign}|\delta_i(t) - \delta_j(t)|^\varsigma \right. \\ &\quad \left. + \frac{1}{2} q_i \text{sign}|\delta_i(t)|^\varsigma \right) \end{aligned} \quad (37)$$

let $\varphi(\tilde{\delta}_i(t)) = \varphi(x_i(t)) - \varphi(x_0(t))$.

By Lemma 1, 2 and 3, the proof process is the same as (27), by taking the derivative of $V(t)$, for simplicity then

$$\begin{aligned} D^+ V(t) &= \sum_{i=1}^N \delta_i(t) \dot{\delta}_i(t) \\ &= \sum_{i=1}^N \delta_i(t) \left(\Xi \delta_i(t) + \varphi(\tilde{\delta}_i(t)) - \varpi \delta_i(t) \right. \\ &\quad \left. - v \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma(t)} \text{sign}|\delta_i(t) - \delta_j(t)|^\varsigma \right. \\ &\quad \left. - \frac{1}{2} v q_i \text{sign}|\delta_i(t)|^\varsigma \right) \\ &\leq -\frac{1}{2} v (4(\lambda_{\min}(\mathcal{H}_{\sigma(t)}))^{\frac{1+\varsigma}{2}} V^{\frac{1+\varsigma}{2}}(t)). \end{aligned} \quad (38)$$

Therefore, from Lemma 4, the settling-time of the leader-following consensus can attain that

$$T_2 \leq T_1 + \frac{4}{v (4\lambda_{\min}(\mathcal{H}_{\sigma(t)}))^{\frac{1+\varsigma}{2}} (1-\varsigma)} V^{\frac{1-\varsigma}{2}}(T_1). \quad (39)$$

Then, this result shows that $V(t) \equiv 0$ for all $t \geq T_2$. Then, the MASs consensus with leader-following case is reached. $\square$

Remark 4 If the impulsive interval $\tau_n = t_{n+1} - t_n$ is a positive constant, we can say that the MASs based on fixed impulsive control can realize consensus within finite time. Supposing that Assumptions 1-4 are met. Theorem 2 of this paper can be rewritten as the same consensus criterion as theorem 1 of [11], that is $(t_{n+1} - t_n)(\lambda + 2\Gamma) + \ln(\theta_n \zeta) \leq 0$. The proof process of the second control stage corresponds to Theorem 2.

Remark 5 Although the TSC problem has been discussed in [20], the MASs consensus via variable impulsive control under switching topology is not considered. Also different from the fixed time impulsive control in [20], [21], [22], and [23], the impulsive interval $\tau_n = t_{n+1} - t_n$ is a fixed positive constant for any n . For the variable impulsive control strategy, by the Theorem 1 and Theorem 2, the impulsive time window can be adjusted within a maximum range $\Pi = \tau_{n+1}^l - \tau_n^l$, where Π components of the free time window and impulsive time window. Consequently, the impulsive instants t_n can occur more flexibly within the maximum impulsive time window, which enables the system to cope with more complex situations in practical systems, such as time delay, temporary interruption of the communication link, etc.

4 Simulation

Two simulation examples of with and without a leader are given in this section, proving the effectiveness and reliability of our control protocol (3) and (31). Subsequently, we employ the Chua's circuit [30] which can be utilized to describe the dynamic behaviour of the MASs

$$\begin{cases} \dot{x}_{i1}(t) = \mu_1(x_{i2}(t)) - x_{i1}(t) - \phi(x_{i1}(t)) \\ \dot{x}_{i2}(t) = x_{i1}(t) - x_{i2}(t) + x_{i3}(t) \\ \dot{x}_{i3}(t) = -\mu_2 x_{i2}(t) \end{cases} \quad (40)$$

where $x_i(t)$ denotes the state of i th agent. μ_1 and μ_2 are known constants, and the function $\phi(\cdot)$ is shown as follows

$$\phi(x_{i1}(t)) = \pi_2 x_{i1}(t) + 0.5(\pi_1 - \pi_2)(|x_{i1} + 1| - |x_{i1} - 1|) \quad (41)$$

where $\pi_1 < \pi_2 < 0$ are two known parameters.

From (1) and (41), we can obtain that

$$\Xi = \begin{bmatrix} -\mu_1(1 + \pi_2) & \mu_1 & 0 \\ 1 & -1 & 1 \\ 0 & -\mu_2 & 0 \end{bmatrix}.$$

Let $\mu_1 = 8.21$, $\mu_2 = 13.145$, $\pi_1 = -1.053$, $\pi_2 = -0.635$. Then, we can yield that $\lambda = (\Xi^T + \Xi) = 13.5264$, $\Gamma = |\pi_1 \mu_1| = 8.64513$.

Fig. 2 The interaction topology of $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$.

(a) case 1

(b) case 2

Fig. 3 The relationship between t_n and the impulsive time window for leaderless case.

Example 1 In the leaderless case consensus, we consider three interaction topologies $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$ of the MASs with four nodes, as indicated in Fig.2. Then, as detailed in Fig.2, the Laplacian matrix of graphs $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$ can be attained as

$$\text{shown } \mathcal{L}_1 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \mathcal{L}_2 = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix},$$

$$\mathcal{L}_3 = \begin{bmatrix} 2 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

From Theorem 1 and the given known parameters, let $b_n = -0.4$, $\zeta = 1.03$, step-length is 0.0001. By simple matrix computation, we have $\rho_n = 0.5200$. The maximum upper bound can be attained that $\tau_{n+1}^l - \tau_n^l \leq -\frac{\ln(\rho_n \zeta)}{\lambda + 2\Gamma} = -\frac{\ln(1.03\rho_n)}{30.81666}$, $n \geq 1$. For the convenience of analysis and calculation, we choose $\Pi = 0.02$. As illustrated by Fig.3, the impulsive instants can occur within the impulsive time win-

Fig. 4 The error value without leader for any initial state under switching topology.

dow. Let $\sigma(t) : [t_0, +\infty)$ presented the switch signal, $\sigma(t) = (n \bmod 4), t \in [t_n, t_{n+1})$.

Next, the initial position state values of the four agent nodes was randomly chosen on the interval $[-3, 3]$. The impulsive control stage is shown in Fig.4, for convenience, we choose five impulsive instants in advance. As a demarcation point, the last impulsive instant of case 1 and case 2 can be described $t_5 = T_1 = 0.1049s$ and $t_5 = T_1 = 0.1062s$, respectively. For the finite-time control stage, from Theorem 1, let $\varpi = 40, v = 2, \eta = 0.7$. As shown in Fig.4, the settling time T_2 can be estimated to be approximately $0.2s$, meanwhile, the consensus error for each node can convergence to zero for all $t \geq 0.2s$. It can be said that the MASs (1) of leader-less case can achieve mean consensus under the control protocol (3) with switching topology. Comparing Fig.4, Fig.5 and Fig.6, we can see that the pure variable impulsive control can alleviate the communication burden between each nodes, but such asymptotic consensus is difficult to achieve in the real world as shown in Fig.5. On the other hand, pure finite-time continuous control can obtain more faster convergence speed while also increasing the communication cost as shown in Fig.6. Therefore, the control scheme of leader-less case proposed in this study is efficient and realistic as shown in Fig.4.

Example 2 In the leader-following case consensus, three interaction topologies with a leader node and four follower nodes as shown in Fig.7. Apparently, the 0 node as a leader and the rest of nodes define the follower agent is consid-

Fig. 5 The error value without leader for any initial state via pure variable impulsive control under switching topology.

Fig. 6 The error value without leader for any initial state via pure finite-time stability under switching topology.

Fig. 7 The interaction topology of $\bar{\mathcal{G}}_1, \bar{\mathcal{G}}_2$ and $\bar{\mathcal{G}}_3$.

ered. Then, we can easy to obtain the pinning gain matrix of graphs $\bar{\mathcal{G}}_1, \bar{\mathcal{G}}_2, \bar{\mathcal{G}}_3$ as follows

$$\mathcal{H}_1 = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \mathcal{H}_2 = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}, \mathcal{H}_3 = \begin{bmatrix} 3 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

The given known parameters are the same as above. By simple calculation, it could be inferred that $\theta_n = 0.9000$, then the maximum upper bound of $\Pi = \tau_{n+1}^l - \tau_n^l$ can be obtain that $\tau_{n+1}^l - \tau_n^l \leq -\frac{\ln(\theta_n \zeta)}{\lambda + 2\Gamma} = -\frac{\ln(1.03\theta_n)}{30.81666}, n \geq 1$. For the convenience, the maximum upper bound is defined $\Pi = 0.0024$. As we can see in Fig.8, the impulsive instants can occur within impulsive time window.

Fig. 8 The relationship between t_n and the impulsive time window for leader-following case.

The initial position state values of the leader agent 0 and the four follower agent nodes was randomly denoted on the interval $[-3,3]$. In Fig.9, the eight impulsive instants were set in advance, where the last impulsive instant of case 1 and case 2 can be expressed $t_{10} = T_1 = 0.0204s$ and $t_{10} = T_1 = 0.0197s$. In the second control stage, from Theorem 2, let $\varpi = 40, v = 2, \eta = 0.7$. From Fig.9, the settling time T_2 can be estimated to be approximately $0.12s$, namely, the state of all follower nodes will in accordance with the leader node for all $t \geq 0.12$. On the whole, the consensus of leader-following case can be reached under protocol (31) with switching topology. As indicated in Fig.10, the variable impulsive control needs to converge to zero only when $t \rightarrow \infty$. Meanwhile, the pure finite-time continuous control can obtain more faster convergence speed while also increasing the communication cost according to Fig.11. Therefore, the consensus technique illustrated by Fig.9 in this investigation is necessary.

Remark 6 It follows from Theorem 1 and 2. A smaller ρ_n value helps obtain a larger Π . Through simple calculation, Fig.4 gets a more enormous maximum upper bound value. Namely, the impulsive interval of Fig.4 is greater than Fig.9. The larger the impulsive interval, the smaller the convergence speed. Comparing Fig.4 with 9, the consensus rate shown as Fig.4 is smaller than Fig.9 because of $\rho_n < \theta_n$. Besides, it should be also mentioned that if $\rho_n = 1$, the maximum upper bound will become negative, thus the control

Fig. 9 The error value with a leader for any initial state under switching topology.

Fig. 10 The error value with a leader for any initial state via pure variable impulsive control under switching topology.

protocol cannot affect the consensus of the MASs. So we should choose $0 < \rho_n < 1$ to obtain a suitable range of Π .

5 Conclusion

This paper concentrates on the problem of finite-time consensus of nonlinear MASs via impulsive time window theory: a TSC strategy. The TSC divide the whole control period into two parts, separately variable impulsive control stage and finite-time consensus control stage. Different from the general single-stage control, this research provides a new investigation perspective for application scenarios under different communication conditions. Meanwhile, the variable impulsive control is also discussed in this paper. Comparing with the traditional fixed impulsive theory, the impulsive sampling time occurs randomly within the impulsive time

Fig. 11 The error value with a leader for any initial state via pure finite-time stability under switching topology.

window, providing much more flexibility to the system. Furthermore, in order to overcome the unreliability of the real network, the switching topology scheme is introduced to improve the stability of the system. Lastly, two numerical simulation examples on the leaderless case and leader-following case are given to demonstrate the theoretical analysis.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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Figures

Figure 1

The TSC schematic diagram

Figure 2

"Please see the Manuscript PDF file for the complete figure caption".

(a) case 1

(b) case 2

Figure 3

The relationship between t_n and the impulsive time window for leaderless case.

(a) case 1

(b) case 2

Figure 4

The error value without leader for any initial state under switching topology.

Figure 5

The error value without leader for any initial state via pure variable impulsive control under switching topology.

Figure 6

The error value without leader for any initial state via pure finite-time stability under switching topology.

Figure 7

"Please see the Manuscript PDF file for the complete figure caption".

(a) case 1

(b) case 2

Figure 8

The relationship between t_n and the impulsive time window for leader-following case.

(a) case 1

(b) case 2

Figure 9

The error value with a leader for any initial state under switching topology.

Figure 10

The error value with a leader for any initial state via pure variable impulsive control under switching topology.

Figure 11

The error value with a leader for any initial state via pure finite-time stability under switching topology.