Mathematical Modelling of Power Skiving for General Profile based on Numerical Enveloping

Kang Jia

Xi'an Jiaotong University

Junkang Guo

Xi'an Jiaotong University

Tao Ma

Xi'an Jiaotong University

Shaoke Wan (✉ skwan2020@xjtu.edu.cn)

Xi'an Jiaotong University

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Title Page

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Authors

Kang Jia^{1,2,3}

Junkang Guo^{1,3}

Tao Ma²

Shaoke Wan^{1,3*}

Corresponding author

Email: skwan2020@xjtu.edu.cn

Affiliations

1. Key Laboratory of Education Ministry for Modern Design & Rotor-Bear System, Xi'an Jiaotong University, Xi'an, P. R. China 710049

2. Key Laboratory of Smart Manufacturing for Special Vehicles and Transmission System, Inner Mongolia, P. R. China 014030

3. School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an, P. R. China 710049

Abstract: Power skiving is an effective generating machining method for internal parts like gears with respect its high productivity. The general mathematic modelling for power skiving is the basis for cutting tools design, machining precision evaluation, and machining process optimization. Currently, mainly studies are focus on the involute gear machining with adopting the analytical enveloping equation. However, these analytical methods have failed to deal with overcutting for general profile skiving tasks. Moreover, little attention has been devoted to investigate the power skiving process with taking variable configuration parameters, which is significant to control the machined surface topography. Herein, we introduce a mathematic modelling method for power skiving with general profile based on the numerical discrete enveloping. Firstly, the basic mathematic model of power skiving is established, in which the center distance is formulated as polynomial of time. With transforming the power skiving into a forming machining of the swept volume of cutting edge, a numerical algorithm is designed to distinguish the machined transverse profile via the discrete enveloping ideology. Especially, the precise instant contact curve is extracted according to the feed motion speed inversely. Finally, simulations for involute gear and cycloid wheel are carried out to verify the effectiveness of this method and investigate the influence of variable radial motions on the machined slot surface topography. The results show this method is capable to simulate the dynamic power skiving process with general profiles and to evaluate the machined results.

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Kang Jia^{1,2,3}, Junkang Guo^{1,3}, Tao Ma^{2*}, Shaoke Wan^{1,3*}

1.Key Laboratory of Education Ministry for Modern Design & Rotor-Bear System, Xi'an Jiaotong University, Xi'an, P. R. China 710049

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Nomenclature

β_c	Helix angle on pitch circle of cutter
β_w	Helix angle on pitch circle of workpiece
$\Delta\varphi_c$	Differential rotation angle of cutter
Δf	Axial feed motion of cutter
φ_c	Rotation angle of cutter
φ_w	Rotation angle of workpiece
Ω	Section angle between axes of cutter and workpiece
E_0	Center distance between cutter and workpiece
E_c	Install eccentricity of cutter
$\mathbf{f}_w$	Section plane of workpiece
N_t	Dividing number of time
N_p	Dividing number of profile
N_s	Dividing number of cutting edge

$K_e(t)$	Polynomial of time for center distance
R_c	Pitch circle radius of cutter
R_w	Pitch circle radius of workpiece
S_R	Rake flank of cutter
$S_c: O_c-X_cY_cZ_c$	Coordinate system of cutter
$S_w: O_w-X_wY_wZ_w$	Coordinate system of workpiece
z_c	The teeth number of cutter
z_w	The teeth number of workpiece

1 Introduction

Power skiving is a typical generating machining method, which provides high productive in machining parts with periodic features like gears. With installing as a pair of cross-axis gear meshing, the cutting edges on the end face of cutter remove a layer of material via the relative axial motion for the part [1]. This working principle indicates that power skiving not only adopts continuously generating motion as gear-hobbing to ensure high machining efficiency, but also retains the advantages of gear-shaping that is capable to manufacture both external and internal part, non-through slots, double-linked gears and so on.

As invented in 1910 by Pittler [2], however, power skiving underwent slow development with subject to the poor stiffness of the machine tools and the short tool life. In recent years, with the development of tool materials, especially, the spindle technology and numerical control systems, the power skiving has performed superiority in gear production. Relevant power skiving solutions [3-5] such as machine tools, cutters etc., have been provided by companies like Gleason, Pittler etc. Meanwhile, theories on power skiving have been studied in difference aspects to enhance its application.

As the basis of successful machining, mathematical model of power skiving and the cutting edge curve were studied widely [6-19]. Jin [6] investigated the analytical theory of gear skiving, and pointed out the action line of skiving is as same as the action line of spiral gears. Li et al. [7-9] analyzed the working principle of skiving, and proposed a cutter design approach to error-free gear skiving. Guo et al. [10,11] investigated the skiving tool design and cutting mechanism of cylindrical gears, and introduced a multiple blades taper skiving tool [12]. Radzevich [13] introduced the design and computation principle of the skiving cutter for gear skiving. Tsai [14] established the mathematical model for design and analysis of power skiving tool for involute gear cutting. Stadtfeld [15] introduced the power skiving technology of Gleason, including the generation kinematics, cutter geometry, chip geometry, machine tool configuration, and processing software. Tomokazu et al. [16] established a calculation model for internal gear skiving with a pinion-type cutter having pitch deviation and run-out. Moriwaki et al. [17,18] investigated the cutting tool parameters of cylindrical skiving cutter with sharpened angle for internal gears to optimize skiving cutter design. Shih and Li [19] proposed an error-free conical power skiving cutter design method via meshing theory with considering the variation of center distance. Jia et al. [20] developed a discrete enveloping-assisted cutting edge curve calculation method for skiving cutter. Li et al. [21] studied the design of power skiving tool with considering the interference and minimizing the machining deviations. These researches provided the general theory and mathematical model for power skiving as well as the calculation method of the cutting edge curve design.

In addition, the simulation of power skiving process is significant to estimate and optimize the

skiving process and cutting tools. Research works are mainly devoted to geometric precision simulation [22-31] and the working condition analysis of cutter [32-37]. Commonly, the geometric simulations include three kinds of manner. First, the analytical ones adopted the meshing equation for geometry calculation, like Guo et al. [22,23] studied the theoretical tooth profile errors of gears in skiving, and investigated the tool setting errors on gear skiving accuracy, further they [24] studied the cutting edge correction method for conical cutter; Zheng et al. [25] generalized the machine kinematics correction and TCA to gear skiving. Secondly, the numerical methods represent the machining process in a discrete way for concrete calculation, like Jia et al. [26] developed a numerical simulation method for non-involute gear power skiving with adopt approximate external enveloping; Zheng et al. [27] developed a novel z-map based numerical method to calculate tooth flank, and investigate the influence of eccentricity error on the surface roughness; Inui et al. [28] developed a triple-dexel based geometric simulation method for power skiving process with GPU computing acceleration. Thirdly, the CAD based methods perform the cutting process as Boolean operation and finished it via commercial CAD package, like Antoniadis et al. [29,30] simulated the kinematics of the cutting process with the aid of commercial CAD software, which allows the precise determination of the non-deformed chips and cutting forces; Tapoglou [31] simulated the chip thickness of skiving with the aid of CAD. In the basis of geometric simulation, cutting force of power skiving is modelled via dividing the cutting edge as a serial of oblique cutting element and integrating the dynamic force of all the elements. McClosky et al [32] and Onozuka et al [33] reported this modelling results are consistent with the experiment cutting force; Li et al [34,35] further discussed the temperature during power skiving process. Besides, some comprehensive simulations were developed such as Klocke et al. [36] investigated the influences of skiving configuration parameters on the working performance like tool wear and chip welding, Schulze et al. [37] studied the kinematic process of skiving and investigated the chip formation mechanisms of skiving with adopting 3D-finite element simulation. Most of the researches are focused on the machining of involute gears with adopting analytical mathematic model. However, it is difficult to process the conditions like self-intersection and overcutting, particularly for universal target profiles. The discrete numerical enveloping method proposed in [26] demonstrates excellent performance to deal with these drawbacks in general profile skiving simulation, but it lacks the capability to investigate the entire skiving process with combining time-varying configuration and kinematic parameters.

Aiming to investigate the dynamic machining process for power skiving with overcoming the shortcoming of [26], this work developed a discrete enveloping based mathematic modelling method for power skiving, which is effect for general profile tasks with strong robustness. The remainder is organized as follows. The basic mathematic model for skiving is introduced in section 2. The numerical simulation method for power skiving is studied in section 3. Then in section 4 the deviation estimation is described. In section 5, several skiving tasks are simulated, and concluded at last.

2 The mathematical model for power skiving

2.1 Configuration of power skiving

In a general power skiving system, the cutter and workpiece is setup as a pair of cross-axis in Figure 1. Workpiece coordinate system S_w : $O_w-X_wY_wZ_w$ is attached to the workpiece, and its Z_w -axis is coincided with the cylindrical workpiece axis; Similarly, cutter coordinate system S_c : O_c-

$X_c Y_c Z_c$ is attached to the cutter, and its Z_c -axis is coincided with the cutter axis; In initial, we define X_w -axis is going through the origin O_c , and X_c -axis is coinciding with X_w -axis. Meanwhile, with respect the meshing of cutter and workpiece, the shaft angle Ω between their axes and the nearest distance E_0 between their axes is given as follows:

$$\Omega = -j_w \beta_w - k_{io} j_c \beta_c \quad (1)$$

$$E_0 = R_w + k_{io} R_c \quad (2)$$

Where, j_w denotes the helix direction of workpiece ($j_w=0$ indicates spur, $j_w=1$ indicates right hand, and $j_w=-1$ indicates left hand), and j_c denotes the helix direction of cutter ($j_c=0$ indicates spur, $j_c=1$ indicates right hand, and $j_c=-1$ indicates left hand); β_w and β_c are the helix angles on the pitch circles respectively for the workpiece and the cutter; k_{io} denotes the type of skiving ($k_{io}=1$ denotes external skiving, and $k_{io}=-1$ denotes internal skiving).

Fig. 1. The model for power skiving system.

Moreover, in practical applications, the cutter takes an eccentricity E_c along the Z_c -axis from the origin O_c , looking for improve the working condition of cutting edges and avoid the interferences during the machining process.

2.2 Motions of power skiving

Power skiving consists of two kinds of coupled motion as illustrate in Fig. 1, i.e. the meshing motion, and the feeding motion. The cutter and workpiece rotate φ_c and φ_w around Z_c -axis and Z_w -axis respectively with constant transmission ratio, which is termed as main meshing motion, contributing to material cut off. Meanwhile, a differential rotation $\Delta\varphi_c$ is implemented on the cutter to ensure the meshing motion when the cutter performs a synchronous linear feed motion f along Z_w -axis, which contributes to produce the complete slot. The rotation angles of the power skiving system are given as below:

$$\varphi_c = -k_{io}\varphi_w \frac{z_w}{z_c} + \Delta\varphi_c \quad (3)$$

Where, z_w and z_c are the slot number of workpiece and teeth number of cutter, respectively; $\Delta\varphi_c$ is the differential rotation angle of the skiving cutter, and it is determined by both the axial feeding Δf of the skiving cutter along Z_w -axis and the configuration of power skiving system as follow:

$$\Delta\varphi_c = k_{io}j_w \frac{\Delta f \sin(\beta_w)}{R_c \cos(\beta_c)} \quad (4)$$

Aiming to satisfy the diverse requirements in practical applications of skiving, such as tooth profile crown correction, machining error sensitivity analysis etc., we involve an additional feed motion K_e to enhance this virtual kinematic model of power skiving as in [38], which expresses the constant radial distance E_0 between the cutter and the part. The variances of feed motion K_e is formulated as polynomial of time as follows:

$$K_e(t) = E_0 + \Delta E = E_0 + \sum_{i=0}^{i=q} c_{e,i} t^i \quad (5)$$

Where q is the polynomial order of the radial feed, c_e is the polynomial coefficients of radial feed, and ΔE is the variance in radial direction.

Furthermore, proper rotation directions of the cutter and the workpiece are crucial to ensure successful material removal during skiving. The rotation of cutter must ensure the cutting edge is sliding inside the slot of workpiece along the feeding direction.

3 Mathematical modelling of skiving process by numerical enveloping

The aforementioned working model of power skiving indicates the engagement of each tooth cut off a layer of material in slot with the help of both meshing motion and feeding motion while the succession of engagements produce the whole slot via the feeding motion. This reveals that the skiving process can be taken as a forming machining process for helical or spur gear like drawn in Fig. 2, in which the swept volume of one cutting edge (SVC) relative to the gear slot during engagement works as the wheel and its instant contact curve generates the slot surface following the feed motion. Consequently, one can deduce the generating process of power skiving as follows: each cutting edge generates one or several points on the desired surface at every engaging moment, and these points further develop a forming curve on the desired slot surface during each engagement of one cutting tooth.

Fig. 2. Generation principle and equivalent process for power skiving.

Commonly, the generating points during power skiving process are determined by enveloping equations. It tells the external normal of generating points on the SVC surface are perpendicular to the velocity vector of their feed motion. However, the analytical method might invalid in cases like singular points, over-cutting, multiple-cutting edge etc. For this reason, this work models the skiving process as a forming machining process in a numerical way. The key points are approximating SVC by a serial of cutting edges, and then obtaining the transversal profile of workpiece and the contact curve via identify the external enveloping curve of the intersection curves between SVC and the workpiece transversal section.

As shown in Fig. 3, the detailed numerical modelling and investigating for the skiving process are arranged in 7 steps.

Step. 1: Dividing the cutting edge curve $\mathbf{r}(u)=[x(u),y(u),z(u),1]^T$ into continuous points for the purpose of maintain the generality of this method, where u is the radial parameter of the cutting edge.

Step. 2: Determining the rotation interval $[\varphi_s, \varphi_e]$ of one tooth engaging cutting by figure out the rotation angles of cutter touching the addendum circle of workpiece.

Step. 3: Generating the cutting edge curve $\mathbf{r}^w(u,t)$ in workpiece system S_w based on the mathematical model of power skiving as given as below:

$$\mathbf{r}^w(u,t) = \mathbf{M}_w^c(t) \cdot \text{trans}(0,0,E_c) \cdot \mathbf{r}^c(u) \quad (6)$$

Where, $\mathbf{M}_w^c(t)$ is the transform matrix from cutter system S_c to workpiece system S_w at t moment. It is given by basic homogeneous coordinate transform matrices according to the kinematic model of skiving.

$$\mathbf{M}_w^c(t) = \text{rot}(z, -\varphi_w) \cdot \text{trans}(K_c(t), 0, 0) \cdot \text{rot}(x, \Omega) \cdot \text{rot}(z, \varphi_c + \Delta\varphi) \quad (7)$$

Step. 4: As shown in Fig. 3(a), establishing the swept volume of one cutting edge SVC through assemble N_t cutting edges $\mathbf{r}^w(u, t_i)$ with rotation angle uniformly within $[\varphi_s, \varphi_e]$.

$$\text{SVC} = \sum_{i=1}^{N_t} \mathbf{r}^w(u, t_i) = \sum_{i=1}^{N_t} \mathbf{M}_w^c(t_i) \cdot \text{trans}(0,0,E_c) \cdot \mathbf{r}^c(u) \quad (8)$$

Where t_i is the time moment of the i_{th} rotation angle. It is given by the angular speed of cutter ω_c as follow:

$$t_i = (\varphi_s(N_t - i) + \varphi_e(i - 1)) / ((N_t - 1)\omega_c) \quad (9)$$

Step. 5: As shown in Fig. 3(b), taking SVC to perform a forming machining on the transversal section of workpiece $\mathbf{f}_w$ ($z = z_0$). Essentially, it is interesting all the cutting edge curve $\mathbf{r}^w(u, t_i)$ of SVC to the transversal section $\mathbf{f}_w$ along the feed motion $f(t)$. In fact, additional rotation $\Delta\varphi_c(t)$ is involved to ensure the intersection curves are symmetrically distributed along the X_w -axis. In this basis, one can figure out the intersect point $\mathbf{E}_{i,j}$ for each i_{th} cutting edge with specified radial parameter u_j satisfies equation (10).

$$\text{rot}(z, \Delta\varphi_c(\tau_0)) \cdot \text{trans}(0,0,f(\tau_0)) \cdot \mathbf{M}_w^c(t_i) \cdot \text{trans}(0,0,E_c) \cdot \mathbf{r}(u_j) \cdot [0,0,1,0]^T - z_0 = 0 \quad (10)$$

Obviously, the proper τ_0 can be ascertained by numerical searching methods since equation (10) is a function of variable τ . Consequently, the point $\mathbf{E}_{i,j}$ can be directly given as:

$$\mathbf{E}_{i,j} = \text{rot}(z, \Delta\varphi_c(t)) \cdot \text{trans}(0,0,f(t)) \cdot \mathbf{M}_w^c(t_i) \cdot \text{trans}(0,0,E_c) \cdot \mathbf{r}(u_j) + [0,0,z_0,0]^T \quad (11)$$

Step. 6: Distinguishing the external envelop profile of all the intersect curves on $\mathbf{f}_w$. The external enveloping profile can be derived numerically as a serial of points along the X_w -axis since the

intersection curves are distributed around X_w -axis. At each radius within the specified radial range $[r_f, r_a]$, setting a line perpendicular to x -axis, then its intersection point for every intersect curve on f_w is easily obtained as the intersection of two straight line segments in x - o - y plane. After all the intersection points between the intersect curves and current line have been identified, the point that taking the maximum / minimum y -component are specified as the enveloping profile points C_k as shown in Fig. 3(c). Concurrently, for each profile point, the time serial number of corresponding cutting edge indicates the cutting occur time of machined profile point C_k .

$$|rot(z, \Delta\phi_c(\tau_p)) \cdot trans(0, 0, f(\tau_p)) \cdot rot(0, 0, -E_c) \cdot \mathbf{M}_w^{c-1}(t_k) \cdot \mathbf{C}_k - \mathbf{S}_R| = 0 \quad (12)$$

$$\mathbf{Q}_k = \mathbf{M}_w^c(t_k) \cdot \text{rot}(0, 0, E_c) \cdot \mathbf{P}_k \quad (13)$$

Fig. 3. Simulation model of power skiving process: (a) swept volume of cutting edge, (b) machined curves on transversal section of workpiece by SVC, (c) discrete enveloping of cutting edges, and (d) extraction of the instant contact points.

The flowchart of this numerical simulation for skiving is provided as in Fig. 4, and it is adopted for following case studies.

In practical machining, the cutting edge and the slot of workpiece are meshing periodically, consequently, the axial displacement of every cutting engagement must comply with this meshing motion. For simplicity, we taking the assumption that the axial position of $\mathbf{f}_w$ can be given arbitrarily in this study, without consider the cusps on the machined slot surface that are produced by the inherent interrupted meshing cutting mechanism.

Fig. 4. The flowchart of numerical simulation for power skiving.

4 Machined error evaluation

In order to evaluate the simulation precision of proposed method, the deviation that between the simulated profile and the theoretical one is calculated. As shown in Fig. 4, in each cross-section of the workpiece, the shortest distance from the point on the simulated profile to the point on the

theoretical profile is specified as machining error, instead of the minimum distance from the simulated point to the theoretical surfaces along its normal vector.

Fig. 5. Machined profile deviation evaluation of power skiving simulation.

5 Numerical examples

Aiming to demonstrate the effectiveness of this method, simulations of power skiving with various configurations for an internal involute gear and a cycloid wheel are performed and the machining errors are investigated.

5.1 Simulation for internal gear power skiving

For the convenience of evaluating the accuracy of this method, an internal helical involute gear power skiving was simulated with the parameters listed in Table. 1, since the cutter and the gear blank are meshing as a pair of standard involute gear with same normal module and normal pressure angle. At first, the standard involute curve like cutting edges for two types of rake flanks were calculated by method in [20]. Then, the cutting edge curve and the enveloping curves on the gear transversal section for case-1 were demonstrated as in Fig. 56(a), in which the distinguished enveloping curves were also involute curves in Fig. 6(b). As shown in Fig. 6(c), the cutters for case-1 and case-2 adopted difference rake flanks. The swept volumes of cutting edge relative to the gear blank for case-1 and case-2, which are demonstrated respectively in Fig. 6(d) and Fig. 6(e), showed difference as well as the geometries of their instant contact curve.

Table. 1. Parameters for internal gear skiving simulation.

Item, symbol [unit]	value
Geometry of gear and cutter	
Number of slot, Z_w	54
Normal module, [mm]	2
Normal pressure angle, [$^\circ$]	20
Helix angle, ϕ_w [$^\circ$]	25
Rotation direction	Right hand
Number of cutting teeth, Z_c	36
Helix angle, ϕ_c [$^\circ$]	10
Rotation direction	Right hand
Normal vector of rake flank: case-1	[0,0,-1]
case-2	[0, 0.2588,-0.9654]
Skiving configuration	
Type of task	Internal skiving

Center distant, E_o [mm]	23.027
Shaft angle, Ω [°]	15
Eccentricity, E_c [mm]	0
Numerical calculation	
Point number of cutting edge	100
Solid number of swept volume	181
Rotation interval	$[-30^\circ, +30^\circ]$
Point number of machined profile	100

Fig. 6. Power skiving of internal involute gear: (a) cutting edge curve for cutter in case-1, (b) enveloping of machined profiles in case-1, (c) cutters in case-1 and case-2, (d) assembly chart of internal power skiving, (e) swept volume of cutting edge and contact curve for case-1 and case-2.

Compare with the standard involute curve, the deviations of machined profile for these two cases were illustrated in Fig. 7, in which the maximum deviations were no more than $0.15 \mu\text{m}$. The accuracy indicates the developed method is capable to simulate the machining error.

Fig. 7. Deviations of machined profile for case-1 and case-2.

In practical machining process, various teeth surface like crowning and conical might be adopted to avoid the interferences on the part gear shoulder and to obtain specific working performance. In common, the teeth surface can be corrected by properly changing the polynomial coefficient of cutting path along the full axial feeding range. Unlike traditional analytical methods, the developed numerical enveloping simulation method was used to study the machined teeth surface by power skiving with two types of center distance $K_e(t)$ for both the two cutting edge configurations.

A linear radial motion $K_e(t)=E_0+0.0067t-0.04$ and a quadratic radial motion $K_e(t)=E_0-0.0025t^2+0.03t-0.09$ were carried out on the power skiving cutter as shown in Fig. 8(a) and (b), respectively. A constant axial feed speed 1mm/s was performed for the full axial length that started at $L=-6$ mm and exited at $L=6$ mm. In Fig. 8, the theoretical tooth surface is represented by the gray grid and the machined tooth surface is defined by the contact curve as blue grid. One can find out the teeth surface performed a taper shape after linear radial motion, in which the deviations were about 24 μm at $L=6$ mm, and the deviations were nearly -23 μm at $L = -6\text{mm}$, and both sides of the tooth surface showed the similar topography but following their contact curve respectively. The simulation for the quadratic radial motion was provided in Fig. 8(b), where the machined tooth surface performed a parabolic variation over the full length of tooth. The deviations were about -34 μm at the two ends of the tooth surface and approached to the minimum nearly 0 μm at the middle. Besides, the small differences of the corresponding grid nodes of tooth surface between case-1 and case-2 are reasonable, since the distributions of contact curve are different in these two cases are different. In all, these results were consistent with the anticipated correction for teeth, i.e. linear polynomial radial motion generates a cone-shaped correction, and quadratic polynomial radial motion produces a drum-like correction.

Fig. 8. Tooth surface topography generated with radial feed motion: (a) linear polynomial motion and (b) quadratic polynomial motion.

5.2 Simulation for cycloid wheel power skiving

Looking for prove the generality of developed method, power skiving for cycloid wheel that adopted in RV reducer were simulated. The parameters for the short epicycloid equation, the power skiving configuration, and the numerical simulation are listed in Table. 2.

Table. 2. Simulation parameters of power skiving for cycloid wheel.

Item, symbol [unit]	value
Geometry of wheel and cutter	
Number of wheel slot, Z_w	10
Radius of base circle, r_b [mm]	100
Radius of generation circle, r_g [mm]	10
Distance from move point to center of generate circle, [mm]	8
Distance of offset, r_o [mm]	18
Helix angle, ϕ_w [°]	0
Number of cutting teeth, Z_c	8
Helix angle, ϕ_c [°]	10
Rotation direction	Right hand
Normal vector of rake flank	[0,0,-1]
Skiving configuration	
Type of task	External skiving
Center distant, E_0 [mm]	180
Shaft angle, Ω [°]	-10

Eccentricity, E_c [mm]	0
Numerical calculation	
Point number of cutting edge	200
Solid number of swept volume	361
Rotation interval, [°]	[-25,25]
Point number of machined profile	200

The whole simulation process is demonstrated as in Fig. 9. Firstly, according to the equidistance line of short epicycloid shown in Fig. 9(a), the cutting edge curve are calculated by [20], and the external power skiving is setup as in Fig. 9(b). Then the machined profile is distinguished through enveloping all the intersection curves on transversal of wheel as in Fig. 9(c). Through inversely tracing the intersection point on the rake flank specified by the recorded time serial number of each profile point, the instant contact point is extracted in work coordinate system. Consequently, the complete contact curve and the swept volume of cutting edge are obtained as in Fig. 9(d).

Fig. 9. Power skiving of cycloid wheel: (a) the mathematical model of the short epicycloid profile, (b) the configuration of power skiving, (c) the machining curves and their enveloping curve on transverse section of wheel, and (d) the swept volume of cutting edge.

Additionally, a cubic radial motion $K_e(t) = 0.000125t^3 - 0.0045t^2 + 0.054t - 0.216$ for $t \in [0, 12]$ and $K_e(t) = -0.000125t^3 + 0.0045t^2 - 0.054t + 0.216$ for $t \in (12, 24]$ was applied on the power skiving cutter for the cycloid wheel machining as shown in Fig. 10. The theoretical tooth surface was represented by gray grid, and the machined surface is illustrated by contact curves in blue grid by sampling nodes. The deviations on the two ends of tooth surface were maximum and were reduced to the minimum near to the middle at each axial path (from A to K). It is consistent with the desired drum-like tooth correction via cubic polynomial radial motion.

Fig. 10. Tooth surface topography generated with cubic polynomial radial feed motion.

As the presented simulations shown, the proposed mathematic model for general profile power skiving is capable to simulate the skiving motion and to evaluate the results of power skiving, and it may be useful for power skiving CNC machine developers as well as the gear manufacturers. Besides, properly changing the polynomial coefficient of feed motions is a solution of gear correction for power skiving.

6 Conclusions

This work developed a mathematical model of power skiving with strong robustness to simulate the instant contact curve for dynamic machining process and general profile tasks via adopting numerical enveloping ideology. This model is verified and applied by simulations with polynomial radial motions. The main conclusions are drawn as follows:

(i) The proposed mathematic model for power skiving includes not only the general configuration parameters but also the center distance formulated as polynomial of time. It is effective for diverse dynamic skiving process modelling rather than the traditional constant kinematic model. Besides, this method takes a strong robustness thanks to the numerical discrete enveloping manner, which ensures this method is success to deal with general profiles like cycloid and to overcome the overcuts in analytical methods.

(ii) This discrete envelop based simulation method can provide the precise instant contact curve of power skiving. At first, the power skiving process is transformed as a forming machining process for the swept volume of cutting edge which is established according to the main generating motion. Secondly, the machined profile is given by distinguishing the external envelop profile of the intersection curves between the swept volume of cutting edge and the transverse of workpiece. Thirdly, the times of each machined profile point are identified to calculate the instant contact curve via translating the corresponding profile points according to their instant feed motions inversely.

(iii) This method simulates the teeth surface topography via the contact curves in axial distribution

since the numerical calculation principle. The simulations for involute gear and cycloid wheel machining validate the machining deviations on teeth surfaces are consistent with the assigned polynomial radial motion on the center distance, i.e. linear, parabolic, cubic, etc., which indicate that properly changing the polynomial coefficient of radial motions is a solution of gear correction for power skiving.

(iv) As a numerical method, its accuracy will be affected by the number of profile point and the number of locus curves. Moreover, this method takes the assumption that cutting tooth performs continuous axial feeding motion along one slot, leading to it lack to identify the residual machining height on teeth surface. Further study will be devoted to involving the periodical meshing of cutting teeth during skiving process.

Declarations

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Authors' contribution Kang Jia proposed the detailed method and carried out the formula derivation and theoretical analysis. Junkang Guo was responsible for the experimental work and part of data analysis. Tao Ma were involved in the discussion and significantly contributed to making the final draft of the article. Shaoke Wan proposed the research idea and technical scheme. All the authors read and approved the final manuscript.

Data availability The authors confirm that the data supporting the findings of this study are available within article.

Compliance with ethical standards

Competing interests The authors declare that they have no competing interests.

Ethical approval Not applicable.

Consent to participate Not applicable.

Consent to publish Not applicable.

Appendix:

The basic homogeneous coordinate matrices are defined as follows:

$$\text{trans}(x, y, z) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

$$\text{rot}(x, \alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

$$\text{rot}(y, \beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

$$\text{rot}(z, \gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

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Figures

Figure 1

The model for power skiving system.

Figure 2

Generation principle and equivalent process for power skiving.

Figure 3

Simulation model of power skiving process: (a) swept volume of cutting edge, (b) machined curves on transversal section of workpiece by SVC, (c) discrete enveloping of cutting edges, and (d) extraction of the instant contact points.

Figure 4

The flowchart of numerical simulation for power skiving.

Figure 5

Machined profile deviation evaluation of power skiving simulation.

Figure 6

Power skiving of internal involute gear: (a) cutting edge curve for cutter in case-1, (b) enveloping of machined profiles in case-1, (c) cutters in case-1 and case-2, (d) assembly chart of internal power skiving, (e) swept volume of cutting edge and contact curve for case-1 and case-2.

Figure 8

Tooth surface topography generated with radial feed motion: (a) linear polynomial motion and (b) quadratic polynomial motion.

Figure 9

Power skiving of cycloid wheel: (a) the mathematical model of the short epicycloid profile, (b) the configuration of power skiving, (c) the machining curves and their enveloping curve on transverse section of wheel, and (d) the swept volume of cutting edge.

Figure 10

Tooth surface topography generated with cubic polynomial radial feed motion.