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Using Caputo-Fabrizio Derivative For The Transmission of Mathematical Model Epidemic Corona Virus

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1 abstract

In just a matter of weeks, a rapidly spreading corona virus that originated in Wuhan, China, has infected more than 20,000 people and killed at least 427 worldwide. The virus is capable of spreading from person to person, and Chinese authorities are scrambling to treat a flood of new patients successfully. The virus has spread from an initial outbreak in Wuhan, China, to 25 other countries, and has resulted in the quarantine of at least 16 Chinese cities. In this article, we considered the mathematical model [1] in which Bats-Hosts-Reservoir-People and their transmission was taken, while we introduced the population of susceptible Bats and visitors to Wuhan city or any country in same mathematical model. Now two types of population are: first in [1] introducing susceptible Bats and second visitors to Wuhan city, china or any country. We used Caputo-Fabrizio derivative with provided result that the addition of susceptible Bats and visitors are not responsible in spread of infection. The numerical result also supposed our model.

Key words : Reservoir to person population(RP), Visitors population(VP), Mathematical model, R_0 , Caputo-Fabrizio derivative, Numerical Simulation.

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Background:

The people of the world seen a new epidemic thread in 2020. After the middle East Respiratory Syndrome(MERS), in middle east, a new virus Corona attacked the "Wuhan city, China". The Corona virus was first emerged in late December and has killed more than 2,500 people there, this means "Wuhan", China alone accounted for nearly 80 percent of the countries with total deaths occur 3,299 or more. Before this, the World Health Organization (WHO) China Country Office informed in 31, December 2019 about pneumonia(unknown cause) detected in "Wuhan" city, Hubei China and WHO announced novel Corona virus (2019-nCoV). The International Committee of Taxonomy assigned it severe acute respiratory syndrome Corona virus-2(SARS-CoV-2) on 11 February, 2020 [2, 18, 23]. All the country then infected, and an perception was drawn that visitors involve in this transmission of new virus mostly.

Scientists of all the world then felt an urgent mathematical model to estimate the transmission of this disease in China. Several researches focusing and developed mathematical models for MERS[3,4,5]. In these mathematical model the scientists trying the reproductive number which responsible for whole model [3, 19, 20]. The people are awarded from different methods, and control strategies to make possible precautions against Corona virus. Here the main aim is to estimate positivity, equilibrium, and Boundedness of these models [6, 21]. One of the best approach towards mathematical models is optimal control [7, 22]. Many different methods are adopted for optimal purpose.

In our work we try to vanish and prove the wrong perception that susceptible Bats and visitors are the spreading agent of Corona virus in "wuhan" or any other country.

Here we focus on the study done in [1] which is Bats-Hosts-Reservoir-People(BHRP) and its different transmission ways of Corona virus in any population. We just interesting in human-to-human transmission, so for that issue we introduced Bats population and visitors population who visit Wuhan China or any other country. We developed a mathematical model for the transmission of Corona virus Reservoir-People-Visitors(RPV) with introducing susceptible bats and estimate the value of R_0 . We apply Caputo-Fabrizio derivative approach towards this issue, and a numerical simulation in the last.

Methodology For Our Model

We using Fixed point theory for the existence for the model. Now the Caputo-Fabrizio fractional derivative for the system (1),

$$\begin{aligned} S_B(t) - S_B(0) &= C_0 F I_t^\tau \{A_B - m_B S_B - \beta_B S_B I_B\}, \\ S_p(t) - S_p(0) &= C_0 F I_t^\tau \{A_B - m_B S_B - \beta_B S_B (I_P + \kappa A_p) - \beta_W S_p W\}, \\ E_p(t) - E_p(0) &= C_0 F I_t^\tau \{\beta_B S_B (I_P + \kappa A_p) + \beta_W S_p W - (1 - \delta_p) W_p E_p - \delta_p W_p E_p - m_p E_p\}, \\ I_p(t) - I_p(0) &= C_0 F I_t^\tau \{(1 - \delta_p) W_p E_p - (\gamma_p + m_p) I_p\}, \\ A_p(t) - A_p(0) &= C_0 F I_t^\tau \{\delta_p W_p E_p - (\gamma_p' + m_p) A_p\}, \\ R_p(t) - R_p(0) &= C_0 F I_t^\tau \{\gamma_p I_p + \gamma_p' A_p - m_p R_p\}, \\ W(t) - W(0) &= C_0 F I_t^\tau \{\mu_p I_p + \mu_p' A_p - \epsilon W\}. \end{aligned} \tag{1}$$

Similarly Caputo-Fabrizio fractional derivative for equation (4) provide the following,

$$\begin{aligned}
S(t) - S(0) &= C_0 F I_t^\tau \{\mu_N - \sigma(t)SI - \alpha S\}, \\
E(t) - E(0) &= C_0 F I_t^\tau \{\sigma(t)SI - \lambda E - \alpha E\}, \\
I(t) - I(0) &= C_0 F I_t^\tau \{\lambda E - \eta I - \alpha I\}
\end{aligned} \tag{2}$$

Now we using the idea in [15] on equation (6) as well as for equation (7), through we obtain equation (8) and (9),

$$\begin{aligned}
S_B(t) - S_B(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{A_B - m_B S_B - \beta_B S_B I_B\} \\
&+ \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{A_B - m_B S_B - \beta_B S_B I_B\} dy, \\
S_p(t) - S_p(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{A_B - m_B S_B - \beta_B S_B (I_P + \kappa A_p) - \beta_W S_p W\} \int_0^t \{A_B - m_B S_B \\
&- \beta_B S_B (I_P + \kappa A_p) - \beta_W S_p W\} dy, \\
E_p(t) - E_p(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{\beta_B S_B (I_P + \kappa A_p) + \beta_W S_p W \\
&- (1 - \delta_p) W_p E_p - \delta_p W_p E_p - m_p E_p\} \int_0^t \{A_B - m_B S_B \\
&- \beta_B S_B (I_P + \kappa A_p) + \beta_W S_p W - (1 - \delta_p) W_p E_p - \delta_p W_p E_p - m_p E_p\} dy, \\
I_p(t) - I_p(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{1 - \delta_p\} W_p E_p - (\gamma_p + m_p) I_p\} \\
&+ \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{1 - \delta_p\} W_p E_p - (\gamma_p + m_p) I_p\} dy, \\
A_p(t) - A_p(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{\delta_p W_p' E_p - (\gamma_p' + m_p) A_p\} \\
&+ \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\delta_p W_p' E_p - (\gamma_p' + m_p) A_p\} dy, \\
R_p(t) - R_p(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{\gamma_p I_p + \gamma_p' A_p - m_p R_p\} \\
&+ \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\gamma_p I_p + \gamma_p' A_p - m_p R_p\} dy, \\
W(t) - W(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{\mu_p I_p + \mu_p' A_p - \epsilon W\} \\
&+ \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\mu_p I_p + \mu_p' A_p - \epsilon W\} dy.
\end{aligned} \tag{3}$$

Now using the new idea of Losada and J.Nieto [15] on the system above, we get,

$$\begin{aligned}
S(t) - S(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{\mu_N - \sigma(t)SI - \alpha S\} + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\mu_N - \sigma(t)SI - \alpha S\}, \\
E(t) - E(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{\sigma(t)SI - \lambda E - \alpha E\} \\
&+ \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\sigma(t)SI - \lambda E - \alpha E\} dy, \\
S_B(t) - S_B(0) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{\lambda E - \eta I - \alpha I\} \\
&+ \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\lambda E - \eta I - \alpha I\} dy.
\end{aligned} \tag{4}$$

We get the simplified form of the equations (10) and (11), after equations (8) and (9),

$$\begin{aligned}
\Phi_1(t, S_B) &= \{A_B - m_B S_B - \beta_B S_B I_B\}, \\
\Phi_2(t, S_p) &= \{A_B - m_B S_B - \beta_B S_B (I_p + \kappa A_p) - \beta_W S_p W\}, \\
\Phi_3(t, E_p) &= \{\beta_B S_B (I_p + \kappa A_p) + \beta_W S_p W - (1 - \delta_p) W_p E_p - \delta_p W_p E_p - m_p E_p\}, \\
\Phi_4(t, I_p) &= \{(1 - \delta_p) W_p E_p - (\gamma_p + m_p) I_p\}, \\
\Phi_5(t, A_p) &= \{\delta_p W_p' E_p - (\gamma_p' + m_p) A_p\}, \\
\Phi_6(t, R_p) &= \{\gamma_p I_p + \gamma_p' A_p - m_p R_p\}, \\
\Phi_7(t, W) &= \{\mu_p I_p + \mu_p' A_p - \epsilon W\}.
\end{aligned} \tag{5}$$

$$\begin{aligned}
\prod_1(t, S) &= \{\mu_N - \sigma(t)SI - \alpha S\}, \\
\prod_2(t, S) &= \{\sigma(t)SI - \lambda E - \alpha E\}, \\
\prod_3(t, S) &= \{\lambda E - \eta I - \alpha I\}.
\end{aligned} \tag{6}$$

Theorem 1.1. *The Kernels of $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6$ and Φ_7 fulfill the Lipschitz condition and contraction if the following inequality hold. $0 \leq (m_B + \beta_m \psi)e + I_B < 1$, where $\psi = 1$ and $e = \frac{1}{2}$.*

Proof: We prove the theorem for $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6$ and Φ_7 respectively. First suppose that S and S_1 are two function for Φ_1 , then,

$$\| \Phi_1(t, S_B) - \Phi_1(t, S_{1B}) \| = \| -m_B S_B \{S_B(t) - S_B(t_1)\} - \beta_B I_B \{S_B(t) - S_B(t_1)\} \|. \tag{7}$$

For equation (12) we apply triangle in-equality, we get,

$$\begin{aligned}
\| \Phi_1(t, S_B) - \Phi_1(t, S_{1B}) \| &\leq \| -m_B S_B \{ S_B(t) - S_B(t_1) \} \| \\
&+ \| \{ -\beta_B I_B \{ S_B(t) - S_B(t_1) \} \} \| . \\
&\leq \| m_B \| + \| \{ \beta_B \| \| I_B \| \{ \| S_B(t) - S_B(t_1) \| \} \} \| . \\
&\leq (m_B + \beta_m \cdot 1) \{ \frac{1}{2} \} + I_B \| S_B(t) - S_B(t_1) \| . \\
&\leq \varphi \| S_B(t) - S_B(t_1) \|
\end{aligned} \tag{8}$$

While here we use $\varphi = (m_B + \beta_m)$ with $\psi = 1$ and $e = \frac{1}{2}$

This shows that the given function is a bounded function, so we have,

$$\| \Phi_1(t, S_B) - \Phi_1(t, S_{1B}) \| \leq \varphi \| S_B(t) - S_B(t_1) \| \tag{9}$$

Hence, we see that the Lipschitz condition for equation (12) is satisfied, also $0 \leq (m_B + \beta_m \psi)e + I_B < 1$, where $\psi = 1$ and $e = \frac{1}{2}$, which also emphasized that the said equation (12) is contraction. The Lipschitz condition for other equations by the similar way are given below,

$$\begin{aligned}
\| \Phi_2(t, S_p) - \Phi_2(t, S_{1p}) \| &\leq \varphi \| S_p(t) - S_p(t_1) \| . \\
\| \Phi_3(t, E_p) - \Phi_2(t, E_{1p}) \| &\leq \varphi_1 \| E_p(t) - E_p(t_1) \| . \\
\| \Phi_4(t, I_p) - \Phi_2(t, I_{1p}) \| &\leq \varphi_2 \| I_p(t) - I_p(t_1) \| . \\
\| \Phi_5 - \Phi_2(t, A_{1p}) \| &\leq \varphi_3 \| A_p(t) - A_p(t_1) \| . \\
\| \Phi_6(t, R_p) - \Phi_2(t, R_{1p}) \| &\leq \varphi_4 \| R_p(t) - R_p(t_1) \| . \\
\| \Phi_7(t, W) - \Phi_2(t, W) \| &\leq \varphi_5 \| W(t) - W(t_1) \| .
\end{aligned} \tag{10}$$

Theorem 1.2. *The Kernels of equation (10) \prod_1, \prod_2 and \prod_3 fulfill the Lipschitz condition and contraction if the following inequality hold. $0 \leq (\sigma(t) + \alpha\psi)e + I < 1$, where $\psi = 1$ and $I = e = \frac{1}{2}$.*

Proof: To prove the concern condition suppose that D and D_1 are any two function then, from \prod_1 we write as,

$$\| \prod_1(t, D) - \prod_1(t, D_1) \| = \| -\sigma(t)SI - \alpha S \| . \tag{11}$$

By triangle inequality equation (15) becomes,

$$\begin{aligned}
\| \prod_1(t, D) - \prod_1(t, D_1) \| &\leq \| -\sigma(t)SI \| + \| -\alpha S \| . \\
&\leq \| \sigma(t)SI \| + \| \alpha S \| . \\
&\leq \| \sigma(t)IS \{ D(t) - D_1(t) \} \| + \| \alpha S \{ D(t) - D_1(t) \} \| . \\
&\leq \| \sigma(t) \| \| I \| + \| \alpha \| \| \{ D(t) - D_1(t) \} \| . \\
&\leq \{ \sigma(t) + \alpha \cdot 1 \} \{ \frac{1}{2} \} \| \{ D(t) - D_1(t) \} \| . \\
&\leq \Omega \| \{ D(t) - D_1(t) \|
\end{aligned} \tag{12}$$

Where $\Omega = \sigma(t) + \alpha$, while again we see that $\psi = 1$ and $I = e = \frac{1}{2}$ showing the same effect of disease spread rate through visitors to "Wahan" city or the people living there in "Wahan". Hence, Lipschitz condition for equation (16) is satisfied, and by similar way we find the remaining equations (10) are below, as,

$$\begin{aligned} \|\prod_1(t, D) - \prod_1(t, D_1)\| &\leq \Omega_1 \|E(t) - E(t_1)\| . \\ \|\prod_2(t, D) - \prod_2(t, D_1)\| &\leq \Omega_2 \|I(t) - I(t_1)\| . \end{aligned} \quad (13)$$

Taking equations (8) and (9) with kernal notation becomes,

$$\begin{aligned} S_B(t) &= S_B(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_1(t, S_B)\} + \frac{2\tau}{(2-\tau)M(\tau)}\int_0^t\{\Phi_1(y, S_B)\}dy, \\ S_p(t) &= S_p(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_2(t, S_p)\} + \frac{2\tau}{(2-\tau)M(\tau)}\int_0^t\{\Phi_2(y, S_p)\}dy, \\ E_p(t) &= E_p(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_3(t, E_p)\} + \frac{2\tau}{(2-\tau)M(\tau)}\int_0^t\{\Phi_3(y, E_p)\}dy, \\ I_p(t) &= I_p(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_4(t, I_p)\} + \frac{2\tau}{(2-\tau)M(\tau)}\int_0^t\{\Phi_4(y, I_p)\}dy, \\ A_p(t) &= A_p(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_5(t, A_p)\} + \frac{2\tau}{(2-\tau)M(\tau)}\int_0^t\{\Phi_5(y, A_p)\}dy, \\ R_p(t) &= R_p(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_6(t, R_p)\} + \frac{2\tau}{(2-\tau)M(\tau)}\int_0^t\{\Phi_6(y, R_p)\}dy, \\ W(t) &= W(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_7(t, W\sigma)\} + \frac{2\tau}{(2-\tau)M(\tau)}\int_0^t\{\Phi_7(y, W)\}dy. \end{aligned} \quad (14)$$

Like equation (19), we obtain net equations of (17) and (18) as a Eqs.(20) and (21),

$$\begin{aligned} S(t) &= S(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\prod_1(t, S)\} + \frac{2}{(2-\tau)M(\tau)}\int_0^t\{\prod_1(y, S)\}dy, \\ E(t) &= E(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\prod_2(t, E)\} + \frac{2}{(2-\tau)M(\tau)}\int_0^t\{\prod_1(y, E)\}dy, \\ I(t) &= I(0) + \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\prod_3(t, I)\} + \frac{2}{(2-\tau)M(\tau)}\int_0^t\{\prod_1(y, I)\}dy. \end{aligned} \quad (15)$$

The recurrence formula for equations (19) and (20) we get,

$$\begin{aligned}
S_{Bn}(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_1(t, S_B(n-1))\} + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\Phi_1(y, S_B(n-1))\}dy, \\
S_{pn}(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_2(t, S_p(n-1))\} + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\Phi_1(y, S_p(n-1))\}dy, \\
E_{pn}(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_3(t, E_p(n-1))\} + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\Phi_1(y, E_p(n-1))\}dy, \\
I_{pn}(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_4(t, I_p(n-1))\} + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\Phi_4(y, I_p(n-1))\}dy, \quad (16) \\
A_{pn}(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_5(t, A_p(n-1))\} + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\Phi_5(y, A_p(n-1))\}dy, \\
R_{pn}(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_6(t, R_p(n-1))\} + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\Phi_6(y, R_p(n-1))\}dy, \\
W_n(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\Phi_7(t, W(n-1))\} + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{\Phi_7(y, W(n-1))\}dy.
\end{aligned}$$

and

$$\begin{aligned}
S_n(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\prod_1(t, S(n-1))\} + \frac{2}{(2-\tau)M(\tau)} \int_0^t \{\prod_1(y, S(n-1))\}dy, \\
E_n(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\prod_2(t, E(n-1))\} + \frac{2}{(2-\tau)M(\tau)} \int_0^t \{\prod_1(y, E(n-1))\}dy, \quad (17) \\
I_n(t) &= \frac{2(1-\tau)}{(2-\tau)M(\tau)}\{\prod_3(t, I(n-1))\} + \frac{2}{(2-\tau)M(\tau)} \int_0^t \{\prod_3(y, I(n-1))\}dy.
\end{aligned}$$

With the initial conditions for Eqs.(21) and (22) given by,

$$S_B^0(t) = S_B(t), S_p^0(t) = S_p(t), E_p^0(t) = E_p(t), I_p^0(t) = I_p(t), A_p^0(t) = A_p(t), R_p^0(t) = R_p(t), W^0(t) = W(t).$$

Now to calculate the successive terms, we use the following difference formula,

$$\begin{aligned}
\Gamma_1 n(t) &= S_B n(t) - S_B(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_1(t, S_B(n-1) - \phi_1(t, S_B(n-2))) \} + \\
&\frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{ \Phi_1(y, S_B(n-1) - \phi_1(y, S_B(n-2))) \} dy, \\
\Gamma_2 n(t) &= S_p n(t) - S_p(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_2(t, S_p(n-1) - \phi_2(t, S_p(n-2))) \} + \\
&\frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{ \Phi_2(y, S_p(n-1) - \phi_2(y, S_p(n-2))) \} dy, \\
\Gamma_3 n(t) &= E_p n(t) - E_p(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_3(t, E_p(n-1) - \phi_3(t, E_p(n-2))) \} + \\
&\frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{ \Phi_3(y, E_p(n-1) - \phi_3(y, E_p(n-2))) \} dy, \\
\Gamma_4 n(t) &= I_p n(t) - I_p(n-1)t = \tag{18} \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_4(t, I_p(n-1) - \phi_4(t, I_p(n-2))) \} + \\
&\frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{ \Phi_4(y, I_p(n-1) - \phi_4(y, I_p(n-2))) \} dy, \\
\Gamma_5 n(t) &= A_p n(t) - A_p(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_5(t, A_p(n-1) - \phi_5(t, A_p(n-2))) \} + \\
&\frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{ \Phi_5(y, A_p(n-1) - \phi_5(y, A_p(n-2))) \} dy, \\
\Gamma_6 n(t) &= R_p n(t) - R_p(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_6(t, R_p(n-1) - \phi_6(t, R_p(n-2))) \} + \\
&\frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{ \Phi_6(y, R_p(n-1) - \phi_6(y, R_p(n-2))) \} dy, \\
\Gamma_7 n(t) &= W n(t) - W(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_7(t, W(n-1) - \phi_7(t, W(n-2))) \} + \\
&\frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{ \Phi_7(y, W(n-1) - \phi_7(y, W(n-2))) \} dy.
\end{aligned}$$

While the recurrence formula for the visitors population in equation (22) are given by,

$$\begin{aligned}
\mathfrak{R}_1 n(t) &= S_n(t) - S(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \left\{ \prod_1(t, S(n-1)) - \prod_1(t, S(n-2)) \right\} + \\
&\frac{2}{(2-\tau)M(\tau)} \int_0^t \left\{ \prod_1(y, S(n-1)) - \prod_1(y, S(n-2)) \right\} dy, \\
\mathfrak{R}_2 n(t) &= E_n(t) - E(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \left\{ \prod_2(t, E(n-1)) - \prod_2(t, E(n-2)) \right\} + \\
&\frac{2}{(2-\tau)M(\tau)} \int_0^t \left\{ \prod_2(y, E(n-1)) - \prod_2(y, E(n-2)) \right\} dy, \\
\mathfrak{R}_3 n(t) &= I_n(t) - I(n-1)t = \\
&\frac{2(1-\tau)}{(2-\tau)M(\tau)} \left\{ \prod_3(t, S(n-1)) - \prod_3(t, I(n-2)) \right\} + \\
&\frac{2}{(2-\tau)M(\tau)} \int_0^t \left\{ \prod_3(y, I(n-1)) - \prod_3(y, I(n-2)) \right\} dy.
\end{aligned} \tag{19}$$

Here we have the following equation (25) with $i = 1, 2, 3, 4, 5, 6, 7$ and equation (26) with $j = 1, 2, 3$ are given,

$$\begin{aligned}
S_B n(t) &= \sum_{i=1}^n \Gamma_1 i(t), S_p n(t) = \sum_{i=1}^n \Gamma_2 i(t), E_p n(t) = \sum_{i=1}^n \Gamma_3 i(t), \\
I_p n(t) &= \sum_{i=1}^n \Gamma_4 i(t), A_p n(t) = \sum_{i=1}^n \Gamma_5 i(t), R_p n(t) = \sum_{i=1}^n \Gamma_6 i(t), \\
W n(t) &= \sum_{i=1}^n \Gamma_7 i(t).
\end{aligned} \tag{20}$$

$$\begin{aligned}
S_n(t) &= \sum_{i=1}^n \prod_1 i(t), E_n(t) = \sum_{i=1}^n \prod_2 i(t), \\
I_n(t) &= \sum_{i=1}^n \prod_3 i(t).
\end{aligned} \tag{21}$$

Here we use the same strategy and assume the following,

$$\begin{aligned}
&\| \Gamma_1 n(t) \| = \| S_B n(t) - S_B(n-1)(t) \| = \\
&\| \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_1(t, S_B(n-1)) - \phi_1(t, S_B(n-2)) \} \\
&+ \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \{ \Phi_1(y, S_B(n-1)) - \phi_1(y, S_B(n-2)) \} \| dy.
\end{aligned}$$

Using triangle inequality for above equation we get,

$$\begin{aligned} & \| S_B n(t) - S_B(n-1)(t) \| \leq \\ & \quad \left\| \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_1(t, S_B(n-1) - \phi_1(t, S_B(n-2))) \} \right\| \\ & + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \| \{ \Phi_1(y, S_B(n-1) - \phi_1(y, S_B(n-2))) \} \| dy. \end{aligned}$$

But we have proved that the kernal satisfied the Lipschits criteria then the above becomes,

$$\begin{aligned} \| S n(t) - S(n-1)(t) \| \leq & \left\| \frac{2(1-\tau)}{(2-\tau)M(\tau)} \{ \Phi_1 \| \{ S_B(n-1) - \right. \\ & \left. S_B(n-2) \} \| \right\| \\ & + \frac{2\tau}{(2-\tau)M(\tau)} \Phi_1 \int_0^t \| \{ S_B(n-1) - S_B(n-2) \} \| dy. \end{aligned} \quad (22)$$

From simplifying form equation (27) implies,

$$\| \Gamma_1 n(t) \| \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \Phi_1 \| \Gamma_1(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \Gamma_1 \int_0^t \| \Gamma_1(n-1)y \| dy. \quad (23)$$

By the same way we get the following result,

$$\begin{aligned} \| \Gamma_2 n(t) \| & \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \Phi_2 \| \Gamma_2(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \Gamma_2 \int_0^t \| \Gamma_2(n-1)y \| dy, \\ \| \Gamma_3 n(t) \| & \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \Phi_3 \| \Gamma_3(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \Gamma_3 \int_0^t \| \Gamma_3(n-1)y \| dy, \\ \| \Gamma_4 n(t) \| & \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \Phi_4 \| \Gamma_4(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \Gamma_4 \int_0^t \| \Gamma_4(n-1)y \| dy, \quad (24) \\ \| \Gamma_5 n(t) \| & \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \Phi_5 \| \Gamma_5(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \Gamma_5 \int_0^t \| \Gamma_5(n-1)y \| dy, \\ \| \Gamma_6 n(t) \| & \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \Phi_6 \| \Gamma_6(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \Gamma_6 \int_0^t \| \Gamma_6(n-1)y \| dy. \end{aligned}$$

Similarly from equation (24) we have below result,

$$\begin{aligned} \| \mathfrak{R}_1 n(t) \| & \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \prod_1 \| \mathfrak{R}_1(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \mathfrak{R}_1 \int_0^t \| \mathfrak{R}_1(n-1)y \| dy, \\ \| \mathfrak{R}_2 n(t) \| & \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \prod_2 \| \mathfrak{R}_2(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \mathfrak{R}_2 \int_0^t \| \mathfrak{R}_2(n-1)y \| dy, \quad (25) \\ \| \mathfrak{R}_3 n(t) \| & \leq \frac{2(1-\tau)}{2-\tau M(\tau)} \prod_3 \| \mathfrak{R}_3(n-1)(t) \| + \frac{2\tau}{2-\tau(M(\tau))} \mathfrak{R}_3 \int_0^t \| \mathfrak{R}_3(n-1)y \| dy. \end{aligned}$$

now considered the theorem given below,

Theorem 1.3. *The model defined in system (1) has exact coupled solution if the condition below hold that is we find that*

$$\frac{2(1-\tau)}{(2-\tau)M(\tau)}\Phi_1 + \frac{2\tau}{(2-\tau)M(\tau)}\Phi_1 < 1$$

Proof: We shown that all the equations in (28) and (29) are bounded and the functions $S_B, S_p, E_p, I_p, A_p, R_p, W$ fulfill the Lipschitz condition, so Eqs. (28) and (29) by recursive method its succeeding relation are given below,

$$\begin{aligned} \|\Gamma_1\| &\leq \|S_B n(0)\| \left[\frac{2(1-\tau)}{(2-\tau)M(\tau)}\Phi_1 + \frac{2\tau}{(2-\tau)M(\tau)}\Phi_1 \right]^n, \\ \|\Gamma_2\| &\leq \|S_p n(0)\| \left[\frac{2(1-\tau)}{(2-\tau)M(\tau)}\Phi_2 + \frac{2\tau}{(2-\tau)M(\tau)}\Phi_2 \right]^n, \\ \|\Gamma_3\| &\leq \|E_p n(0)\| \left[\frac{2(1-\tau)}{(2-\tau)M(\tau)}\Phi_3 + \frac{2\tau}{(2-\tau)M(\tau)}\Phi_3 \right]^n, \\ \|\Gamma_4\| &\leq \|I_p n(0)\| \left[\frac{2(1-\tau)}{(2-\tau)M(\tau)}\Phi_4 + \frac{2\tau}{(2-\tau)M(\tau)}\Phi_4 \right]^n, \\ \|\Gamma_5\| &\leq \|A_p n(0)\| \left[\frac{2(1-\tau)}{(2-\tau)M(\tau)}\Phi_5 + \frac{2\tau}{(2-\tau)M(\tau)}\Phi_5 \right]^n, \\ \|\Gamma_6\| &\leq \|R_p n(0)\| \left[\frac{2(1-\tau)}{(2-\tau)M(\tau)}\Phi_6 + \frac{2\tau}{(2-\tau)M(\tau)}\Phi_6 \right]^n, \\ \|\Gamma_7\| &\leq \|W n(0)\| \left[\frac{2(1-\tau)}{(2-\tau)M(\tau)}\Phi_7 + \frac{2\tau}{(2-\tau)M(\tau)}\Phi_7 \right]^n. \end{aligned} \tag{26}$$

which shows that the existence and as well as the continuity of the concern solutions is valid and proved. Furthermore, to ensure that the above function is a solution of Eq. (3), we proceed as follows:

$$\begin{aligned} S_B(t) - S_B(0) &= S_B n(t) - T_1(t), \\ S_p(t) - S_p(0) &= S_p n(t) - T_2(t), \\ E_p(t) - E_p(0) &= E_p n(t) - T_3(t), \\ I_p(t) - I_p(0) &= I_p n(t) - T_4(t), \\ A_p(t) - A_p(0) &= A_p n(t) - T_5(t), \\ R_p(t) - R_p(0) &= R_p n(t) - T_6(t), \\ W(t) - W(0) &= W n(t) - T_7(t). \end{aligned} \tag{27}$$

Where the terms $T_1(t), T_2(t), T_3(t), T_4(t), T_5(t), T_6(t)$ and $T_7(t)$ are classified as below,

$$\begin{aligned}
& \| T_1 n(t) \| = \left\| \frac{2(1-\tau)}{2-\tau M(\tau)} \Phi_1(t, S_B n) - \Phi_1(t, S_B(n-1)) \right\| + \\
& \quad \frac{2\tau}{2-\tau M(\tau)} \int_0^t (\Phi_1(y, S_B n) - \Phi_1(y, S_B(n-1))) \| dy. \\
& \| T_1(t) \| \leq \left\| \frac{2(1-\tau)}{2-\tau M(\tau)} \Phi_1(t, S_B n) - \Phi_1(t, S_B(n-1)) \right\| + \\
& \quad \frac{2\tau}{2-\tau M(\tau)} \int_0^t (\Phi_1(y, S_B n) - \Phi_1(y, S_B(n-1))) \| dy. \\
& \leq \frac{2(1-\tau)}{(2-\tau)M(\tau)} \Gamma_1 \| S_B - S_B(n-1) \| + \frac{2\tau}{(2-\tau)M(\tau)} \Gamma_1 \| S_B - S_B(n-1) \| \quad (28)
\end{aligned}$$

In recurrence manner we write as

$$\| T_1(t) \| \leq \left(\frac{2(1-\tau)}{2-\tau M(\tau)} + \frac{2\tau}{2-\tau M(\tau)} t_0 \right)^{n+1} \cdot \Gamma_1^{n+1} a \quad (29)$$

Now using limit $n \rightarrow \infty$ on Eq.(34)

$$\| T_1(t) \| \rightarrow 0.$$

The same procedure using for Eq.(32) we get,

$$\begin{aligned}
& \| T_2(t) \| \rightarrow 0, \| T_3(t) \| \rightarrow 0, \| T_4(t) \| \rightarrow 0, \\
& \| T_5(t) \| \rightarrow 0, \| T_6(t) \| \rightarrow 0, \| T_7(t) \| \rightarrow 0.
\end{aligned}$$

To show system (3) having unique solution, we suppose that there exists another solution of system (3) are $S_{1B}(t), S_{1p}(t), E_{1p}(t), I_{1p}(t), A_{1p}(t), R_{1p}(t)$ and $W_1(t)$, such that,

$$S_B(t) - S_{1B}(t) = \frac{2(1-\tau)}{(2-\tau)M(\tau)} \Phi_1(t, S_B) - \Phi_1(t, S_{1B}) + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t \Phi_1(y, S_B) - \Phi_1(y, S_{1B}) dy \quad (30)$$

Now for Eq(34) using $\| \cdot \|$, and applying Lipschitz condition of kernel we,

$$\| S_B(t) - S_{1B}(t) \| \left(1 - \frac{2(1-\tau)}{(2-\tau)M(\tau)} \Gamma_1 - \frac{2\tau}{(2-\tau)M(\tau)} \Gamma_1 t \right) \leq 0 \quad (31)$$

Theorem 1.4. *The model (3) solution will unique if*

$$\left(1 - \frac{2(1-\tau)}{(2-\tau)M(\tau)} \Gamma_1 - \frac{2\tau}{(2-\tau)M(\tau)} \Gamma_1 t \right) > 0$$

Proof: If condition defined in above theorem hold then Eq.(36) written as,

$$\| S_B(t) - S_{1B}(t) \| = 0. \quad (32)$$

We easily get that,

$$S_B(t) = S_{1B} \quad (33)$$

Provided that the following solution of all concern,

$$\begin{aligned} S_p(t) &= S_{1p}(t), E_p(t) = E_{1p}(t), I_p(t) = I_{1p}(t), \\ A_p(t) &= A_{1p}(t), R_p(t) = R_{1p}(t), W(t) = W_1(t), \end{aligned}$$

Similarly apply the same theorem (above) and procedure we also find the visitors population as below, provided that the following solution of all concern,

$$S(t) = S_1(t), E(t) = E_1(t), I(t) = I_1(t).$$

Results

In this subsection we derived two types of results that both the population showing the same effect on the new disease **"Corona Virus"** spread in **China "Wahan"** city. The equations (13), (14), (16), and (17) providing the same values i.e, $\psi = 1$ and $e = \frac{1}{2}$, indicated that if the visitors visit the particular city or no visitors there in the concern city the spreading ratio of the disease remain same from the fixed values taken both population. These fixed values of both the population ψ and e create a perception that visitors have not a key role in this epidemic disease which spread in particular region of the **"Wahan" city**.

The model defined in system (4) by using the strategies of Eqs.(28) and (29), we see that the succeeding relation with continuity like in Eq.(32)the terms $\hat{T}_1, \hat{T}_2, \hat{T}_3$, showing same behavior of Eq.(34) with assigning new supposition of solution $\hat{S}_1, \hat{S}_2, \hat{S}_3$, provides us Eq.(36) of the form of Eq.(35) revealed that the spreading of Corona virus in **"Wahan city"** do not effected by visitors.

Conclusion

considering the published data with calculating all parameters, we concluded that the models **"BHRP"**, and **"RP"** showed that the spread of Corona virus is very high then MERS in any population. But the addition of our model to published data model showed that the susceptible Bats and visitors to Wuhan or any country having same estimation as that population, more specially visitors of any country are not responsible in the spread of more infection in that area. Since the objective of this mathematical model is to estimate the role of susceptible Bats and visitors in spread of **"Corona virus"** in any population.

2 Simulation methods and statistical analysis

For curve fitting we used Runge-Kutta-fourth-order method. the reproductive number was considered 0.5 for (BHRP) model while for Bats and visitors we considered $R_0 = 1$.

3 Mathematical Model And transmission of Bats-Hosts-Reservoir-People(BHRP)

On 19 January, 2020 [9] published BHRP mathematical transmission model in bioRxiv, with the following certain assumptions:

Bats population: 1:) Divided into four section, susceptible, exposed, infected, and removed are $(S_B), (E_B), (I_B)$ and (R_B) respectively. But in our model we considered susceptible Bats (S_B) only, and ignored the remaining because of no concern in human population. While n_B and m_B are taken the birth and death rate. B is the rate of infection from S_B when contact with I_B .

Host Population: 2:) Population of host is also divided in four compartments: $(S_H), (E_H), (I_H)$ and (R_H) denoted susceptible, exposed, infected, and removed hosts. Here we leave all the compartments of the Hosts population, due to no role in human population infection.

Reservoir Population: Here W denoted SARS-CoV-2 in reservoir (the seafood market). In our study the rate of asymptomatic infected people and symptomatic infected people export virus with μ_p and $\hat{\mu}_p$ respectively from markets or any other source.

People Population: This population is divided in to five classes, there five compartments are: $(S_p), (E_p), (I_p), (A_p)$ and (R_p) denoted susceptible, exposed, symptomatic infected, asymptomatic infected, and removed peoples compartments. Here m_p represent death rate. δ_p was defined for the proportion of asymptomatic infection, also the S_p infected from W and I_p with the transmission rate β_W and β_p .

4 The New Bats And Visitors Model

Here we drawn some more assumptions for our model:

Assumption 1: In our study we considered the Bats population only.

Assumption 2: We ignored the transmission of Bats-Host population due to unknown spreading source.

Assumption 3: Hosts population of Bats is totally ignored in this study because of its unknown relation and infection.

Assumption 4: We leave other related terms and contact in reservoir population in this article.

Assumption 5: We taken the class of susceptible Bats to check weather this class spread any infection in the concern model.

Assumption 6: In this study we introduced visitors population compartment separately. During in any outbreak the death rate is normally high, therefore, the model BHRP converted

into BRP and then in BRV(Bats-reservoir-visitors) model by the following,

$$\begin{aligned}
S_B^\bullet &= A_B - m_B S_B - \beta_B S_B I_B, \\
S_p^\bullet &= A_B - m_B S_B - \beta_B S_B (I_P + \kappa A_p) - \beta_W S_p W, \\
E_p^\bullet &= \beta_B S_B (I_P + \kappa A_p) + \beta_W S_p W - (1 - \delta_p) W_p E_p - \delta_p W_p E_p - m_p E_p, \\
I_p^\bullet &= (1 - \delta_p) W_p E_p - (\gamma_p + m_p) I_p, \\
A_p^\bullet &= \delta_p W_p' E_p - (\gamma_p' + m_p) A_p, \\
R_p^\bullet &= \gamma_p I_p + \gamma_p' A_p - m_p R_p, \\
W^\bullet &= \mu_p I_p + \mu_p' A_p - \epsilon W.
\end{aligned} \tag{34}$$

In outbreak the visitors and hosts interaction was very slow in "Wuhan" or other country. Now introducing dynamic population of the visitors to any city or country is as in separate for estimation of infection,

$$\begin{aligned}
S^\bullet &= \mu_N - \sigma(t) S I - \alpha S, \\
E^\bullet &= \sigma(t) S I - \lambda E - \alpha E, \\
I^\bullet &= \lambda E - \eta I - \alpha I.
\end{aligned} \tag{35}$$

5 Transmissibility of Corona Virus or SARS-CoV-2 Based on the BRP And Bats-visitors model

In the article, R_0 is assessed for transmissibility of Corona virus or SARS-CoV-2. The, R_0 we defined as, the expected number of secondary infections with introducing any single infected individual to susceptible population [10, 11, 12]. The value of $R_0 > 1$, or $R_0 < 1$, showing different characteristics of the model in outbreak to in control or out of control. In this study, R_0 was deduced from the BRP model and Bats-visitors population by next generation matrix approach [13].

6 Some Results

Here in this subsection, we given some basic definition of the fractional calculus, which we used later,

Definition A;

Let $H \in G^1(c, d)$ and d is greater then c , also $\tau \in [0, 1]$ then CFF derivative [14] is given as,

$$P_t^\tau(h(t)) = \frac{M(\tau)}{(1-\tau)} \int_0^\tau \hat{h}(x) e^{-\tau(\frac{\tau-x}{1-\tau})} dx.$$

Here $M(\tau)$, which implies normality with $M(0) = M(1) = 1$ [14]. But if H is not contain in $G^1(c, d)$ then we get,

$$P_t^\tau(h(t)) = \frac{\tau M(\tau)}{(1-\tau)} \int_a^\tau (h(t) - h(x)) e^{-\tau(\frac{\tau-x}{1-\tau})} dx.$$

Definition B:

When $v = \frac{(1-\tau)}{\tau}\varepsilon[0, \infty)$ and $\tau = \frac{1}{1+v}\varepsilon[0, 1]$ we have the following result,

$$P_t^\tau(h(t)) = \frac{K(v)}{v} \int_a^t \hat{h}(x)e^{(-\frac{t-x}{v})} dx.$$

With

$$K(0) = K(\infty) = 1.$$

Applying $\lim v \rightarrow 0$ we get,

$$P_t^\tau(h(t)) = \lim_{v \rightarrow 0} \frac{1}{v} e^{(-\frac{t-x}{v})} dx = v(x-t).$$

This integral definition was provided by Losada and J.Nieto [15].

Definition C:

Suppose that " $0 < \tau < 1$ " then CFFD integral of given function h is as,

$$I_t^\tau h(t) = \frac{2(1-\tau)}{(2-\tau)M(\tau)} h(\tau) + \frac{2\tau}{(2-\tau)M(\tau)} \int_0^t h(s) ds, t \geq 0.$$

The above equation will also be written as,

$$\frac{2(1-\tau)}{(2-\tau)M(\tau)} + \frac{2\tau}{(2-\tau)M(\tau)} = 1. \quad (36)$$

This gives that $M(\tau) = \frac{2}{(2-\tau)}$, with $0 < \tau < 1$. From Eq.(3) a new form of the "*Caputo–Fabrizio fractinal derivative*" of order $0 < \tau < 1$ is further investigated by Losada and J.Nieto [15].

$$P_t^\tau h(t) = \frac{1}{(1-\tau)} \int_0^t \hat{h}(x)e^{\tau(\frac{t-x}{1-\tau})} dx.$$

This above form is used in mathematical model of HBV and diabetes model [16, 17].

7 Formulation of Model BVP from BRPV

Now we replacing system (1) by new CF fractional derivative, given below as,

$$\begin{aligned} C_0FD_t^\tau S_B &= A_B - m_B S_B - \beta_B S_B I_B, \\ C_0FD_t^\tau S_p &= A_B - m_B S_B - \beta_B S_B (I_P + \kappa A_p) - \beta_W S_p W, \\ C_0FD_t^\tau E_p &= \beta_B S_B (I_P + \kappa A_p) + \beta_W S_p W - (1 - \delta_p) W_p E_p - \delta_p W_p E_p - m_p E_p, \\ C_0FD_t^\tau I_p &= (1 - \delta_p) W_p E_p - (\gamma_p + m_p) I_p, \\ C_0FD_t^\tau A_p &= \delta_p W_p' E_p - (\gamma_p' + m_p) A_p, \\ C_0FD_t^\tau R_p &= \gamma_p I_p + \gamma_p' A_p - m_p R_p, \\ C_0FD_t^\tau W &= \mu_p I_p + \mu_p' A_p - \epsilon W. \end{aligned} \quad (37)$$

Similarly replacing equation (2) by CF fractional derivative as,

$$\begin{aligned}
C_0FD_t^\tau S &= \mu_N - \sigma(t)SI - \alpha S, \\
C_0FD_t^\tau E &= \sigma(t)SI - \lambda E - \alpha E, \\
C_0FD_t^\tau I &= \lambda E - \eta I - \alpha I.
\end{aligned} \tag{38}$$

8 Basic Reproductive Number

The basic reproductive number of the system (1) is given by,

$$R_0 = \frac{A_B(1 - \delta_p)W_p + (\gamma_p I_p)A_p}{\mu_p + \mu_p A_p - W} + \frac{\beta_W A_p}{(\gamma_p' + m_p)}.$$

9 Table1: Parameters Used In BHRP And BRP Visitors Model

Notation	Parameter description
n_B	Bats Birth rate
n_H	Hosts Birth rate
n_p	People Birth rate
m_B	Bats death rate
m_H	hosts death rate
$1/\omega_B$	Bats incubation period
$1/\omega_H$	Host incubation period
$1/\omega_p$	People incubation period
$1/\hat{\omega}_B$	Latent people period
$1/\gamma_B$	Bats infection period
$1/\gamma_H$	Hosts infection period
$1/\gamma_p$	The symptomatic people infectious period
$1/\hat{\gamma}_p$	The asymptomatic people infectious period
β_B	I_B to S_B transmission rate
β_{BH}	I_B to S_H transmission rate
β_H	I_H to S_H transmission rate
β_p	I_p to S_p transmission rate
β_W	W to S_p transmission rate

10 Declarations

Ethical Approval and Consent to participate

The article is approved and all participants are agree.

Consent for publication

All the participants are agree for its publication.

Availability of data and materials

All the data will be available.

Conflict of Interest

The authors shows no conflict of interest for the article.

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References

- [1] Tian-Mu Chen¹, Jia Rui, Qiu-Peng Wang, Ze-Yu Zhao, Jing-An Cui and Ling Yin, A mathematical model for simulating the phase-based transmissibility of a novel coronavirus, *Infectious Diseases of Poverty*, <https://doi.org/10.1186/s40249-020-00640-3>.
- [2] World Health Organization. Coronavirus. World Health Organization, cited January 19, 2020. Available: <https://www.who.int/health-topics/coronavirus>.
- [3] Muhammad Tahir, Syed Inayat Ali Shah, Gul Zaman and Tahir Khan, A Dynamic Compartmental Mathematical Model Describing The Transmissibility Of MERS-CoV Virus In Public, *Punjab University Journal of Mathematics (ISSN 1016-2526) Vol. 51(4)(2019)* pp. 57-71.
- [4] Li Q, Guan X, Wu P, Wang X, Zhou L, Tong Y, et al. Early transmission dynamics in Wuhan, China, of novel coronavirus-infected pneumonia. *N Engl J Med*. 2020. <https://doi.org/10.1056/NEJMoa2001316>
- [5] M.Tahir et al. Stability Behaviour of Mathematical Model MERS Corona Virus Spread in Population, *Filomat* 33:12 (2019), 39473960 <https://doi.org/10.2298/FIL1912947T>.
- [6] M. De la Sen, Ravi P. Agarwa, A. Ibeas, and S. Alonso-Quesada, On the Existence of Equilibrium Points, Boundedness, Oscillating Behavior and Positivity of a SVEIRS Epidemic Model under Constant and Impulsive Vaccination, *Advances in Difference Equations* Volume 2011, Article ID 748608, 32 pages [doi:10.1155/2011/748608](https://doi.org/10.1155/2011/748608).

- [7] Junli Liu , Tailei Zhang, Global stability for a tuberculosis model, *Mathematical and Computer Modelling* 54 (2011) 836845.
- [8] Li Q, Guan X, Wu P, Wang X, Zhou L, Tong Y, et al. Early transmission dynamics in Wuhan, China, of novel coronavirus-infected pneumonia. *N Engl J Med*. 2020. <https://doi.org/10.1056/NEJMoa2001316>.
- [9] Chen T, Rui J, Wang Q, Zhao Z, Cui J-A, Yin L. A mathematical model for simulating the transmission of Wuhan novel Coronavirus. *bioRxiv*. 2020: 2020.2001.2019.911669. Accessed 13 Feb 2020.
- [10] Chen T, Rui J, Wang Q, Zhao Z, Cui J-A, Yin L. A mathematical model for simulating the transmission of Wuhan novel Coronavirus. *bioRxiv*. 2020: 2020.2001.2019.911669. Accessed 13 Feb 2020.
- [11] Zhang S, Hu Q, Deng Z, Hu S, Liu F, Yu S, et al. Transmissibility of acute haemorrhagic conjunctivitis in small-scale outbreaks in Hunan Province, China. *Sci Rep*. 2020;10:119.
- [12] Tahir et al., Ebola virus epidemic disease its modeling and stability analysis required abstain strategies, *Cogent Biology* (2018), 4: 1488511 <https://doi.org/10.1080/23312025.2018.1488511>.
- [13] Cui J-A, Zhao S, Guo S, Bai Y, Wang X, Chen T. Global dynamics of an epidemiological model with acute and chronic HCV infections. *Appl Math Lett*. 2020;103:106203.
- [14] Caputo, M., Fabrizio, M.: A new definition of fractional derivative without singular kernel. *Prog. Fract. Differ. Appl.* 1(2), 113 (2015).
- [15] Losada, J., Nieto, J.J.: Properties of a new fractional derivative without singular kernel. *Prog. Fract. Differ. Appl.* 1(2), 8792 (2015).
- [16] Singh, J., Kumar, D., Baleanu, D.: On the analysis of fractional diabetes model with exponential law. *Adv. Differ. Equ.* 2018(1), 231 (2018).
- [17] Muhammad Tahir, Syed Inayat Ali Shah and Gul Zaman, Prevention strategy for superinfection mathematical model tuberculosis and HIV associated with AIDS, *Cogent Mathematics and Statistics* (2019), 6: 1637166 <https://doi.org/10.1080/25742558.2019.1637166>.
- [18] Cui J-A, Zhao S, Guo S, Bai Y, Wang X, Chen T. Global dynamics of an epidemiological model with acute and chronic HCV infections. *Appl Math Lett*. 2020;103:106203.
- [19] Longini IM Jr, Nizam A, Xu S, Ungchusak K, Hanshaoworakul W, Cummings DA, et al. Containing pandemic influenza at the source. *Science*. 2005;309: 10837.
- [20] Chen T, Ka-Kit Leung R, Liu R, Chen F, Zhang X, Zhao J, et al. Risk of imported Ebola virus disease in China. *Travel Med Infect Dis*. 2014;12:6508.

- [21] Chen TM, Chen QP, Liu RC, Szot A, Chen SL, Zhao J, et al. The transmissibility estimation of influenza with early stage data of small-scale outbreaks in Changsha, China, 2005-2013. *Epidemiol Infect.* 2017;145:42433.
- [22] Tahir et. al, Ebola virus epidemic disease its modeling and stability analysis required abstain strategies, *Cogent Biology* (2018), 4: 1488511 <https://doi.org/10.1080/23312025.2018.1488511>.
- [23] M.Tahir, Nousheen Anwar, syed Inayat Ali shah and Tahir Khan, Modeling and stability analysis of epidemic expansion disease Ebola virus with implications prevention in population, *Cogent Biology* (2019), 5: 1619219 <https://doi.org/10.1080/23312025.2019.1619219>.

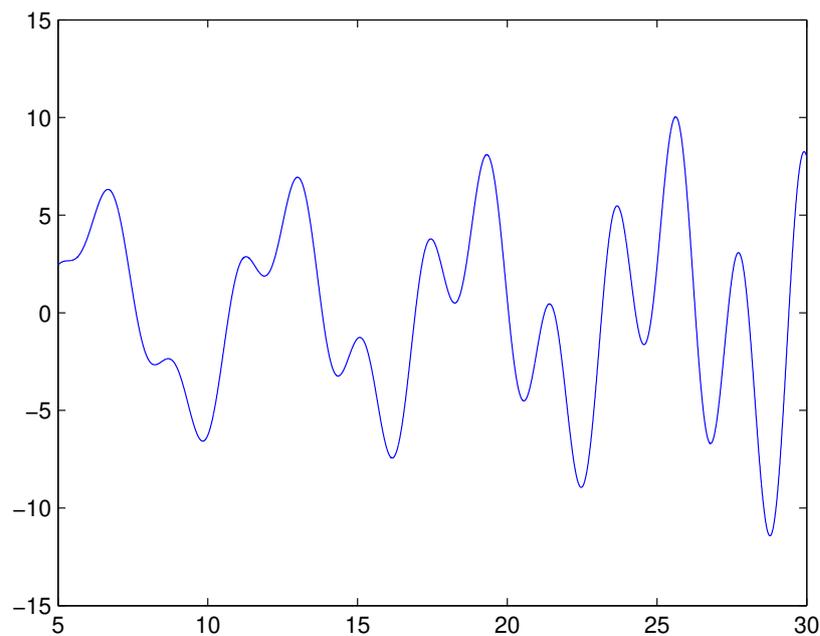


Figure 1: The Plot Show A Random Behavior of Both population model.

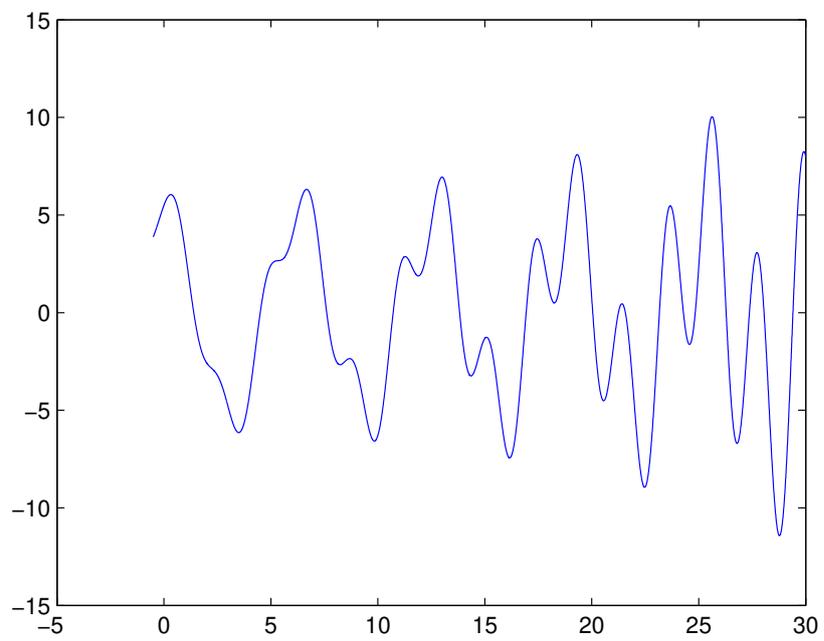


Figure 2: The Plot Show A Random Behavior of Both population model.

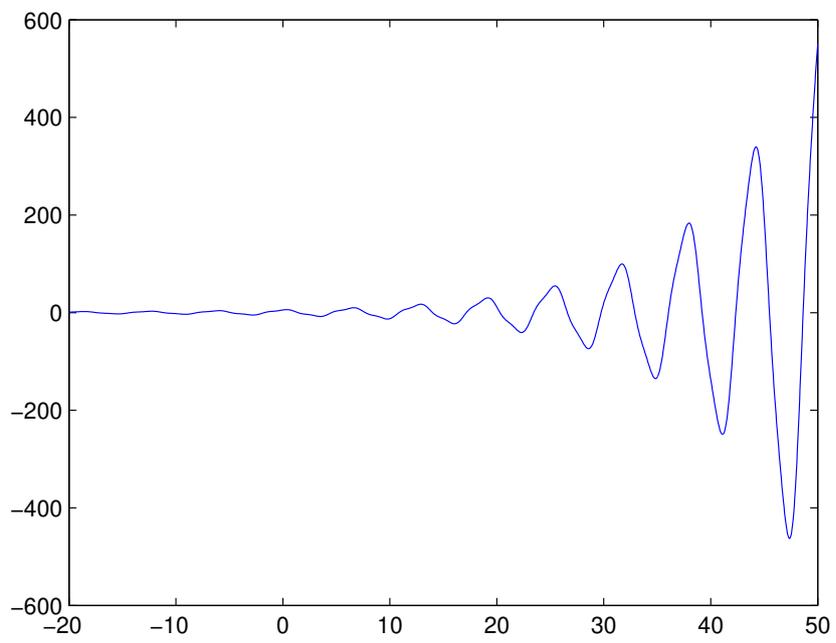


Figure 3: The Plot Show A Random Behavior of Both population model.

Figures

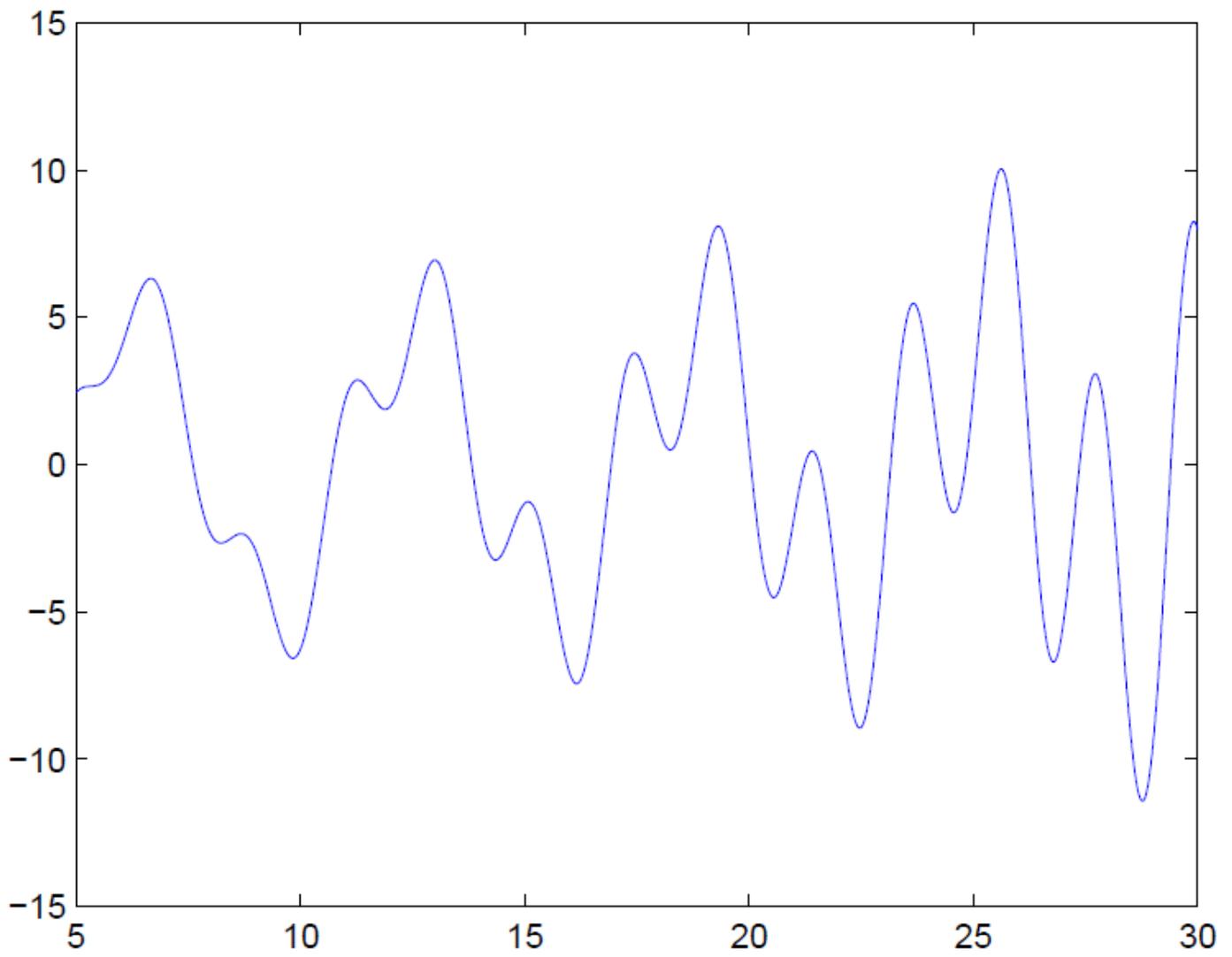


Figure 1

The Plot Show A Random Behavior of Both population model.

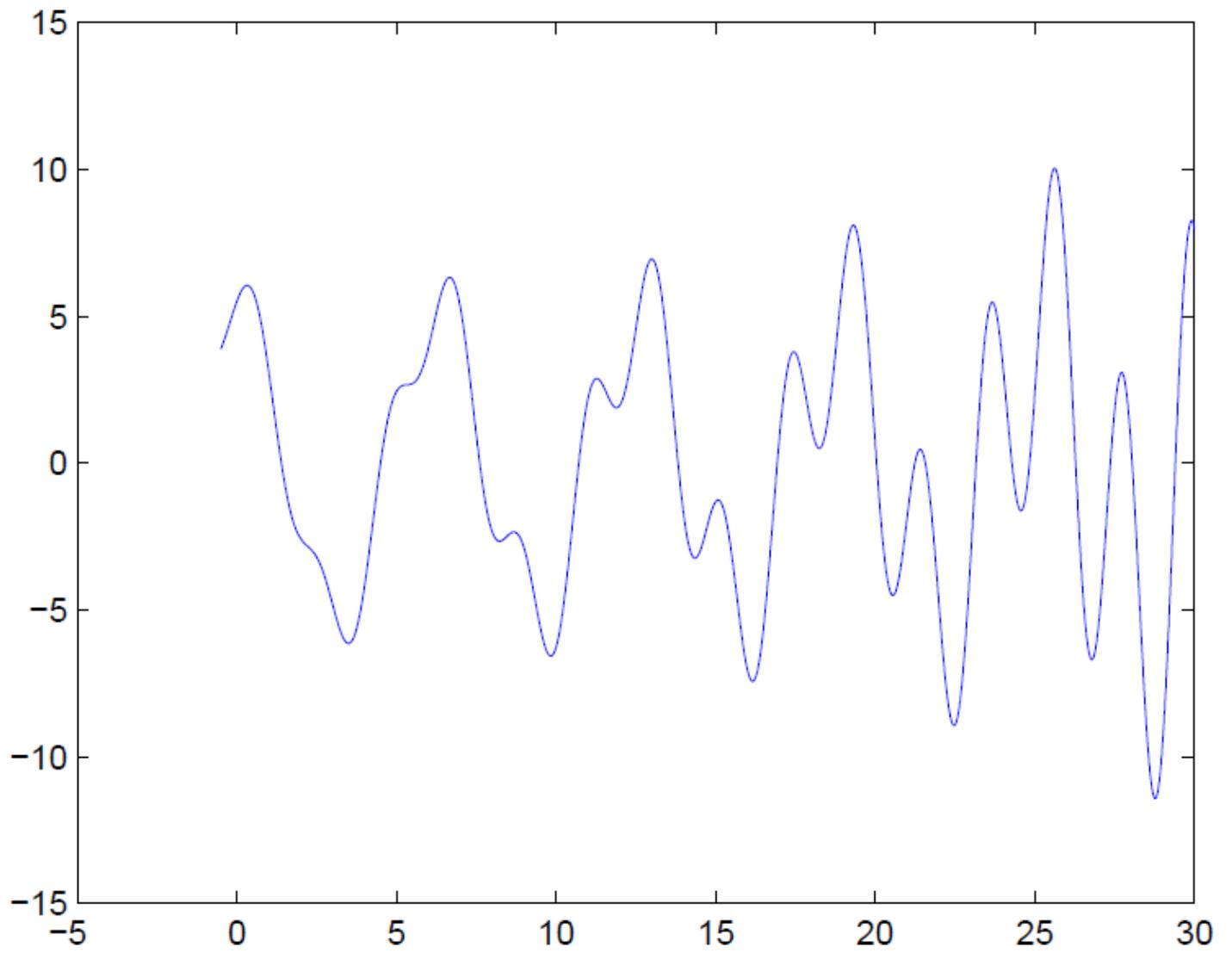


Figure 2

The Plot Show A Random Behavior of Both population model.

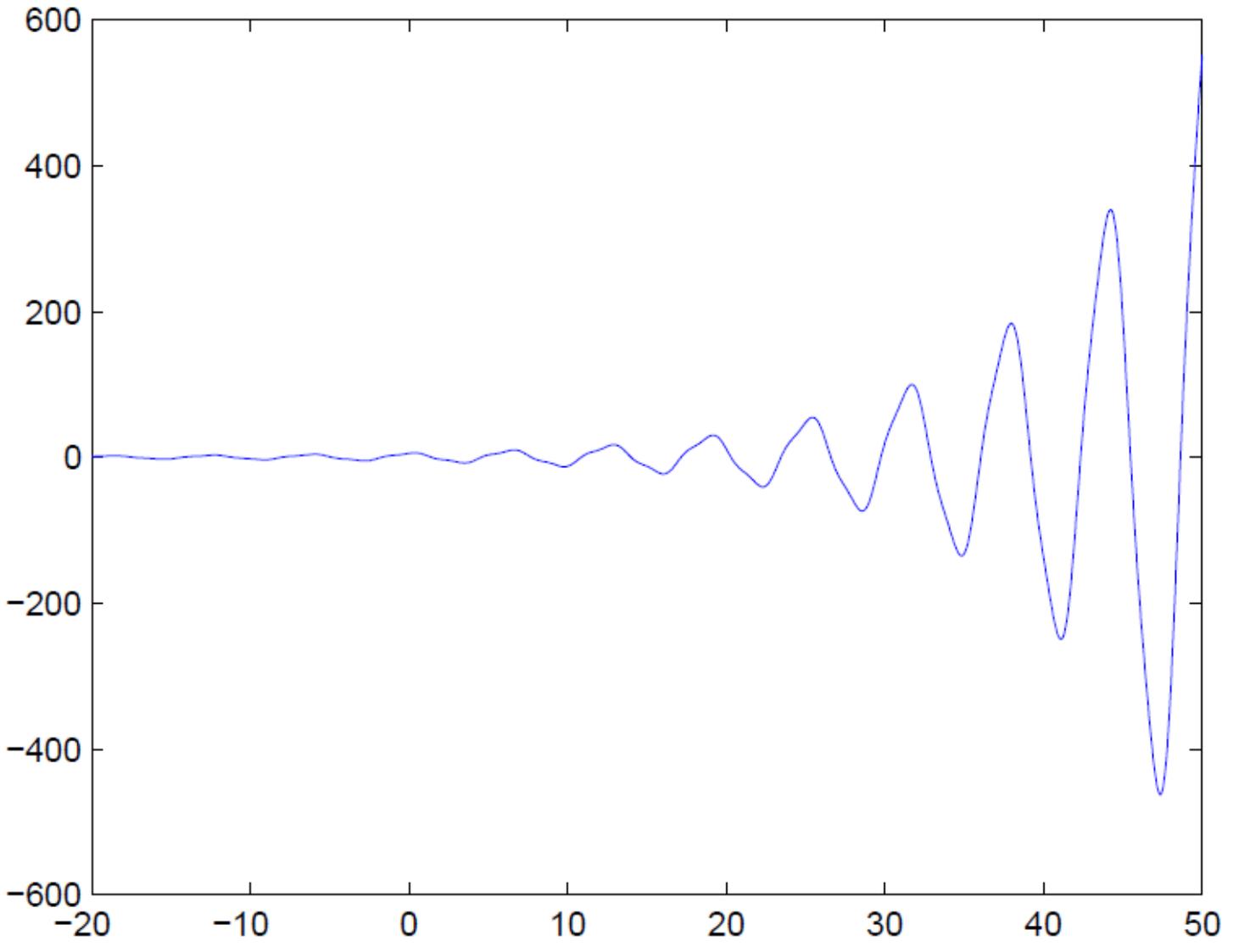


Figure 3

The Plot Show A Random Behavior of Both population model.