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A Single-Phase Modelling for the Oxygen Uptake Rate in Excess Post-Exercise Oxygen Consumption

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Abstract

A two-parameters [a rate constant k and a factor f ($0 \leq f < 1$)] modelling, describing satisfactorily the post-exercise oxygen uptake rate (V_{O2}) as a function of the recovery time (t), is presented. f controls the rate equation dV_{O2}/dt, particularly at $t = 0$ where $(dV_{O2}/dt)_{t=0} \propto -k(1-f)$, a less abrupt decay than $(dV_{O2}/dt)_{t=0} \propto -k$ expected from an exponential. Fitting the model to a set of experimental V_{O2} vs t data after a 3MT it was found a set of values with f close to 0 and another with $f > 1/2$, with a narrow distribution of values for the half-recovery time $\tau_{1/2}=(1/k)\ln[(2-f)/(1-f)]$ ($\langle\tau_{1/2}\rangle$=0.641 min, σ=0.062 min), very similar to that (T) found by fitting a model based on a logit transform ($\langle T\rangle$=0.672 min, σ=0.081 min). The parameter f is a reliable index of the initial acceleration of the oxygen uptake rate recovery (and likely of the heart rate recovery) and, together with the half-recovery time $\tau_{1/2}$, may be a useful method in characterizing and monitoring performs and exercise forms, very important in the physiology area.

Introduction

Three-minute all-out test (3MT) is a maximal effort evaluation protocol based on the original model of critical power whose metabolic parameters of aerobic (EP) and anaerobic (WEP) capacities are obtained through the mathematic analysis of a subject's output power kinetics. The kinetics of output power of 3MT can determine important parameters for training prescription such as some transition limits when it comes to effort intensity[1]. EP is the threshold that measures the transition from intense to severe exercise and uses mainly oxidative metabolism[1,2], while WEP uses anaerobic sources and is a finite amount of work carried until it runs out in severe intensity exercises[3]. The post-exercise oxygen consumption (EPOC) after 3MT can provide important information about the maximum effort performed by the individual.

Direct physiological parameters such as blood lactate concentration and oxygen uptake measure the domains of intensity transition during exercise[4]. Researchers have been using such physiological markers, collecting them right after exercise, to determine the contribution of the aerobic and anaerobic energetic metabolisms[5]. EPOC may physiologically represent the reestablishment of blood and muscular O_2 to ATP and PCr resynthesis, as well as removing lactate from high intensity efforts[6]. Blood lactate concentration and EPOC can reliably estimate the anaerobic capacity in 3MT[7].

The oxygen uptake rate ($\dot{V}O_2$) recovery is conventionally assumed to proceed in a biphasic manner, consisting of "fast" and "slow" components. The overall process has been described by two simple exponentials and it has been acknowledged as the standing model[8-11].

However, it has been argued[12] that application of two-exponential model to post-exercise oxygen uptake rate and heart rate (HR) might be questionable, since metabolic processes like lactate elimination could follow rather the mass action law, instead an exponential decay. Besides that, the procedure by curve fitting to a first order exponential decay in HR recovery, following max exercise, seem not to confirm that a first order equation is the optimal model to establish the most appropriate exercise protocol[13]. The time decaying obtained by this procedure varied unacceptably with small changes in onset of monitoring and the residuals of the fitted curve were non-random[13].

Therefore, it is opportune to consider alternative models based on the mass action law, for instance, to fit the experimental data. The mass action law requires that

the rate equation for V_{O2} decaying takes into account two components: the residual "oxygen debt" ($V_{O2} - V_{O2rest}$) and the "oxygen debt" already paid ($V_{O2peak} - V_{O2}$), in a total net peak post-exercise uptake ($V_{O2peak} - V_{O2rest}$)[12]. Stupnicki *et al.*[12], by applying a logit transform, $\log[X_i/(X_{peak}-X_i)]$, to the recorded variables X_i [post-exercise heart rate (HR) and V_{O2}] and relating them to the decimal logarithm of the recovery time t, obtained single-phase course of changes for both variables, rendering a linear relationship, which facilitated various comparisons. This logit–log transformation was also used with success to model horse post-exercise heat rate recovery[14]. However, Stupnicki *et al.*[12] did not give an analytical form to fit the data nor provided the rate equation for the V_{O2} decaying.

In this work, the rate equation for the oxygen uptake rate decaying, dV_{O2}/dt, is considered as being first-order on the fraction X of the residual "oxygen debt" to decay, and first-order on $(1–fX)$, a function of the fraction $(1–X)$ of the "oxygen debt" already paid, where f $(0 \le f < 1)$ is a factor introduced to control the recovery start. The approach provides an analytical form for the oxygen uptake rate as a recovery time function, without loss of generality, since, by setting $f = 0$, the model becomes just the standing mono-exponential one.

Theoretical aspects

The *f*-single-phase modelling

The rate equation, dV_{O2}/dt, for the oxygen uptake rate (V_{O2}) at the recovery instant t was assumed as being first-order on the fraction X of the residual "oxygen debt" to decay, $X = (V_{O2} - V_{O2rest})/(V_{O2peak} - V_{O2rest})$, where V_{O2peak} is oxygen uptake rate at $t = 0$ and V_{O2rest} the oxygen uptake rate at rest ($t \to \infty$), and first-order on $(1–fX)$, a function of the fraction $(1–X)$ of the "oxygen debt" already paid, $(1–X) = (V_{O2peak} - V_{O2})/(V_{O2peak} - V_{O2rest})$, where f, with the constraint $0 \le f < 1$, is a factor introduced to control the recovery start. Since $dV_{O2}/dt \propto dX/dt$, it follows

$$dX/dt = -kX(1-fX) \quad , \qquad (1)$$

where k (min^{-1}) is a rate constant. The parameter f controls the rate equation (equation (1)), particularly at $t = 0$ when $X = 1$, so

$$(dX/dt)_{t=0} = -k(1-f) \quad . \qquad (2)$$

The solution of equation (1), under the conditions $X = 1$ at $t = 0$ and $X = 0$ at $t \rightarrow \infty$, is

$$X = \exp(-kt)/\{1-f[1-\exp(-kt)]\} \quad . \tag{3}$$

If $f = 0$, the decaying process is a simple exponential $X = \exp(-kt)$ and $(dX/dt)_{t=0} = -k$, a more abrupt decaying than $(dX/dt)_{t=0} = -k(1-f)$ when $f > 0$.

The half-recovery time ($\tau_{1/2}$), the time for X to decay to half of its initial value, can be evaluated from equation (3) as

$$\tau_{1/2} = (1/k)\ln[(2-f)/(1-f)] \quad . \tag{4}$$

Restoring the definition $X = (V_{O2} - V_{O2rest})/(V_{O2peak} - V_{O2rest})$ in equation (3) yields

$$V_{O2} = V_{O2rest} + (V_{O2peak} - V_{O2rest})\exp(-kt)/\{1-f[1-\exp(-kt)]\} \quad . \tag{5}$$

Equation (5), named here f-single-phase (f1p) model, can be fitted to the experimental V_{O2} vs t data to obtain the parameters k, f, V_{O2peak} and V_{O2rest}. Equivalently, equation (5) could be expressed in terms of the parameters k and $\tau_{1/2}$, since f can be solved from equation (4) as $f = [\exp(k\tau_{1/2})-2]/[\exp(k\tau_{1/2})-1)]$, with the constraint $k\tau_{1/2} \geq \ln 2$. Therefore, among the parameters k, f and $\tau_{1/2}$, only two of them are independent in the f1p model.

The integral of equatio (3) between $t = 0$ and t, $X_{integ}(t)$ (in min), when multiplied by the quantity $(V_{O2peak} - V_{O2rest})$, represents the total net volume of oxygen uptake up to the instant t, $VT_{O2net}(t)$, excluded the contribution of the rest component. $X_{integ}(t)$ can be evaluated by

$$X_{integ}(t) = -(1/k)(1/f)\ln\{1-f[1-\exp(-kt)]\} \quad , \tag{6}$$

so, the total integrated quantity, X_{integ}, between $t = 0$ and $t = \infty$, is

$$X_{integ} = -(1/k)(1/f)\ln(1-f) = -\tau_{1/2}(1/f)\ln(1-f)/\ln[(2-f)/(1-f)] \quad . \tag{7}$$

The constraint equation (4) has been used to write X_{integ} indistinctly in terms of k and f or $\tau_{1/2}$ and f in equation (7). If $f = 0$, equation (7) becomes $X_{integ} = 1/k = \tau_{1/2}/\ln 2$ just as in case of an exponential decay, since the limit of $(1/f)\ln[(1-f)]$ is -1 when $f \to 0$.

The total net volume of oxygen uptake up to complete recovery ($t = \infty$), VTO_{2net}, the so-called EPOC, is obtained by multiplying equation (7) by the quantity ($V_{O2peak} - V_{O2rest}$), or

$$VTO_{2net} = (V_{O2peak} - V_{O2rest})X_{integ} \quad . \tag{8}$$

To better illustrate the effect of the parameter f on the experimental V_{O2} vs t curve, Figure 1 shows plots of equation (3) for several values of k (or f) for a fixed value of $\tau_{1/2} = \ln 2$. The inclination $(dX/dt)_{t=0} = -k(1-f)$ at $t=0$ is effectively less abrupt with increasing f (insert in Figure 1), and particularly less abrupt than an exponential decay where $(dX/dt)_{t=0} = -k$. The increase of the parameter f retards effectively the decaying of V_{O2} at the beginning of the recovery, however, the rate of decaying at $t = \tau_{1/2}$, $(dX/dt)_{\tau_{1/2}} = -(k/2)[1-f/2]$, becomes more abrupt with increasing f, as shown by the tangent dash lines plotted with slopes $(dX/dt)_{\tau_{1/2}}$ at $t=\tau_{1/2}$ in Figure 1 for several values of k (or f, since $\tau_{1/2} = \text{cte}$).

(insert Figure 1)

About the logit transformation

It is opportune to compare the present $f1p$ model with that could be inferred from a graphical procedure proposed by Stupnicki *et al.*[12]. They applied a logit transformation, $\log[X_i/(X_{peak}-X_i)]$, to the net recorded variables X_i ($X_i = V_{O2} - V_{O2rest}$ and $X_{peak} = V_{O2peak} - V_{O2rest}$), and observed an unique regime of linear plot, with a negative slope ($-\alpha$), in the whole domain of the logarithm of the recovery time t. Then, the relation $\log[X_i/(X_{peak}-X_i)] = -\alpha \log t + B$, where B is a constant, should be kept. By writing $B = \log T^{\alpha}$, where T is a characteristic time, it is concluded that $X_i = X_{peak}\{1/[(t/T)^{\alpha}+1]\}$, or

$$X = 1/[(t/T)^{\alpha}+1] \quad , \tag{9}$$

to keep the same notation for the fraction $X = X_i / X_{peak} = (V_{O2} - V_{O2rest})/(V_{O2peak} - V_{O2rest})$ of the present work. So equation (9) could be written as

$$V_{O2} = V_{O2rest} + (V_{O2peak} - V_{O2rest})\{1/[(t/T)^{\alpha} + 1]\} \quad . \tag{10}$$

Equation (10), named here Stu model, could be fitted to the experimental V_{O2} vs t data and the parameters α, T, V_{O2peak} and V_{O2rest} determined. T represents the half-recovery time, the time for X in equation (9) to decay to 1/2, the same meaning of the parameter $\tau_{1/2}$ of the f1p model.

The rate equation that governs the decaying law of equation (9) is given by

$$dX/dt = -(\alpha/T)X^{(1+1/\alpha)}(1-X)^{(1-1/\alpha)} \quad , \tag{11}$$

being, therefore, of $(1+1/\alpha)$-order on X and $(1-1/\alpha)$-order on $(1-X)$. The overall order with respect to both, X and $(1-X)$, remains constant and equal to $(1-1/\alpha) + (1+1/\alpha) = 2$, independent on α. The parameter α should be ≥ 1 to avoid negative-order on $(1-X)$. With $\alpha > 1$, equation (11) gives always $(dX/dt)_{t=0} = 0$ at $t = 0$, which is different from $(dX/dt)_{t=0} = -k(1-f)$ of the f1p model. If $\alpha = 1$, the rate equation (11) would be a second-order on X and zero-order on $(1-X)$, and $(dX/dt)_{t=0} = -(1/T)$ at $t = 0$, as the unique exception.

Material and methods

Ten recreationally-trained male runners (24 ± 4 years; 80.3 ± 8.7 kg; 179 ± 5 cm; 9.0 ± 2.5% body fat) participated in this study performing 3MT, after signing an informed consent form approved by the Human Research Ethics Committee of the University of Campinas (protocol CAAE: 61934516.5.0000.5404), in accordance with the Declaration of Helsinki.

3MT was carried out on a non-motorized treadmill (NMT) after a five-minute warm-up consisting in walking on a motorized treadmill (MT) at 6 km/h. Each participant ran on a NMT tethered to an adjustable-height pole by a steel cable attached to a load cell (CSL/ZL-250, MK Control and Instrumentation)[15]. Despite participants were constantly encouraged during the test, no information in regards of time was given. The test only was considered successful after 3 minutes of non-stoppable running. The mechanical output power generated by each subject during the 3MT test

was recorded and analyzed against time to find the aerobic capacity (EP) and anaerobic work capacity (WEP) values of the power output graph[2].

The oxygen uptake and carbon dioxide production were measured by using a gas analyzer (Cosmed Italy K4b²) that was calibrated after each session. The gas analyzer was integrated to an online system of breath-by-breath data caption. The oxygen uptake rate (V_{O2}) was measured as a function of the recovery time (t) up to about 10 min after the start of the 3MT recovery. The models were fitted to the experimental V_{O2} vs t data.

The fitting processes were carried out using a routine of least-squares method based on the Levenberg-Marquardt algorithm. When was the case, the similarity between the parameters given by different models was statistically tested for equal means using One-Way ANOVA.

Results

Figure 2 shows the f1p model fitting well the experimental V_{O2} vs t. The fitted parameters V_{O2peak}, V_{O2rest}, k, and f (or $\tau_{1/2}$) are shown in Table 1 (f1p model) with the respective standard errors. The data were organized in order of increasing f. When the fitting process converged rapidly to the lower constraint value $f = 0$ (subjects 1 and 2 in Table 1), f was set to 0 and k obtained by fitting, and $\tau_{1/2}$ evaluated through $\tau_{1/2} = (1/k)\ln 2$. The mean value of $\tau_{1/2}$ was $\langle \tau_{1/2} \rangle = 0.641$ min with standard deviation $\sigma = 0.062$ min.

(insert Figure 2)

(insert Table 1)

Figure 3(a) shows the fitting of the Stu model to the experimental data of the subject number 8, as an instance, in comparison with that of the f1p model. The Stu model fitted well all the present set of experimental data. Table 1 (Stu model) shows the fitted parameters V_{O2peak}, V_{O2rest}, α, and T with the standard errors. Figure 3(b) shows the adjusted R-square and the reduced sum of squares χ^2 obtained by fitting both the models (Stu and f1p) to the experimental data. The mean value of the half-recovery time T was $\langle T \rangle = 0.672$ min with standard deviation $\sigma = 0.081$ min. Figure 3(c) shows the distribution of T and $\tau_{1/2}$ plotted as normal Gaussian distributions $F(\tau,\sigma) = [1/(2\pi\sigma^2)^{1/2}]\exp[-((t-\tau)/\sigma)^2/2]$, using the mean values $\langle \tau_{1/2} \rangle$ and $\langle T \rangle$ with the respective standard deviations (σ).

(insert Figure 3)

The standard biphasic two-exponential model, which can be cast as $V_{O2} = V_{O2rest} + A_1\exp(-t/\tau_1) + A_2\exp(-t/\tau_2)$, where A_1 and A_2 are constants and τ_1 and τ_2 are the "fast" and the "slow" decaying constant times, respectively, was also fitted to the present experimental data. In order to minimize the parameters to be fitted, V_{O2rest} was fixed as the mean value of V_{O2} in the last minute of recovery and the parameters A_1, τ_1, A_2, and τ_2 were obtained by fitting. Table 1 (two-exponential model) shows the fitted parameters with the standard errors.

The constant times τ_1 and τ_2 were yet converted (in order to compare them with $\tau_{1/2}$) to the half-recovery times $\tau_{1/2}^{(1)}$ and $\tau_{1/2}^{(2)}$ by using $\tau_{1/2}^{(1)} = \tau_1 \ln 2$ and $\tau_{1/2}^{(2)} = \tau_2 \ln 2$. The mean values for $\tau_{1/2}^{(1)}$ and $\tau_{1/2}^{(2)}$ were $\langle \tau_{1/2}^{(1)} \rangle = 0.476$ min, with standard deviation $\sigma = 0.082$ min, and $\langle \tau_{1/2}^{(2)} \rangle = 3.94$ min, with standard deviation $\sigma = 1.91$ min.

Figure 4(a) shows the reduced residual sum of square (χ^2) and the adjusted R-square values obtained by fitting the standard biphasic two-exponential model and the $f1p$ model. Figure 4(b) shows normal Gaussian distributions $F(\tau,\sigma) = [1/(2\pi\sigma^2)^{1/2}]\exp[-((t-\tau)/\sigma)^2/2]$ plotted by using the mean value $\langle \tau_{1/2} \rangle$ ($f1p$ model) and the mean values $\langle \tau_{1/2}^{(1)} \rangle$ and $\langle \tau_{1/2}^{(2)} \rangle$ (standard biphasic model), with the respective standard deviations (σ).

(insert Figure 4)

Discussion

The values of f obtained by fitting the $f1p$ model define a group of individuals for which f was zero or lightly greater than zero [subjects 1 to 5, Table 1 ($f1p$ model)] and another for which $f > 1/2$ [subjects 6 to 10, Table 1 ($f1p$ model)]. $f > 0$ means that the rate equation at $t = 0$ [$(dX/dt)_{t=0} = -k(1-f)$] is not so abrupt as in case of an exponential. This apparent bimodal distribution of f in this set of recreationally trained subjects may be characteristic of the individuals. Though this hypothesis could not be conclude by analyzing only this set of experimental data, it is clear that the present method opens large possibilities of investigation about exercises and individual performs through the parameter f.

Figure 5(a) shows that the values of $\tau_{1/2}$ obtained by the $f1p$ model, when analyzed as function of the parameter f, are randomly distributed along the horizontal line described by $\tau_{1/2} = \langle \tau_{1/2} \rangle = 0.641$ min (within the standard error $\sigma = 0.062$ min).

Indeed, the linear fitting of $\tau_{1/2}$ vs f in Figure 5(a) yields the fairly horizontal line given by $\tau_{1/2}(\text{min}) = 0.644 - 0.0066 \times f$. The constancy of $\tau_{1/2}$ may mean a characteristic of the exercise, opening other possibilities for further investigations. Under constant $\tau_{1/2}$, $k = (1/\tau_{1/2})\ln[(2-f)/(1-f)]$, which follows from equation (4), also fitted well to the experimental k vs f data [Figure 5(b)], yielding $\tau_{1/2} = 0.636$ min with standard error $\sigma = 0.022$ min, in good agreement with $\langle \tau_{1/2} \rangle = 0.641$ min. In addition, X_{integ} (the second part of equation (7)) also fitted reasonably the experimental X_{integ} vs f data with constant $\tau_{1/2}$ [Figure 5(c)], yielding $\tau_{1/2} = 0.646$ min with standard error $\sigma = 0.122$ min, also in good agreement with $\langle \tau_{1/2} \rangle$.

Figure 5(c) means that X_{integ} diminishes effectively with the increase of f, and so VT_{O2net}, except by the amplitude factor $(V_{\text{O2peak}} - V_{\text{O2rest}})$ in equation (8). By keeping higher the rate V_{O2} at the beginning of the recovery, which means higher f, the total integrated oxygen uptake is paid faster at the beginning of the process. This statement could be supported by computing the time τ_{Q} necessary for $X_{\text{integ}}(t)$ (equation (6)) to reach the fraction $[1-\exp(-1)]$ (~63%) of the total X_{integ} (equation (7)). Evaluating equation (6) for $t = \tau_{\text{Q}}$ and rending it to $[1-\exp(-1)]X_{\text{integ}}$ yields

$$\tau_{\text{Q}} = -t_{1/2}\ln\{[(1-f)[1-\exp(-1)]-(1-f)]/f\}/\ln[(2-f)/(1-f)] \quad . \qquad (12)$$

Equation (12) fitted reasonably well the experimental τ_{Q} vs f data with constant $t_{1/2}$ [Figure 5(d)], yielding $\tau_{1/2} = 0.648$ min and standard error $\sigma = 0.135$ min, also in good agreement with $\langle \tau_{1/2} \rangle$.

The narrow distribution of $t_{1/2}$ in the f1p model ($\langle \tau_{1/2} \rangle = 0.641$ min, $\sigma = 0.062$ min) is in agreement with that of T in the Stu model ($\langle T \rangle = 0.672$ min, $\sigma = 0.081$ min) [Figure 3(c)]. The distributions for $t_{1/2}$ and T were statistically tested for equal means using One-Way ANOVA yielding $F = 0.9223$ and $p = 0.3496$, which means that, at the 0.05 level, the population means are not significantly different. However, the Stu model fitted a bit better the experimental V_{O2} vs t data [minor χ^2 and better R values in Figure 3(b)]. Nevertheless, the integration of equation (9) from $t = 0$ up to $t = T_{\text{f}}$ (the experimental final time of measurement) to rend X_{integ} by the Stu model (in analogy to equation (7) for the f1p model) gives a complex result: T_{f}Hypergeometric2F1[1, $1/\alpha$, $1+1/\alpha$, $-(T_{\text{f}}/T)^{\alpha}$], where Hypergeometric2F1[1, $1/\alpha$, $1+1/\alpha$, $-(T_{\text{f}}/T)^{\alpha}$] is a complex function evaluable numerically only in advanced routines.

Figure 4(a) shows that χ^2 of the *f*1p model were larger than those of the standard two-exponential model for individuals of the group with $f = 0$ or lightly greater than zero (subjects 1 to 5), while χ^2 were found minor than those of the standard model for the group of individuals with $f > 1/2$ (subjects 6 to 10). The same general aspects of quality apply to the *R*-square values from fitting both models (Figure 4(a)). This suggests that the standard two-exponential model could be not quite appropriated to fit experimental data that exhibit a not so high decaying of the rate V_{O2} at the beginning of recovery, where the *f*1p model previews a less abrupt decaying $[(dX/dt)_{t=0} = -k(1-f)]$ when $f > 0$. This seems to be the case for the recovery following peak exercise, for which a first order equation is far to confirm to be the optimal model to establish the most appropriate exercise protocol[13].

Apart from the fact that the standard two-exponential model to have more parameters to be fitted with respect to the single-phase models (*f*1p or Stu), Figure 5(b) shows that the distribution of the "fast" half-recovery time $\tau_{1/2}^{(1)}$ was even larger ($\langle \tau_{1/2}^{(1)} \rangle = 0.476$ min, $\sigma = 0.082$ min) than $\tau_{1/2}$ of the *f*1p model, and the distribution of the "slow" half-recovery time $\tau_{1/2}^{(2)}$ was too large ($\langle \tau_{1/2}^{(2)} \rangle = 3.94$ min, $\sigma = 1.91$ min), likely meaningless, even by fixing the rest parameter V_{O2rest} in the fitting process.

Conclusions

The present *f*-single-phase modelling is a simple and powerful method able to fit satisfactorily with only two parameters, a rate constant (k) and a factor f ($0 \leq f < 1$), experimental data of oxygen uptake rate in recovery process. The parameter f controls the rate equation dV_{O2}/dt, particularly at $t = 0$ where $(dV_{O2}/dt)_{t=0} \propto -k(1-f)$, a decay effectively less abrupt than $(dV_{O2}/dt)_{t=0} \propto -k$, expected from an exponential. Fitting the modelling to a set of experimental V_{O2} vs t data after a 3MT yielded a bimodal set of values for f (a set with $f = 0$ or close to zero and a set with $f > 1/2$) and a narrow distribution of values for the half-recovery time $\tau_{1/2} = (1/k)\ln[(2-f)/(1-f)]$, with a mean value $\langle \tau_{1/2} \rangle = 0.641$ min and standard deviation $\sigma = 0.062$ min. The distribution of $\tau_{1/2}$ was very similar to that found for the half-recovery time T ($\langle T \rangle = 0.672$ min, $\sigma = 0.081$ min) obtained by fitting a single-phase model, inferred from a graphical procedure based on a logit-log transformation. The parameter f is a reliable index of the initial acceleration of the oxygen uptake rate recovery (and likely of the heart rate recovery) and, together with the half-recovery time $\tau_{1/2}$, may be a useful method in characterizing and

monitoring performs and exercise forms.

Author contributions

The study was conceived and designed by C.A.G. and M.C.T.G. Supporting materials and analysis tools were provided by C.A.G. Experimental data were collected and processed by M.C.T.G. The single-phase 1/*f*p model was conceived by D.R.V. All the authors contributed writing the paper.

Disclosure statement

The authors declare that they have no competing interest.

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Figure captions

Figure 1. Plots illustrating the effect of varying k (or f) for a fixed value of the half-recovery time $\tau_{1/2} = \ln 2$, according to the f-single-phase model. The zoom inserted illustrates the inclination $(\mathrm{d}X/\mathrm{d}t)_{t=0} = -k(1-f)$ at $t = 0$ being less abrupt as f increases. The strait dash lines are tangent to the curves at $t = \tau_{1/2}$ so their inclinations $(\mathrm{d}X/\mathrm{d}t)_{t=\tau_{1/2}} = -(k/2)(1-f/2)$ are more abrupt as f (or equivalently k) increases.

Figure 2. Fitting the f-single-phase model (full lines) to the experimental data (points). The inserts show residuals of the fitting.

Figure 3. (a) Illustration of fitting the $f1p$ model and the Stu model to the experimental data of the subject 8. The inserts show residuals of the fittings. (b) Reduced residual sum of square (χ^2) and adjusted R-square (R) from fitting both models to the set of experimental data. (c) Normal Gaussian distributions of the half-recovery times $\tau_{1/2}$ and T projected using the mean values $\langle \tau_{1/2} \rangle$ and $\langle T \rangle$ and the respective standard deviations (σ).

Figure 4. (a) Reduced residual sum of square (χ^2) and adjusted R-square (R) from fitting the $f1p$ model and the standing biphasic two-exponential model to the experimental data. (b) Normal Gaussian distributions for the half-recovery times projected using the mean value $\langle \tau_{1/2} \rangle$ (for the $f1p$ model), and the mean values "fast" $\langle \tau_{1/2}^{(1)} \rangle$ and "slow" $\langle \tau_{1/2}^{(2)} \rangle$ (for the standard biphasic two-exponential model), with the respective standard deviations (σ).

Figure 5. Parameters and other quantities evaluated as a function of the parameter f for the $f1p$ model. (a) $\tau_{1/2}$: the linear fitting $\tau_{1/2} = 0.644 - 0.0066 \times f$ supporting that $\tau_{1/2}$ is practically a constant ($\tau_{1/2} = \langle \tau_{1/2} \rangle = 0.641$ min). (b) k: it was well fitted by $k = (1/\tau_{1/2}) \ln[(2-f)/(1-f)]$ (from equation (4)) with constant $\tau_{1/2} = 0.636$ min ($\sigma = 0.022$ min). (c) X_{integ}: it was reasonably fitted by equation (7) with constant $\tau_{1/2} = 0.646$ min ($\sigma = 0.122$ min). (d) τ_Q: it was reasonably fitted by equation (12) with constant $\tau_{1/2} = 0.648$ min ($\sigma = 0.135$ min).

386 Table 1. Fitted parameters for the *f*1p model, the Stu model, and the standing biphasic
387 two-exponential model. Numbers between the brackets are standard errors estimated by
388 the fitting.

			*f*1p model		
Subject	V_{O2peak} (ml/min·kg)	V_{O2rest} (ml/min·kg)	k (min^{-1})	$\tau_{1/2}$ (min)	f
1	29.9 (0.8)	9.09 (0.19)	0.99 (0.06)	0.700 (0.045)	0
2	36.8 (0.5)	7.83 (0.09)	1.25 (0.04)	0.554 (0.015)	0
3	27.6 (0.8)	7.84 (0.14)	1.02 (0.16)	0.678 (0.057)	0.01 (0.22)
4	40.3 (0.9)	8.70 (0.16)	1.15 (0.12)	0.641 (0.039)	0.08 (0.19)
5	41.9 (1.2)	11.1 (0.2)	1.13 (0.11)	0.657 (0.052)	0.09 (0.20)
6	45.8 (1.2)	9.73 (0.19)	2.00 (0.22)	0.610 (0.037)	0.58 (0.10)
7	37.8 (0.7)	9.45 (0.12)	2.71 (0.23)	0.658 (0.024)	0.73 (0.06)
8	35.1 (0.9)	9.04 (0.15)	2.55 (0.26)	0.737 (0.037)	0.82 (0.04)
9	28.5 (1.0)	9.30 (0.17)	2.82 (0.40)	0.689 (0.057)	0.83 (0.03)
10	33.0 (1.2)	6.97 (0.19)	3.58 (0.49)	0.574 (0.038)	0.85 (0.02)
			Stu model		
	V_{O2peak} (ml/min·kg)	V_{O2rest} (ml/min·kg)	α	T (min)	
1	32.0 (1.5)	7.08 (0.61)	1.08 (0.13)	0.622 (0.077)	
2	36.5 (0.6)	6.81 (0.16)	1.46 (0.06)	0.554 (0.022)	
3	25.9 (0.7)	7.50 (0.20)	1.84 (0.15)	0.775 (0.046)	
4	37.6 (0.8)	8.22 (0.21)	1.89 (0.11)	0.728 (0.031)	
5	39.0 (0.9)	10.7 (0.3)	1.95 (0.15)	0.759 (0.040)	
6	42.9 (0.9)	9.25 (0.23)	2.19 (0.15)	0.665 (0.028)	
7	35.8 (0.5)	9.21 (0.13)	2.43 (0.13)	0.597 (0.018)	
8	33.9 (0.7)	8.62 (0.18)	2.41 (0.17)	0.748 (0.030)	
9	27.8 (0.8)	9.00 (0.21)	2.39 (0.29)	0.696 (0.047)	
10	31.9 (1.0)	6.72 (0.20)	2.63 (0.25)	0.576 (0.030)	
			two-exponential model		
	A_1 (ml/min·kg)	τ_1 (min)	A_2 (ml/min·kg)	τ_2 (min)	
1	16 (2)	0.47 (0.09)	8.3 (1.7)	2.9 (0.5)	
2	26 (1)	0.57 (0.04)	6.0 (1.0)	3.3 (0.4)	
3	18 (2)	0.85 (0.11)	2.2 (1.7)	4.4 (2.7)	
4	29 (5)	0.79 (0.12)	3.7 (5.5)	2.4 (1.8)	
5	29 (2)	0.77 (0.08)	4.6 (1.8)	5.1 (1.9)	
6	37 (2)	0.69 (0.06)	2.8 (1.9)	4.4 (2.8)	
7	31 (1)	0.65 (0.03)	0.8 (0.6)	9.9 (9.7)	
8	28 (1)	0.76 (0.06)	3.1 (0.9)	10.1 (4.8)	
9	20 (1)	0.74 (0.10)	2.3 (1.3)	7.1 (4.9)	
10	29 (1)	0.59 (0.06)	2.6 (1.1)	7.2 (4.1)	

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Figures

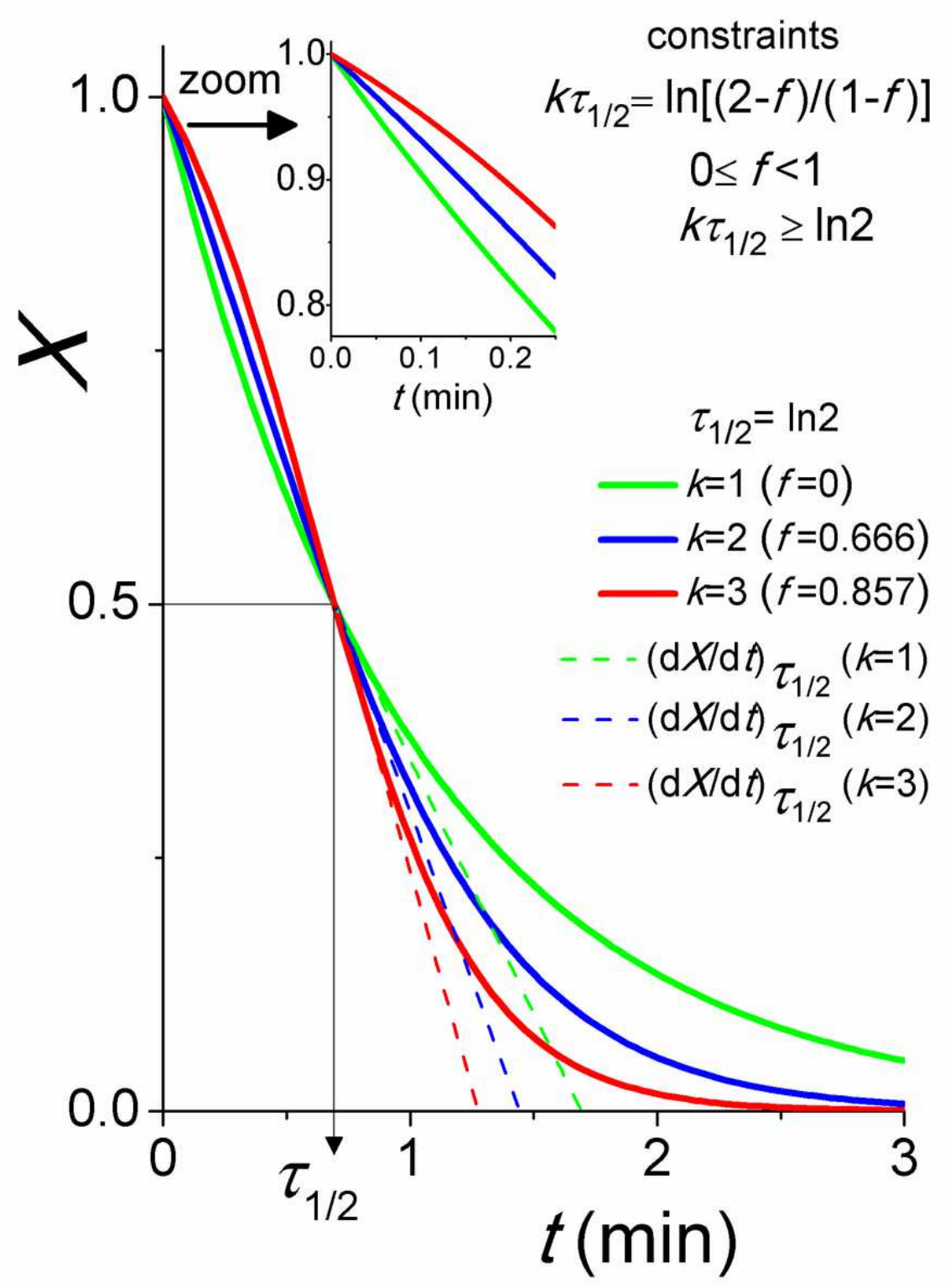

Figure 1

Plots illustrating the effect of varying k (or f) for a fixed value of the half-recovery time t1/2 = ln2, according to the f-single-phase model. The zoom inserted illustrates the inclination (dX/dt)t=0 = -k(1-f) at

t = 0 being less abrupt as f increases. The strait dash lines are tangent to the curves at t = t1/2 so their inclinations (dX/dt)t=t1/2 = -(k/2)(1-f/2) are more abrupt as f (or equivalently k) increases.

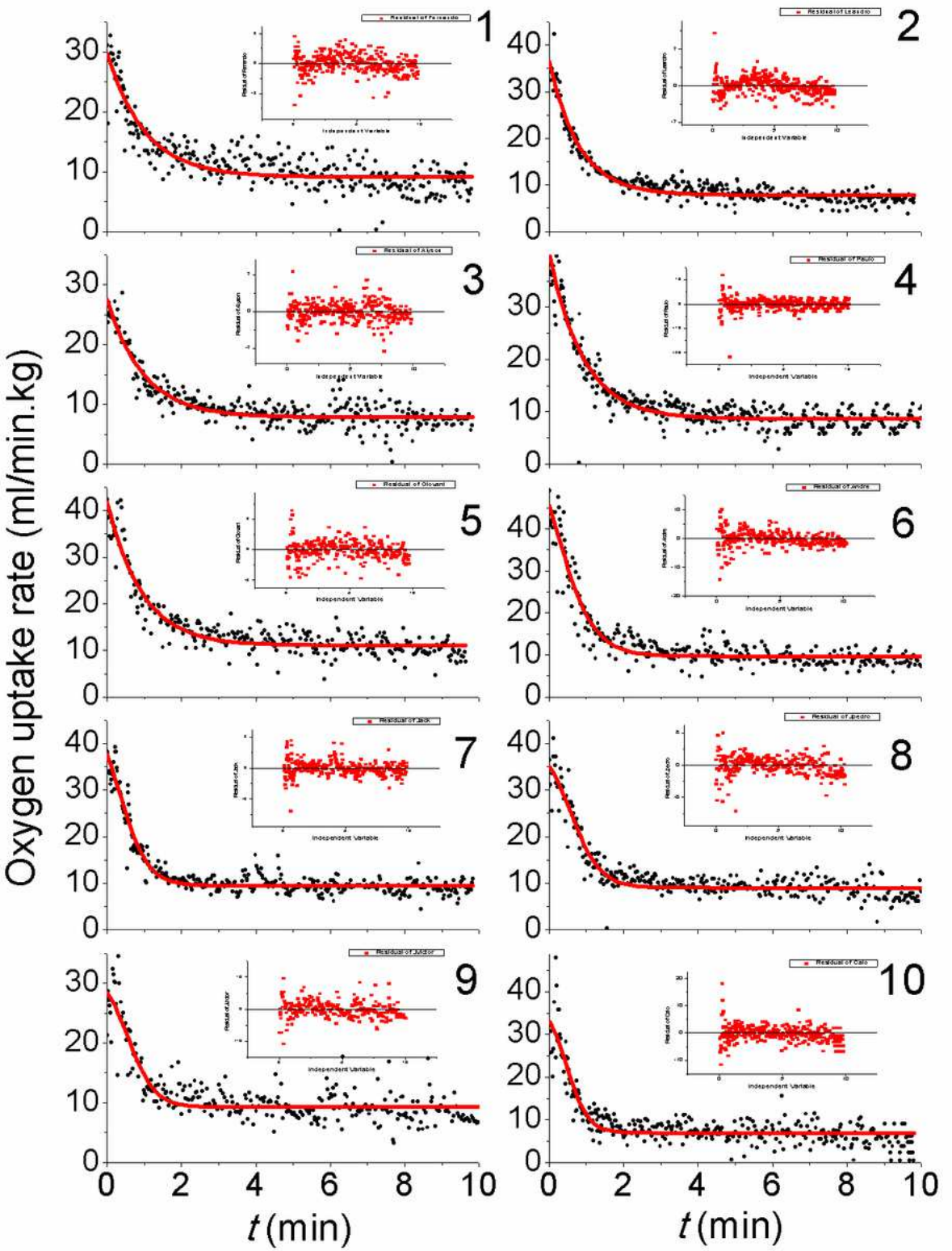

Figure 2

Fitting the f-single-phase model (full lines) to the experimental data (points). The inserts show residuals of the fitting.

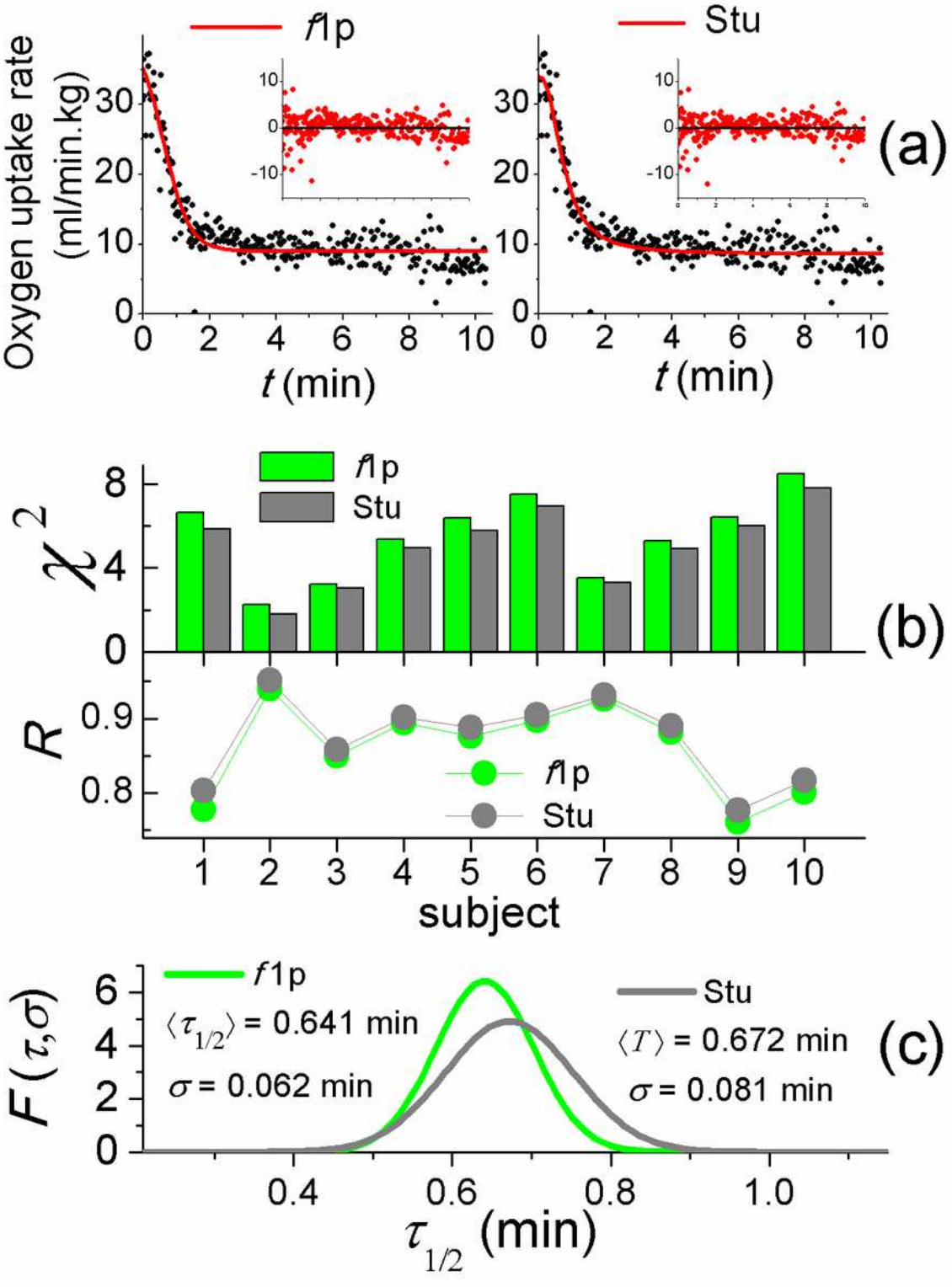

Figure 3

Please see the Manuscript Doc file for the complete figure caption.

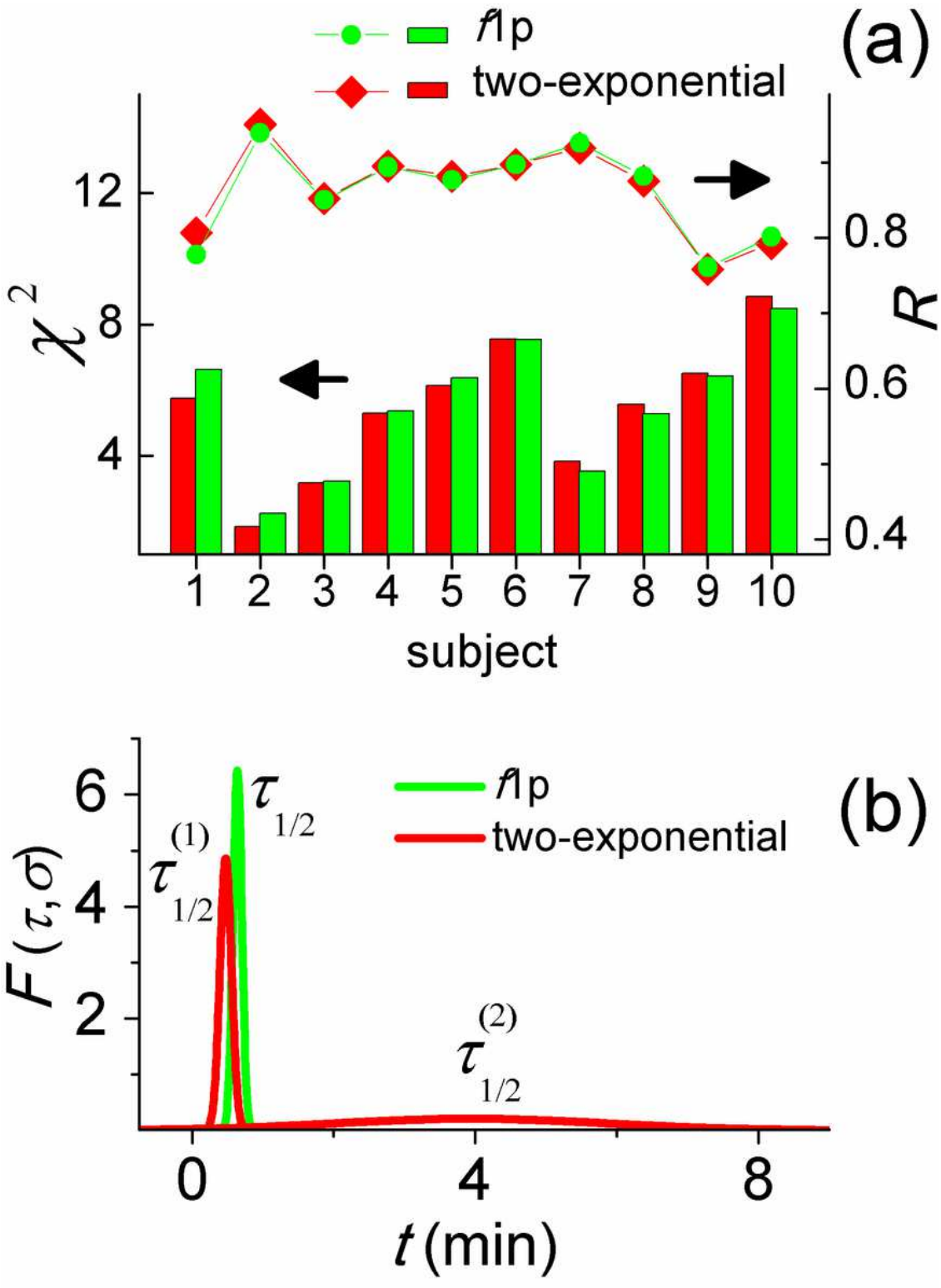

Figure 4

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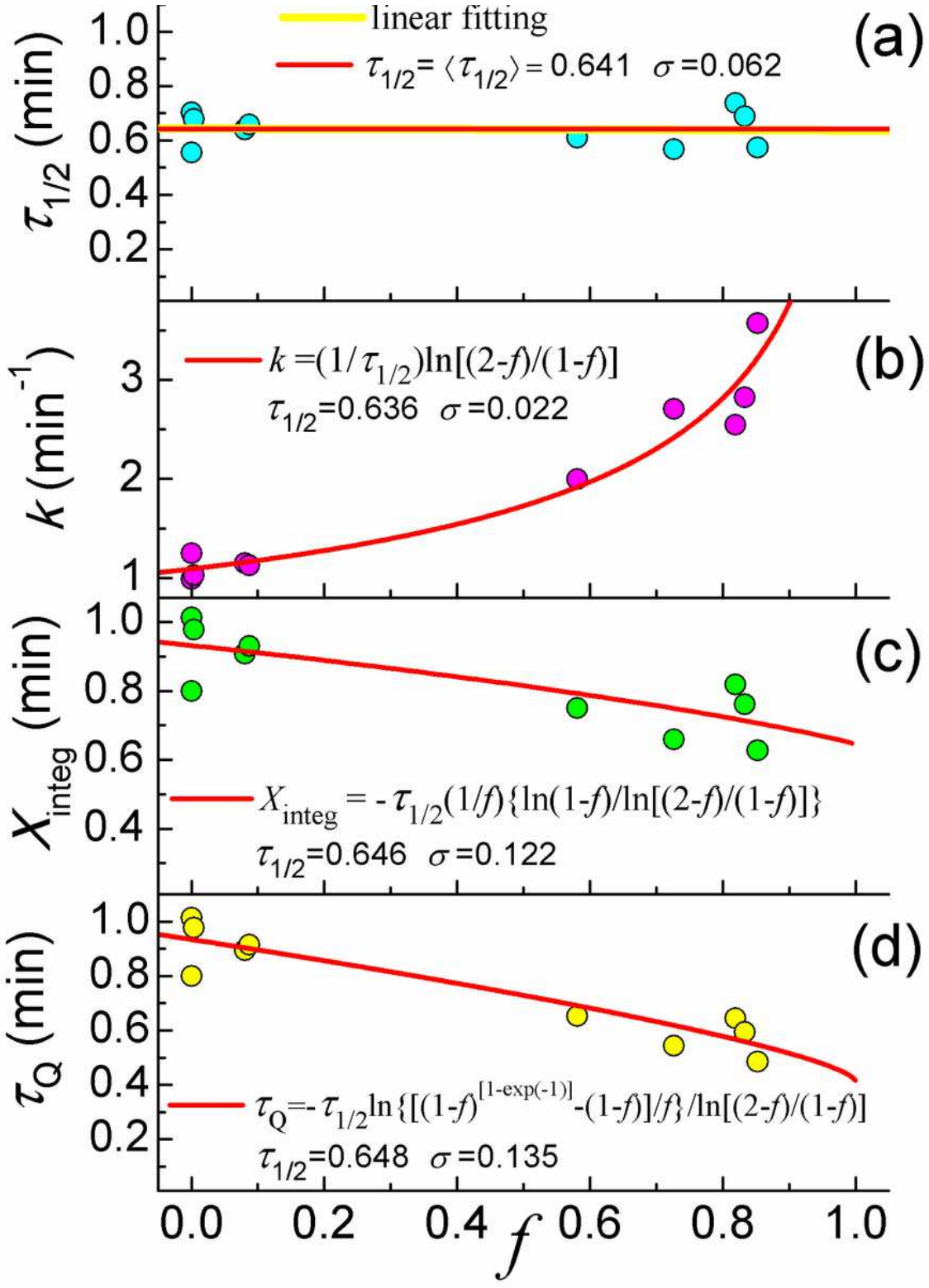

Figure 5

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