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Research Article

Keywords: Online firm, price discrimination, urban/rural markets, location, zoning

Posted Date: November 3rd, 2022

DOI: https://doi.org/10.21203/rs.3.rs-2221894/v1

Additional Declarations: No competing interests reported.

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October 31, 2022

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Abstract

We contribute to the previous offline spatial price discrimination by adding an online entrant that results in partially equalized prices, and the urban (rural) segments are served by the offline (online) firms. Online competition induces the offline firms to move closer to the market center, which causes the equilibrium locations to no longer be the social optimum due to the online price distortion. The greater the online advantage is, the higher the online price, the less dispersed the offline locations, and the lower the offline prices will be in urban areas. Finally, zoning policies and two extensions of online price discrimination and multiple offline firms with free entry are offered.

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JEL Classification Numbers. R30, L13

1 Introduction

Spatial price discrimination means that firms can set locational discriminatory prices, which has been studied by Hoover (1937), followed by Hurter and Lederer (1985), Thisse and Vives (1988) and Vogel (2011), and this classical issue recently has attracted the attention of policy makers due to the rapid development of online-offline competition in the past two decades.[1] We explore the issue in further depth by incorporating online competition and establishing a novel urban/rural framework to explore the relocation tendency of offline retailers and partially equalized price patterns, as well as corresponding zoning implications.

Consider an urban/rural economy where the total population is distributed along a unit-length linear market with the highest population density at the market center (urban area), and lower population density appears in two end segments (rural areas). Two offline firms (retailers) engage in spatial price discrimination, and bear the delivery costs. They also face competition

[1] The influence of online competition on geographical price variations has received public attention. For instance, the US Federal Trade Commission (FTC) previously found that at Staples, prices were 12% higher with only one office superstore in 1997 after the Staples/Office Depot merger, but now the FTC has less concern over the same merger, because most of the variation in prices has been reduced due to the presence of online retailing.

from the entry of a location-irrelevant online firm. Our setting highlights two major practical online-offline differences. First, online firms usually enact uniform pricing, while offline firms may choose spatially discriminatory pricing with less opposition from customers.[2,3] Second, consumers will suffer an additional distaste cost in an online transaction.[4] The entry of an online retailer is shown to reduce the locational price differentiation and result in equalized pricing except in the most urban area, which has seldom been discussed in previous studies to the best of our knowledge. The urban consumers will benefit from lower offline prices due to competition between less dispersed offline firms after the online entry, while the rural consumers also benefit from the above equalized lower price.

Our price pattern explains the empirical evidence in Brynjolfsson and Smith (2000), who analyzed the prices of two homogenous products of books and CDs and found that price dispersion is lower in internet channels than in conventional channels. Our theory shows that a newly entering online retailer not only plays a role as a new competitor, but also brings in locational irrelevant pricing, leading to a kinked equilibrium price pattern in the urban area. This thus shows a considerable reduction in overall price levels, and a reduction in price variations as well.

We assume the online firm has different market power in setting prices compared to the offline firms, because it must set its prices uniformly for consumers across areas. This pricing behavior is empirically observed and has decisively changed the nature of traditional spatial price competition, where the online-offline economy does not follow the traditional discrimi-

[2]Online retailing tends to be locational irrelevant for consumers due to the nature of cyberspace, in contrast to offline transactions, in which distance is relevant for consumers. Technically, data mining on personal geographical information could allow online firms to enact spatial price discrimination. However, our setting can be justified, because in practice online spatial price discrimination is limited for various reasons: consumers can easily compare and see through discriminatory prices on line, consumer loyalty may be eroded if they discover they are the targets of such discrimination, and there may be governmental regulations in place to restrict data processing and profiling due to privacy and data protection which discourage or disallow such discrimination.

[3]We also provide an alternative analysis in Subsection 4.1 where the online firm enacts spatial price discrimination, with an assumption that it will not irritate customers due to unfair treatment.

[4]Consumers are unable to fully inspect products in person beforehand, could suffer from transaction risks, normally have to wait longer, sometimes much longer, for delivery, and are usually subject to shipping and handling fees, all of which are summarized as a distaste cost for online transactions in our framework. On the other hand, online firms may provide broader and faster search and comparisons between prices and quality of products (in addition to lower prices due to lower overhead, wider product ranges, and even generous return policies to offset the online purchasing distaste cost), which are not embedded in the current model for simplicity. See Goldfarb and Tucker (2019) for reviewing studies on other online-offline differences.

natory price rule, namely that the lowest-cost firm sets its price equal to the second-lowest firm's marginal cost. Our comparative statics reveals that the less the online distaste cost is, the higher the online price is, the closer the offline firms are, and the lower/higher the offline prices in urban/non-urban areas are. Intuitively, a greater online advantage induces offline firms to move closer to the market center to avoid online vs offline competition, and thus the online price increases because of the raised demand. Moreover, after the online entry, the urban offline prices decline due to competition between more concentrated offline firms, but the non-urban offline prices increase due to the increased online price.

Our online-offline competition is shown to have new implications for socially optimal locations and zoning policies. The traditional wisdom in Hurter and Lederer (1985) and Lederer and Hurter (1986) is that the equilibrium locations are socially optimal under spatial price discrimination when the social welfare is the sum of firms' profits and consumer surplus with equal weights. We find that this property is no longer valid with online competition, because of the online price distortion effect, in which the online firm has market power to set a distorted uniform price which induces the equilibrium locations to be less far apart than the social optimum.

When the social welfare has a large weight in firms' profits, in addition to the offline vs offline competition avoidance in previous offline studies, we reveal the other two opposite influences of the online firm, namely that online price distortion appears, and that the waste of delivery cost is reduced for the remote consumers from offline locations. Since the relationship between the social weight on consumer surplus and the optimal offline locations is no longer monotone, we obtain different zoning policies from various numerical illustrations. The optimal zoning policies will depend on the social optimal offline locations and relocation tendency, which depend on the above two opposite influences. For instance, when the social planner only cares about firms' profits, the optimal zoning policy may restrict offline firms to locate in the urban area and leave the rural consumers to be served by the online firm if the online advantage is not large, contrary to the previous pure offline frameworks where the optimal zoning is setting two end-point locations for firms. This is because the online firm has some market power and sets an above-cost price, so the offline prices can also rise, at the same time, less dispersed offline locations can reduce the waste delivery costs for remote consumers, because they are now served by the online firm. Our results will be associated with online-offline and offline-offline competition and the trade-off between the distaste cost and delivery costs. It can be compared with the previous finding in Bárcena-Ruiz and Casado-Izaga (2014) that restricting offline firms in the most urban area is beneficial to consumers due to more intense competition.[5] In our online vs offline and urban vs rural framework, it may be optimal to limit the location of the offline firms to moderately rural areas, which creates more intense competition between online and offline firms, which is mostly beneficial to consumers.

Among the literature on spatial price discrimination with oligopoly competition, Hurter and Lederer (1985) and Lederer and Hurter (1986) further included a location stage with duopolistic competition, and show that one firm will locate at the first quartile, and the other firm locates at the third quartile in equilibrium. In a sequential entry game, Heywood and Ye (2009) revealed that a public firm can restrict its market in order to induce earlier private entrants to locate near the welfare-maximizing points. Vogel (2011) further considered an entry game with heterogeneous-cost firms to demonstrate that more productive firms will be more isolated. In contrast to the previous location and pricing patterns, we demonstrate less dispersed locations and more equalized prices after the online entry. Another related issue is to compare spatial price discrimination with simple mill pricing, given firms located at the endpoints of a Hotelling line or uniformly distributed in a circular market. For instant, Thisse and Vives (1988) examined the pricing strategies among firms and suggested that spatial discriminatory pricing is the unique Nash equilibrium, which provides lower prices and higher consumer surplus in comparison with simple mill pricing. Taylor and Wagman (2014) further discussed the privacy implication and revealed that some consumers near the firms could be worse off without privacy. Esteves and Shuai (2022) considered an elastic demand and demonstrated that spatial price discrimination (personalized pricing) reduces consumer surplus when the demand elasticity is not small.

Numerous studies have addressed the influences of online competition. Balasubramanian (1998) constructed a circular model and analyzed price competition among one direct channel (mail order) and conventional offline stores. The shipping cost is locationally irrelevant and fixed for the direct channel, while it increases with distance for conventional offline stores. He found that the direct channel may choose partial coverage when sending its catalog to consumers, and each retailer can compete with the direct channel instead of its nearby retailers. Loginova (2009) analyzed the price and welfare among offline retailers and competitive online retailers with addition of the stipulation that consumers may visit offline stores to collect information and come back home for their online purchasing. She showed that conventional

[5]The optimal zoning analysis under oligopoly competition was investigated using different frameworks, such as the Hotelling model (Lai and Tsai, 2004; Matsumura and Matsushima, 2011; Bárcena-Ruiz and Casado-Izaga, 2014), Cournot competition (Chen and Lai, 2008; Colombo, 2012), circular model (Hamondi and Risueno, 2012), and connected cities (Bárcena-Ruiz and Casado-Izaga, 2017, 2018, 2020).

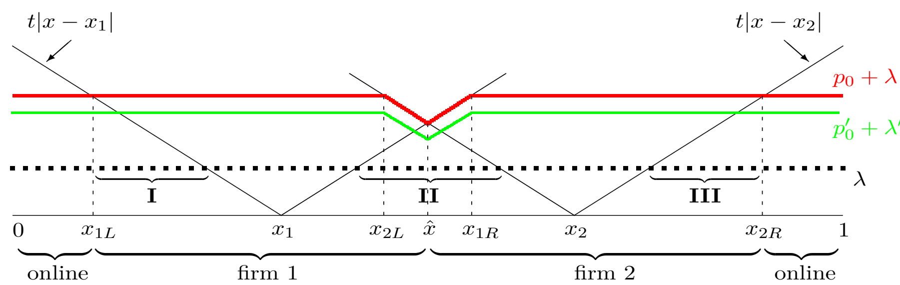
stores may raise their prices unexpectedly in response to the entry of online firms. Recently, Guo and Lai (2017) constructed a location and non-discriminatory pricing game, and found that online competition induces offline retailers to move toward urban areas, occupying non-overlapping and contiguous market segments, and the online retailer serves the remote rural areas. Chen *et al.* (2017) endogenized firms' online-offline market choices according to product quality and suggested that an entrant firm with a higher/lower quality may sell online/offline. Guo and Lai (2022) further revealed that third-degree online price discrimination under online-offline competition may improve social welfare when the online-offline cross-demand parameter is large. Moreover, there are various empirical studies on competition between offline stores and online firms such as Clay *et al.* (2002), Brynjolfsson and Smith (2000), Goolsbee (2001), Forman et al. (2009) and Duch-Brown et al. (2017). The current paper complements this stream of literature by providing the first analysis of the role of online competition in the classical framework of spatial price discrimination.

The rest of this paper is organized as follows. Section 2 describes the model, and the benchmark scenario without online competition is presented in Section 3.1. Section 3.2 discusses the equilibrium locations and prices, and the socially optimal locations are also offered. Moreover, the analysis on average price level and price variation are provided. The implied various zoning policies are presented in Section 3.3. Two extensions allowing online price discrimination and multiple offline firms with free entry are discussed in Section 4. We provide some empirical and policy implications in Section 4.3. Some concluding remarks are offered in Section 5. All proofs are listed in the Appendix.

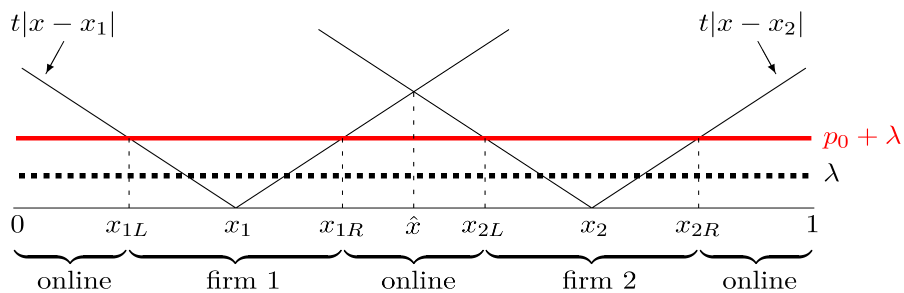
2 The Model

Suppose there are two offline firms (1 and 2) producing homogeneous goods and engaging in spatial price discrimination on consumers who are non-uniformly distributed along a linear market (urban in the central area and rural in the remaining areas) with a unit length, and these two firms locate at $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$ with $x_1 \leq x_2$, respectively. For tractability, let the population density be $f(x) = a - b|2x - 1|$, which is symmetric around the market center, where b represents the spatial gradient of population density with $0 < b \leq 2$ to ensure non-negative population segments in the whole market, and $a = \frac{b}{2} + 1$ to normalize the total population mass as one unit.[6] Denote the shipping rate and the price for firm 1 (2)

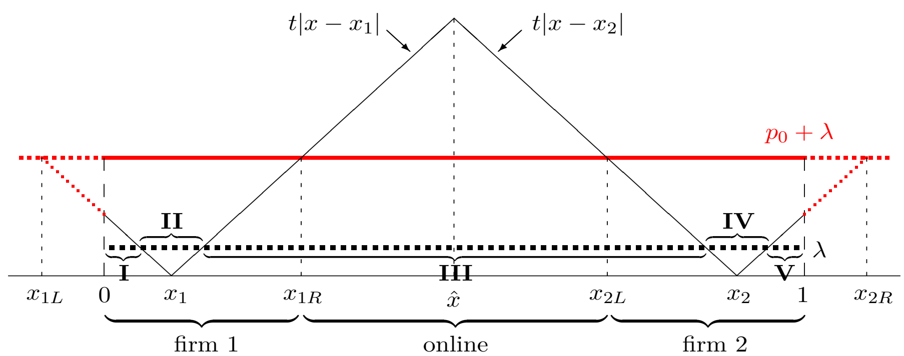
[6]If the population density is uniform along a linear market, an online firm entering the market will make offline firms be indifferent about where, within some ranges of the linear market, to locate their stores, and



(a) $x_{1L} > 0$, $x_{1R} < \hat{x}$, $\lambda' < \lambda$.



(b) $x_{1L} \geq 0$, $x_{1R} \leq \hat{x}_j$.



(c) $x_{1L} < 0$, $x_1 < \hat{x}$.

Figure 1: The equilibrium prices with online competition under various location dispersions.

as t and p_1 (p_2), respectively, and the delivery costs are paid by offline firms. Suppose each consumer has inelastic demand for one unit of the product and yields a constant reservation price $\bar{v}$, which is large enough to ensure that each consumer buys one from the firm with the lowest price. A consumer who is indifferent between firm 1 and firm 2 is denoted by $\hat{x}$. It is worth mentioning that asymmetric locations are allowed, although our following figures describe symmetric locations just for convenience. Assume both offline firms have zero production cost for simplicity. Consider a two-stage game as a benchmark scenario in which firms simultaneously choose their locations in the first stage, while prices are simultaneously set in the second stage.

Next, consider the entry of an online firm with zero production cost which has no location choice and sets a uniform price (p_0) for all consumers in the first stage. That is, p_0, x_1, and x_2 are determined in the first stage, while $p_1(x)$ and $p_2(x)$ are set in the second stage. Any online buyer suffers an overall distaste cost λ that includes a constant shipping fee, the inconvenience of checking the product quality beforehand, transaction risks, and waiting several days for shipping.[7,8] Assume that λ is not too large ($0 < \lambda < t/4$), to ensure the entry of an online firm. The setting of our game structure can be justified by two reasons. First, in general, the online firm may be a dominant firm in a specific industry, such as Amazon, which is the largest online firm in the global book market. Second, online price discrimination is not overwhelming in reality, because consumers can see through the discrimination easily. If the online firm is

multiple uniform price equilibria appear. This location pattern has been discussed in Guo and Lai (2014), where firms set mill prices as in typical Hotelling models.

[7]When the online firm sets its price in the second stage, we will encounter an undercutting problem and then there exists no price equilibrium in the second stage in this alternative setting. Specifically, if the online firm sets a positive price p_0 and the offline firm i occupies a positive market share and sets a constant price $p_i = p_0 + \lambda$, then the online firm can benefit from reducing its price a little to undercut the offline firms in the intervals where the serving cost for consumers $p_0 + \lambda$ is between the two providing costs of offline firms $t|x - x_1|$ and $t|x - x_2|$. In order to rule out the above undercutting, in equilibrium the online firm should not set positive prices. However, a zero online price is not optimal for the online firm, since it can set a positive price to keep some positive market share and get a positive profit. Therefore, there appears no equilibrium in this alternative scenario.

[8]Practically, an online firm (say Amazon) may charge the same shopping cost to any place inside a city. In fact, Amazon posted that the shipping costs in apparel, beauty, jewelry and watches, were 8.99 per shipment and 0.99 per item equally inside the mainland USA as of June 2022. We follow Balasubramanian (1998) and Guo and Lai (2014, 2017) to assume that there is a fixed distaste cost paid by consumers in any location of the market for online purchasing. The above settings can be justified by the fact that the online business has scale economy and can control the cost of shipping, and specifically has an advantage for some electronic versions of products such as music, books, and software.

allowed to set up discriminatory prices in the second stage, then the equilibrium prices become a simple pattern where the lowest-cost firm (including the online firm) at each point sets its price equal to the cost of the second-lowest firm, which will be discussed in Subsection 4.1.

3 The Equilibrium Analysis

3.1 The benchmark scenario without online competition

We first solve the price equilibrium in the second stage. Spatial price discrimination under duopoly competition results in an equilibrium price pattern such that at any geographical point, the firm with the lowest delivery cost sets up a discriminatory price equal to the delivery cost of its rival. The equilibrium prices are $p_1(x) = t|x - x_2|$ for $x \in [0, \hat{x}]$ and $p_2(x) = t|x - x_1|$ for $x \in [\hat{x}, 1]$; recall that $\hat{x}$ is the indifferent consumer. The profit functions for firm 1 and firm 2 are:

$$\pi_1 = \int_0^{\hat{x}} (p_1(x) - t|x - x_1|)\, f(x)dx,$$
$$\pi_2 = \int_{\hat{x}}^1 (p_2(x) - t|x - x_2|)\, f(x)dx.$$

Then, solving $\partial\pi_1/\partial x_1 = 0$ and $\partial\pi_2/\partial x_2 = 0$ simultaneously in the first stage yields the equilibrium locations. The consumer surplus is defined as $CS = \int_0^1 (\bar{v} - \min\{p_1(x), p_2(x)\})\, f(x)dx$, and the social welfare (W) is defined as the weighted sum of profits and consumer surplus with a weighted parameter α, $W = \alpha(\pi_1 + \pi_2) + (1 - \alpha)CS$. The larger the value of α, the more concern of the social planner on the interests of firms. We can solve $\partial W/\partial x_1 = 0$ and $\partial W/\partial x_2 = 0$ simultaneously for the socially optimal locations x_1^w and x_2^w. Notably, the case $\alpha = 1/2$ represents the typical setting for the social welfare, meaning equal weights on firms and consumers. The following result provides several benchmark properties without online competition which are generally consistent with the previous findings in Hurter and Lederer (1985) and Bárcena-Ruiz and Casado-Izaga (2014), while our setting is more general than their models by considering urban/rural population distributions.

Benchmark results. (1) *In the benchmark scenario where there are only two competing offline firms in the market, there exists a unique location equilibrium* $(x_1^*, x_2^*) = (\frac{1}{4} + \frac{\sqrt{b^2+4}-2}{4b}, 1 - x_1^*)$. (2) *The socially optimal locations are* $\left((x_1^w(\alpha) = \frac{1}{4} - \frac{1}{2b} + \frac{\sqrt{4\alpha(\alpha+b-2\alpha b)+\alpha^2 b^2}}{4\alpha b}, x_2^w(\alpha) = 1 - x_1^w(\alpha)\right)$ *when* $1/3 < \alpha < 1$, *and minimal location differentiation* ($x_1^w(\alpha) = x_2^w(\alpha) = 1/2$) *appears when* $\alpha \leq 1/3$. *Maximal location differentiation* ($x_1^w = 1 - x_2^w = 0$) *appears only* *when* $\alpha = 1$. (3) *The socially optimal locations are less (more) diverse than the equilibrium locations when* $\alpha < 1/2$ ($\alpha > 1/2$). *When* $\alpha = 1/2$, *the equilibrium locations are also socially desirable.* (4) *When* $\alpha \neq 1/2$, *the socially optimal locations can be achieved by restricting firms' locations in the middle point* $x_1^w(\alpha) = x_2^w(\alpha) = 1/2$ *if* $\alpha \leq 1/3$, *allowing firms' locations only within the area* $[x_1^w(\alpha), x_2^w(\alpha)]$ *if* $1/3 < \alpha < 1/2$, *and forcing firms to locate outside the area* $(x_1^w(\alpha), x_2^w(\alpha))$ *if* $\alpha > 1/2$.

Benchmark result (1) shows that the location equilibrium is unique and symmetric, but is not socially desirable except $\alpha = 1/2$. When $\alpha = 1/2$, the equivalence between the equilibrium and social optimum is consistent with Hurter and Lederer (1985), even though we consider a nonuniform population distribution here. Notably, b converges to zero and $\alpha = 1/2$, locations converge to $x_1^* = x_1^w = 1/4$ and $x_2^* = x_2^w = 3/4$ in Hurter and Lederer (1985). Our equilibrium locations herein are $x_1^* = \frac{1}{4} + \frac{\sqrt{b^2+4}-2}{4b} \in [\frac{1}{4}, \frac{1}{2}]$ and $x_2^* = 1 - x_1^*$, by the assumption $b \in (0, 2]$, and $\partial x_1^* / \partial b > 0$, because of higher population densities in areas near the market center. Intuitively, when population distribution becomes more uneven, the central market is more attractive, inducing offline firms to move closer to the market center.

Benchmark results (2)–(3) also demonstrate the inconsistency between the equilibrium locations and the socially optimal locations when $\alpha \neq 1/2$, which departs from the findings in Hurter and Lederer (1985) and Lederer and Hurter (1986), due to different social weights of consumers and firms. The social optimum $x_1^w(\alpha)$ converges to $x_1^w(\alpha) = \frac{1}{4} + \frac{1-2\alpha}{4\alpha}$ as in Bárcena-Ruiz and Casado-Izaga (2014) when b converges to zero. When $\alpha = 1/2$, the equilibrium locations are also socially desirable, in line with the traditional understanding, because consumers are always served by the lowest-cost firms, and allowing general population distributions herein does not change this. The socially optimal locations $(x_1^w(\alpha), x_2^w(\alpha))$ are less dispersed than the equilibrium locations when $\alpha < 1/2$, and more dispersed than the equilibrium locations when $\alpha > 1/2$. Intuitively, when $\alpha < 1/2$, meaning that the social planner places more weight on consumer surplus than producer surplus, it forces firms to choose closer locations that enhance competition and lead to lower prices and thus benefits consumers. Notably, when $\alpha \leq 1/3$, the socially optimal locations agglomerate at the center point. In contrast, when $\alpha > 1/2$, meaning that the social planner more cares about the interests of firms, the offline firms are allowed to choose more dispersed locations to avoid competition and increase their profits and prices. Moreover, comparative statics reveal $\frac{\partial x_1^w(\alpha)}{\partial \alpha} < 0$ and $\frac{\partial x_1^w(\alpha)}{\partial b} > 0$ when $\alpha > 1/3$. In the degenerated case, that b converges to zero, $x_1^w(\alpha)$ converges to zero when α approaches 1, which is maximally differentiated. Notably, whenever $\alpha < 1$,

$x_1^w(\alpha)$ is always positive, meaning that the socially optimal locations are finitely differentiated. Moreover, when the population distribution becomes more uneven, the socially optimal locations clearly become less dispersed.

Benchmark result (4) suggests some implication on zoning policies which are consistent with Bárcena-Ruiz and Casado-Izaga (2014), but our uneven population distribution is a more general setting than theirs. Specifically, when b converges to zero, our model will be degenerated to Bárcena-Ruiz and Casado-Izaga (2014). Our optimal zoning policy depends on the parameters α and b. To be precise, limiting locations to the center of the market, restricting them within an interval, and allowing them only in an outside interval are socially optimal zoning policies for $\alpha \leq 1/3$, $1/3 < \alpha < 1/2$, and $\alpha > 1/2$, respectively. Note that the corresponding zoning ranges will be affected by parameter b. Since $\frac{\partial x_1^w(\alpha)}{\partial b} > 0$, the optimal locations become less dispersed as b increases, because the population distribution is more concentrated at the market center, so the socially optimal locations are also closer to the market center. Therefore, when $1/3 < \alpha < 1/2$, the zoning interval $[x_1^w(\alpha), 1-x_1^w(\alpha)]$ becomes narrower as the population distribution becomes more uneven. However, when $\alpha > 1/2$, the zoning intervals $\left\{[0, x_1^w(\alpha)], [1-x_1^w(\alpha), 1]\right\}$ become wider as b increases.

3.2 Price discrimination with online competition

Now we analyze the equilibrium when there is an online firm (denoted by a subscript "0") entering the market and both offline firms are free to change their locations. There exist three types of spatial discriminatory prices in the second-staged equilibria described by symmetric cases for convenience in Figure 1, where the online firm occupies the rural markets (Figure 1(a)), both the most urban market and the rural markets (Figure 1(b)), and only the urban market (Figure 1(c)), respectively. These three types sequentially correspond to less dispersed offline locations, moderately dispersed locations, and most dispersed locations. The spatial price patterns (red lines) are different from the traditional wisdom and are either partially equalized or fully equalized across locations due to online competition.

Among the above three types, we may find the unique symmetric subgame perfect equilibrium type described in Figure 1(a). Precisely, the online firm will take the market segments in $[0, x_{1L}]$ and $[x_{2R}, 1]$ with a uniform price $p_0 + \lambda$, where $x_{1L} = x_1 - \frac{p_0+\lambda}{t}$ and $x_{2R} = x_2 + \frac{p_0+\lambda}{t}$ are the consumers who are indifferent between online and offline purchases. The most urban areas (x_{2L}, x_{1R}) are served by offline firms with different spatial prices equal to the offline rival's delivery cost, and the two non-urban segments $[x_{1L}, x_{2L}]$ and $[x_{1R}, x_{2R}]$ are also served by offline firms but priced uniformly as $p_0 + \lambda$. The profits of firms become:

$$\pi_0 = \int_0^{x_{1L}} p_0 f(x)dx + \int_{x_{2R}}^{1} p_0 f(x)dx, \tag{1}$$

$$\pi_1 = \int_{x_{1L}}^{x_{2L}} (p_0 + \lambda - t|x - x_1|) f(x)dx + \int_{x_{2L}}^{\hat{x}} t(|x - x_2| - |x - x_1|) f(x)dx, \tag{2}$$

$$\pi_2 = \int_{x_{1R}}^{x_{2R}} (p_0 + \lambda - t|x - x_2|) f(x)dx + \int_{\hat{x}}^{x_{1R}} t(|x - x_1| - |x - x_2|) f(x)dx. \tag{3}$$

The profits for Figure 1(b) and 1(c) can be calculated similarly, provided in the appendix (page 29). For the case in Figure 1(a), solving $\partial\pi_0/\partial p_0 = 0$, $\partial\pi_1/\partial x_1 = 0$, and $\partial\pi_2/\partial x_2 = 0$ simultaneously yields the implicit equilibrium functions

$$\tilde{p}_0 = \frac{2t(1 + 2b\tilde{x}_1) - b(t + 4\lambda) - \sqrt{\phi_1}}{6b}, \tag{4}$$

$$\tilde{x}_1 = 1 - \tilde{x}_2 = \frac{-t(2 - b) - 4b(\tilde{p}_0 + \lambda) + \sqrt{t^2(b + 2)^2 + 32b^2(\tilde{p}_0 + \lambda)^2}}{4bt}, \tag{5}$$

where $\phi_1 = 2t^2b^2\tilde{x}_1(2\tilde{x}_1 - 1) + t^2(2 - b)^2 + 2b\left(2t^2\tilde{x}_1 - (2 - b)t\lambda - 2b\lambda(2t\tilde{x}_1 - \lambda)\right)$, and $\tilde{x}_2 = 1 - \tilde{x}_1$; the "$\sim$" superscript represents the symmetric equilibrium in this online-offline scenario. Notably, the above equilibrium locations and the online price have no simple analytical solutions. With online competition, the socially optimal locations are solved by maximizing the social welfare $W = \alpha(\pi_1 + \pi_2 + \pi_0) + (1 - \alpha)CS$. Assume the government has a limited calculation ability such that its socially optimal locations policy is only related to the locations of offline firms for simplicity. The social optimum is solved by a Nash equilibrium between the government and the online firm.[9] We will discuss the robustness later for the case when the government may fully consider the reaction of the online price $p_0(x_1, x_2)$, which is generally consistent with the major equilibrium properties.

The following proposition describes the influence of online competition on the equilibrium and social optimum.

Proposition 1. (1) *There exists a unique type of equilibrium ($\widetilde{x}_1^*, \widetilde{x}_2^* = 1 - \widetilde{x}_1^*, \widetilde{p}_0^*$) such that the online firm occupies two end segments, and offline firms enact discriminatory pricing in the central (urban) area, while their prices are equalized as $p_0^* + \lambda$ for the less densely populated (rural) segments.* (2) *The equilibrium locations are generally not socially desirable even when $\alpha = 1/2$. In the standard case, $\alpha = 1/2$, the socially optimal locations are more agglomerated than the equilibrium under the sufficient condition that b is not large and λ is not small.*

[9] Solving the social optimum by a Nash equilibrium is also adopted by minimum quality standard literature such as Ecchia and Lambertini (1997).

(3) *The equilibrium locations are neither minimal nor maximal differentiation. However, the socially optimal locations are minimally differentiated when* α *is small, while when* α *is close to 1, the social optimum is (not) maximal differentiation when* λ *is small (large).* (4) *Online competition induces offline firms to move closer to the market center and causes all prices to decline, and consumers living in the urban area will pay even lower prices than the prices in other areas.*

Proposition 1(1) shows Figure 1(a) as a unique type of equilibrium that is symmetric in which the online firm occupies two end segments of the market, and the offline firms engage in discriminatory pricing for consumers living in the center segment. The equilibrium prices are equalized for some areas, as there are lower prices in the center market, and a uniform price $\widetilde{p}_0 + \lambda$ for other segments. The intuition is that the offline delivery costs for the end segments are relatively high and will be occupied by the online firm. Moreover, offline firms have locational advantages for their nearby consumers, and this advantage is enlarged in the central area, because this area is highly populated. Those cases in Figure 1(b) and 1(c) cannot be equilibria, because the offline firms have the incentive to move slightly toward the market center to serve more densely clustered consumers.

Proposition 1(2) describes the distortion appearing in both the location and price stages. The socially optimal price pattern is that each consumer x is served by the firm with the lowest unit cost among $t|x - x_1|$, $t|x - x_2|$, and λ for two offline firms and the online firm, respectively. If the online price is given as zero, it represents the case with no online price distortion. When $\alpha = 1/2$ and $p_0 = 0$, the socially optimal locations will be equivalent to minimizing the total shipping costs $t|x - x_1|$ or $t|x - x_2|$ for offline purchases and the distaste cost λ for online purchases and yield

$$x_1 = 1 - x_2 = \frac{1}{4} - \frac{1}{2b} - \frac{\lambda}{t} + \frac{\sqrt{t^2(b+2)^2 + 32b^2\lambda^2}}{4tb}, \tag{6}$$

which is increasing in b. This will be the case of Figure 1(a). This implies the equilibrium locations obtained from Eq. (5) will be equal to the socially optimal locations in Eq. (6) when $\alpha = 1/2$ and there is no online price distortion ($p_0 = 0$), which is consistent with Hurter and Lederer (1985) and Lederer and Hurter (1986). However, the equilibrium online price is always positive because of its market power, and thus the equilibrium locations are not socially desirable, even when $\alpha = 1/2$. When $\alpha \neq 1/2$, the equilibrium locations are also generally not socially desirable due to both the online price distortion and different social weights of consumers and firms. This result can be compared to Benchmark result (3). Moreover, in the standard case $\alpha = 1/2$, Proposition 1(2) also reveals that the socially optimal locations are more agglomerated than the equilibrium, which contrasts with the previous spatial pricing models and our benchmark result that the equilibrium locations are socially desirable without online competition. Notably, although we need the sufficient condition that b is not large to prove this property, various numerical calculations suggest that this finding generally holds for all $b \in [0, 2]$.

Intuitively, for the other two types in Figure 1(b) and Figure 1(c), less dispersed locations always reduce the summation of delivery costs and distaste cost, because more urban consumers are served by the offline firms, which can reduce their delivery costs, relative to the change in distaste cost. When α is not close to 1, the optimal location pattern is that offline firms serve the urban area, while the online firm serves two separated rural areas, because the shipping costs for those consumers sufficiently close to offline firms are less than the distaste cost (λ).

Recall that the pricing rule in the traditional spatial price discrimination literature (with $\alpha = 1/2$) is that the lowest-cost firm in any market point sets its price equal to the second-lowest cost, which induces socially optimal locations, because all consumers are served by the associated lowest-cost firms. In contrast, the online firm does not follow the above pricing rule, but sets its prices uniformly for all consumers, which results in consumers belonging to segments I, II, and III in Figure 1(a) not being served by the lowest-cost firms. For example, the consumers belonging to segment I are distant from both offline firms, so $t|x - x_1| > \lambda$, and they are still served by firm 1 at a price $p_0 + \lambda$. Moreover, segment II in Figure 1(a) describes those consumers living in the urban area who are served by offline firms with a shipping cost $t|x - x_i|$ that is higher than the distaste cost λ for purchasing online.

Proposition 1(3) suggests that the equilibrium locations are neither minimal nor maximal differentiation, because of the location result in Proposition 1(1). Moreover, the socially optimal locations depend on the social weight α. When α is small, the social optimum requires minimal differentiation in locations, because consumer surplus is best enhanced by increasing offline competition. However, when α is close to 1, the social optimum depends on the parameter λ. If λ is small, maximal differentiation is socially optimal, while if λ is large, the social optimum is never maximal differentiation. Intuitively, large location differentiation induces less competition between offline firms and among online and offline firms that raises the total profits of firms, but reduces consumer surplus. The associated social optimum will be the case in Figure 1(c). Two effects appear as the locations become more and more differentiated. The first effect is competition avoidance among firms, which is greater when λ is small, because the online firm can setup a higher online price that benefits all firms due to overall price increases.

Competition avoidance benefits firms but hurts consumers, and may either increase or decrease the social welfare, depending on the parameter α. The second effect is the waste of additional shipping costs, because the offline firms must pay for shipping costs that are less than the distaste costs for consumers close to the offline firms. For instance, those consumers in segments I and V in Figure 1(c) should be served by online firm from the social perspective, but are served by offline firms in this case. That is, when locations become more differentiated, the sum of total shipping costs and the distaste costs increase due to the urban-rural population distribution. The second effect reduces the social welfare. Henceforth, when λ is large, the second effect is large and the social optimum cannot be maximal differentiation. However, when λ is small, the second effect is small, and maximal differentiation is socially desirable. This result is different from our Benchmark result (3).

Compared with the benchmark case, introducing the online firm also results in the above two effects on offline locations. With online competition, the offline firms move closer to the urban area due to the online vs offline competition avoidance. Moreover, the wasting shipping costs comes from online price distortion, a market power to set a uniform distorted price (instead of the second lowest delivery cost for offline prices) that is associated with more dispersed offline locations. We find that the above roles result in neither maximal non-minimal differentiation in the equilibrium, but either maximal/minimal or non-maximal/non-minimal differentiation in the social optimum. In the benchmark case, maximal differentiation is socially desirable only when $\alpha = 1$, because the dispersed locations are associated with higher offline prices due to that the competition avoidance effect leads to the highest profits for firms. Therefore, only the extreme case $\alpha = 1$, when the social interest setting does not take into account the welfare of consumers at all, will lead to maximal differentiation. With online competition, the socially optimal locations are still minimally differentiated when α is small, but may be either maximal or non-maximal differentiation when α is close to one. The above second part is different from the benchmark result, depending on the online disadvantage (λ).

Proposition 1(4) reveals that online competition induces all prices to be lower than that under the benchmark scenario. The online firm plays a role as the third competitor and induces a more competitive environment in the spatial price discrimination framework. Since the cost of the online firm must be lower than at least one offline firm, and the equilibrium spatial price discrimination is determined as the second highest cost, the online entry reduces all prices. Moreover, offline firms move toward the market center after the online entry, which induces a further price reduction, because both offline firms are less separated and competition is thus more intense than that under the benchmark scenario. Therefore, all consumers are better off due to lower prices resulting from online competition.[10]

The following proposition demonstrates comparative statics for the distaste parameter λ:

Proposition 2. *When λ decreases, given that b is not large,* (1) *offline firms become more concentrated;* (2) *the equilibrium online price $\widetilde{p}_0^*$ becomes higher, but $\widetilde{p}_0^* + \lambda$ becomes lower;* (3) *the offline prices become lower in urban areas, but higher in other areas.*

Intuitively, the above results reveal that when the online firm has more advantage, offline firms will move closer to the market center in order to avoid the competition from the online firm. Hence, the online firm benefits from higher demand due to reducing the distaste cost for consumers, and from the higher concentration of the offline firms. Both induce the online firm to set a higher price because of the increased market power. However, the increase of the online price only partially offset the decrease in λ, leading to a lower $p_0^* + \lambda$. In contrast, the offline prices in the urban area decline due to competition between more concentrated offline firms, and also decreased in the non-urban areas due to the decreased $\widetilde{p}_0^* + \lambda$. This price pattern is shown as the green line in Figure 1(a). Notably, while Proposition 2 requires the sufficient condition that b is not large to ensure the above property of comparative statics for convenience of technical derivation, our various numerical illustrations reveal that this property is still valid for all $b \leq 2$.

Proposition 2 suggests that the offline firms move closer to the market center as the online distaste cost becomes lower, because the online advantage is increased. In reality, the online distaste cost has been, in general, decreasing in recent years due to the fast development of logistics technology, so offline firms tend to move closer to the market center and concentrate in densely populated areas (for instance, Guo and Lai (2017) described the bookstore phenomenon in the USA).

The comparison between the equilibrium and the social optimum with online competition is complicated. Overall numerical illustrations will be shown later in Table 1, where Panel A includes cases under exogenous p_0 and Panel B includes cases under endogenous $p_0(x_1)$. The social optimum when $\alpha = 1/2$ (cases 6, 7, and 8 in Table 1) is equivalent to minimizing the

[10] This finding is based on the assumption that shipping costs are paid by firms. Consumers only care about the offered prices plus their online distate costs. However, when consumers must pay the shipping costs, Guo and Lai (2017) showed that some consumers may be worse off after the online entry, because the distance between consumers and offline firms increases due to either relocation of the survived offline firms or the exit of those high-cost offline firms.

total online distaste costs, and offline shipping costs:

$$\min_{x_1} 2\left[\int_0^{x_{1L}} \lambda f(x)dx + \int_{x_{1L}}^{\frac{1}{2}} t|x-x_1|f(x)dx\right], \tag{7}$$

where $x_{1L} = x_1 - \frac{p_0(x_1)+\lambda}{t}$ is a function of x_1 and p_0, which are complicated and there are no analytical solutions. Therefore, numerical analysis on social optimal locations and zoning policies will be employed later in Table 1, where the social optimal locations for offline firms may be either more agglomerated (eg. cases 7A and 7B) or more dispersed (eg. cases 6A and 6B) than the equilibrium locations. Our finding is different from that in Hurter and Lederer (1985), where their equilibrium locations are socially desirable. Intuitively, Proposition 1(2) and Figure 1(a) show that the consumers in segments I, II, and III are not efficiently served by the lowest-cost firms. For instance, when b is small and λ is large as in cases 7A and 7B, segment II becomes large, and the more agglomerated socially optimal locations than the equilibrium locations will reduce the inefficiency from segment II.

Moreover, the online price in the equilibrium can be lower or higher than that of the social optimum. Our numerical illustrations reveal the influence of parameters λ and b. Intuitively, when λ decreases, the online firm's advantage becomes more significant, and thus offline firms become more concentrated (see a comparison between cases 1A, 2A, and 3A in Table 1), so the online price becomes higher. That is, a lower distaste cost implies a larger online market share from the social interest function in Eq.(7). However, the effect of λ on the socially optimal locations for offline firms are mixed. Locations may be more concentrated (comparing 8A with 7A, or 8B with 7B when the optimal pattern remains as in Figure 1(a)), although there appears another price effect in which p_0 increases along a smaller λ that leads to a lower $p_0 + \lambda$ and smaller offline market shares. Nevertheless, this price effect is dominated by the previous location effect from more agglomerated locations of the offline firms. Locations may also be more separated when the optimal pattern switches from Figure 1(a) to Figure 1(b) (see a comparison between 7A and 6A, or 7B and 6B). This is because a small λ gives more online advantage to serve urban consumers from a social aspect. In the case of Figure 1(b), both the most urban consumers and rural consumers are served by the online firm, because the online distaste cost (λ) is smaller than the offline transportation costs.

When b increases, consumers are more agglomerated at the central area, inducing offline firms to locate closer to the market center both in the equilibrium and the social optimum cases. However, the influence of b on p_0 has two contrary effects. As b increases, the first effect is from a lower demand in rural areas that clearly induces the online firm to set a lower price, while the second effect is from competition with offline firms who choose less dispersed locations due to more concentrated urban consumers, which results in less competition between the online and offline channels, and thus the online price increases. Our result reveals that the first effect dominates the second one and thus p_0 decreases as b increases (see a comparison between 2A and 3A in Table 1). We also find that the socially optimal locations become more concentrated as b increases if the optimal pattern remains as in Figure 1(a).

The above results are generally robust if we consider an alternative three-stage game structure such that offline firms choose their locations in the first stage, then the online firm sets its price in the second stage $p_0(x_1, x_2)$, and finally two offline firms engage in spatial price discrimination in the third stage.[11] Intuitively, offline firms have more strategic incentive to choose less dispersed locations in stage one under this setting, because the more concentrated location pattern leaves a larger online market share in the rural markets, which induces the online firm to set a higher price that is beneficial to the offline firms eventually. Therefore, the optimal locations in this setting are less dispersed than those in the previous two-stage game.

Under our two-staged game with p_0, x_1, and x_2 are decided simultaneously. We further compare the population-weighted average price (EP) and the standard division of price (SD): $\text{EP} = \int_0^1 p(x) \cdot f(x)dx$ and $\text{SD}^2 = \int_0^1 (p - \text{EP})^2 f(x)dx$, where $p(x)$ is the equilibrium prices. In the benchmark case, the EP and SD^2 are: $\text{EP} = \frac{t}{2} - t\big(b^2 + 6(\sqrt{b^2+4} - 2)\big)/24b$ and $\text{SD}^2 = -\frac{t^2(b^2-12)}{576}$. The average price and price variation will both be greatly reduced when the competition is more intense due to the online participation. Numerical illustration using $t = 1$, $b = 1, .., 2$, $\lambda = 0.1, .., 0.3$ reveals EP and SD drop by 15%–48%, and 50%–99%, respectively. This result provides an empirical implication to test the influence of online competition on the average price level and price variations. Our theoretical results might be consistent with, but are not directly implied by previous empirical studies such as Brynjolfsson and Smith (2000), which compared price patterns between internet channels and conventional channels, and showed that price dispersion is lower in internet channels.

[11] In detail, the offline firms have no incentives to choose overly dispersed locations such that the online firm occupies the urban area described as in Figures 1(b) and 1(c). The optimal online price p_0 is equal to the reaction function (5). However, there appears an additional strategic incentive for offline firms in the first stage, which considers the reaction of the online price in the second stage.

Table 1: Illustrations for zoning policy (p_0 is given / $p_0(x_1)$ is considered).

Cases	Parameter values	x_1^*	p_0^*	$\tilde{x}_1^w (= 1 - \tilde{x}_2^w)$	$\tilde{p}_0$	$\frac{\partial \pi_1}{\partial x_1}$	Zoning policy
Panel A: when p_0 is given							
1A	$\alpha \leq 0.381$, $b = 1$, $\lambda = 0.15$ (λ is large)	0.335	0.085	0.5 (Figure 1(a))	0.153	-0.363	$x_1 = x_2 = 1/2$
2A	$\alpha = 0$, $b = 1$, $\lambda = 0.1$ (λ is small)	0.346	0.111	0.5 (Figure 1(a))	0.173	-0.335	$x_1 = x_2 = 1/2$
3A	$\alpha = 0$, $b = 1.5$, $\lambda = 0.1$ (λ is small)	0.363	0.106	0.5 (Figure 1(a))	0.154	-0.348	$x_1 = x_2 = 1/2$
4A	$\alpha = 1$, $b = 1$, $\lambda = 0.08$ (λ is very small)	0.351	0.121	0 (Figure 1(c))	0.227	0.247	$x_1 \leq 0$, $x_2 \geq 1$
5A	$\alpha = 1$, $b = 1.9$, $\lambda = 0.21$ (b is large and λ is large)	0.358	0.053	0.315 (Figure 1(a))	0.039	0.104	$x_1 \in [0, 0.315]$, $x_2 \in [0.685, 1]$
6A	$\alpha = 1/2$, $b = 1$, $\lambda = 0.1$ (λ is small)	0.346	0.111	0.345 (Figure 1(b))	0.055	0.048	$x_1 \in [0, 0.345]$, $x_2 \in [0.655, 1]$
7A	$\alpha = 1/2$, $b = 1$, $\lambda = 0.15$ (λ is moderate)	0.335	0.085	0.373 (Figure 1(a))	0.101	-0.075	$x_1, x_2 \in [0.373, 0.627]$
8A	$\alpha = 1/2$, $b = 1$, $\lambda = 0.2$ (λ is large)	0.326	0.059	0.349 (Figure 1(a))	0.070	-0.046	$x_1, x_2 \in [0.349, 0.651]$
Panel B: when $p_0(x_1)$ is considered							
1B	$\alpha \leq 0.0423$, $b = 1$, $\lambda = 0.15$ (λ is large)	0.335	0.085	0.5 (Figure 1(a))	0.153	-0.363	$x_1 = x_2 = 1/2$
2B	$\alpha = 0$, $b = 1$, $\lambda = 0.1$ (λ is small)	0.346	0.111	0.326 (Figure 1(b))	0.059	0.050	$x_1 \in [0, 0.326]$, $x_2 \in [0.674, 1]$
3B	$\alpha = 0$, $b = 1.5$, $\lambda = 0.1$ (λ is small)	0.363	0.106	0.336 (Figure 1(b))	0.049	0.067	$x_1 \in [0, 0.336]$, $x_2 \in [0.664, 1]$
4B	$\alpha = 1$, $b = 1$, $\lambda = 0.08$ (λ is very small)	0.351	0.121	0 (Figure 1(c))	0.215	0.257	$x_1 \leq 0$, $x_2 \geq 1$
5B	$\alpha = 1$, $b = 1.9$, $\lambda = 0.21$ (b is large and λ is large)	0.358	0.053	0.315 (Figure 1(a))	0.039	0.104	$x_1 \in [0, 0.315]$, $x_2 \in [0.685, 1]$
6B	$\alpha = 1/2$, $b = 1$, $\lambda = 0.1$ (λ is small)	0.346	0.111	0.326 (Figure 1(b))	0.059	0.050	$x_1 \in [0, 0.326]$, $x_2 \in [0.674, 1]$
7B	$\alpha = 1/2$, $b = 1$, $\lambda = 0.15$ (λ is moderate)	0.335	0.085	0.355 (Figure 1(a))	0.094	-0.039	$x_1, x_2 \in [0.355, 0.645]$
8B	$\alpha = 1/2$, $b = 1$, $\lambda = 0.2$ (λ is large)	0.326	0.059	0.337 (Figure 1(a))	0.065	-0.023	$x_1, x_2 \in [0.337, 0.663]$

3.3 Zoning policies

This section provides the implication of zoning policies when the online retailer is included. Now consider the case where a regulator enacts his zoning policy at stage 0.[12] We can find the optimal zoning policy is to restrict the offline locations in some areas in order to maximize the social welfare function $W(\alpha)$. However, the optimal zoning is complicated and behaves non-monotonically relatively to our benchmark case. The complication is because zoning policies can be enacted as industrial instruments to balance multiple aspects on the trade-off between urban and rural, online and offline, and consumers and firms in order to raise the social welfare.

Proposition 3. *Given p_0 is exogenous for social optimum.* (1) *When α is small,* (*i.e., the regulator cares more about consumers*)*, the optimal zoning policy is limiting both firms to locate in the middle of the market.* (2) *When α is close to one and λ is small, the optimal zoning policy is to force firms to locate outside the interval* $(0, 1)$*.* (3) *When α is around* $1/2$*, the social optimal locations are finitely differentiated, and the optimal zoning policy can be either allowing firms' locations to be only within a middle interval or limiting both firms to locate outside a middle interval, depending on parameter settings.*

The above proposition reveals that our result is consistent with Bárcena-Ruiz and Casado-Izaga (2014) when α is small. That is, the optimal zoning is restricting offline firms to be located at the market center. However, when α is not small, the socially optimal locations and the implied zoning policy are generally complicated, and we are unable to obtain analytical solutions. When α is close to 1, the socially optimal locations and the zoning policy depend on the online disadvantage parameter (λ). If λ is small, we find that the socially optimal locations are maximal differentiation and the zoning policy is restricting them outside the market interval $[0, 1]$, which is consistent with previous studies such as Bárcena-Ruiz and Casado-Izaga (2014). However, if λ is not small, the socially optimal locations may be finitely differentiated and the zoning policy is restricting them outside the urban area such that $x_1 \in [0, x_1^w]$ and $x_2 \in [x_2^w, 1]$ in the 5A and 5B cases in Table 1. For other cases of α, we find various zoning policies. For instance, when $\alpha = 1/2$, if λ is not small, and Figure 1(a) is thus the socially optimal pattern, the zoning policy is restricting them to locating inside the urban area such that $x_1, x_2 \in [x_1^w, x_2^w]$, because $x_1^w > x_1^*$. Moreover, if λ is small such that Figure 1(b) is

[12] In order to emphasize the impacts on the strategic interactions in an oligopoly market, the zoning policy herein does not include the negative production externalities of firms such as the impact of pollution and noise on the resident environment. If we consider the effect of non-extremely negative externalities on neighboring consumers, then the socially optimal locations of firms will be even more remote.

socially desirable, then the zoning policy is restricting them outside the urban area such that $x_1 \in [0, x_1^w]$ and $x_2 \in [x_2^w, 1]$.

Intuitively, when the regulator is very concerned about the consumer surplus (α is small), the socially optimal zoning is to create competition among these three firms by proper regulations on firms' locations; that is, the social optimum is to restrict two offline retailers agglomerated at the center point, which is consistent with the optimal locations in Bárcena-Ruiz and Casado-Izaga (2014) and our benchmark scenario in the Benchmark result. However, when α approaches one, meaning that the regulator mostly cares about the profits of firms, we find that maximal differentiation may not be socially optimal when the online disadvantage parameter λ is not small. For moderate α, either restricting offline firms inside or outside the urban area could be possible. In particular, when λ is small enough to induce a socially desirable outcome as in Figure 1(b), in which the most urban area is served by the online firm, and thus the offline firms have an incentive to move toward to the market center. Therefore, the zoning policy should restrict the location of offline firms to outside the urban interval $[x_1^w, x_2^w]$.

Proposition 4 reveals the robustness of our zoning policy when $p_0(x_1)$ is considered by the government. However, the welfare analysis for general α is complicated.

Proposition 4. *Given p_0 is a function of x for social optimum.* (1) *When α is small* (*i.e., the regulator cares more about consumers*)*, the optimal zoning policy will be to limit both firms to locate in the middle of the market.* (2) *When α is close to one and λ is small, the optimal zoning policy is to force firms to locate outside the interval* $(0, 1)$.

We can summarize several cases along α. First, when α is small, the social planner will enforce offline firms to be agglomerated that create drastic competition between offline firms and benefits consumers. Second, when α is close to one and λ is small, the social planner will regulate the offline firm to be out of the market, so the offline locations are maximal differentiated and all firms are benefited from a market with less competitive environment overaly. These two cases are consistent with our benchmark case. Third, when α is moderate, the optimal trade-off between the interests of firms and consumers may result in moderately separated locations as Figure 1(b) by numerical calculations. The optimal zoning has no simple solution, depending on relative values of parameters. The full patterns of the zoning regulation and associated internal interaction effects under a moderate α cannot be clearly characterized.

Table 1 reports the socially optimal locations and zoning policies, which illustrates our findings in Propositions 3 and 4. First, the optimal locations and zoning show different patterns from those in Hurter and Lederer (1985) and Bárcena-Ruiz and Casado-Izaga (2014).

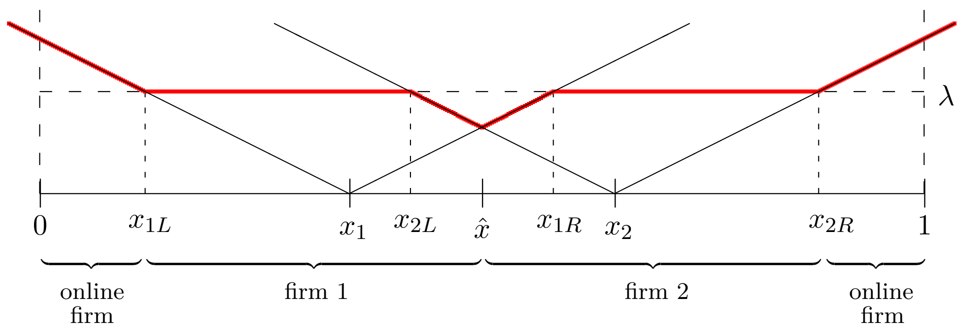
Since the equilibrium locations are not socially desirable, the socially optimal locations can be either more agglomerated (in cases 7A and 7B) or more separated (in cases 6A and 6B) than the equilibrium locations, even when $\alpha = 1/2$, consistent with Proposition 1(2). Second, when the social weight on firms (α) is small, the optimal locations in the cases under exogenous p_0 are agglomerated and the zoning policies are to restrict firms at the market center (in 1A, 2A, and 3A cases) in order to maximize competition between offline firms. However, when $p_0(x_1)$ is considered by the social planner, the cases 2B and 3B reveal the possibility that the socially optimal locations are not agglomerated when λ is small. This is because the strong online advantage may dominate the previous argument of maximizing offline competition. This novel finding highlights how digital business may affect the zoning policy. Third, when $\alpha = 1$, meaning that the social planner maximizes the interest of firms, the previous studies suggest that maximal offline differentiation appears due to competition avoidance. With online competition, we find that maximal differentiation may not be optimal when λ is large, specifically online competition is considered in 5A and 5B. However, when λ is small, maximal differentiation is still socially desirable. Intuitively, when the online firm has a strong advantage (λ is small), letting the online firm serve the central market is proper for the social optimum.

4 Extensions

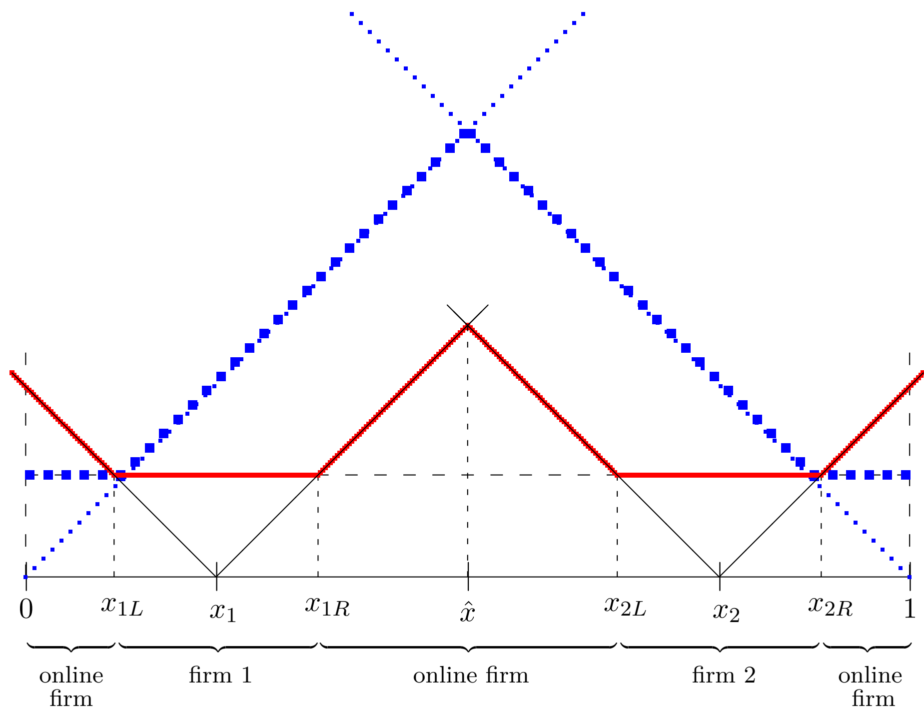
4.1 Online price discrimination

Online firms recently have gained an advantage due to their ability to collect data and use data mining, one use of which could be to enact spatial price discrimination.[13] However, online price discrimination may not be overwhelming, because consumers may discover and see through online firms' price discrimination, and react negatively if they discover they are the recipients of unequal treatment. In this subsection, we consider a modified game structure, where offline firms choose their locations in the first stage, and then in the second stage, all three firms (one online and two offline firms) engage in spatial price discrimination. The price equilibrium in the second stage herein can be described as in Figure 2(a) ($\frac{1}{2} - \frac{\lambda}{t} < x_1 \leq \frac{1}{2}$) and Figure 2(b) ($0 \leq x_1 \leq \frac{1}{2} - \frac{\lambda}{t}$). In particular, the case in Figure 2(b) describes less dispersed offline firms and that the online firm has no advantage in the urban area when $\lambda > t|\frac{1}{2} - x_i|$. According to this modified setting, the socially optimal locations are determined before price choices.

[13] Firms may also use it to earn profits from selling data to advertisers, to suggest related products or those which might be of interest to consumers, and so on, which can be further discussed in a framework of two-sided market.



(a) The equilibrium prices when the online firm engages in spatial price discrimination in the case of $\frac{1}{2} - \frac{\lambda}{t} < x_1 \leq \frac{1}{2}$.



(b) The price equilibrium when the locations are more dispersed in the case of $0 \leq x_1 \leq \frac{1}{2} - \frac{\lambda}{t}$ (where the blue-dotted lines represent the case when $x_1 = 1 - x_2 = 0$).

Figure 2: Online spatial price discrimination.

The following proposition demonstrates that Figure 2(a) is the only equilibrium pattern, and describes the influence of online competition on the equilibrium prices, locations, and the social optimum when the online firm engages in spatial price discrimination. Note that a superscript "$\approx$" represents the cases with online price discriminatory.

Proposition 5. (1) *In the equilibrium with online price discrimination, the online firm occupies two end segments, and offline firms enact discriminatory pricing in the central area. The equilibrium prices are piecewise decreasing as they approach to the market center.* (2) *There exists a unique location equilibrium* $\tilde{\tilde{x}}_1^* = \frac{-2t+tb-4b\lambda+\sqrt{t^2(b+2)^2+32b^2\lambda^2}}{4bt}$, $\tilde{\tilde{x}}_2^* = 1 - \hat{x}_1^*$, *which is less dispersed than in the benchmark scenario.* (3) *The equilibrium locations are (not) socially desirable, when* $\alpha = 1/2$ ($\alpha \neq 1/2$). (4) *When* $\alpha \leq 1/2$, *the socially optimal locations* ($\tilde{\tilde{x}}_1^{*w}, \tilde{\tilde{x}}_2^{*w} = 1-\hat{x}_1^{*w}$) *are interior solutions such that the online firm occupies two end segments, and offline firms enact discriminatory pricing in the central area. When* $\alpha \geq 2/3$, *the socially optimal locations are maximally differentiated.*

In this scenario, the consumers in the remote areas pay higher prices and are served by the online firm, while the consumers in the central area benefit from the competition between offline firms with lower prices. Proposition 5(1) suggests a modified finding that the online/offline firms occupy rural/urban segments, respectively. Proposition 5(2) reveals that allowing the online firm to enact spatial price discrimination will reduce the offline price $p_0^* + \lambda$ to λ for the consumers in (x_{1L}, x_{2L}) and (x_{1R}, x_{2R}) due to more competition between the online firm and each offline firm. However, for the most rural areas, competition will be reduced and prices are raised due to online price discrimination as shown in the comparison between Figure 1(a) and Figure 2(b). Henceforth, both offline firms tend to move closer to the market center to compete for the dense customers and the locations of firms are less dispersed than those in the benchmark scenario. That is, $\tilde{\tilde{x}}_1^* > x_1^*$. Proposition 5(3) confirms that the equilibrium locations are not socially desirable for $\alpha \neq 1/2$, which is consistent with Proposition 1(2). However, in the case $\alpha = 1/2$, the equilibrium locations are socially desirable. This is because when the online firm is allowed to set spatially discriminatory prices, the inefficient segments (I, II, III) in Figure 1(a) no longer exist. That is, online spatial discrimination does not lead to online price distortion in Subsection 4.1, and when $\alpha = 1/2$, the socially optimal locations are equivalent to the equilibrium locations, which is consistent with Hurter and Lederer (1985) in this scenario. This is because when the online firm can also engage in spatial price discrimination, it is equivalent to the case of three offline firms who all engage in spatial price discrimination.

Proposition 5(4) demonstrates that the maximal differentiation ($\tilde{\tilde{x}}_1^* = 0, \hat{x}_2^* = 1$) can be socially optimal under the sufficient condition of $\alpha \geq 2/3$, which can be compared with the benchmark scenario of $\alpha = 1$. Intuitively, the online firm plays a role of reducing the prices when the offline firms are not too dispersed. When α is large ($\alpha \geq 2/3$), the social planner is more concerned about the profits of firms, and locational maximal differentiation for the offline firms induces higher prices, which can be described similarly as in Figure 2(b). This price increasing effect from locational dispersion is higher than the benchmark scenario, in which no online firm enters the market. The socially optimal locations in other cases for $1/2 < \alpha < 3/2$ depend on the values of parameters. This finding can be compared to our previous analysis in Section 3.2, where the online firm cannot engage in price discrimination. In Section 3.2, the socially optimal locations are maximally differentiated when α is close to 1. This is because the price-increasing effect is small when we compare Figure 1(c) and Figure 2(b). That is, when the online firm cannot enact spatial price discrimination, more dispersed offline locations cannot increase prices greatly, because the online firm occupies the urban area and sets a uniform online price. However, Proposition 5(4) reveals that when α is large ($\alpha > 2/3$), in contrast to α being close to 1 in Proposition 3, where finite differentiation is possible when λ is not small, the socially optimal locations herein are always maximally differentiated, which leads to much higher prices described as the blue-dotted lines in Figure 2(b) and the online firm occupying most market areas except the two end segments. That is, the range of α for maximal differentiation as the social optimum here is larger than that without online price discrimination in Section 3. Notably, our optimal locations are the same as Bárcena-Ruiz and Casado-Izaga (2014) when α approaches one. Both their result and ours are due to avoiding competition between offline firms. The implied optimal zoning policy also depends on the value of α. When $\alpha = 1/2$, the equilibrium locations are socially desirable, and zoning is unnecessary. When $\alpha \geq 2/3$, the socially optimal locations are maximal differentiation (as the blue-dotted lines in Figure 2(b)), and the optimal zoning policy is restricting firms at the boundaries $x_1 = 0$ and $x_2 = 1$. However, when $\alpha < 1/2$, various numerical illustrations suggest that the zoning policies may be either restricting both firms in the urban segment or in the two rural segments, depending on the values of parameters.

4.2 Multiple offline firms with free entry

In the above discussion, we only analyzed duopolistic offline firms, but the intuition of how online competition affects strategic spatial price discrimination may be extended to the case of multiple firms. Consider a modified framework following Vogel (2011), where there exist numerous potential heterogenous offline firms, $i = 1, 2, \ldots, N$, with associated different unit production cost, $c_1 < c_2 < \ldots < c_N$, which will enter the market if their expected profits are non-negative. Let c_0 be the unit cost of the online firm. In contrast to the uneven distribution of consumers in previous sections, we have assumed the consumers with a mass of one are uniformly distributed in a circular market à la Salop (1979). Further analysis can show the equilibrium pattern similar to the case of Figure 1(b). The profit function of the online firm becomes

$$\pi_0 = \left(1 - \sum_{i=1}^{N^*} |x_{iR} - x_{iL}|\right)(p_0 - c_0) = \left(1 - \sum_{i=1}^{N^*} \frac{2(p_0 + \lambda - c_i)}{t}\right)(p_0 - c_0),$$

where N^* is the equilibrium number of offline firms, which are allowed to enter the market freely. There appear multiple location equilibria due to the assumption of uniform population density such that each offline firm occupies a separate market segment and only competes directly with the online firm.[14] This setting leads to completely equalized prices $p_0^* + \lambda$ for all offline firms, which are generally much lower than the case without online competition. Further calculations from the first-order condition $\partial\pi_0/\partial p_0 = 0$ show

$$p_0^* = \frac{1}{2}(c_0 - \lambda) + \frac{1}{4N^*}t + \frac{1}{2N^*}\sum_{i=1}^{N^*} c_i.$$

The profit of firm i becomes

$$\pi_i = \int_{x_{iL}}^{x_{iR}} (p_0^* + \lambda - c_i - t|x - x_i|)dx = \frac{(p_0^* + \lambda - c_i)^2}{t}.$$

Therefore, N^* is the number satisfying $p_0^* + \lambda - c_{N^*} \geq 0$ and $p_0^* + \lambda - c_{N^*+1} < 0$. The low-cost offline firms occupy larger market segments than those of the high-cost firms. The online firm takes over those consumers who live away from neighboring offline firms, which are spatially discrete segments along the circular market. Further comparative statics will reveal that the more the online advantage (λ is smaller) there is, the fewer the offline firms will exist, and the lower the market shares and profits are. Moreover, we find that $\partial p_0^*/\partial\lambda < 0$ and $\partial(p_0^* + \lambda)/\partial\lambda > 0$ when N^* is unchanged. That is, when the online firm has more competition advantage, it can set a higher price, but $p_0^* + \lambda$ becomes lower due to an inelastic demand assumption in this model.

[14] Since we have uniformly distributed consumers in this modified framework, online competition plays a role to equalize the equilibrium price, and then a slight move of location for each firm does not change market shares and profits. Therefore, there exist infinitely multiple location equilibria.

4.3 Empirical and policy implications

This subsection attempts to provide numerous implications for how online competition may affect both spatial price discrimination and zoning policy from our theoretical findings. The proposed theory predicts that online competition induces more equalized and lower prices across rural and urban regions, which leads to empirical implications that the entry of online retailers may induce more price convergence, either across areas or along the time periods, and may induce significantly lower prices.

The rural consumers who are orginally far away from offline locations greatly benefit from falling prices which were previously much higher than that in the urban area before the online entry. The urban consumers also benefit from lower prices due to more competition between offline firms, who become more concentrated in the urban area after the online entry. Another empirical implication may be associated with the fact that the impact of online competition depends on the depth of online popularity, and there may be different for urban and rural people. Among numerous empirical studies, Duch-Brown *et al.* (2017) examined EU country-level data for several consumer electronic products, and found evidence that consumer surplus increases after the introduction of online competition. However, there are large differences across countries, which may reflect the fact that e-commerce is at different stages of development across European countries. Although there does not appear to be international price convergence across European countries, empirical studies about spatial price dispersion and the role of online competition using more detailed regional data within a country are still limited. Besides, our model suggests that distaste costs matters for the equilibrium pattern. Forman *et al.* (2009) discussed how the entry of offline stores such as Wal-Mart Stores, Inc., Target, and Barnes and Nobles in local areas affects online purchasing and how that the changes in distance to offline stores mitigates the sensitivity to online price discounts. Both Duch-Brown *et al.* (2017) and Forman *et al.* (2009) also found the importance of disutility costs of purchasing online.

Our theory also suggests policy implications about zoning. Consider the most usual setting about social welfare, which has equal weights for producer surplus and consumer surplus. When there is no online competition, zoning is not necessary because equilibrium locations are also socially optimal. However, online competition leads a rationale of zoning such that the optimal zoning policy may be either (A) regulating firms' locations to be inside the urban area when the online firm has relatively competitive advantage or the population distribution is more even, or (B) forcing firms to locate outside the urban area for other cases.

5 Conclusions

Online-offline competition has become an issue of great concern for researchers, practitioners, and policy makers in recent years. We contribute to this literature by adding the role of online competition into offline spatial price discrimination. This paper provides a novel framework that combines online competition, spatial price discrimination, zoning, and urban-rural population distribution to demonstrate a new insight into the current digital world phenomenon. Before the online entry, our benchmark result shows that the socially optimal locations of duopolists and the implied zoning policy are affected by the distribution of population, but the coincidence between the equilibrium and social optimal locations still holds when the government has equal weight on consumers and firms still holds. With online competition, the offline firms serve the central urban area of the market, while the remote rural areas are served by the online firm. The equilibrium locations can never be socially optimal due to the online market power, and online competition induces all prices to decline to be partially equalized; both findings are in contrast to the previous literature on spatial price discrimination. We reveal various zoning policies, depending on online-offline and offline-offline competition, the online advantage and the social weights on producer surplus and consumer surplus. The intuition behinds our result is that the online firm sets a uniform price to induce equalized spatial prices, which eliminates the effect of competition avoidance between offline firms by choosing maximally differentiated locations. Two extensions analyze scenarios in which the online firm can enact spatial price discrimination, and in which multiple offline firms with different production costs compete with one online firm. Our model also provides some empirical and policy implications. A possible future study is to explore either a two-sided market with advertisement or a dual-channel situation in which offline firms can also run their online channels.

Proof of Appendix

Proof of Benchmark result

(1) We first rule out all asymmetric locations in equilibria. First, suppose $x_1+x_2>1$, meaning that $\hat{x}>1/2$.

$$\begin{aligned}\pi_1 &= \int_0^{\frac{1}{2}} (p_1(x)-t|x-x_1|)\,(1-\frac{b}{2}+2bx)dx + \int_{\frac{1}{2}}^{\hat{x}} (p_1(x)-t|x-x_1|)\,(1+\frac{3}{2}b-2bx)dx \\ &= \frac{t\left(b(2-3x_1-x_2)(2(3x_1^2+x_2^2)-3x_1-5x_2+2)+6(x_2-x_1)(3x_1+x_2)\right)}{24},\end{aligned} \tag{A.1}$$

$$\begin{aligned}\pi_2 &= \int_{\hat{x}}^{1} (p_2(x)-t|x-x_2|)\,(1+\frac{3}{2}b-2bx)dx \\ &= \frac{t(x_2-x_1)\left[b(2(x_1+2x_2)^2-9(x_1+3x_2)+6(x_2^2+2))+6(4-x_1-3x_2)\right]}{24}.\end{aligned} \tag{A.2}$$

Solving $\partial\pi_1/\partial x_1=0$ and $\partial\pi_2/\partial x_2=0$ leads to first-ordered conditions: $9bx_1^2+(2bx_2-7b+6)x_1+bx_2^2-(3b+2)x_2+2b=0$, $bx_1^2-(3b+2(1-bx_2))x_1-7bx_2^2+3(3b+2)x_2-2(b+2)=0$ which yields four interior solutions. These solutions all lead to either $x_1+x_2=1$ or violating second-ordered conditions by detailed calculations and thus there are no interior solutions. Second, in the case where $x_1+x_2<1$, there also exist no interior solutions. Third, the corner solution ($x_1=0$ or $x_2=1$) when $x_1+x_2\neq 1$ can also be ruled out. Consider the case $x_2=1$ for the situation $x_1+x_2>1$, and then firm 1 will response to choose $x_1=(5b-6+\sqrt{25b^2+12b+36})/(18b)\in[1/3,1/2)$. However, given this x_1, firm 2 will always has an incentive to move left at $x_2=1$. Henthforth, any corner location with $x_1+x_2\neq 1$ cannot be an equilibrium, and we need only consider the symmetric case ($x_1+x_2=1$). The profit functions become

$$\pi_1 = \frac{t(x_2-x_1)\left(6(3x_1+x_2)+b\left(2(7x_1^2+x_2^2)-3(3x_1+x_2)+8x_1x_2\right)\right)}{24}, \tag{A.3}$$

$$\pi_2 = \frac{t(x_2-x_1)\left(6(4-x_1-3x_2)+b\left(2(x_1^2+7x_2^2)-9(x_1+3x_2)+8x_1x_2+12\right)\right)}{24}. \tag{A.4}$$

Solving $\partial\pi_1/\partial x_1=0$ and $\partial\pi_2/\partial x_2=0$ simultaneously yields the unique solution of locations ($x_1^*=\frac{1}{4}+\frac{\sqrt{b^2+4}-2}{4b}, x_2^*=1-x_1^*$), which also satisfies the second-order condition $\left.\frac{\partial^2\pi_i}{\partial x_i^2}\right|_{x_1=x_1^*,x_2=x_2^*}<0$, $i=1,2$. Notably, the first-order condition also yields another solution $\{x_1=\frac{1}{4}+\frac{\sqrt{b^2+4}+2}{4b}, x_2=1-x_1\}$, which in fact violates the second-order condition.

(2) The consumer surplus CS can be calculated by

$$\begin{aligned}
CS &= V - \int_0^{\hat{x}} t(x_2 - x)f(x)dx - \int_{\hat{x}}^1 t(x - x_1)f(x)dx \\
&= V - \frac{tb}{24}(6(x_2^3 - x_1^3 + x_1x_2(x_2 - x_1)) - 3(3x_2 - 5x_1)(x_2 + x_1) - 2(6x_1 - 1) \\
&\quad + 6(x_1 + x_2)^2 + 12(1 - 2x_1)). \qquad \text{(A.5)}
\end{aligned}$$

Then, we maximize $SW(\alpha) = \alpha(\pi_1 + \pi_2) + (1 - \alpha)CS$ to solve the first-order conditions $\partial SW(\alpha)/\partial x_1 = 0$ and $\partial SW(\alpha)/\partial x_2 = 0$, which yield the first-ordered conditions: $4\alpha bx_1^2 + 2\alpha(2 - b)x_1 + \alpha - 1 = 0$ and $x_2 = 1 - x_1$. Then, the solutions are $x_2^w(\alpha) = 1 - x_1^w(\alpha)$, and $x_1^w(\alpha) = \frac{1}{4} - \frac{1}{2b} + \frac{\sqrt{4\alpha(\alpha+b-2\alpha b)+\alpha^2b^2}}{4\alpha b}$ when $\alpha > 1/3$ and $x_1^w(\alpha) = 1/2$ when $\alpha \leq 1/3$. The boundary solution $x_1^w = 1 - x_2^w = 0$ appears only when $\alpha = 1$.

(3) The comparative statics yields

$$\frac{\partial x_1^w(\alpha)}{\partial \alpha} = \frac{-1}{2\alpha\sqrt{\alpha(4\alpha + b - 2\alpha b) + \alpha b^2}} < 0, \qquad \text{(A.6)}$$

$$\frac{\partial x_1^w(\alpha)}{\partial b} = \frac{\sqrt{\alpha(4\alpha + b - 2\alpha b) + \alpha b^2} - 2\alpha - b(1 - 2\alpha)}{2b^2\sqrt{\alpha(4\alpha + b - 2\alpha b) + \alpha b^2}} > 0, \quad \text{when } \alpha > 1/3. \qquad \text{(A.7)}$$

In the case of $\alpha = 1/2$, $x_1^w(\alpha = 1/2) = x_1^*$.

(4) We derive the incentive of relocation for firm 1 at the social optimum (x_1^w, x_2^w) when $\alpha > 1/3$:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{-t}{4}\Big(b\big(x_1(7x_1 - 2x_2 - 3) + x_2(1 - x_2)\big) + 2(3x_1 - x_2)\Big), \qquad \text{(A.8)}$$

$$\text{when } x_1 \geq x_1^w, \quad \text{where } x_1 + x_2 \geq 1,$$

which leads to $\frac{\partial \pi_1}{\partial x_1}\Big|_{x_2=1-x_1} = \frac{-t}{2}(4x_1 - 1 + 2bx_1(2x_1 - b))$ and

$$\frac{\partial \pi_1}{\partial x_1}\Big|_{(x_1^w(\alpha), x_2^w(\alpha))} = \frac{t(2\alpha - 1)}{2\alpha} \gtreqless 0 \quad \text{if} \quad \alpha \gtreqless \frac{1}{2}. \qquad \text{(A.9)}$$

Thus, if $\alpha \leq 1/3$, then the government should assign the market center ($x = 1/2$) to be the locations for both firms, and if $1/3 < \alpha < 1/2$, then the government should enact a zoning policy that only allows firms to locate within $[x_1^w(\alpha), x_2^w(\alpha)]$. As similar proof applies for $\alpha > 1/2$. While the above proofs analyze local properties for x_1 near x_1^w, the conclusion applies to having global properties. When $1/3 < \alpha < 1/2$, $\partial\pi_1/\partial x_1 = -t(2bx_2 + b(1 + 3bx_1) - 7b)/4$ is a decreasing function in x_1 for $x_1 + x_2 > 1$ since the zoning policy is $x_1 \in [x_1^w, x_2^w]$. When $\alpha > 1/2$, it can be shown that it is optimal for firm 1 to choose $x_1 = x_1^w$ under the zoning policy $x_1 \in [0, x_1^w]$ and $x_2 = x_2^w$ by detailed calculations.

□

Proof of Proposition 1

(1) There are three cases in the equilibrium: $x_{1R} < \hat{x}$, $x_{1R} = \hat{x}$, and $x_{1R} > \hat{x}$, where x_{1R} is the right marginal consumer indifferent between the online firm and firm 1, which can be described as Figure 1. Similar to (1)–(3), in the case of Figure 1(b),

$$\pi_0 = p_0 \left(\int_0^{x_{1L}} f(x)dx + \int_{x_{1R}}^{x_{2L}} f(x)dx + \int_{x_{2R}}^{1} f(x)dx \right), \tag{A.10}$$

$$\pi_1 = \int_{x_{1L}}^{x_{1R}} (p_0 + \lambda - t|x - x_1|) f(x)dx, \tag{A.11}$$

which can be reduced to the following equations under symmetric locations

$$\pi_0\Big|_{\text{Fig1(b)}} = \frac{p_0}{t}\Big(2\big(b(1-4x_1)-2\big)(p_0+\lambda)+t\Big), \tag{A.12}$$

$$\pi_1\Big|_{\text{Fig1(b)}} = \frac{(p_0+\lambda)^2\big(b(4x_1-1)+2\big)}{2t}, \tag{A.13}$$

and consumer surplus

$$CS\Big|_{\text{Fig1(b)}} = \overline{v} - p_0 - \lambda, \tag{A.14}$$

and in the case of Figure 1(c),

$$\pi_0 = p_0 \int_{x_{1R}}^{x_{2L}} f(x)dx, \tag{A.15}$$

$$\pi_1 = \int_0^{x_{1R}} (p_0 + \lambda - t|x - x_1|) f(x)dx. \tag{A.16}$$

Under symmetric condition $x_2 = 1 - x_1$, Eqs. (A.12) and (A.13) can be reduced to the following equations

$$\pi_0\Big|_{\text{Fig1(c)}} = \frac{p_0\big(t - 2(p_0+\lambda+tx_1)\big)\big(b(tx_1+p_0+\lambda)+t\big)}{t^2}, \tag{A.17}$$

$$\begin{aligned}\pi_1\Big|_{\text{Fig1(c)}} = \frac{1}{12t^2}\Big(&4b\lambda^3 + 3\lambda^2\big(4b(p_0+tx_1)+t(2-b)\big)\\ &+ 6\lambda(p_0+tx_1)\big(2b(p_0+tx_1)+t(2-b)\big) + 4bp_0^3\\ &+ 3tp_0^2(4bx_1+2-b) - 4bt^3x_1^3 - 3t^3(2-b)x_1^2\Big),\end{aligned} \tag{A.18}$$

$$CS\Big|_{\text{Fig1(c)}} = \overline{v} - p_0 - \lambda. \tag{A.19}$$

We can exclude the case $x_{1R} < \hat{x}$, since firm 1 obviously earns more profits from moving closer to the center with more urban customers. Moreover, the second case $x_{1R} = \hat{x}$ is a special case of $x_{1R} \geq \hat{x}$ and can be excluded later. Both asymmetric cases $x_1 \neq 1 - x_2$ in Figure 1(b) and Figure 1(c) are still not equilibria, by the same argument. Therefore, the only equilibrium case is described as Figure 1(a). Then, the first-order conditions $\partial \pi_0 / \partial p_0 = 0$, $\partial \pi_1 / \partial x_1 = 0$, and $\partial \pi_2 / \partial x_2 = 0$ lead to

$$\begin{aligned} t^2 \cdot \left. \frac{\partial \pi_0}{\partial p_0} \right|_{x_2 = 1 - x_1} &= 6bp_0^2 - 2\big(t(2-b) - 4b(\lambda - tx_1)\big)p_0 \\ &\quad + (tx_1 - \lambda)\big(t(2-b) - 2b(\lambda - tx_1)\big) = 0, \end{aligned} \tag{A.20}$$

$$\begin{aligned} 2t \cdot \left. \frac{\partial \pi_1}{\partial x_1} \right|_{x_2 = 1 - x_1} &= -2bt^2 x_1^2 - t\big(t(2-b) + 4b(p_0 + \lambda)\big)x_1 + 2b(p_0 + \lambda)^2 \\ &\quad + t\big(t - (2-b)(p_0 + \lambda)\big) = 0. \end{aligned} \tag{A.21}$$

The above two conditions are quadratic in p_0 and in x_1, respectively, and lead to the reaction functions in (4) and (5) by the second-order condition. Notably, those cases $\hat{x} \neq 1/2$ cannot be equilibria since the firm more distant from the market center has an incentive to move closer to the market. Specifically, when $\hat{x} < 1/2$, firm 1 will choose a larger x_1 to serve more urban consumers. Hence, the equilibrium is the case $x_{1R} > \hat{x}$, which is depicted as Figure 1(a).

(2) Consider a case in which $\alpha = 1/2$ and λ is not small so that Figure 1(a) is socially optimal; solving $\partial SW / \partial x_1 = 0$ and $\partial SW / \partial x_2 = 0$ yields the first-ordered condition of the social optimum

$$\begin{aligned} 4t \cdot \left. \frac{\partial SW}{\partial x_1} \right|_{x_2 = 1 - x_1} &= -2bt^2 x_1^2 + (bt - 2t - 4b\lambda)\, tx_1 - 2bp_0^2 - (2-b)\lambda t \\ &\quad + 2b\lambda^2 + t^2 = 0, \end{aligned} \tag{A.22}$$

and $x_2 = 1 - x_1$.

We may compare the above (A.22) and the first-ordered condition (A.21) in equilibrium. We need to multiply (A.21) by $-(p_0 - 1)$ to equalize the second-order terms x_1^2 of (A.21) and (A.22), which leads to a difference in the first-order terms of (A.21) and (A.22): $p_0\big(- 4p_0 b - tb - 4b\lambda + 4btx_1 + 2t\big)$, which is zero iff $x_1 = \frac{1}{4} - \frac{1}{2b} + \frac{(p_0 + \lambda)}{t}$. This does not generally hold and thus the equilibrium is generally not socially desirable even in the case $\alpha = 1/2$.

When $\alpha \neq 1/2$, similar arguments apply and thus the equilibrium locations $(\tilde{x}_1, \tilde{x}_2)$ are not socially desirable.

(3) When $\alpha = 0$, $\frac{\partial SW}{\partial x_1}\Big|_{x_2=1-x_1, x_1=1/2} = \frac{(p_0+\lambda)(t(2+b)-2b(p_0+\lambda))}{2t}$, which is positive since $p_0 + \lambda < t/2$ to ensure positive online market shares. Therefore, when α is close to zero, the social optimal locations are $\tilde{x}_1^w = 1 - \tilde{x}_2^w = 1/2$.

When $\alpha = 1$, consider the more dispersed case described in Figure 1(c). Then, all offline prices are $\tilde{p}_0 + \lambda$. The social welfare $SW = \pi_0 + \pi_1 + \pi_2$. We then have

$$\frac{\partial SW}{\partial x_1}\Big|_{(x_1=0,x_2=1)} = -\frac{1}{t}\left(2b(p_0^2 - \lambda^2) - \lambda t(2-b)\right). \tag{A.23}$$

When $x_1 = 0$ and $x_2 = 1$, the optimal choice of p_0 for the online firm is to maximize

$$\pi_0 = p_0 \int_{x_1^R}^{x_2^L} f(x)dx = \frac{p_0}{t^2}\Big(t - 2(p_0+\lambda)\Big)\Big(t + b(p_0+\lambda)\Big), \tag{A.24}$$

which yields the solution

$$\tilde{p}_0|_{x_0=0,x_1=1} = \frac{t}{6} - \frac{2}{3}\lambda + \frac{-2t + \sqrt{t^2(b+2)^2 - 2b(t-2\lambda)(t+b\lambda)}}{6b}. \tag{A.25}$$

Then, $\frac{\partial SW}{\partial x_1}\Big|_{x_1=0,x_2=1,p_0=\tilde{p}_0(x_1=0,x_2=1)}$ is positive (negative) when λ is large (small) by detailed calculations. For instance, when $b = 1$

$$18t \cdot \frac{\partial SW}{\partial x_1}\Big|_{x_1=0,x_2=1,p_0=\tilde{p}_0(x_1=0,x_2=1)} = 4(\lambda+t)(2\lambda - t) + (4\lambda + t)\sqrt{4\lambda^2 + 2\lambda t + 7t^2}, \tag{A.26}$$

which is positive iff $\lambda > \frac{3\sqrt{5}-5}{20}t \approx 0.0854t$. Henceforth, when λ is large, the socially optimal locations are not maximal differentiation, while maximal differentiation appears when λ is small.

(4) Given any $\tilde{p}_0$

$$\tilde{x}_1 - x_1^* = \frac{1}{4bt}\left(\sqrt{t^2(b+2)^2 + 32b^2(\tilde{p}_0+\lambda)^2} - 4b(\tilde{p}_0+\lambda) - t\sqrt{b^2+4}\right), \tag{A.27}$$

which is positive if and only if $\psi = 4b(\tilde{p}_0+\lambda)^2 + t^2 - 2t\sqrt{b^2+4}(\tilde{p}_0+\lambda) > 0$. We have $\tilde{p}_0 + \lambda < \frac{t}{4}$, since the online firm will be dominated by the offline firm if $\tilde{p}_0 + \lambda > \frac{t}{4}$. This is because the offline firm may choose $\tilde{x}_1 = 1/4$ to dominate the online firm. Since ψ is decreasing in $\tilde{p}_0$ from $\tilde{p}_0 + \lambda < \frac{t}{4}$, and ψ is positive when $\tilde{p}_0 + \lambda = 0$, we prove that ψ is always positive, and therefore, $\tilde{x}_1 > x_1^*$. From (5), the reaction function $\tilde{x}_1$ satisfies

$\partial\tilde{x}_1/\partial p_0 < 0$, meaning that offline firms move close to the market center as the online price decreases. This is because

$$\frac{\partial \tilde{x}_1}{\partial p_0} = \frac{8b(p_0+\lambda)}{\sqrt{t^2(b+2)^2+32b^2(p_0+\lambda)^2}} - \frac{1}{t}, \tag{A.28}$$

which is negative iff

$$32b^2(p_0+\lambda)^2 - t^2(2+b)^2 < 0. \tag{A.29}$$

The last inequality holds from $p_0+\lambda < \frac{t}{4}$. Therefore, we have $\partial\tilde{x}_1/\partial p_0 < 0$. Further from

$$\tilde{x}_1\Big|_{p_0=\frac{t}{4}-\lambda} - x_1^* = -\frac{1}{4} + \frac{\sqrt{3b^2+4b+4}}{4b} - \frac{\sqrt{b^2+4}}{b} > 0, \tag{A.30}$$

which leads to $\tilde{x}_1 > x_1^*$.

The equilibrium prices under the benchmark scenario are $p^* = \max\{t|x-x_1^*|, t|x-x_2^*|\}$, which are higher than the equilibrium prices with online competition. That is, $p_0^*(x) > \tilde{p}_0$ when $x < x_{2L}$ or $x > x_{1R}$, and $\tilde{p}_0^*(x) > \tilde{p}^*(x) = \max\{t|x-\tilde{x}_1^*|, t|x-\tilde{x}_2^*|\}$ when $x_{2L} < x < x_{1R}$, since $x_1^* < \tilde{x}_1^* < \tilde{x}_2^* < x_2^*$.

$\square$

Proof of Proposition 2

From (4) and (5), solving the equations of implicit differentiation yields

$$\frac{d\tilde{p}_0}{d\lambda} = \frac{-2\big(\sqrt{\phi_1}-4b(\tilde{p}_0+\lambda)\big)\big(t(2-b)+4(bt\tilde{x}_1-\sqrt{\phi_2}-bz)\big)}{\varphi}, \tag{A.31}$$

$$\frac{d\tilde{x}_1}{d\lambda} = \frac{\big(\sqrt{\phi_1}-8b(\tilde{p}_0+\lambda)\big)\big(t(2-b)+4(bt\tilde{x}_1-\sqrt{\phi_2}-bz)\big)}{\varphi}, \tag{A.32}$$

where $\phi_2 = t^2(b+2)^2+32b^2(\tilde{p}_0+\lambda)^2$ and $\varphi = -8b(\tilde{p}_0+\lambda)\big(t(2-b)+4\big(b(t\tilde{x}_1-\lambda)-\sqrt{\phi_2}\big)\big) - \sqrt{\phi_1}\big(t(2+b)-4b(t\tilde{x}_1-\lambda)+10\sqrt{\phi_2}-4t\big)$. Substituting $\sqrt{\phi_1}$ and $\sqrt{\phi_2}$ from (4) and (5) into the above $d\tilde{p}_0/d\lambda$ and $d\tilde{x}_0/d\lambda$ leads to

$$\frac{d\tilde{p}_0}{d\lambda} = \frac{2t\big(t(2-b)+4b(t\tilde{x}_1-2p_0-\lambda)\big)(2-b+4b\tilde{x}_1)}{\varphi_2} \tag{A.33}$$

$$\frac{d\tilde{x}_1}{d\lambda} = \frac{\big(t(2-b)+4b(t\tilde{x}_1-p_0-\lambda)\big)^2}{(t\varphi_2)}, \tag{A.34}$$

where $\varphi_2 = -3t^2(2-b)^2 + 8b\big(b(3\tilde{x}_1 t^2 + 2\lambda^2) + 4t(\tilde{p}_0 + \lambda b\tilde{x}_1) - (2-b)\lambda t - 6\tilde{x}_1 t^2(1 + b\tilde{x}_1) + 2b\tilde{p}_0\big(\tilde{p}_0 + 2\lambda + t(4\tilde{x}_1 - 1)\big)\big)$. Then, $\varphi_2 < 0$ when b is not too large. This is because $\partial\varphi_2/\partial\lambda > 0$, and thus

$$\varphi_2 < \varphi_2\Big|_{z=\frac{t}{4}-\tilde{p}_0,\tilde{x}_1=x_1^*} = -2t^2(3 + b^2 - 2b\sqrt{b^2+4}) < 0 \quad \text{when } b < 1.6914. \tag{A.35}$$

The second term of the numerator of $d\tilde{p}_0/\partial\lambda$ is positive when b is not large. That is, since this term $t(2-b) + 4b(t\tilde{x}_1 - 2\tilde{p}_0 - \lambda)$ is increasing in $\tilde{x}_1$ and decreasing in $\tilde{p}_0$, it is greater than when we replace $\tilde{x}_1 = x_1^*$ and $\tilde{p}_0 = \frac{1}{4}t - \lambda$ into this term: $t(2-b) + 4b(t\tilde{x}_1 - 2\tilde{p}_0 - \lambda) < -4b\lambda - t\sqrt{b^2+4} + 2 + b < 0$ when b is not large. Notably, even $\lambda = 0$, $b < 1.1547$ ensure this term is negative. Therefore, $d\tilde{p}_0/d\lambda < 0$, and $d\tilde{x}_1/d\lambda < 0$ is obtained. $d(\tilde{p}_0 + \lambda)/d\lambda < 0$ can be further shown by detailed calculations: $d(\tilde{p}_0+\lambda)/d\lambda = -(1/\varphi_2)(4b(\tilde{p}_0 + \lambda + tx_1) + t(2 - b))(4b(tx_1 - \tilde{p}_0 - \lambda) + t(2-b)) > 0$. Finally, since a smaller λ induces more agglomerated x_1 and x_2, it leads to lower offline prices for the urban area à la spatial price discrimination as shown by the green line in Figure 1(a). However, the offline prices for the non-urban areas are $p_0 + \lambda$, which become higher.

□

Proof of Proposition 3

(1) When $\alpha = 0$ (Figure 1(a)),

$$2t \cdot \frac{\partial SW}{\partial x_1}\bigg|_{x_2=1-x_1,\, x_1=1/2} = (p_0+\lambda)\big(t(2+b) - 2b(p_0+\lambda)\big) > 0. \tag{A.36}$$

Therefore, the optimal zoning policy is $x_1 = x_2 = 1/2$. Thus, this optimal zoning policy applies for the cases with small α.

(2) When $\alpha = 1$, following the proof of Proposition 1(3) when λ is small, the social optimum is maximal differentiation ($x_1^w = 0, x_2^w = 1$):

$$2t \cdot \frac{\partial \pi_1}{\partial x_1}\bigg|_{x_2=1-x_1,\, x_1=0} = 2b\,(p_0+\lambda)^2 + t^2 + t\,(p_0+\lambda)(b-2) > 0. \tag{A.37}$$

Therefore, the optimal zoning policy is $x_1 = 0$ and $x_2 = 1$. When $\alpha = 1$ and λ is not small, if still at Figure 1(c), from the condition $2t \cdot \frac{\partial SW}{\partial x_1}\Big|_{x_2=1-x_1} = 0$, we have $2b(\lambda^2 - p_0^2) + t(2bx_1 + 2 - b)(\lambda - tx_1) + 2btx_1\lambda = 0$. Thus,

$$\begin{aligned} 2t \cdot \frac{\partial \pi_1}{\partial x_1}\bigg|_{x_2=1-x_1,\, x_1=x_1^w} = &\ \big(\underbrace{2b(\lambda^2 - p_0^2) + t(2bx_1 + 2 - b)(\lambda - tx_1) + 2btx_1\lambda}_{=0} \\ &+ 4bp_0^2 + 4bp_0(tx_1+\lambda) + tp_0(2-b) > 0. \end{aligned} \tag{A.38}$$

Therefore, the optimal zoning policy is $x_1 \leq x_1^w$ and $x_2 \geq x_2^w (= 1 - x_1^w)$. Thus, the above zoning policies apply to the case when α is close to one.

(3) Consider the case where $\alpha = 1/2$ and λ is not small, so that Figure 1(a) is socially optimal, the first-order condition of the social optimum is $-2bt^2x_1^2 + (bt - 2t - 4b\lambda)\,tx_1 - 2bp_0^2 - (2-b)\lambda t + 2b\lambda^2 + t^2 = 0$ from the proof of Proposition 1(2). At the socially optimal location, we have

$$\begin{aligned} 2t \cdot \left.\frac{\partial \pi_1}{\partial x_1}\right|_{x_2 = 1 - x_1,\, x_1 = x_1^w} &= \underbrace{-2bt^2x_1^2 + (bt - 2t - 4b\lambda)\,tx_1 - 2bp_0^2 - (2-b)\lambda t + 2b\lambda^2 + t^2}_{=0} \\ &\quad + \; p_0 \left(t(b-2) + 4b(p_o + \lambda - tx_1)\right) \\ &= -\big(tp_0\,(2-b) + 4b(tx_1 - p_0 - \lambda)\big) < 0. \end{aligned} \tag{A.39}$$

Therefore, when $\alpha = 1/2$, the optimal zoning policy is asking firms to locate in $(x_1, x_2) \in [x_1^w, x_2^w(1 - x_1^w)]$. Now consider the case in which $\alpha = 1/2$ and λ is small such that Figure 1(b) is the socially optimal pattern. We have $\left.\frac{\partial SW}{\partial x_1}\right|_{x_2 = 1 - x_1} = \frac{2b(\lambda - p_0)(p_0 + \lambda)}{t}$, which is negative (positive) when $\lambda < p_0$ ($\lambda > p_0$). Thus, the socially optimal location for firm 1 is the either smallest $x_1 = \frac{p_0 + \lambda}{t}$ or the largest $x_1 = \frac{1}{2} - \frac{p_0 + \lambda}{t}$ for the pattern Figure 1(b). Then,

$$\left.\frac{\partial \pi_0}{\partial x_1}\right|_{\text{Figure 1(b)}} = \frac{b\big(2\lambda(2 - b(1 - 4x_1)) + t\big)^2}{8t(2 - b + 4bx_1)^2} > 0. \tag{A.40}$$

So the optimal zoning policy is $[0, x_1^w]$ and $[x_2^w, 1]$. Thus, the above discussion applies for the cases that α is around 1/2.

□

Proof of Proposition 4

(1) When α is small, the socially optimal locations will be the case of $1/2 - \lambda/t \leq x_1 \leq 1/2$ as Figure 1(a). Substituting the best response $\tilde{p}_0(x_1)$ in Eq.(4) into the social welfare function $\widetilde{SW}$ yields

$$\begin{aligned} &\left.\frac{\partial \widetilde{SW}}{\partial x_1}\right|_{x_1 = \frac{1}{2},\, x_2 = 1 - x_1} \\ &= \frac{\Big(11t^2(b+4) + 32b\big(t(2b+t) - 2z(t - bz)\big)\Big)\tilde{\phi}}{108tb\tilde{\phi}} \\ &\quad + (4bz - t(2+b))\left(32bz(bz - t) + t^2(44 - 28b + 11b^2) - 16b^2zt\right), \end{aligned} \tag{A.41}$$

where $\tilde{\phi} = \sqrt{(2bz - t)^2 + t^2(b-1)^2 - 2t(b^2z - t)}$. This differential can be shown as positive when even $0 \le b \le 2$, and $0 \le z \le t/4$. Therefore, the socially optimal locations are $x_1 = 1/2$ and $x_2 = 1/2$ when α is small.

$$\left.\frac{\partial \pi_1}{\partial x_1}\right|_{x_1=\frac{1}{2},\, p_0=\tilde{p}_0} = \frac{1}{18}\Big((2t - b(4z-t))\tilde{\phi} - 4t^2(1+b^2) - 4bzt(b+2) - 10bt^2 + 8b^2z^2\Big), \tag{A.42}$$

which is negative by detailed calculations. Henceforth, firm 1 has incentive to move to the left at the market center point. We thus prove that the optimal zoning policy is limiting both firms to locate in the middle of the market when α is small.

(2) When α is close to one, the social optimal locations may satisfy either $1/2 - \lambda/t \le x_1 \le 1/2$ or $0 \le x_1 \le 1/2 - \lambda/t$. Consider $\alpha = 1$ for convenience. In the case $0 \le x_1 \le 1/2 - \lambda/t$,

$$\frac{\partial \widetilde{SW}}{\partial x_1} = -2bt^2x1^2 + ((b-2)t^2 + 4b\lambda t)x_1 - b\lambda(2b\lambda + (2-b)t) - 2bp_0^2, \tag{A.43}$$

which leads to $\partial\pi_1/\partial x_1$ being either positive or negative. Similarly, in the case $1/2 - \lambda/t \le x_1 \le 1/2$, it can be shown by detailed calculations that the same conclusion is obtained.

□

Proof of Proposition 5

(1) We can exclude the case $x_{1R} < \hat{x}$ described in Figure 2(b), since firm 1 obviously earns more profits from moving closer to the center with more urban customers. Moreover, the case $x_{1R} = \hat{x}$ is a special case of $x_{1R} \ge \hat{x}$ as following analysis and described in Figure 2(a). Since the online firm is allowed to engage in spatial price discrimination, the equilibrium price at each point becomes the second-lowest unit cost, provided by the lowest-cost firm. The unit costs for offline firms and the online firm are $x_1 + t|x - x_1|$, $x_2 + t|x - x_2|$, and λ, respectively. The profit of the online firm becomes

$$\pi_0 = \int_0^{x_{1L}} \big(t|x - x_1| - \lambda\big) f(x)dx + \int_{x_{2R}}^{1} \big(t|x - x_2| - \lambda\big) f(x)dx, \tag{A.44}$$

where $t|x - x_i|$, $i = 1, 2$ are the unit costs for consumers and $t|x - x_i| - \lambda$ is the price received by the online firm, while the offline firms have the profit functions:

$$\pi_1 = \int_{x_{1L}}^{x_{2L}} (\lambda - t|x - x_1|) f(x)dx + \int_{x_{2L}}^{\hat{x}} t\big((x_2 - x) - (x - x_1)\big) f(x)dx, \tag{A.45}$$

$$\pi_2 = \int_{x_{1R}}^{x_{2R}} (\lambda - t|x - x_2|) f(x)dx + \int_{\hat{x}}^{x_{1R}} t\big(|x - x_1| - |x - x_2|\big) f(x)dx. \tag{A.46}$$

(2) Thus,

$$\frac{\partial \pi_1}{\partial x_1} = \frac{1}{4t}\left(-bt^2(3x_1 + x_2 - 1)(x_1 - x_2) + 2t^2(x_2 - x_1) - 2\lambda\big(2b(2tx_1 - \lambda) + t(2-b)\big)\right), \tag{A.47}$$

$$\frac{\partial \pi_2}{\partial x_2} = \frac{1}{4t}\left(bt^2(3x_2 + x_1 - 3)(x_2 - x_1) - 2t^2(x_2 - x_1) - 2\lambda\big(2b(2tx_2 + \lambda) - t(3b+2)\big)\right). \tag{A.48}$$

The above two first-ordered conditions yield

$$\tilde{\tilde{x}}_1^* = \frac{-2t + tb - 4b\lambda + \sqrt{t^2(b+2)^2 + 32b^2\lambda^2}}{4bt}, \tag{A.49}$$

and $\tilde{\tilde{x}}_2^* = 1 - \tilde{\tilde{x}}_1^*$. Comparing this equilibrium with the benchmark scenario yields

$$\tilde{\tilde{x}}_1^* - x_1^* = \frac{1}{4bt}\left(\sqrt{t^2(b+2)^2 + 32b^2\lambda^2} - t\sqrt{b^2+4} - 4b\lambda\right), \tag{A.50}$$

which is positive, since $0 < b \leq 2$ and $0 < \lambda < t/4$.

(3) The social welfare function can be presented (under the symmetric case) as two different cases, $1/2 - \lambda/t \leq x_1 \leq 1/2$ and $0 \leq x_1 \leq 1/2 - \lambda/t$ (described in Figure 2(a) and Figure 2(b), respectively):

$$\begin{aligned} &SW_{x_1 \geq 1/2 - \lambda/t} \\ &= 2\int_0^{x_{1L}} (\alpha t(x_1 - x) + (1-\alpha)(V - t(x_1 - x) - \lambda))f(x)dx \\ &+ 2\int_{x_{1L}}^{x_{2L}} (\alpha(\lambda - t|x - x_1|) + (1-\alpha)(V - \lambda))f(x)dx \\ &+ 2\int_{x_{2L}}^{1/2} (\alpha(t(x_2 - x) - t(x - x_1)) + (1-\alpha)(V - (t(x_2 - x) - t(x - x_1)))f(x)dx, \end{aligned} \tag{A.51}$$

$$\begin{aligned} &SW_{x_1 \leq 1/2 - \lambda/t} \\ &= 2\int_0^{x_{1L}} (\alpha t(x_1 - x) + (1-\alpha)(V - t(x_1 - x) - \lambda))f(x)dx \\ &+ 2\int_{x_{1L}}^{x_{1R}} (\alpha(\lambda - t|x - x_1|) + (1-\alpha)(V - \lambda))f(x)dx \\ &+ 2\int_{x_{1R}}^{1/2} (\alpha t(x - x_1) + (1-\alpha)(V - t(x - x_1) - \lambda)f(x)dx. \end{aligned} \tag{A.52}$$

Both are cubic polynomial functions of x_1, and their three-ordered terms are both $2bt(3\alpha - 2)/3x_1^3$.

For the case $x_1 \geq 1/2 - \lambda/t$, the first-order condition of social welfare maximization under the symmetric condition $x_2 = 1 - x_1$ yields

$$2bt^2(3\alpha-2)x_1^2 - t\big(\alpha t(7b+2) - 4b(t - z + \alpha z)\big)x_1 + (t-2\lambda)(2tb+3t-\lambda)\alpha \\ - t(tb - 3b\lambda - 2\lambda), \tag{A.53}$$

which yields the optimal location

$$\tilde{\tilde{x}}_1^{*w} = \frac{\alpha b(7t+4\lambda) - 2t(2b-\alpha) - \sqrt{\phi_w}}{4bt(3\alpha-2)}, \tag{A.54}$$

where $\phi_w = \big(\alpha^2(2-b)^2 + 8b(5\alpha-2)(1-\alpha)\big)t^2 + 32b\lambda(2\alpha-1)^2(2+b)t - 32b^2\lambda^2(7\alpha^2 - 8\alpha + 2)$. These socially optimal locations are different from the equilibrium locations, except when $\alpha = 1/2$ by detailed calculations.

(4) The social welfare $SW_{x_1 \geq 1/2 - \lambda/t}$ always has a unique interior solution $\tilde{\tilde{x}}_1^{*w}$ above or $\tilde{\tilde{x}}_1^{*w} = 1/2 - \lambda/t$ when $\alpha \leq 1/2$, because $SW_{x_1 \geq 1/2 - \lambda/t}$ is a cubic polynomial with a negative three-ordered term, and

$$\left.\frac{\partial SW_{x_1 \geq 1/2-\lambda/t}}{\partial x_1}\right|_{x_1 = 1/2} = -t + \lambda(b+2) + \alpha\Big(2t - 3\lambda(b+2) + \frac{2b\lambda^2}{t}\Big) < 0, \tag{A.55}$$

$$\left.\frac{\partial^2 SW_{x_1 \geq 1/2-\lambda/t}}{\partial x_1^2}\right|_{x_1 = 1/2} = -4b\lambda(1-\alpha) - \alpha t(b+2) < 0. \tag{A.56}$$

However, when $\alpha = 1$, the social welfare function $SW_{x_1 \geq 1/2 - \lambda/t}$ is a cubic polynomial with a positive three-ordered term $4/3btx_1^3$, and there are two solutions for this cubic polynomial $x_1 = (2b - 4 - 2\sqrt{b^2+4})/(8b) < 0$ and $x_1 = (2b - 4 + 2\sqrt{b^2+4})/(8b) > 1/2 - \lambda/t$. Since $\partial(SW_{x_1 \geq 1/2-\lambda/t})/\partial x_1|_{\alpha=1, x_1=0} = -t < 0$, $SW_{x_1 \geq 1/2 - \lambda/t}$ has the maximal value at $x_1 = 0$ within the interval $0 \leq x_1 \leq 1/2 - \lambda/t$. Henceforth, the social optimum $(\tilde{\tilde{x}}_1^{*w}, \tilde{\tilde{x}}_2^{*w}) = (0, 1)$ if α is close to 1. A similar but more technical proof applies for all $\alpha \geq 2/3$ by detailed calculations.

□

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