Stability of switched systems with admissible switching signals

Hui-Ting Wang

China University of Geosciences School of Automation

Yong He (✉ heyong08@cug.edu.cn)

China University of Geosciences (Wuhan)

Qing-Guo Wang

Beijing Normal University-Hong Kong Baptist University United International College

Chuan-Ke Zhang

China University of Geosciences School of Automation

Min Wu

China University of Geosciences School of Automation

Research Article

Keywords: switched systems, admissible edgedependent, mode-dependent, bounded maximum average dwell time, divergence time

Posted Date: March 1st, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-225426/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Stability of switched systems with admissible switching signals

Hui-Ting Wang · Yong He · Qing-Guo Wang · Chuan-Ke Zhang ·
Min Wu

Received: date / Accepted: date

Abstract In this paper, stability of switched systems is investigated for a class of switching signals which meet some admissibility conditions. Firstly, the admissible edge-dependent divergence time is defined in terms of admissible transition edges and it will vary with the compensation bounds. Then the admissible edge-dependent bounded maximum average dwell time (BMADT) is imposed on switching signals. As a result, a sufficient condition is obtained for globally uniformly exponential stability of switched nonlinear systems with such switching signals. Secondly, by setting the equal compensation bounds for the same reaching subsystems, the mode-dependent divergence time is defined, and then the mode-dependent BMADT is proposed. A stability condition under the mode-dependent BMADT is established. These stability results are then applied to switched linear systems. The numerical example is presented to show that the proposed techniques are less restrictive and more flexible in application, compared with the BMADT.

Keywords switched systems · admissible edge-dependent · mode-dependent · bounded maximum average dwell time · divergence time

1 Introduction

Switched systems have attracted great attention, due to their widespread applications in various fields [1–4]. Switched systems may be classified to three types: all subsystems are stable, all subsystems are unstable, and some subsystems are unstable [5–11]. Note that stability of subsystems does not guarantee stability of a switched system and a switching signal governing selection of subsystems is crucial for stability of the system. A switching signal is classified by causes determining whether to switch: state of subsystems or time. When a switched system with all subsystems being unstable is stabilized under a state-dependent switching sequence, it is common to find a stable convex combination [12–14]. However, designing a state-dependent switching signal requires the full state information.

In contrast, time-dependent switching signals are easier to be designed [15–18]. When there are stable and unstable subsystems, slow switching and fast switching are used. The representative techniques include the dwell time (DT) [19], the average dwell time (ADT) [20], the mode-dependent ADT [21] and the admissible edge-dependent ADT [22, 23]. The main idea behind them is to activate stable subsystems for sufficiently long to suppress the state divergence made by unstable subsystems. However, the stability conditions involve priori knowledge of stability of subsystems, and they fail on switched systems with all subsystems being unstable.

For switched systems with all subsystems being unstable, Mao *et al.* [11] proposed the divergence time and the bounded maximum average dwell time (BMADT). The time spans which are less than lower compensation bound or more than upper compensation bound have divergence time, then the BMADT is put forward based

Hui-Ting Wang · Yong He · Chuan-Ke Zhang · Min Wu
School of Automation, China University of Geosciences,
Wuhan 430074, PR China
Hubei Key Laboratory of Advanced Control and Intelligent
Automation for Complex Systems, Wuhan 430074, PR China
E-mail: heyong08@cug.edu.cn

Qing-Guo Wang
Institute of Artificial Intelligence and Future Networks, Bei-
jing Normal University, BNU-HKBU United International
College, Zhuhai 519087, PR China

on the divergence time. Under the BMADT switching, the state divergence caused by switches with divergence time are compensated for by those without divergence time. It means that there is no need to know stability of subsystems in advance and the BMADT can be applied to any switched system regardless of whether or not its subsystems are stable or not. However, the BMADT switching ignores the mapping from time to subsystems, resulting in conservativeness. Relationships between switching time instants and activated subsystems are involved [24] in admissible transition edges, which also reveals the change of activated subsystems before and after a switching time instant. It has been shown that use of admissible transition edges is effective in generalizing the stability criteria of switched systems [22, 23].

In view of the above observations, we think that admissible transition edges can relax the restrictions of the BMADT switching for better stability conditions. Firstly, compensation bounds are allowed to be local for different admissible transition edges. Then the admissible edge-dependent divergence time is defined, by which the admissible edge-dependent BMADT is established. Then, a stability condition is proposed via the admissible edge-dependent BMADT, where the suppression of divergent state vary in different admissible transition edges, contributing to fewer restrictions and more application flexibility than that in [11]. In addition, the case that ignores the differences of starting subsystems is considered, then compensation bounds are only consistent with reaching subsystems. Hence, the admissible edge-dependent divergence time is turned into the mode-dependent divergence time. Thus, the mode-dependent BMADT is proposed, which is utilized to derive a stability criterion of switched nonlinear systems. Based on these results, stability criteria of switched linear systems are established to better illustrate the effectiveness of proposed admissible switching signals.

The rest of the paper is organized as follows. Section 2 introduces descriptions of switched systems and the definition of the BMADT. Section 3 contains definitions of the admissible edge-dependent BMADT and mode-dependent BMADT, and stability conditions for nonlinear and linear switched systems. Section 4 presents a numerical example. Conclusions are given in Section 5.

2 Problem Formulation and Preliminaries

Consider the switched nonlinear system described by

$$\dot{x}(t) = f_{\sigma(t)}(x(t)), \quad x(t_0) = x_0, \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system state, $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$, is a locally Lipschitz nonlinear function, $\sigma(t) : [t_0, \infty) \rightarrow \mathbb{M} = \{1, 2, \dots, m\}$, is the switching signal, m is the number of subsystems, switching time instants are denoted by $t_s, s = 1, 2, \dots$, $\Delta_s = t_s - t_{s-1}$ is the dwell time of s -th switching. Assume that σ is continuous from the right everywhere, the system (1) has the solution for certain σ and the initial condition x_0 , which is denoted by $x(t)$, and the origin is an equilibrium of the system (1), i.e., $f_{\sigma}(0) = 0$. The stability of system (1) is studied in this paper.

Definition 1 [21] The equilibrium $x = 0$ of system (1) is globally uniformly exponentially stable (GUES) under certain switching signal σ and the initial condition x_0 if $\|x(t)\| \leq ke^{-\eta(t-t_0)}\|x_0\|, k > 0, \eta > 0$.

The stability of system (1) depends on the switching signal $\sigma(t)$. Choose $\underline{t}$ and $\bar{t}$ with $\bar{t} > \underline{t} > 0$. Define

$$\dot{T}(s, \underline{t}, \bar{t}) = \begin{cases} \Delta_s, & \Delta_s < \underline{t}, \\ 0, & \underline{t} \leq \Delta_s \leq \bar{t}, \\ \Delta_s - \bar{t}, & \Delta_s > \bar{t}, \end{cases} \quad (2)$$

and

$$T(t, \underline{t}, \bar{t}) = \sum_{s=1}^l \dot{T}(s, \underline{t}, \bar{t}) + t - t_l, \quad t \in [t_l, t_{l+1}). \quad (3)$$

$T(t, \underline{t}, \bar{t})$ is called [11] the divergence time over the interval $[t_0, t]$. Let $N(t)$ be the number of switchings in $[t_0, t]$. Choose $\tau_{\underline{t}, \bar{t}} > 0$. If the switching signal satisfy

$$N(t) \geq \frac{t - t_0}{\tau_{\underline{t}, \bar{t}} + \bar{t}} - N_0, \quad N_0 \geq 1, \quad (4)$$

$$T(t, \underline{t}, \bar{t}) \leq \frac{\tau_{\underline{t}, \bar{t}}(t - t_0)}{\tau_{\underline{t}, \bar{t}} + \bar{t}} + T_0, \quad T_0 \geq t_1 - t_0, \quad (5)$$

$\tau_{\underline{t}, \bar{t}}$ is called [11] the bounded maximum average dwell time (BMADT).

Example 1 This example is given to illustrate how BMADT works when restricting switching signals. Suppose that there are three subsystems in the system (1), namely, $\mathbb{M} = \{1, 2, 3\}$, and four switches over the time interval $[t_0, t]$, then switching time instants are denoted by $t_s, s = 1, 2, 3, 4$. Let $\sigma(t_0) = \sigma(t_1^-) = 1, \sigma(t_1^+) = \sigma(t_2^-) = 2, \sigma(t_2^+) = \sigma(t_3^-) = 1, \sigma(t_3^+) = \sigma(t_4^-) = 3, \sigma(t_4^+) = \sigma(t) = 2$. Assume that $t_0 = 0, N_0 = 1$ and $T_0 = t_1 - t_0$. Choose $\underline{t}, \bar{t}$ and $\tau_{\underline{t}, \bar{t}}$, then the switching signal $\sigma(t)$ is restricted by (2)-(5). The first switching time instant satisfies $t_1 \in (0, 2(\tau_{\underline{t}, \bar{t}} + \bar{t})]$. Let $t_1 = \bar{t}$, then $t_2 \in (\bar{t}, 3(\tau_{\underline{t}, \bar{t}} + \bar{t})]$. Choose $t_2 = \underline{t} + 2\bar{t}$, $t_3 \in (\underline{t} + 2\bar{t}, 4(\tau_{\underline{t}, \bar{t}} + \bar{t})]$. Let $t_3 = \underline{t} + 3\bar{t}$, then $t_4 \in (\underline{t} + 3\bar{t}, 5(\tau_{\underline{t}, \bar{t}} + \bar{t})]$. It is seen that $\underline{t}, \bar{t}$ and $\tau_{\underline{t}, \bar{t}}$ are assigned globally over the entire time horizon, ignoring the information of the mapping: $\sigma : [t_0, t] \rightarrow \mathbb{M} = \{1, 2, 3\}$.

Thus, admissible transition edges are introduced in [24], which look into how switches happen among subsystems. Let $\sigma(t_{s-1}^+) = \sigma(t_s^-) = i$, indicate that the i -th subsystem is activated at t_{s-1} , and $\sigma(t_s^+) = \sigma(t_{s+1}^-) = j$, indicate that the j -th subsystem is activated at t_s . Then (i, j) denotes the admissible transition edge whenever the switching from the i -th subsystem to the j -th subsystem is admissible, where i and j represent the starting and reaching subsystem, respectively.

3 Stability Analysis

Consider a given switching sequence. Choose $\underline{t}_{i,j}$ and $\bar{t}_{i,j}$ with $\bar{t}_{i,j} > \underline{t}_{i,j} > 0, i, j \in \mathbb{M}, i \neq j$. Define

$$\dot{T}(s, \underline{t}_{i,j}, \bar{t}_{i,j}) = \begin{cases} \Delta_s, & \Delta_s < \underline{t}_{i,j}, \\ 0, & \underline{t}_{i,j} \leq \Delta_s \leq \bar{t}_{i,j}, \\ \Delta_s - \bar{t}_{i,j}, & \Delta_s > \bar{t}_{i,j}, \end{cases} \quad (6)$$

and

$$T(t, \underline{t}_{i,j}, \bar{t}_{i,j}) = \sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{M}, i \neq j} \sum_{s=1}^l \dot{T}(s, \underline{t}_{i,j}, \bar{t}_{i,j}) + t - t_l. \quad (7)$$

$T(t, \underline{t}_{i,j}, \bar{t}_{i,j})$ is called the admissible edge-dependent divergence time of the admissible transition edge (i, j) in the interval $[t_0, t]$. Let $N_{i,j}(t)$ be the number of switchings, $T_{i,j}(t)$ be the running time, and $\Delta_{1,i,j}$ be the first time interval of (i, j) in $[t_0, t]$. Choose $\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} > 0$. If the switching signal satisfy

$$N_{i,j}(t) \geq \frac{T_{i,j}(t)}{\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} + \bar{t}_{i,j}} - N_{0,i,j}, \quad N_{0,i,j} \geq 1, \quad (8)$$

$$T(t, \underline{t}_{i,j}, \bar{t}_{i,j}) \leq \frac{\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} T_{i,j}(t)}{\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} + \bar{t}_{i,j}} + T_{0,i,j}, \quad T_{0,i,j} \geq \Delta_{1,i,j}. \quad (9)$$

$\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j}$ is called the admissible edge-dependent BMADT of the admissible transition edge (i, j) .

To see the difference of the admissible edge-dependent BMADT from BMADT, consider Example 1 again. Assume that $t_0 = 0, N_{0,i,j} = 1$ and $T_{0,i,j} = \Delta_{1,i,j}$. Choose $\underline{t}_{i,j}, \bar{t}_{i,j}$ and $\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j}$, $(i, j) = (1, 2), (2, 1), (1, 3), (3, 2)$. Then the switching signal $\sigma(t)$ is restricted by (6)-(9). The first switching time instant satisfies $t_1 \in (0, 2(\tau_{\underline{t}_{1,2}, \bar{t}_{1,2}}^{1,2} + \bar{t}_{1,2})]$. Let $t_1 = \bar{t}_{1,2}$, then $t_2 \in (\bar{t}_{1,2}, 2(\tau_{\underline{t}_{2,1}, \bar{t}_{2,1}}^{2,1} + \bar{t}_{2,1}) + \bar{t}_{1,2}]$. Choose $t_2 = \underline{t}_{2,1} + \bar{t}_{1,2}$, $t_3 \in (\underline{t}_{2,1} + \bar{t}_{1,2}, 2(\tau_{\underline{t}_{1,3}, \bar{t}_{1,3}}^{1,3} + \bar{t}_{1,3}) + \underline{t}_{2,1} + \bar{t}_{1,2}]$. Let $t_3 = \bar{t}_{1,3} + \underline{t}_{2,1} + \bar{t}_{1,2}$, then $t_4 \in (\bar{t}_{1,3} + \underline{t}_{2,1} + \bar{t}_{1,2}, 2(\tau_{\underline{t}_{3,2}, \bar{t}_{3,2}}^{3,2} + \bar{t}_{3,2}) + \underline{t}_{2,1} + \bar{t}_{1,2})$.

$\bar{t}_{1,3} + \underline{t}_{2,1} + \bar{t}_{1,2}$. It is observed that $\underline{t}_{i,j}, \bar{t}_{i,j}, \tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j}$ are local for different admissible transition edges.

In order to study stability of the system (1), Lyapunov-like functions are applied [24]. Suppose that there exist Lyapunov-like functions $V_j(x(t)), j = 1, 2, \dots, m$, positive constant $\alpha_j > 0, j = 1, 2, \dots, m$, positive constants $0 < \lambda_{i,j} < 1, \mu_{i,j} > 1, i, j \in \mathbb{M}, i \neq j$, and two functions, κ_1 and κ_2 in class κ_∞ , such that

$$\kappa_1(\|x(t)\|)^2 \leq V_j(x(t)) \leq \kappa_2(\|x(t)\|)^2, \quad (10)$$

$$\dot{V}_j(x(t)) \leq \alpha_j V_j(x(t)), \quad (11)$$

$$V_j(x(t_s)) \leq \begin{cases} \lambda_{i,j} e^{\alpha_j T(s, \underline{t}_{i,j}, \bar{t}_{i,j})} V_i(x(t_{s-1})), & \Delta_s \geq \underline{t}_{i,j}, \\ \mu_{i,j} e^{\alpha_j T(s, \underline{t}_{i,j}, \bar{t}_{i,j})} V_i(x(t_{s-1})), & \Delta_s < \underline{t}_{i,j}. \end{cases} \quad (12)$$

Let $N(t) = \sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{M}, i \neq j} N_{i,j}(t) = \underline{N}(t) + \bar{N}(t) = \sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{M}, i \neq j} \underline{N}_{i,j}(t) + \bar{N}_{i,j}(t)$, where $\underline{N}_{i,j}(t)$ and $\bar{N}_{i,j}(t)$ denote the number of switchings whose corresponding dwell time is less or not less than $\underline{t}_{i,j}$ of (i, j) in $[t_0, t]$, respectively. Suppose $\gamma_{i,j} = \frac{\underline{N}_{i,j}(t)}{\bar{N}_{i,j}(t)}$, such that

$$\gamma_{i,j} \leq \frac{\ln \lambda_{i,j}}{\ln \lambda_{i,j} - \ln \mu_{i,j}}, \quad (13)$$

and $\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j}$ satisfies

$$\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} \leq \tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{a*, i,j} = \frac{(1 - \gamma_{i,j}) \ln \lambda_{i,j} + \gamma_{i,j} \ln \mu_{i,j}}{-\alpha_j}. \quad (14)$$

Theorem 1 *If there are Lyapunov-like functions $V_j(x(t)), j = 1, 2, \dots, m$, which satisfy (10)-(12), then the system (1) is GEUS under the switching signal satisfying (8), (9) and (13), for any admissible edge-dependent BMADT, $\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} \leq \tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{a*, i,j}, i, j \in \mathbb{M}, i \neq j$.*

Proof: Let $t_p, t_q \in \{t_s\}$, be the switching time instant whose corresponding dwell time is not less than $\underline{t}_{\sigma(t_{p-1}), \sigma(t_p)}$ or less than $\underline{t}_{\sigma(t_{q-1}), \sigma(t_q)}$, respectively. Then, for any t satisfying $t_0 < \dots < t_l \leq t < t_{l+1}, l = N(t)$. It is seen from (11) and (12) that

$$\begin{aligned} & V_{\sigma(t)}(x(t)) \\ & \leq e^{-\alpha_{\sigma(t)}(t-t_l)} e^{-\alpha_{\sigma(t_l)} T(l, \underline{t}_{\sigma(t_{l-1}), \sigma(t_l)}, \bar{t}_{\sigma(t_{l-1}), \sigma(t_l)})} \\ & \quad \times \lambda_{\sigma(t_{l-1}), \sigma(t_l)} V_{\sigma(t_{l-1})}(x(t_{l-1})) \\ & \leq e^{-\alpha_{\sigma(t_l)}(t-t_l + T(l, \underline{t}_{\sigma(t_{l-1}), \sigma(t_l)}, \bar{t}_{\sigma(t_{l-1}), \sigma(t_l)})} \\ & \quad \times e^{-\alpha_{\sigma(t_{l-1})} T(l-1, \underline{t}_{\sigma(t_{l-2}), \sigma(t_{l-1})}, \bar{t}_{\sigma(t_{l-2}), \sigma(t_{l-1})})} \end{aligned}$$

$$\begin{aligned}
& \times \lambda_{\sigma(t_{l-1}), \sigma(t_l)} \mu_{\sigma(t_{l-2}), \sigma(t_{l-1})} V_{\sigma(t_{l-2})}(x(t_{l-2})) \\
& \leq \dots \\
& \leq e^{-\alpha_{\sigma(t)}(t-t_l)} \prod_{s=0}^l e^{-\alpha_{\sigma(t_s)} T(s, \underline{t}_{\sigma(t_{s-1})}, \sigma(t_s), \bar{t}_{\sigma(t_{s-1})}, \sigma(t_s))} \\
& \times \prod_{i \in \mathbb{M}}^{\bar{N}(t)} \lambda_{\sigma(t_{p-1}), \sigma(t_p)} \prod_{j \in \mathbb{M}}^{\underline{N}(t)} \mu_{\sigma(t_{q-1}), \sigma(t_q)} V_{\sigma(t_0)}(x(t_0)).
\end{aligned} \tag{15}$$

Let i, j be the values of $\sigma(t)$ on $[t_{s-1}, t_s)$ and $[t_s, t_{s+1})$, respectively. Then (15) becomes

$$\begin{aligned}
& V_{\sigma(t)}(x(t)) \\
& \leq \exp \left(\sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{M}, i \neq j} \bar{N}_{i,j}(t) \ln \lambda_{i,j} \right) V_{\sigma(t_0)}(x(t_0)) \\
& \times \exp \left(\sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{M}, i \neq j} \underline{N}_{i,j}(t) \ln \mu_{i,j} \right) \\
& \times \exp \left(\sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{M}, i \neq j} \alpha_j T(t, \underline{t}_{i,j}, \bar{t}_{i,j}) \right).
\end{aligned}$$

Define

$$\begin{aligned}
\psi_{i,j} &= \alpha_j T_{0,i,j} - ((1 - \gamma_{i,j}) \ln \lambda_{i,j} + \gamma_{i,j} \ln \mu_{i,j}) N_{0,i,j}, \\
\varsigma_{i,j} &= - \frac{\left(\alpha_j \tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} + (1 - \gamma_{i,j}) \ln \lambda_{i,j} + \gamma_{i,j} \ln \mu_{i,j} \right)}{\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} + \bar{t}_{i,j}},
\end{aligned}$$

then we have $\psi_{i,j}, \varsigma_{i,j} \geq 0$ from (13) and (14). By (8) and (9)

$$\begin{aligned}
V_{\sigma(t)}(x(t)) & \leq \exp \left(\sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{M}, i \neq j} \psi_{i,j} \right) V_{\sigma(t_0)}(x(t_0)) \\
& \times \exp \left(\sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{M}, i \neq j} \varsigma_{i,j} T_{i,j}(t) \right).
\end{aligned}$$

Finally, it follows from (10) that

$$\|x(t)\| \leq \sqrt{\frac{\kappa_2}{\kappa_1}} e^{\prod_{i \in \mathbb{M}} \prod_{j \in \mathbb{M}, i \neq j} \psi_{i,j}} e^{-\frac{\prod_{i \in \mathbb{M}} \prod_{j \in \mathbb{M}, i \neq j} \varsigma_{i,j} T_{i,j}(t)}{2}} \|x_0\|,$$

and thus the system (1) is GUES. The proof is completed.

From (12), it is seen that if $\Delta_s < \underline{t}_{i,j}$, $i, j \in \mathbb{M}, i \neq j$, $\mu_{i,j}$ is applied, which means energy functions increase at switching instants. When $\Delta_s \geq \underline{t}_{i,j}$, $\lambda_{i,j}$ is utilized, which is effective in dissipating upward energy functions. Apparently, the involvement of admissible transition edges contributes to more application flexibility than that in [11] as the compensations of energy functions vary in different admissible transition edges.

In order to better demonstrate the effectiveness of the proposed admissible edge-dependent BMADT, we simplify the system (1) to a switched linear system:

$$\dot{x}(t) = A_{\sigma(t)} x(t), \tag{16}$$

where $A_{\sigma(t)}, \sigma(t) \in \mathbb{M}$, are known real constant matrices with appropriate dimensions.

The linear matrix inequalities (LMIs) are required to obtain the stability criteria of the system (16). Suppose there exist matrices $P_j > 0, j = 1, 2, \dots, m$, positive constant $\alpha_j > 0, j = 1, 2, \dots, m$, positive constants $0 < \lambda_{i,j} < 1$, and $\mu_{i,j} > 1, i, j \in \mathbb{M}, i \neq j$, such that

$$\alpha_j P_j \geq P_j A_j + A_j^T P_j, \tag{17}$$

$$P_j \leq \mu_{i,j} P_i, \tag{18}$$

$$\lambda_{i,j} P_i \geq e^{A_i^T s} P_j e^{A_i s}, s \in [\underline{t}_{i,j}, \bar{t}_{i,j}], \tag{19}$$

and

$$\gamma_{i,j} \leq \frac{\ln \lambda_{i,j}}{\ln \lambda_{i,j} - \ln \mu_{i,j}}, \tag{20}$$

$$\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} \leq \tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{a*, i,j} = \frac{(1 - \gamma_{i,j}) \ln \lambda_{i,j} + \gamma_{i,j} \ln \mu_{i,j}}{-\alpha_j}. \tag{21}$$

Corollary 1 *If there are positive matrices $P_j, j = 1, 2, \dots, m$, which satisfy (17)-(19), the system (16) is GEUS under the switching signal satisfying (8), (9) and (20), for any admissible edge-dependent BMADT, $\tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j} \leq \tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{a*, i,j}, i, j \in \mathbb{M}, i \neq j$.*

Proof: Suppose that there are $P_j, j = 1, 2, \dots, m$, satisfying (17)-(19). Choose the Lyapunov-like functions as

$$V_j(x(t)) = x^T(t) P_j x(t).$$

Then, (11) and (12) are obtained and

$$\kappa_1 = \min_{j \in \mathbb{M}} \{\lambda_{\min}(P_j)\}, \quad \kappa_2 = \max_{j \in \mathbb{M}} \{\lambda_{\max}(P_j)\}.$$

It follows from a similar argument to the proof of Theorem 1 that

$$\|x(t)\| \leq \sqrt{\frac{\kappa_2}{\kappa_1}} e^{\prod_{i \in \mathbb{M}} \prod_{j \in \mathbb{M}, i \neq j} \psi_{i,j}} e^{-\frac{\prod_{i \in \mathbb{M}} \prod_{j \in \mathbb{M}, i \neq j} \varsigma_{i,j} T_{i,j}(t)}{2}} \|x_0\|,$$

and thus the system (16) is GUES. The proof is completed.

If the differences of starting subsystems are ignored: $\underline{t}_{i,j} = \underline{t}_{r,j}, \bar{t}_{i,j} = \bar{t}_{r,j}, i, r, j \in \mathbb{M}, i \neq r$, then choosing $\underline{t}_j$ and $\bar{t}_j$ with $\bar{t}_j > \underline{t}_j > 0, j = 1, 2, \dots, m$. Define

$$\dot{T}(s, \underline{t}_j, \bar{t}_j) = \begin{cases} \Delta_s, & \Delta_s < \underline{t}_j, \\ 0, & \underline{t}_j \leq \Delta_s \leq \bar{t}_j, \\ \Delta_s - \bar{t}_j, & \Delta_s > \bar{t}_j, \end{cases} \tag{22}$$

and

$$T(t, \underline{t}_j, \bar{t}_j) = \sum_{j \in \mathbb{M}} \sum_{s=1}^l \dot{T}(s, \underline{t}_j, \bar{t}_j) + t - t_l. \tag{23}$$

$T(t, \underline{t}_j, \bar{t}_j)$ is called the mode-dependent divergence time over the interval $[t_0, t]$. $\underline{t}_j, \bar{t}_j$ are named lower and upper compensation bounds of subsystem j . Let $N_j(t)$ be the number of switchings, $T_j(t)$ be the running time, and $\Delta_{1,j}$ be the first time interval of subsystem j in $[t_0, t]$. Choose $\tau_{\underline{t}_j, \bar{t}_j}^j > 0$. If the switching signal satisfy

$$N_j(t) \geq \frac{T_j(t)}{\tau_{\underline{t}_j, \bar{t}_j}^j + \bar{t}_j} - N_{0,j}, \quad N_{0,j} \geq 1, \quad (24)$$

$$T(t, \underline{t}_j, \bar{t}_j) \leq \frac{\tau_{\underline{t}_j, \bar{t}_j}^j T_j(t)}{\tau_{\underline{t}_j, \bar{t}_j}^j + \bar{t}_j} + T_{0,j}, \quad T_{0,j} \geq \Delta_{1,j}. \quad (25)$$

$\tau_{\underline{t}_j, \bar{t}_j}^j$ is called the mode-dependent BMADT.

Compared with the admissible edge-dependent BMADT, the mode-dependent BMADT omits the differences of starting subsystems. $\tau_{\underline{t}_j, \bar{t}_j}^j, \underline{t}_j$ and $\bar{t}_j$ are only relevant to reaching subsystems.

To study stability of the system (1) under the mode-dependent BMADT switching, suppose that there exist Lyapunov-like functions $V_j(x(t)), j = 1, 2, \dots, m$, positive constants $\alpha_j > 0, 0 < \lambda_j < 1, \mu_j > 1, j = 1, 2, \dots, m$, and two functions, κ_1 and κ_2 in class κ_∞ , such that

$$\kappa_1(\|x(t)\|)^2 \leq V_j(x(t)) \leq \kappa_2(\|x(t)\|)^2, \quad (26)$$

$$\dot{V}_j(x(t)) \leq \alpha_j V_j(x(t)), \quad (27)$$

$$V_j(x(t_s)) \leq \begin{cases} \lambda_j e^{\alpha_j T(s, \underline{t}_j, \bar{t}_j)} V_i(x(t_{s-1})), & \Delta_s \geq \underline{t}_j, \\ \mu_j e^{\alpha_j T(s, \underline{t}_j, \bar{t}_j)} V_i(x(t_{s-1})), & \Delta_s < \underline{t}_j. \end{cases} \quad (28)$$

Let $N(t) = \sum_{j \in \mathbb{M}} N_j(t) = \sum_{j \in \mathbb{M}} \underline{N}_j(t) + \bar{N}_j(t)$, where $\underline{N}_j(t)$

and $\bar{N}_j(t)$ denote the number of switchings whose corresponding dwell time is less or not less than $\underline{t}_j$ of subsystem j in $[t_0, t]$, respectively. Suppose $\gamma_j = \frac{\underline{N}_j(t)}{\bar{N}_j(t)}$, such that

$$\gamma_j \leq \frac{\ln \lambda_j}{\ln \lambda_j - \ln \mu_j}, \quad (29)$$

and $\tau_{\underline{t}_j, \bar{t}_j}^j$ satisfies

$$\tau_{\underline{t}_j, \bar{t}_j}^j \leq \tau_{\underline{t}_j, \bar{t}_j}^{a*,j} = \frac{(1 - \gamma_j) \ln \lambda_j + \gamma_j \ln \mu_j}{-\alpha_j}. \quad (30)$$

Corollary 2 *If there are Lyapunov-like functions $V_j(x(t)), j = 1, 2, \dots, m$, which satisfy (26)-(28), the system (1) is GEUS under the switching signal satisfying (24), (25) and (29), for any mode-dependent BMADT, $\tau_{\underline{t}_j, \bar{t}_j}^j \leq \tau_{\underline{t}_j, \bar{t}_j}^{a*,j}, j = 1, 2, \dots, m$.*

The compensations of energy functions differ in diverse subsystems. However, the differences of starting subsystems are omitted, μ_j and λ_j only vary with reaching subsystems. It results in less application flexibility than Theorem 1.

Similarly, to study stability of the system (16) under the mode-dependent BMADT switching, suppose there exist matrices $P_j > 0, j = 1, 2, \dots, m$, positive constants $\alpha_j > 0, 0 < \lambda_j < 1$ and $\mu_j > 1, j = 1, 2, \dots, m$, such that

$$\alpha_j P_j \geq P_j A_j + A_j^T P_j, \quad (31)$$

$$P_j \leq \mu_j P_i, \quad (32)$$

$$\lambda_j P_i \geq e^{A_i^T s} P_j e^{A_i s}, s \in [\underline{t}_j, \bar{t}_j], \quad (33)$$

and

$$\gamma_j \leq \frac{\ln \lambda_j}{\ln \lambda_j - \ln \mu_j}, \quad (34)$$

$$\tau_{\underline{t}_j, \bar{t}_j}^j \leq \tau_{\underline{t}_j, \bar{t}_j}^{a*,j} = \frac{(1 - \gamma_j) \ln \lambda_j + \gamma_j \ln \mu_j}{-\alpha_j}. \quad (35)$$

Corollary 3 *If there are positive matrices $P_j, j = 1, 2, \dots, m$, which satisfy (31)-(33), the system (16) is GEUS under the switching signal satisfying (24), (25) and (34), for any mode-dependent BMADT, $\tau_{\underline{t}_j, \bar{t}_j}^j \leq \tau_{\underline{t}_j, \bar{t}_j}^{a*,j}, j = 1, 2, \dots, m$.*

Remark 1 Note that (19) and (33) are true only under the condition $\underline{t}_{i,j} \leq s \leq \bar{t}_{i,j}$ and $\underline{t}_j \leq s \leq \bar{t}_j$, respectively, which cannot be solved by the traditional LMI toolbox. Whereas, setting sample points in the time interval is able to solve it.

4 Numerical Example

Consider the system (16). Let $\sigma(t) : [0, \infty] \rightarrow \mathbb{M} = \{1, 2, 3\}$, $A_1 = \begin{bmatrix} -1.9 & 0.6 \\ 0.6 & -0.1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.1 & -0.9 \\ 0.1 & -1.4 \end{bmatrix}$, $A_3 = \begin{bmatrix} -0.7 & -0.3 \\ 0.8 & -1.7 \end{bmatrix}$. The switching signal is constructed with Corollary 1 to stabilize the system. Choose $\alpha_j, j = 1, 2, 3$

$$\alpha_1 = 0.37, \alpha_2 = 0.31, \alpha_3 = 0.29,$$

choose $\mu_{i,j}, i, j \in \mathbb{M}, i \neq j$

$$\mu_{2,1} = 2.77, \mu_{1,2} = 2.71, \mu_{1,3} = 2.64,$$

$$\mu_{3,1} = 2.66, \mu_{3,2} = 2.61, \mu_{2,3} = 2.70,$$

choose $\lambda_{i,j}, i, j \in \mathbb{M}, i \neq j$

$$\lambda_{2,1} = 0.71, \lambda_{1,2} = 0.69, \lambda_{1,3} = 0.63,$$

$$\lambda_{3,1} = 0.73, \lambda_{3,2} = 0.65, \lambda_{2,3} = 0.68,$$

choose compensation bounds $\underline{t}_{i,j}, \bar{t}_{i,j}$, $i, j \in \mathbb{M}, i \neq j$

$$\underline{t}_{2,1} = 0.50, \underline{t}_{1,2} = 0.51, \underline{t}_{1,3} = 0.45,$$

$$\underline{t}_{3,1} = 0.50, \underline{t}_{3,2} = 0.48, \underline{t}_{2,3} = 0.52,$$

$$\bar{t}_{2,1} = 2.00, \bar{t}_{1,2} = 2.10, \bar{t}_{1,3} = 1.90,$$

$$\bar{t}_{3,1} = 2.15, \bar{t}_{3,2} = 1.82, \bar{t}_{2,3} = 1.84,$$

and choose $\gamma_{i,j}, \tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j}$, $i, j \in \mathbb{M}, i \neq j$

$$\gamma_{2,1} = 0.15, \gamma_{1,2} = 0.18, \gamma_{1,3} = 0.20,$$

$$\gamma_{3,1} = 0.21, \gamma_{3,2} = 0.20, \gamma_{2,3} = 0.25,$$

$$\tau_{\underline{t}_{2,1}, \bar{t}_{2,1}}^{2,1} = 0.32, \tau_{\underline{t}_{1,2}, \bar{t}_{1,2}}^{1,2} = 0.37, \tau_{\underline{t}_{1,3}, \bar{t}_{1,3}}^{1,3} = 0.40,$$

$$\tau_{\underline{t}_{3,1}, \bar{t}_{3,1}}^{3,1} = 0.10, \tau_{\underline{t}_{3,2}, \bar{t}_{3,2}}^{3,2} = 0.40, \tau_{\underline{t}_{2,3}, \bar{t}_{2,3}}^{2,3} = 0.14.$$

The feasible solutions are obtained by Corollary 1. Then an admissible edge-dependent BMADT switching signal is constructed under $\underline{t}_{i,j}, \bar{t}_{i,j}, \gamma_{i,j}, \tau_{\underline{t}_{i,j}, \bar{t}_{i,j}}^{i,j}$. The resulting divergence time and the switching signal are shown in Fig 1.

Fig. 1 The divergence time and the switching signal under admissible edge-dependent BMADT switching

The switching system with the initial state $x(0) = [3, -5]^T$ is simulated, and the state trajectories are depicted in Fig 2. The state converges to zero as $t \rightarrow \infty$, which indicates that the admissible edge-dependent BMADT switching signal can stabilize the system (16).

The mode-dependent BMADT switching signal is constructed with Corollary 3 to stabilize the system (16). Choose α_j , $j = 1, 2, 3$

$$\alpha_1 = 0.37, \alpha_2 = 0.31, \alpha_3 = 0.29,$$

choose μ_j , $j = 1, 2, 3$

$$\mu_1 = 2.77, \mu_2 = 2.71, \mu_3 = 2.64,$$

Fig. 2 The state trajectories under admissible edge-dependent BMADT

choose λ_j , $j = 1, 2, 3$

$$\lambda_1 = 0.71, \lambda_2 = 0.69, \lambda_3 = 0.63,$$

choose compensation bounds $\underline{t}_j, \bar{t}_j$, $j = 1, 2, 3$

$$\underline{t}_1 = 0.50, \underline{t}_2 = 0.51, \underline{t}_3 = 0.45,$$

$$\bar{t}_1 = 2.00, \bar{t}_2 = 2.10, \bar{t}_3 = 1.90,$$

and choose $\gamma_j, \tau_{\underline{t}_j, \bar{t}_j}^j$, $j = 1, 2, 3$

$$\gamma_1 = 0.15, \gamma_2 = 0.18, \gamma_3 = 0.20,$$

$$\tau_{\underline{t}_1, \bar{t}_1}^1 = 0.32, \tau_{\underline{t}_2, \bar{t}_2}^2 = 0.37, \tau_{\underline{t}_3, \bar{t}_3}^3 = 0.40.$$

The feasible solutions are obtained by Corollary 3. It is observed that the choose of parameters is much less than that in Corollary 1. Simultaneously, there are $\underline{t}_j = \underline{t}_{i,j}, \bar{t}_j = \bar{t}_{i,j}, \lambda_j = \lambda_{i,j}, \mu_j = \mu_{i,j}, i, j \in \mathbb{M}, i \neq j$.

A mode-dependent BMADT switching signal is generated under $\underline{t}_j, \bar{t}_j, \lambda_j$ and μ_j . The divergence time and the switching signal are shown in Fig 3.

The state trajectories with initial state $x_0 = [3, -5]$, are shown in Fig 4. It is clear that the mode-dependent BMADT switching signal can stabilize the switched system.

When it comes to the BMADT switching in [11], all the parameters are global for different admissible transition edges

$$\alpha = 0.37, \lambda = 0.71, \mu = 2.77,$$

$$\underline{t} = 0.50, \bar{t} = 2.00, \gamma_1 = 0.15, \tau_{\underline{t}, \bar{t}} = 0.32.$$

Namely, there are $\lambda = \lambda_j = \lambda_{i,j}, \mu = \mu_j = \mu_{i,j}, \alpha = \alpha_j, \underline{t} = \underline{t}_j = \underline{t}_{i,j}, \bar{t} = \bar{t}_j = \bar{t}_{i,j}, i, j \in \mathbb{M}, i \neq j$.

Fig. 3 The divergence time and the switching signal under mode-dependent BMADT switching

Fig. 4 The state trajectories under mode-dependent BMADT

5 Conclusion

Stability of switched systems with admissible switching signals is studied in this paper. Stability results under the admissible edge-dependent BMADT switching are general in that compensation bounds and admissible edge-dependent BMADT are not confined to be global. In addition, there is a large range of choosing parameters in stabilizing switched systems. With those parameters, the compensations of divergent state are different for diverse admissible transition edges. Besides, stability criteria under the mode-dependent BMADT are more restrictive than those under the admissible edge-dependent BMADT, because the differences of starting subsystems are omitted.

Acknowledgements The research was supported by National Natural Science Foundation of China under Grant with No.61973284 and No.62022074 and the 111 Project under Grant with No.B17040.

Conflict of interest

The authors declare that they have no conflict of interest.

References

- [1] Williams, S.M., Hoft, R.G.: Adaptive frequency domain control of PWM switched power line conditioner. *IEEE Transactions on Power Electronics*. 6(4), 665–670 (1991)
- [2] Varaiya, P.: Smart cars on smart roads: problems of control. *IEEE Transactions on Automatic Control*. 38(2), 195–207 (1993)
- [3] Joos, S., Bitzer, M., Karlemeyer, R., Graichen, K.: Constrained online trajectory planning for nonlinear flat SISO systems using a switched state variable filter. *Automatica*. 110, 108583 (2019)
- [4] Niu, B., Zhao, P., Liu, J.D., Ma, H.J., Liu, Y.J.: Global adaptive control of switched uncertain nonlinear systems: An improved MDADT method. *Automatica*. 115, 108872 (2020)
- [5] Koru, A.T., Delibasi, A., Ozbay, H.: Dwell time-based stabilisation of switched delay systems using free-weighting matrices. *International Journal of Control*. 91(1), 1–11 (2018)
- [6] Li, Y., Zhang, H.B.: Dwell time stability and stabilization of interval discrete-time switched positive linear systems. *Nonlinear Analysis-Hybrid Systems*. 33, 116–129 (2019)
- [7] Xiang, W.M.: Stabilization for continuous-time switched linear systems: A mixed switching scheme. *Nonlinear Analysis-Hybrid Systems*. 36, 100872 (2020)
- [8] Kundu, A.: A new condition for stability of switched linear systems under restricted minimum dwell time switching. *Systems & Control Letters*. 135, 104597 (2020)
- [9] Lian, J., Zhang, K.: Exponential stability for switched Cohen-Grossberg neural networks with average dwell time. *Nonlinear Dynamics*. 63, 331–343 (2011)
- [10] Xu, X.Z., Mao, X., Zhang, H.B.: Stability analysis of switched system with all subsystems unstable under novel average dwell time criteria. *IEEE Access*. 7, 44959–44965 (2019)
- [11] Mao, X., Zhu, H., Chen, W., Zhang, H.B.: New results on stability of switched continuous-time systems with all subsystems unstable. *ISA Transactions*. 87, 28–33 (2019)
- [12] Li, Z.G., Wen, C.Y., Soh, Y.C.: Stabilization of a class of switched systems via designing switching laws. *IEEE Transactions on Automatic Control*. 46(4), 665–670 (2001)
- [13] Pettersson, S., Lennartson, B.: Stabilization of hybrid systems using a min-projection strategy. in *Proceedings of the 2001 American Control Conference*. (223–228) (2001)
- [14] Xu, S.S.D., Chen, C.C.: On existence of stabilizing switching laws within a class of unstable linear systems. *Abstract and Applied Analysis*. 681523, (2013)
- [15] Platonov, A.V.: On the global asymptotic stability and ultimate boundedness for a class of nonlinear switched systems. *Nonlinear Dynamics*. 92, 1555–1565 (2018)
- [16] Huang, C.X., Cao, J., Cao, J.D.: Stability analysis of switched cellular neural networks: A mode-dependent average dwell time approach. *Neural Networks*. 82, 84–99 (2016)
- [17] Wang, R.H., Jiao, T.C., Zhang, T., Fei, S.M.: Improved stability results for discrete-time switched systems: A multiple piecewise convex Lyapunov function

- approach. *Applied Mathematics and Computation*. 353, 54–65 (2019)
- [18] Wang, R.H., Hou, L.L., Zong, G.D., Fei, S.M., Yang, D.: Stability and stabilization of continuous-time switched systems: A multiple discontinuous convex Lyapunov function approach. *International Journal of Robust and Nonlinear Control*. 29(5), 1499–1514 (2019)
- [19] Morse, A.S.: Supervisory control of families of linear set-point controllers-part I: Exact matching. *IEEE Transactions on Automatic Control*. 41(10), 1413–1431 (1996)
- [20] Hespanha, J.P., Morse, A.S.: Stability of switched systems with average dwell-time. *Proceedings of the 38th IEEE Conference on Decision and Control*. 3, 2655–2660 (1999)
- [21] Zhao, X.D., Zhang, L.X., Shi, P., Liu, M.: Stability and stabilization of switched linear systems with mode-dependent average dwell time. *IEEE Transactions on Automatic Control*. 57(7), 1809–1815 (2012)
- [22] Yang, J.Q., Zhao, X.D., Bu, X.H., Qian, W.: Stabilization of switched linear systems via admissible edge-dependent switching signals. *Nonlinear Analysis-Hybrid Systems*. 29, 100–109 (2018)
- [23] Hou, L.L., Zhang, M.Z., Zhao, X.D., Sun, H.B., Zong, G.D.: Stability of discrete-time switched systems with admissible edge-dependent switching signals. *International Journal of Systems Science*. 49(5), 974–983 (2018)
- [24] Kundu, A., Chatterjee, D.: Stabilizing switching signals for switched systems. *IEEE Transactions on Automatic Control*. 60(3), 882–888 (2015)

Figures

Figure 1

The divergence time and the switching signal under admissible edge-dependent BMADT switching

Figure 2

The state trajectories under admissible edge-dependent BMADT

Figure 3

The divergence time and the switching signal under mode-dependent BMADT switching

Figure 4

The state trajectories under mode-dependent BMADT