A rapidly evolving high-amplitude δ Scuti star crossing the Hertzsprung Gap

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1 **A rapidly evolving high-amplitude δ Scuti star crossing the**
2 **Hertzsprung Gap**

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11 People cannot witness the stellar evolution process of a single star obviously
 12 in most cases because of its extremely secular time-scale, except for some spe-
 13 cial time nodes in it (such as the supernova explosion [1]). But in some specific
 14 evolutionary phases, we have the chances to witness such process gradually on
 15 human times-scales. When a star evolved leaving from the main sequence, the
 16 hydrogen nuclei fusion in its core is gradually transferring into the shell. In the
 17 Hertzsprung–Russell diagram, its evolutionary phase falls into the Hertzsprung
 18 gap, which is one of the most rapidly evolving phases in the life of a star [2].
 19 Here we report a discovery of a rapidly evolving high-amplitude δ Scuti star
 20 KIC6382916 (J19480292+4146558) which is crossing the Hertzsprung gap. Ac-
 21 cording to the analysis of the archival data, we find three independent pulsation
 22 modes of it, whose amplitudes and frequencies are varying distinctly in 4 years.
 23 The period variation rates of the three pulsation modes are one or two orders
 24 larger than the best seismic model constructed by the standard evolution the-
 25 ory, which indicates the current theory cannot precisely describe the evolution
 26 process in this rapidly evolving phase and needs further upgrades. Moreover,
 27 the frequency and amplitude interactions between the three independent pulsa-
 28 tion modes and their harmonics/combinations open a new window to the future
 29 asteroseismology.

30 δ Scuti stars are a class of short-period pulsating variable stars with period between
 31 0.02 and 0.25 day and the spectral classes A-F, which locate on the main sequence or post
 32 main sequence evolutionary stage at the bottom of the classical Cepheid instability strip
 33 and excited by the κ mechanism [3]. High-amplitude δ Scuti stars (hereafter HADS) are
 34 a subclass of δ Scuti stars, who always have larger amplitudes ($\Delta V \geq 0^m3$) and slower
 35 rotations [3]. Most of the HADS show single or double radial pulsation modes, and some
 36 of them have three radial pulsation modes or even some non-radial pulsation modes [4–7].
 37 Theoretically, the period variation rates ($(1/P)(dP/dt)$) of HADS caused by the evolution
 38 should have a value from 10^{-10} yr^{-1} on the main sequence to 10^{-7} yr^{-1} on the post-main
 39 sequence phase [8]. According to the seismic models, the observed period variation rates
 40 of some HADS (such as XX Cyg, AE UMa, and VX Hya [5, 6, 9]) can be interpreted by

the evolutionary effect self-consistently, and each of these stars should be located in the Hertzsprung gap in the H-R diagram with a helium core and a hydrogen-burning shell. In these researches, the period variation rates are obtained from the ground based time-series photometric data accumulated in several decades, but only that of the fundamental pulsation modes have a sufficient precision to be confirmed and tested because of the quality of the data. The high precision photometric data from *Kepler space telescope* lasted for about 4 years could provide us excellent opportunity to determine the period and amplitude variation rates not only for the fundamental mode, but also for the other pulsation modes. These observed quantities would tell us more secrets about the stellar evolution in this special rapidly evolving phase.

KIC6382916 (J19480292+4146558, also known as GSC 03144-595) is a HADS to be monitored extensively by the *Kepler space telescope* for its pulsation properties, which shows three independent frequencies (pulsation modes) in its light curves [10]. These three pulsation modes have been confirmed as the fundamental, the first overtone and the second overtone radial p-modes [11], and $T_{\text{eff}} = 6950 \pm 100$ K, $\log g = 3.7 \pm 0.1$ and $v \sin i = 50 \pm 10$ km s⁻¹ have been estimated by the high-dispersion spectra [10]. Using the long-cadence (LC) photometric data of KIC6382916 lasted from BJD 2454953 to 2456424 (Quarter 0-17), we extract the first 23 frequencies with largest amplitudes (which have a cut-off of 1.3 mmag, see Photometric Data Reduction in Methods and Extended Data Table II for more details) to study the variations and interactions of them. These 23 frequencies are composed as that of the fundamental (f_0), the first overtone (f_1), and the second overtone (f_2) modes, together with their harmonics/combinations. The IDs of these frequencies are also identified as the labels of the corresponding pulsation modes in this work.

In order to extract the variation information of the amplitudes and frequencies over time, we use the Short-Time Fourier Transformation to deal with the LC photometric data. The pre-whiting process in a time window of 150 days is performed when the window is moving from the start to the end time of the LC data, with a step of 30 days. In each step, the frequencies are fixed as that have been extracted in the complete LC data (see Extended Data Table II), while the amplitudes and phases are obtained by the non-linear least square fitting. Then, we get the amplitude and phase variations of the 23 frequencies. Using the Fourier-Phase Diagram method [6, 12, 13], the variations of the frequencies can be derived from the variations of the phases.

73 The amplitude and phase variations of f_0 , f_1 , and f_2 are presented in Figure 1, and
 74 each one of them is performed by a quadratic fitting.¹ In Figure 1, we find the amplitudes
 75 of f_0 (A_{f_0}) is relatively stable, that of f_1 (A_{f_1}) is slightly decreasing while A_{f_2} is distinctly
 76 increasing by about 24% in 4 years. On the other hand, the phases (subtracted by its average)
 77 of f_0 , f_1 , and f_2 variate with a same trend but different levels, which is corresponding to
 78 the increasing periods of them with different period variation rates: $(1/P_0)(dP_0/dt) =$
 79 $(3.0 \pm 0.6) \times 10^{-7} \text{ yr}^{-1}$, $(1/P_1)(dP_1/dt) = (3.0 \pm 0.5) \times 10^{-7} \text{ yr}^{-1}$, and $(1/P_2)(dP_2/dt) =$
 80 $(2.3 \pm 0.2) \times 10^{-6} \text{ yr}^{-1}$ (see Methods for more details).

81 Noting the decrement of A_{f_1} and the increment of A_{f_2} , it is quit interesting to study
 82 the amplitude interactions between the 23 pulsation modes, which also indicate the energy
 83 transformation between them. Here, we introduce the Interaction Diagram (see Figure 2)
 84 to show the amplitude interactions between the pulsation modes. In Figure 2, the color of
 85 a small square represents the correlation coefficient² between the amplitudes of the labeled
 86 pulsation modes whose column and row intersect at the square. The dendrograms on the
 87 upper and left represent the Agglomerative Hierarchical Clustering (AHC) process [14] in
 88 the amplitude interaction space which is spanned by the vectors (corresponding to each of
 89 the pulsation modes) consisting of the correlation coefficients between a specific pulsation
 90 mode to all of them. In general, the AHC process reorders the pulsation modes in order to
 91 cluster those who have similar behaviors in the interaction space together.

92 At first glance of Figure 2, the dominating interactions occur between the pulsation modes
 93 including f_1 and f_2 . In detail, all the pulsation modes including f_2 are clustered together
 94 and have increasing amplitudes, while the pulsation modes of f_1 , $2f_1$, $2f_0 + f_1$, $f_0 + 2f_1$, and
 95 $f_0 + 3f_1$ are anti-correlated with the above f_2 related modes and have decreasing amplitudes.
 96 What's more interesting is that the $3f_0$, $3f_1$, $-2f_0 + 2f_1$, and $-f_0 + 2f_1$ modes, which are
 97 not f_2 related, but have increasing amplitudes like that of the f_2 related modes. All these
 98 relations and structures shown in the Interaction Diagram of amplitudes indicate that there
 99 exists interactions or energy transformations between the independent pulsation modes and
 100 their harmonics/combinations, and this opens a new window for exploring the interior of
 101 the stars in future researches.³

¹ The variations of all the 23 pulsation modes are listed in Extended Data Figure 2, 3, and 4.

² Here we use the Spearman's rank correlation coefficient, which is a measure of how well the relationship
 between two variables can be described by a monotonic function.

³ Comparing with the complicated interactions shown in Figure 2, the Interactions Diagram of the phases
 (see Extended Data Figure 5) shows that almost all the pulsation modes have the same variation trend

Figure 1. Variation of the amplitudes and phases (subtracted by their averages) of f_0 , f_1 , and f_2 . In each of the subfigures, the data points are fitted by a quadratic fitting, the best-fit result is presented by a solid black line, and the corresponding residuals are plotted in the lower panel. The 2σ (deep red) and 3σ (light red) bounds of the fitting are also shown in the subfigures. In the lower panel of each subfigures, the σ_{eff} is defined as $(Q_{\text{obs}} - Q_{\text{cal}})/\sigma$, where Q_{obs} and Q_{cal} are the values which come from the observation and model calculation, respectively; σ is the uncertainty of the observed points.

except the $-2f_0 + 2f_1$, $f_0 + f_1 - f_2$, and $3f_0$ modes, which leads to the decreasing periods of these three modes.

Figure 2. Interaction diagram of amplitudes of the 23 pulsation modes.

102 The single star evolutionary models are constructed by using different initial masses with
 103 three groups of $[\text{Fe}/\text{H}]$ and two different values of equatorial rotation velocities (v_{eq}), from
 104 the pre-main sequence to the red giant branch. At each of the steps on the evolutionary
 105 tracks, the pulsation frequencies are calculated based on the corresponding stellar structure.
 106 Figure 3 shows the best-fit seismic models of the observed independent frequencies (f_0 , f_1 ,
 107 and f_2) together with the corresponding evolutionary tracks for specific combinations of
 108 $([\text{Fe}/\text{H}], v_{\text{eq}})$. The detailed information of the best-fit seismic models is collected in Table I.

109 The results of the seismic models re-confirm the conclusion that f_0 , f_1 , and f_2 are the
 110 fundamental, first overtone, and second overtone radial p-modes, respectively [11]. Moreover,
 111 it shows that this star locates in the later evolutionary phase of the Hertzsprung gap, which
 112 is a more rapidly evolving phase compared with those stars in the earlier evolutionary phase
 113 of the Hertzsprung gap (such as AE UMa [5] and VX Hya [6]). The most incredible thing
 114 is that the observed period variation rates of f_0 and f_1 are an order of magnitude larger
 115 than the theoretical predicted ones, and two orders of magnitude larger for the case of f_2 . If
 116 we ascribe these observed period variation rates to the stellar evolution (which seems to be
 117 the most reasonable case), it indicates that the star evolving more rapidly than theoretical
 118 prediction and the standard stellar evolution theory cannot precisely describe a single star's
 119 evolution process in this rapidly evolving phase.

121 **Figure 3.** The best-fit seismic models for KIC6382916 along with the corresponding
 122 evolutionary tracks from the main-sequence to the red giant branch. The solid lines
 123 and the dash lines represent the evolutionary tracks of $v_{\text{eq}} = 40 \text{ km s}^{-1}$ and $v_{\text{eq}} =$
 124 120 km s^{-1} , respectively. The different colors represent different value of $[\text{Fe}/\text{H}]$. The
 125 colored pluses and crosses represent the best-fit seismic models of different value of
 126 v_{eq} .

127 For KIC6382916, following the current variation rates, the amplitude of f_2 will exceed
 128 that of f_0 at about BJD 2460331 (Jan, 2024), and exceed that of f_1 at about BJD 2460343

Table I. The frequencies and period variation rates of the three independent pulsation modes from the best-fit seismic models and observations (which are listed in the last column).

f_0 (c days ⁻¹)	f_1 (c days ⁻¹)	f_2 (c days ⁻¹)	$(1/P_0)(dP_0/dt)$ (yr ⁻¹)	$(1/P_1)(dP_1/dt)$ (yr ⁻¹)	$(1/P_2)(dP_2/dt)$ (yr ⁻¹)	Mass $M_\odot$	[Fe/H] dex	Rotation km s ⁻¹
4.9099157	6.4314933	8.0886817	3.97×10^{-8}	3.64×10^{-8}	3.12×10^{-8}	1.81	0.266	40
4.9084637	6.4334156	8.1142769	3.59×10^{-8}	3.09×10^{-8}	2.38×10^{-8}	1.79	0.266	120
4.9106769	6.4315033	8.0896883	3.85×10^{-8}	3.56×10^{-8}	3.10×10^{-8}	1.83	0.111	40
4.9097575	6.4319370	8.1102328	3.51×10^{-8}	3.06×10^{-8}	2.40×10^{-8}	1.81	0.111	120
4.9092663	6.4320905	8.1067212	3.41×10^{-8}	3.14×10^{-8}	2.73×10^{-8}	1.87	0.040	40
4.9087663	6.4328544	8.1284759	3.10×10^{-8}	2.68×10^{-8}	2.09×10^{-8}	1.85	0.040	120
4.909845(5)	6.431886(9)	8.03541(4)	$(3.0 \pm 0.6) \times 10^{-7}$	$(3.0 \pm 0.5) \times 10^{-7}$	$(2.3 \pm 0.2) \times 10^{-6}$	—	—	—

(Feb, 2024). This prediction can be tested in the near future by the following photometric observations, which also could provide us an opportunity to witness the stellar evolution process of a single star gradually in this special evolutionary phase. Moreover, the current continuous photometric data from *Kepler* can sufficiently support us to carry out the searches on the interactions between the pulsation modes via the Interaction Diagram, which could not only be used as a new tool to classify the different pulsation modes of a pulsating star or classify the different kinds of pulsating stars, but also opens a new window for the future asteroseismology.

137 METHODS

138 Photometric Data Reduction

139 The long-cadence (LC) photometric data of KIC6382916 from the *Kepler space telescope*
140 were used in this work, which covers from BJD 2454953 to 2456424 (Quarter 0-17) (publicly
141 available PDC data [15, 16]). We downloaded the light curves (in the format of reduced
142 BJD and magnitudes) of KIC6382916 from Mikulski Archive for Space Telescope (MAST)
143 ⁴, which were then normalized to be zero in the mean for each quarter. An overview of all
144 the normalized LC data in time domain and frequency domain are shown in Extended Data
145 Figure 1.

146 All the above normalized LC data were pre-whiten to extract the frequencies, amplitudes,
147 and phases of the pulsation modes. This process was cut-off until the amplitude smaller than
148 1.3 mmg and we totally got 23 pulsation modes (see in Extended Data Table II), which is
149 much higher than the typical noise level in *Kepler* data of this star (~ 0.03 mmag). This
150 choice ensured that the uncertainties of the amplitudes and phases were typically smaller
151 than the variations of some interested pulsation modes in 4 years, which was determined
152 by the subsequent Short-Time Fourier Transformation results. In each step in the pre-
153 whiting process, the Lomb-Scargle algorithm [17] was used to help to find the initial value of
154 frequency with largest amplitude, and then a non-linear least square fitting was performed
155 to get the final values of the frequency, amplitude, and phase. The frequencies within
156 $1.0 \leq f \leq 24.4 \text{ d}^{-1}$ were considered in this work, which was determined by the LC Nyquist
157 frequency [13].

158 The Short-Time Fourier Transformation [13, 18–20] was then performed to the normalized
159 LC data to get the variation of the amplitudes and phases. In this process, a time window of
160 150 days was moving from the start to the end time of the LC data, with a step of 30 days.
161 In each step, the pre-whiting process was performed to extract the amplitudes and phases
162 of the specific 23 pulsation modes, while the frequencies were fixed as the values obtained
163 in the complete LC data in Extended Data Table II.

164 At last, the amplitude and phase (subtracted by its average value) for each of the 23
165 pulsation modes in each of the moving window were collected (with the times which were

⁴ <http://archive.stsci.edu/kepler>

defined as the midpoints of the window). After removing the periodic signals between 350 to 400 days which is related to the 372.5 day Earth-trailing orbit of the *Kepler Space Telescope*, the variations of the amplitudes and phases of the 23 pulsation modes were presented in Extended Data Figure 2, 3, and 4.

In this work, the pre-whiting process was performed by the Fourier decomposition which can be presented by the formula

$$m = m_0 + \sum A_i \sin [2\pi(f_i t + \phi_i)] , \quad (1)$$

where m_0 is the shifted value, A_i is the amplitude, f_i is the frequency and ϕ_i is the corresponding phase.

The uncertainties all through the work were estimated by the following expressions [21, 22]:

$$\sigma_A = D\sqrt{\frac{2}{N}}\sigma_N, \sigma_f = \frac{D\sqrt{6}\sigma_N}{\pi\sqrt{NAT}}, \sigma_\phi = \frac{D\sigma_N}{\pi\sqrt{2NA}}. \quad (2)$$

In these expressions, σ_A , σ_f , and σ_ϕ are the uncertainties of the amplitude, frequency, and phase, respectively; σ_N can be approximated by the standard deviation of the final residual light curve; A is the amplitude; N and T are the total number of data points and the total time base-line employing in the pre-whiting process, respectively; D can be estimated as the square-root of the average number of consecutive data points of the same sign in the final residual light curves. In this work, the residual light curve were conservatively considered as the light curves in which the specific 23 pulsating signals had been removed.

Theoretical Model Calculation

In order to determine the stellar mass and evolutionary stage based on the single star evolutionary models, the open source 1D stellar evolution code Modules for Experiments in Stellar Astrophysics (MESA [23–27]) was used to construct the structural and evolutionary models. At each step along with the evolutionary tracks, the pulsation frequencies of the specific structure were calculated by the stellar oscillation code GYRE [28] (see, e.g., Refs. [5, 6, 29]).

The initial parameters used to construct pre-main sequence evolutionary models of KIC6382916 were configured as follows. Different metallicity [Fe/H] with values of 0.266, 0.111, and 0.040 dex were considered as the initial metallicity of the evolutionary models

(see Extended Data Table I for more details). The following formulas were used to calculate the initial heavy element abundance Z and initial hydrogen abundance X :

$$[\text{Fe}/\text{H}] = \log \frac{Z}{X} - \log \frac{Z_{\odot}}{X_{\odot}}, \quad (3)$$

$$Y = 0.24 + 3Z, \quad (4)$$

$$X + Y + Z = 1, \quad (5)$$

where $X_{\odot} = 0.7381$ and $Z_{\odot} = 0.0134$ [30]. Equation (4) was provided by Ref. [31]. Based on the given values of $[\text{Fe}/\text{H}]$, we got $(X = 0.672, Z = 0.022)$, $(X = 0.695, Z = 0.016)$, and $(X = 0.704, Z = 0.014)$ as the initial inputs of the evolutionary models. The initial mass of the models was set in the interval from $1.5 M_{\odot}$ to $2.5 M_{\odot}$ with a step of $0.01 M_{\odot}$, covering the typical mass range of HADS. The rotation of the star had also been considered in the model calculation. Because Ref. [10] provided us the projected rotational velocity $v \sin i = 50 \pm 10 \text{ km s}^{-1}$ from high-resolution spectroscopic observation, the equatorial rotation velocities $v_{\text{eq}} = 40 \text{ km s}^{-1}$ and $v_{\text{eq}} = 120 \text{ km s}^{-1}$ were set to be the inputs in the model calculation, which covered a reasonable range of v_{eq} . The mixing-length parameter was set to be a value of $\alpha_{\text{MLT}} = 1.89$ [9]. All the evolutionary tracks were calculated from the pre-main sequence to red giant branch.

Based on the pulsation frequencies calculated in every step along with the evolutionary tracks, we got the best-fit seismic models (which have the smallest χ^2 with respect to the observed values of f_0 , f_1 , and f_2) with the different combinations of $([\text{Fe}/\text{H}], v_{\text{eq}})$ (see in Figure 3 and Table I).

The possibility that f_0 , f_1 , and f_2 could be the non-radial pulsation modes was also tested in our calculation. It can be excluded based on two reasons: (i) we explored the frequency spectrum carefully and did not find any hints of rotation splits of the identified frequencies; (ii) according to the model calculation, the non-radial pulsation modes represented negative period variation rates in post-main sequence phase, which was opposite to the observed values.

Fitting on the Three Independent Pulsation Modes

The amplitudes and phases of the three independent pulsation modes (f_0 , f_1 , and f_2) were fitted by the quadratic polynomial $a + b \cdot t + c \cdot t^2$. The Markov Chain Monte Carlo (MCMC) algorithm [32] was used to determine the coefficients and their uncertainties in above expression, and the fitted coefficients are listed in Extended Data Table III.

Based on the Fourier Phase Diagram method [6], the period variation rates can be directly derived via the fitted coefficients of the quadratic term of the phases.

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Competing Interests The authors declare no competing interests.

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314 **EXTENDED DATA**

317 **Extended Data Figure 1. Overview of the normalized LC data in time domain and**
 318 **frequency domain of KIC6382916.**

Extended Data Figure 2. Variation of the amplitudes and phases of the 23 pulsation modes, Part I.

Extended Data Figure 3. Variation of the amplitudes and phases of the 23 pulsation modes, Part II.

Extended Data Figure 4. Variation of the amplitudes and phases of the 23 pulsation modes, Part III.

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Extended Data Figure 5. Interaction diagram of phases of the 23 pulsation modes.

Extended Data Table I. The observed stellar parameters of KIC6382916.

Parameter	Value	Reference
[Fe/H] (dex)	0.266 ± 0.021	Ref. [33]
	0.111 ± 0.021	Ref. [34]
	$0.040^{+0.25}_{-0.35}$	Ref. [35]
T_{eff} (K)	7075 ± 281	Ref. [36]
	6950 ± 100	Ref. [10]
	6923.39 ± 24.99	Ref. [33]
	6737.92 ± 15.29	Ref. [34]
	6548^{+148}_{-181}	Ref. [35]
	6557 ± 229	Ref. [37]
$v \sin i$ (km s ⁻¹)	50 ± 10	Ref. [10]

Extended Data Table II. Frequency solution of the first 23 frequencies with largest amplitudes.

ID	Frequency (c days ⁻¹)	Amplitude (mmag)	Phase (rad/2 π)
f_0	4.9098455 ± 0.0000002	79.88 ± 0.05	-0.0218 ± 0.0001
f_1	6.4318869 ± 0.0000003	77.84 ± 0.05	-0.2356 ± 0.0001
$f_0 + f_1$	11.3417324 ± 0.0000007	28.38 ± 0.05	0.2002 ± 0.0003
$-f_0 + f_1$	1.522043 ± 0.000001	16.53 ± 0.05	0.0338 ± 0.0005
$2f_0$	9.819690 ± 0.000001	16.11 ± 0.05	-0.0881 ± 0.0005
f_2	8.035414 ± 0.000002	12.54 ± 0.05	-0.0011 ± 0.0007
$2f_1$	12.863775 ± 0.000002	11.71 ± 0.05	-0.3292 ± 0.0007
$2f_0 + f_1$	16.251578 ± 0.000002	8.51 ± 0.05	0.376 ± 0.001
$-f_0 + 2f_1$	7.953928 ± 0.000003	6.94 ± 0.05	-0.348 ± 0.001
$2f_0 - f_1$	3.387805 ± 0.000004	5.26 ± 0.05	0.251 ± 0.002
$f_0 + 2f_1$	17.773618 ± 0.000005	4.23 ± 0.05	0.291 ± 0.002
$f_0 + f_2$	12.945257 ± 0.000006	3.37 ± 0.05	-0.034 ± 0.003
$2f_0 + 2f_1$	22.683465 ± 0.000006	3.24 ± 0.05	0.025 ± 0.003
$f_1 + f_2$	14.467308 ± 0.000006	3.19 ± 0.05	-0.041 ± 0.003
$-f_1 + f_2$	1.603525 ± 0.000007	2.97 ± 0.05	-0.060 ± 0.003
$-f_0 + f_2$	3.125577 ± 0.000008	2.58 ± 0.05	-0.371 ± 0.003
$f_0 + f_1 - f_2$	3.306303 ± 0.000009	2.18 ± 0.05	-0.108 ± 0.004
$3f_0$	14.72954 ± 0.00001	2.10 ± 0.05	-0.153 ± 0.004
$-f_0 + f_1 + f_2$	9.55745 ± 0.00001	1.99 ± 0.05	-0.191 ± 0.004
$3f_1$	19.29566 ± 0.00001	1.93 ± 0.05	-0.212 ± 0.004
$f_0 + 3f_1$	24.20551 ± 0.00001	1.87 ± 0.05	0.286 ± 0.005
$3f_0 + f_1$	21.16143 ± 0.00001	1.37 ± 0.05	-0.227 ± 0.006
$-2f_0 + 2f_1$	3.04408 ± 0.00001	1.38 ± 0.05	0.041 ± 0.006

321 **Extended Data Table III. Quadratic fitting results of the amplitudes and phases of f_0 ,**
322 **f_1 , and f_2 .**

Variable	a	b	c	$\chi^2/\text{d.o.f.}$
A_{f_0}	$79.7(\pm 0.1)$	$7(\pm 3) \times 10^{-4}$	$-4(\pm 2) \times 10^{-7}$	43.02/41
A_{f_1}	$78.1(\pm 0.1)$	$-6(\pm 30) \times 10^{-5}$	$-2.4(\pm 1.8) \times 10^{-7}$	44.96/41
A_{f_2}	$12.0(\pm 0.1)$	$-1.8(\pm 0.3) \times 10^{-3}$	$2.3(\pm 0.2) \times 10^{-6}$	143.34/41
$\phi_{f_0} - \langle \phi_{f_0} \rangle$	$-1.1(\pm 0.2) \times 10^{-3}$	$3.4(\pm 0.6) \times 10^{-6}$	$-2.0(\pm 0.4) \times 10^{-9}$	34.34/41
$\phi_{f_1} - \langle \phi_{f_1} \rangle$	$-1.6(\pm 0.2) \times 10^{-3}$	$-4.6(\pm 0.6) \times 10^{-6}$	$-2.6(\pm 0.4) \times 10^{-9}$	15.39/41
$\phi_{f_2} - \langle \phi_{f_2} \rangle$	$-1.3(\pm 0.1) \times 10^{-2}$	$4.1(\pm 0.4) \times 10^{-5}$	$-2.5(\pm 0.2) \times 10^{-8}$	34.34/41

Figures

Figure 1

Variation of the amplitudes and phases (see the Manuscript file for full figure legend)

Figure 2

Interaction diagram of amplitudes of the 23 pulsation modes.

Figure 3

The best-fit seismic models for KIC6382916 along with the corresponding 122 evolutionary tracks from the main-sequence to the red giant branch. The solid lines and the dash lines represent the evolutionary tracks of $v_{eq} = 40 \text{ km s}^{-1}$ and $v_{eq} = 123 \text{ 120 km s}^{-1}$, respectively. The different colors represent different value of $[\text{Fe}/\text{H}]$. The 124 125 colored pluses and crosses represent the best-fit seismic models of different value of 126 v_{eq} .