# Logarithmic Spiral Gear Design And Production Of With The 3D-Printer

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**Research Article**

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## ABSTRACT

In this study, a software has been developed for parametric design and modeling of logarithmic spiral gears and gear pairs, which are a member of non-circular gears, using complex numbers in the CAD (Computer Aided Design) environment. In software development, the widely used AutoLISP programming language, a software programming language specially designed for AutoCAD, was used. In the program, parameters such as modules and number of teeth are entered by the user interactively, and the design and modeling of logarithmic spiral gears in the AutoCAD environment is performed automatically. The production of this designed gear pair has been completed with a 3D printer. This work contributes to the designer for the functional and precise production of logarithmic spiral gears to the design and manufacture of these gears. This work contributes to the designer and manufacturer in the functional and precise design and production of logarithmic spiral gears.

**Keywords:** Non-circular gear, AutoLISP, Logarithmic spiral, AutoCAD, 3D-Printer

## 1. INTRODUCTION

Non-circular gears are often used to achieve irregular movements. Non-circular gear mechanisms are used to convert stable angular or linear input speed to a variety of angular or linear output speeds.

In Century 17-18 Non-circular gears, clock mechanisms, celestial object inspection devices, musical instruments, automatic gaming instruments, key opening looms, Geneva mechanisms and pumps have been used in such as devices. Non-circular gears have been found to be simple to use in versatile mechanical applications.

In the early 20th century, non-circular gears were used in electromechanical systems to control and operate non-linear potentiometers in various literature. Although non-circular gears have long been known as machine parts, they have not been widely used due to the complex algebraic design of their design and technological difficulties in manufacturing. These gears perform circular or linear periodic motion change. This simplicity in use, price, mechanical power, overload tolerance and lifetime are considered as a competitive alternative to electric servo drives in any case [1]. This increases the importance of non-circular gears despite the development of digital technology.

When the literature is examined, it is seen that computer programs are used to obtain the tooth profile of the comb cutters, which include numerous theoretical and experimental studies, and non-circular gears, the bottom section of the tooth profile is examined and the gears produced by wire erosion and involute method are compared. In the production of non-circular gears, theoretical model development studies have been found in which only a rolling motion on the section diameter is considered [2]. Advances in computer-aided design software and CNC looms have made the design and manufacture of non-circular gears more economical and more efficient [3-5].

It is known that non-circular gears are used in various mechanisms to provide unusual movement or speed characteristics [6]. It is seen that these non-circular gear driven mechanisms offer simple, safe and precise solutions [7,8] compared to cam mechanisms [7,8] and can be manufactured less expensive than cams [9] and are extremely economical compared to specialized servo systems.

In recent years, research on non-circular gears has varied, including basic mathematical analysis, design, manufacturing and application fields [10-12], and some general design criteria that can be applied to all different types of non-circular gears have been reported in the literature [13-14].

In the researches that effectively utilized CAD / CAM software, methods have been developed for integrated effective design and production of different types of tooth bodies. Examples of industrial applications for precision design and production of non-circular gear wheels include machining, casting and special hobbing methods on CNC machines [15]. In this study, 2D and 3D dimensional design of logarithmic spiral gear systems has been realized with the software prepared in AutoLISP language in AutoCAD environment by using the literature studies. These designed gears were produced with a 3D printer.

## 2. DESIGN PARAMETERS OF LOGARITHMIC SPIRAL GEAR

The most common use of non-circular gears is to transmit rotational motion between two parallel axes. The basic principle of non-circular gears is that the two planar curves are rounded without slipping.

The trajectory formed by drawing the course of motion of the relativistic moving two gear centers is clearly visible in the figure 1. The uncertainty of the forces and torques between the tooth surfaces of the gear wheels permits the practical application of the teeth.

The angular velocity ratio and distance function of the central angles of the driven gears 1 and 2 are shown in figure 1.

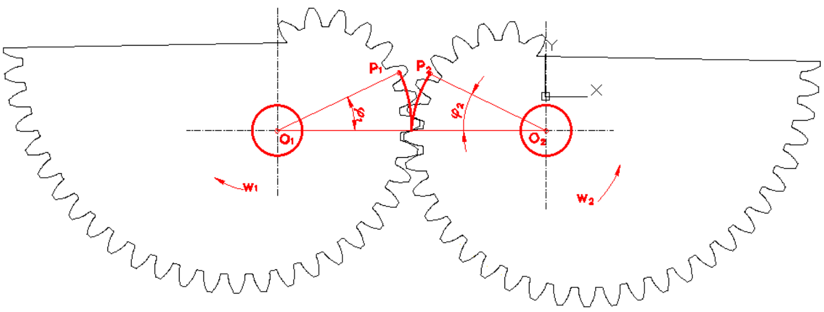

**Figure 1.** Point of departure of the two-relative center of gear wheels

$$\eta = \frac{w_2}{w_1} = \frac{\frac{d\varphi_2}{dt}}{\frac{dw_1}{dt}} = \frac{\overline{O_1 P_0}}{\overline{O_2 P_0}} \tag{1}$$

If the rotated angular velocity is $w_1 = 1$ by the rotated centrode 1 in the course of time $t = \tau$ then the rotation angle is:

$$\Phi_1 = w_1 . \tau \tag{2}$$

The rotation angle of rotation centrode 2 is calculated by the following formula;

$$\Phi_2 = \int_0^{\tau} \eta . dt \tag{3}$$

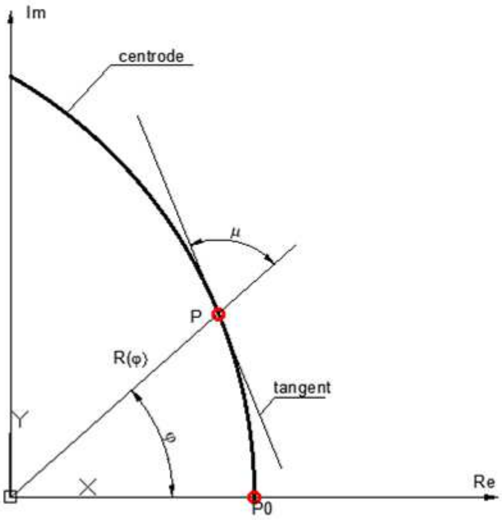


**Figure 2.** Non-slip rolling line of the cutter [16]

If the distance of the axes is “a”, the formulas used for the radii of centrodes $R_1 = \overline{OP_1}$ and $R_2 = \overline{OP_2}$ are calculated as follows;

$$R_1 = \frac{a\eta(\tau)}{1+\eta(\tau)} \qquad R_2 = \frac{a)}{1+\eta(\tau)} \tag{4}$$

Cartesian coordinates of points $P_1$ and $P_2$ are curves in fixed axis systems are

The Cartesian co-ordinates of $P_1$ and $P_2$ points are the curves in the systems of fixed axes are

$$r_1 = [R_1 \cos(\tau), R_1 \sin(\tau)], r_2 = [R_2 \cos(\Phi_2),\ R_2 \sin(\Phi_2)] \tag{5}$$

The common length of the rounded curves from the original position to the actual contact position is $\widehat{P_0P_1} = \widehat{P_0P_2}$. The arc length between $L$ points $P_0$ and $P$ is the curve given as $R=R(\varphi)$.

The general pole equation of the curve is calculated as follows;

$$L = \int_0^{\varphi} \sqrt{R^2 + \left(\frac{dR}{d\varphi}\right)^2 d\varphi} \tag{6}$$

The $OP$ angle of the polar radius and the tangent of the center $R = R(\varphi)$ are calculated as follows

$$\mu = arctan\left(\frac{R}{\left(\frac{dR}{d\varphi}\right)}\right) \tag{7}$$

## 3. CUTTER AND GEAR DESIGN

The coordinates of the cutter used to form the profiles of the circular and non-circular gears made by the rolling method are calculated with complex numbers as follows.

$$
\left.\begin{aligned}
p_{c0} &= m\left(-\frac{\pi}{4} - h\tan(\alpha) - Ih\right)\\
p_{c1} &= m\left(-\frac{\pi}{4} + (h+c)\tan(\alpha) - I(h+c)\right)\\
p_{c2} &= m\left(\frac{\pi}{4} - (h+c)\tan(\alpha) + I(h+c)\right)\\
p_{c3} &= m\left(\frac{\pi}{4} + h\tan(\alpha) - Ih\right)\\
p &= p_{i-4} + m\pi\\
i &= 4,5,\ldots\ldots,
\end{aligned}\right\} \tag{8}
$$

In Fig. 3, the imaginary unit $I = \sqrt{-1}$, the tooth profile angle α, the modulus "m", dedendum and addendum are "h" and "h + c", the fillet radius of ) the cutter is 0'.

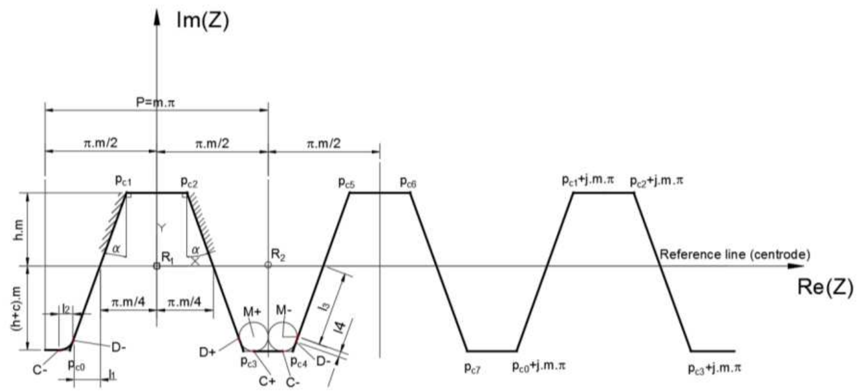


**Figure 3.** Reference profile of the rack-cutter

The production of tooth profiles is achieved by non-slip rolling of the cutter along the center of the gear wheel. P is the common point of the gear center line and the cutter rolling line. The pitch line of the cutter and the rolling centerline of the gear is tangential to point P.

If the following equations are complied with for the corner points tj and $t_{j+1}$, the cutter will be rounded off the center.

$$
w_j = \left(p_{cj} - L\right)e^{\left(I(\emptyset+\mu)\right)} + Re^{(I\emptyset)}, \quad w_{j+1} = \left(p_{cj+1} - L\right)e^{\left(I(\emptyset+\mu)\right)} + Re^{I\emptyset} \tag{9}
$$

The overall movement of the cutter consists of three parts from the original position to the actual position;

1. Displacement with (- L) in the negative direction of the actual axes
2. Rotation with angle $\varphi + \mu$ around of point (O is the axis of the gear and origin of the complex coordinate system fixed to the gear
3. Replacement with $R.e^{I\varphi}$

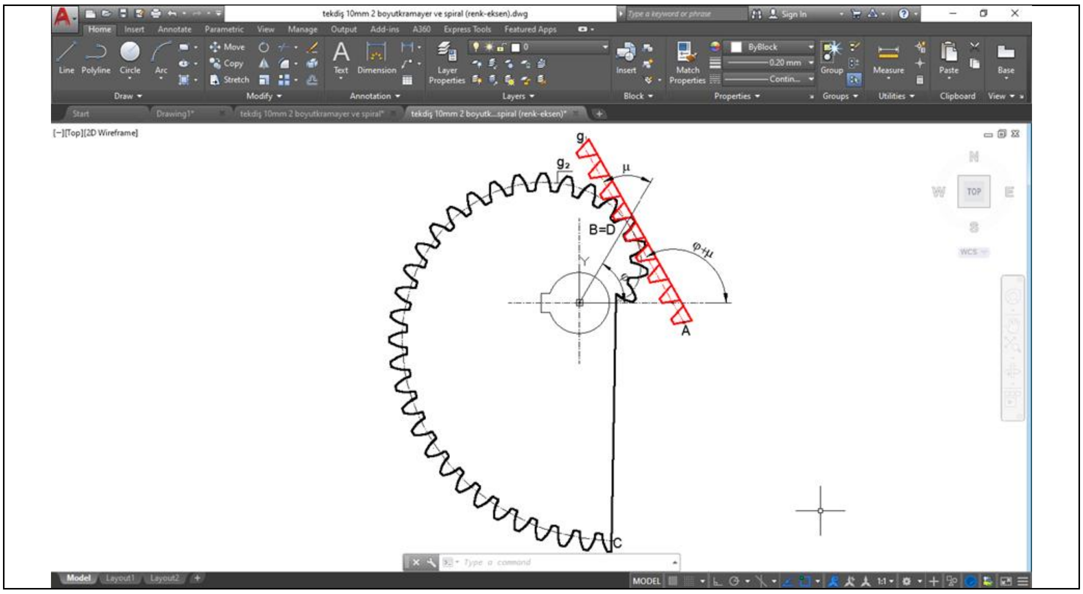


**Figure 5.** Non-slip rolling of the cutter on the logarithmic spiral axis

Figure 5 Shows the successive separate positions of the cutter calculated in formula (9) by the rolling method.

## 3. THE CORRECT PART OF INVOLUTE TEETH

The normal SP line of the cutter profile and the normal point S formed of the threaded profile is the point P of the common axis of the rolling curve (Figure 6).

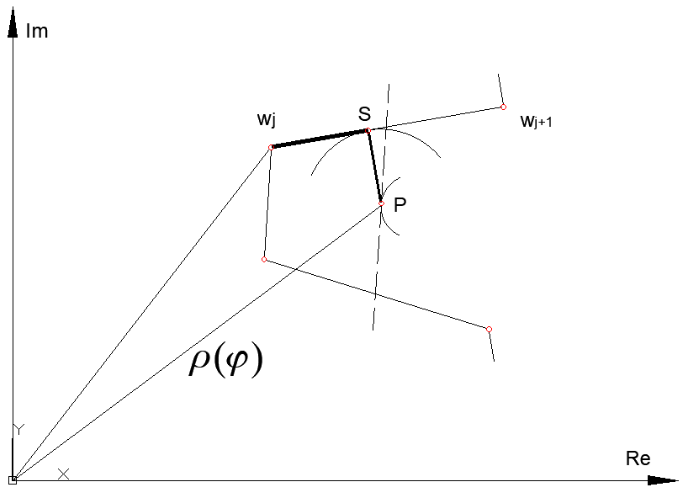


**Figure 6.** Non-slip rolling points of the cutter teeth profile [16].

From equation 9, the complex unit vector is calculated as follows;

$$\Delta = \frac{w_{j+1} - w_j}{|w_{j+1} - w_j|} \tag{10}$$

The coordinates of the cutter corner points are calculated as follows

$$x_j = Re\,(p_{cj}),$$

$x_{j+1} = Re\,(p_{cj+1}),$

$y_{\,j} = Im\,(p_{cj}),$

$y_{j+1} = Im\,(p_{cj+1})$

$\Delta x = x_{j+1} - x_j$

$\Delta y = y_{j+1} - y_j$

When we replace the cutting corner points in equation 10, the following equation is obtained.

$$\Delta = \frac{(\Delta x + I\Delta y).e^{I(\varphi+\mu)}}{\sqrt{\Delta x^2} + \Delta y^2} \tag{11}$$

Equation 10 represents the rolling equation of the breaker. The non-slip rounding of the cutter profile in the gear axis, the complex expression in the *S* point with the parameters $\lambda$, $\eta$ is as follows;

$$Re^{(I\emptyset)} + I\Delta\lambda = w_j + \Delta\eta \tag{12}$$

The lengths are $\lambda = \overline{SP}$ ve $\eta = \overline{w_j S}$ . The *SP* cutter profile and the teeth profile are common normal. The complex equation (12) has a real and imaginary part, so two scalar solutions of the equation (12) can be obtained in a closed manner:

$$\left.\begin{aligned} \lambda &= \frac{y_j + x_{j+1} - y_{j+1}x_j - y_jL + y_{j+1}L}{\sqrt{(x_{j+1}-x_j)^2 + (y_{j+1}-y_j)^2}} \\ \eta &= \frac{-y_j y_{j+1} + x_j^2 - x_jL + x_{j+1}L + y_j^2 - x_j x_{j+1}}{\sqrt{(x_{j+1}-x_j)^2 + (y_{j+1}-y_j)^2}} \end{aligned}\right\} \tag{13}$$

The complex coordinates of the exact *S* point are expressed in closed form as follows.

$$S = e^{I\varphi}\left[R + \frac{y_j x_{j+1} - y_{j+1}x_j - L\Delta y}{\Delta x^2 + \Delta y^2}(\Delta x + I\Delta y)e^{I\mu}\right] \tag{14}$$

## 4. LOGARITHMIC SPIRAL GEAR DESIGN

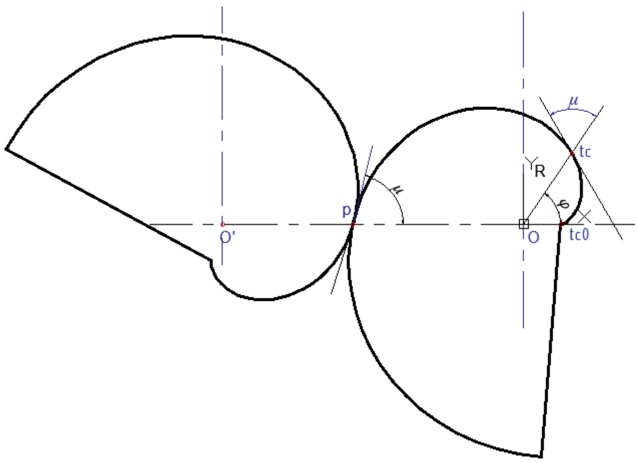


**Figure 6.** Non-slip rolling of two logarithmic spirals

In Figure 6, the centrodes are given two identical logarithmic spirals. The polar equation of the logarithmic spiral curves is expressed as follows.

$$R = a.e^{b\varphi} \tag{15}$$

The angle between the pole radius and the tangent when put in place in equation 7 is expressed as follows.

$$\mu = arctan\left(\frac{1}{b}\right) \tag{16}$$

and when we replace the spring length equation 6, which is considered in closed form, is expressed as follows.

$$L = \frac{a\sqrt{1+b^2}(e^{(b\phi)}-1)}{b} \tag{17}$$

When we replace the real and imaginary parts of the teeth in equation 11, we can simply define them as follows.

For the right side of the gear teeth.

$$\Delta x = (2h + c)mtan(\alpha) \tag{18}$$

$$\Delta y = (2h + c)m \tag{19}$$

For the left side of the tooth.

$$\Delta x = -(2h + c)mtan(\alpha) \tag{20}$$

$$\Delta y = -(2h + c)m \tag{21}$$

General equation in both sides

$$\Delta x^2 + \Delta y^2 = (2h + c)^2m^2(1 + \tan(\alpha)^2 \tag{22}$$

Developed with AutoLISP programming language using the above equations, and the user interactively runs the program in AutoCAD environment with the gear parameters on the logarithmic spiral $a = 20$, $b = 0.25$, $m = 2$, $\alpha = 200$, $h = 1$, $c = 0.25$, $Z = 30$ The logarithmic spiral gear wheel is designed in 2D and 3D dimensions and is shown in Figure 7.

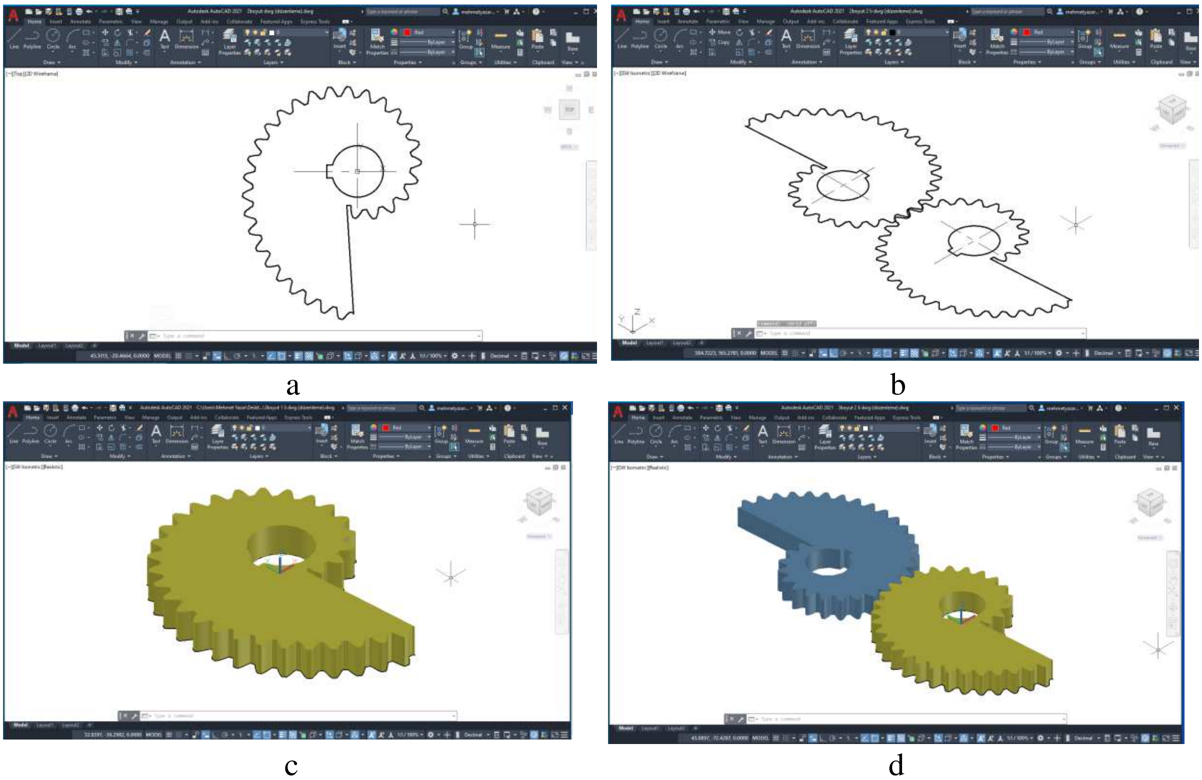


**Figure 7**. 3D dimensional logarithmic spiral gear design

## 5. MANUFACTURE OF LOGARITHMIC SPIRAL GEAR IN 3D PRINTER

The logarithmic spiral gear Creality Ender-3 Pro 3D printer, which was designed in AutoCAD environment in 2 dimensions and 3 dimensions, was produced by making the printer print settings given in Table 1 and from the material given in Table 2 (Figure 8). It has been determined that the gear wheel runs smoothly after production.

**Table 1.** 3D-Printer print settings

| | |
|---|---|
| Print quality | Standard Quality-0.2 mm |
| Default printing Temperature | 200 $^0$C |
| Default Build Plate Temperature | 60 $^0$C |
| Standby Temperature | 175 $^0$C |
| Retraction Distance | 6.50 mm |
| Retraction Speed | 25 mm/s |
| Fan Speed | 100 % |

**Table 2.** Material properties

| | |
|---|---|
| Display Name | PLA |
| Brand | Generic |
| Material Type | PLA |
| Color | Generic |
| **Properties** | |
| Density | 1.24 g/cm$^3$ |
| Diameter | 1.75 mm |

**Figure 8.** Logarithmic spiral gear pair produced by 3D-Printer.

## 6. CONCLUSION AND DISCUSSION

In this study, logarithmic spiral gear pairs, which are a member of non-circular gear wheels, were designed in CAD environment and produced with 3D-printer instead of classical gears, which are an indispensable component of mechanical systems.

With this study, an internationally accepted approach has been applied to the design and manufacture of logarithmic spiral gears, and the knowledge and skill infrastructure has been prepared for the parametric realization of the design and manufacture of logarithmic spiral gears.

By using the polar equations and complex numbers of the design curves of logarithmic spiral gear wheels, design equations can be designed in CAD environment by means of “AutoLISP” programming language.

In this study, the design of logarithmic spiral gears in the desired number of teeth and modules has been designed interactively in AutoCAD environment and produced with a 3D printer.

The possibility of production with wire erosion machine which is one of the non-traditional methods, metal and plastic extrusion, plastic injection method has led to the production of logarithmic spiral gear wheel.

Based on the literature studies, it is expected that the density of the studies on the production of logarithmic spiral gears, kinematic analysis of curves and computer aided design will increase in the following years.

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# Figures

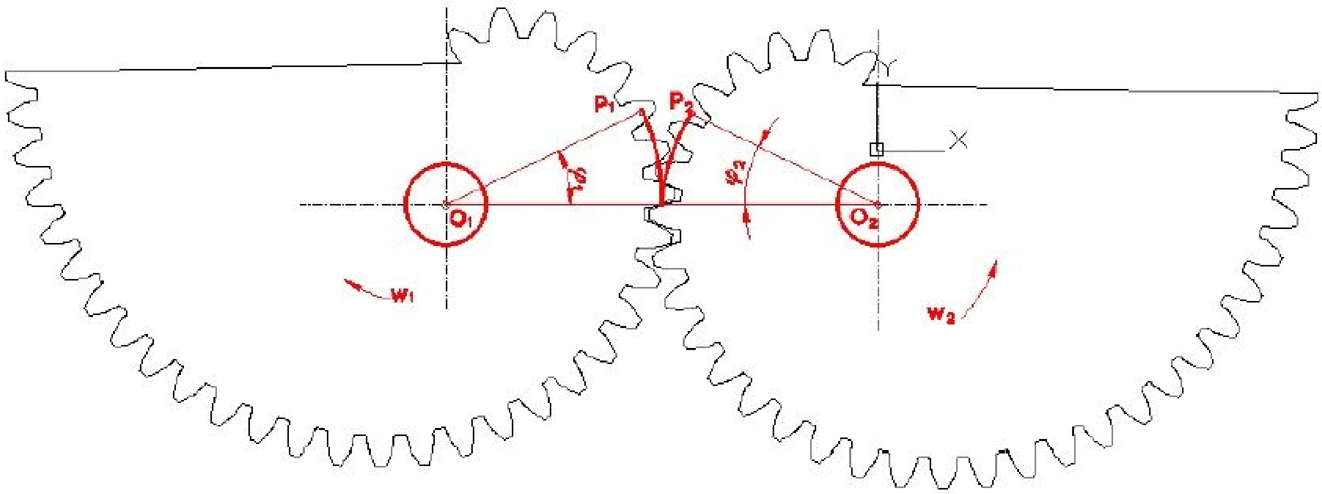



**Figure 1**

Point of departure of the two-relative center of gear wheels

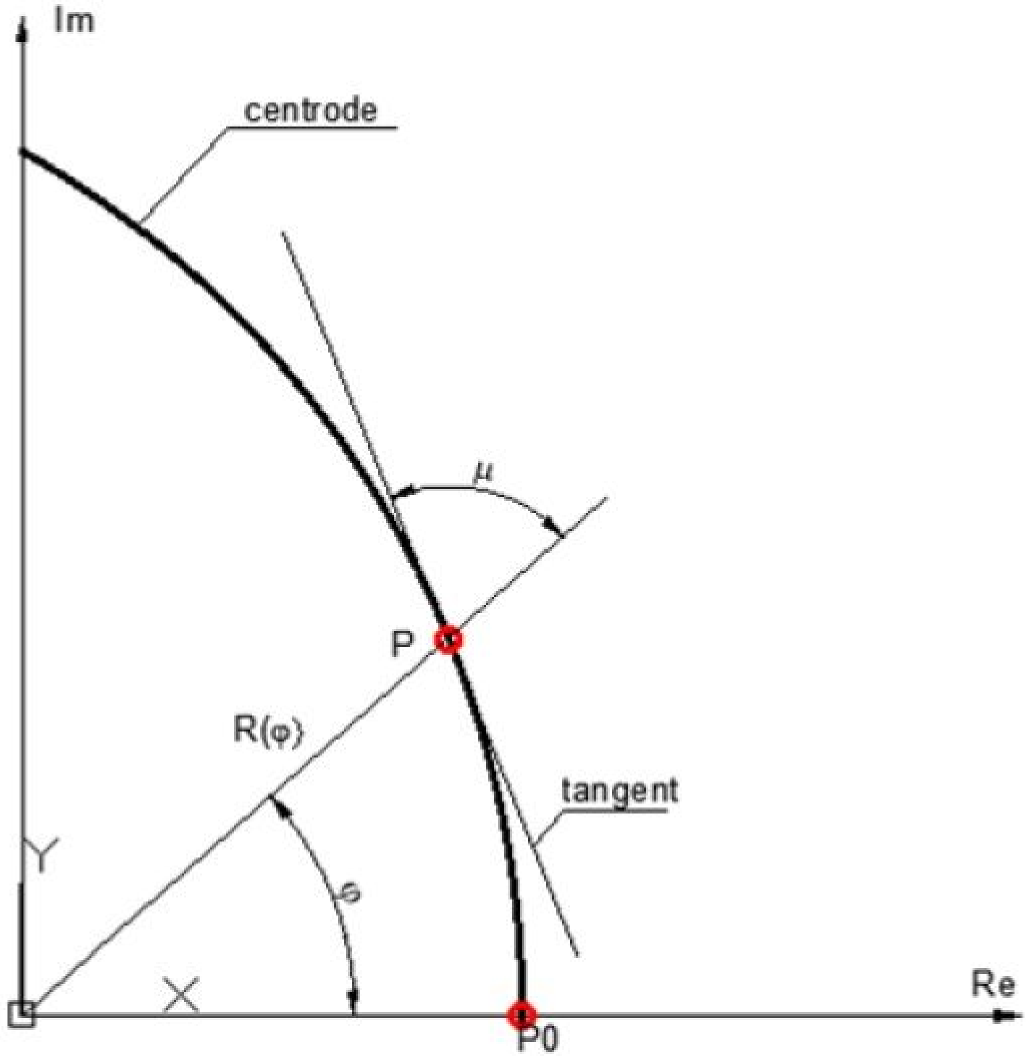
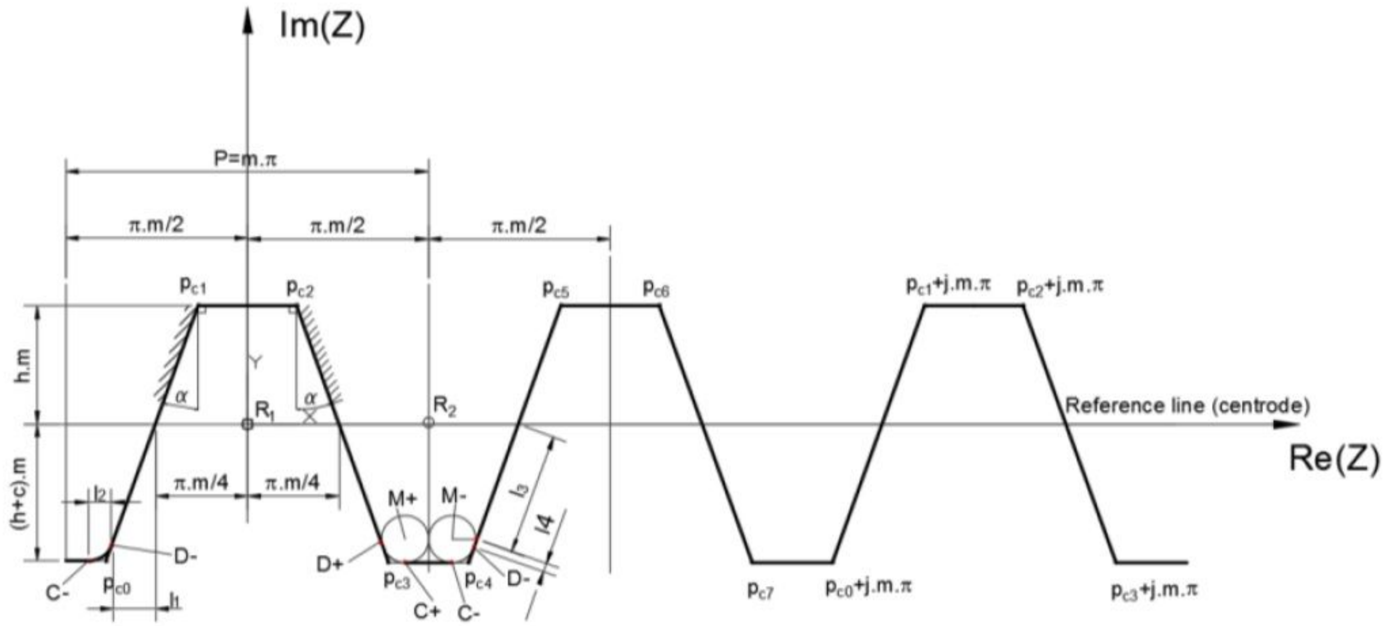


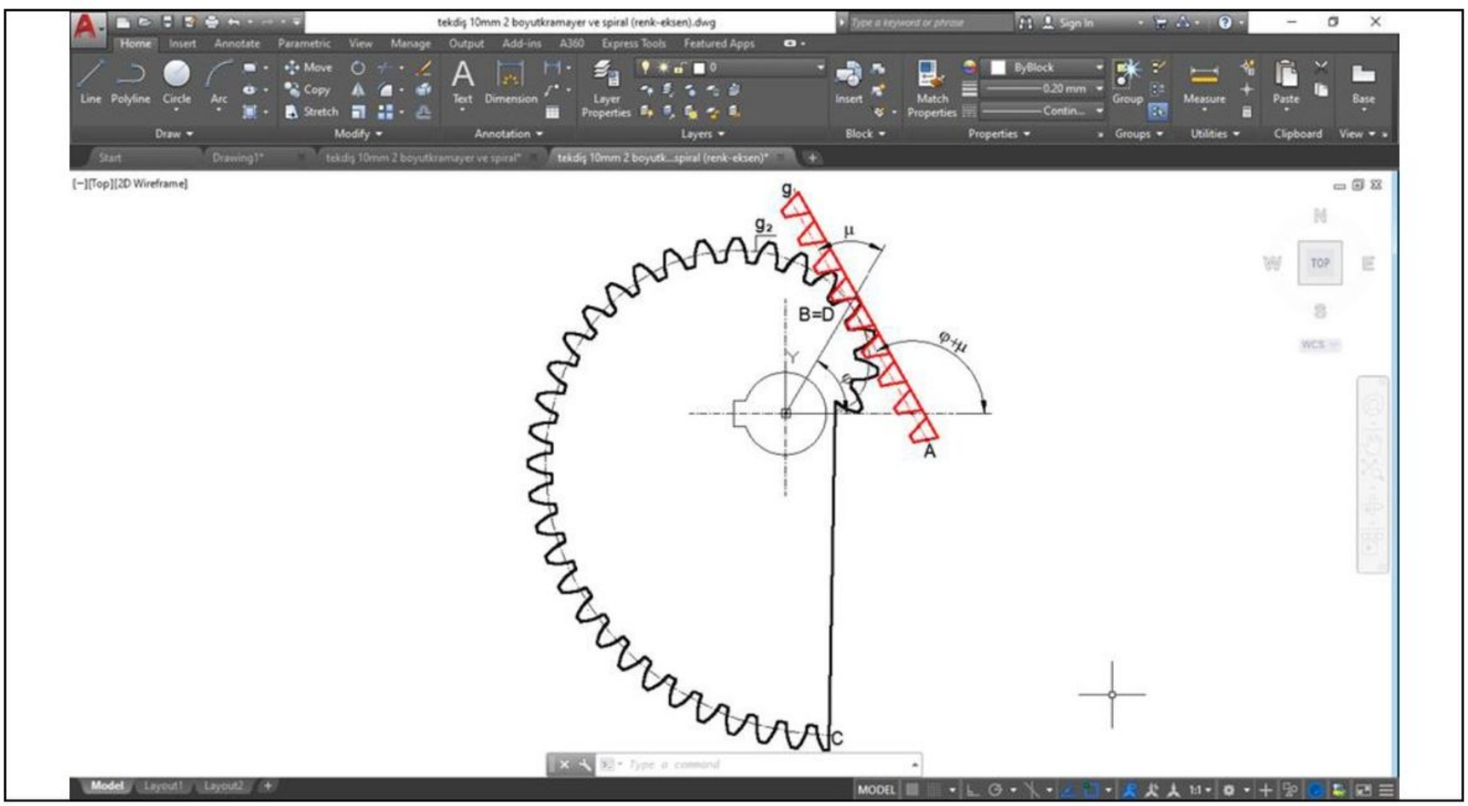

**Figure 2**

Non-slip rolling line of the cutter [16]




## Figure 3

Reference profile of the rack-cutter




## Figure 4

Non-slip rolling of the cutter on the logarithmic spiral axis

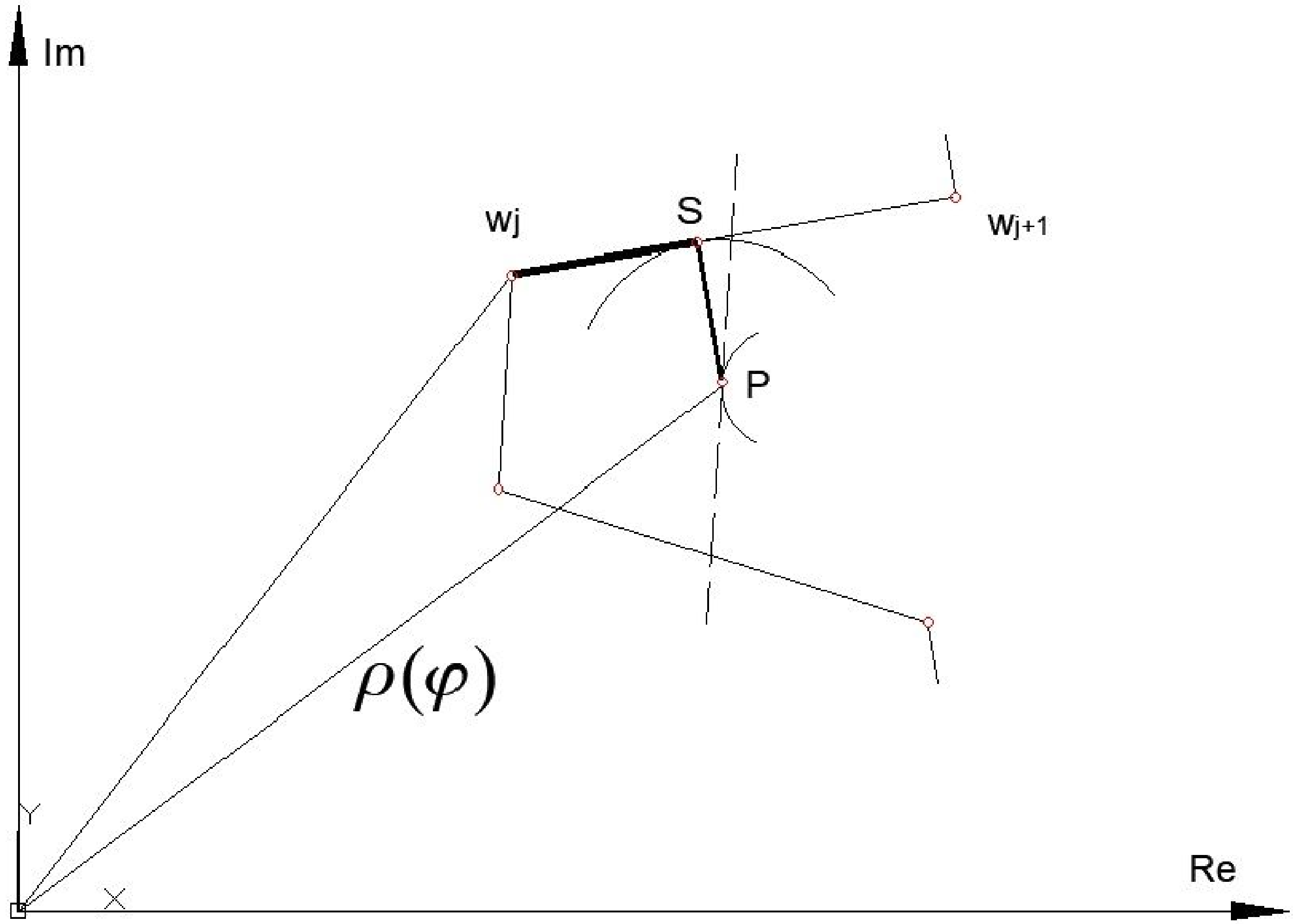



**Figure 5**

Non-slip rolling points of the cutter teeth profile [16].

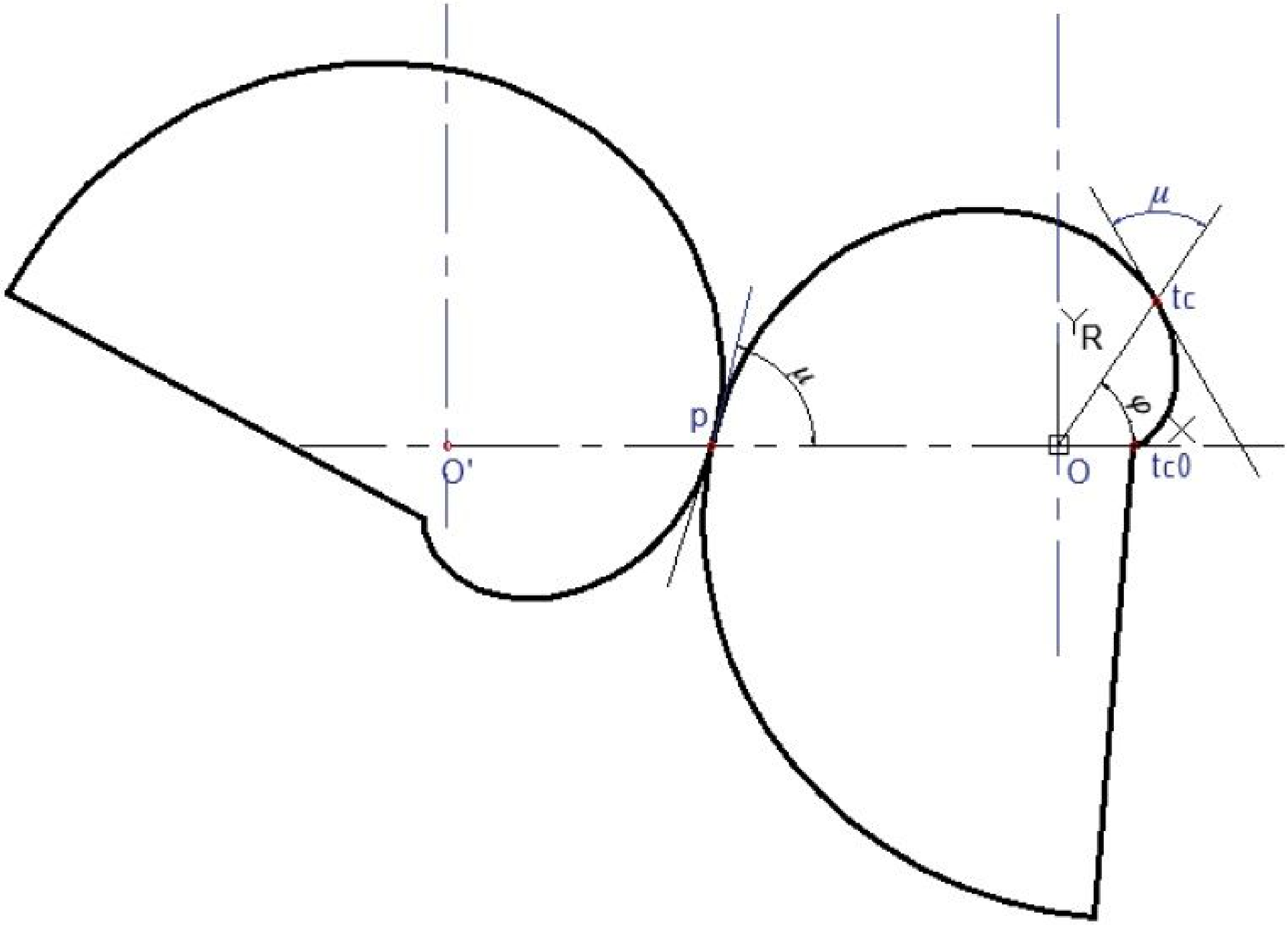



**Figure 6**

Non-slip rolling of two logarithmic spirals

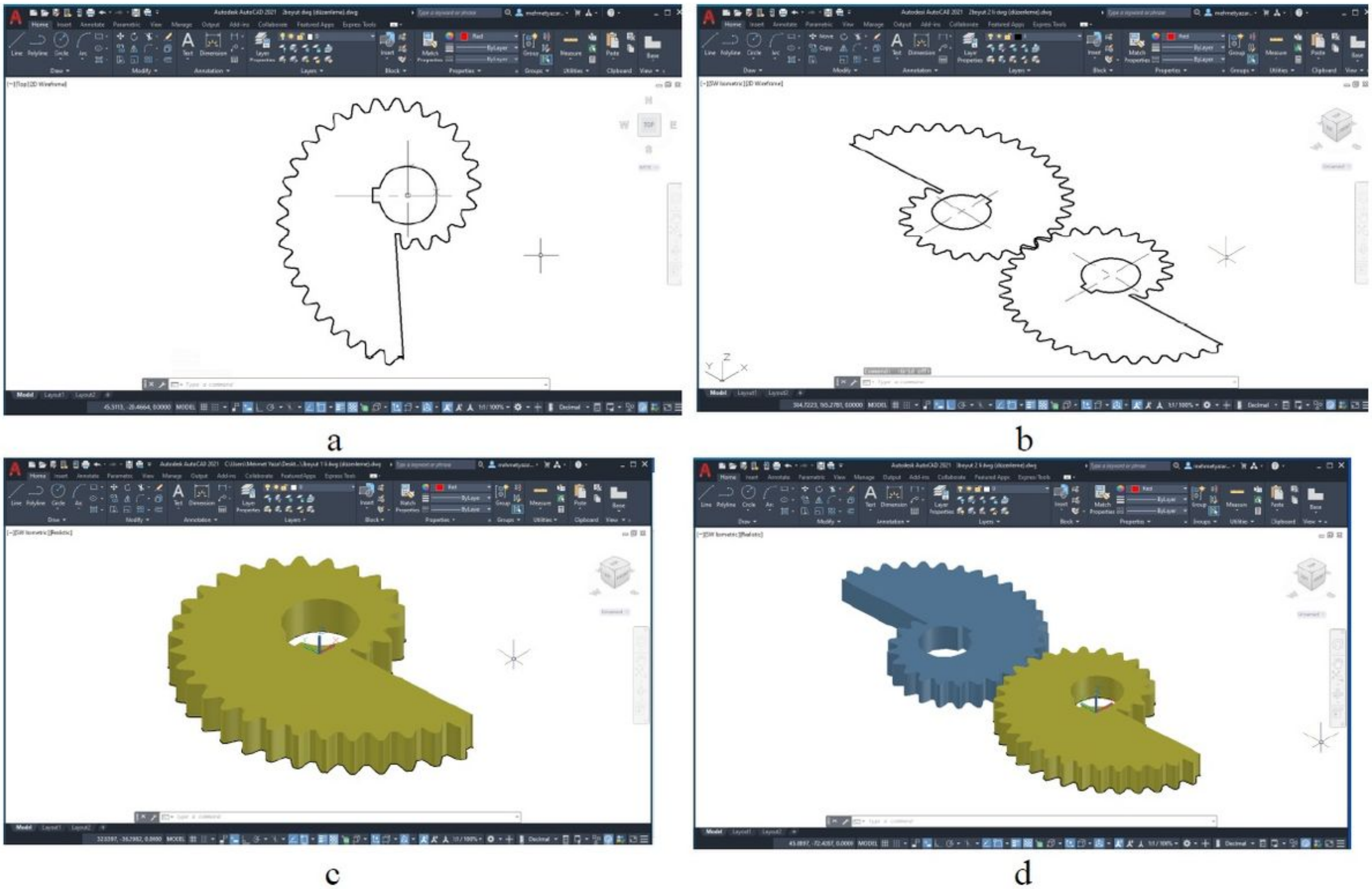



**Figure 7**

3D dimensional logarithmic spiral gear design

**Figure 8**

Logarithmic spiral gear pair produced by 3D-Printer.