Singular Soliton, Shock-wave, Breather-stripe Soliton, Hybrid Solutions and Numerical Simulations for a (2+1)-Dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada System in Fluid Mechanics

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Singular soliton, shock-wave, breather-stripe soliton,
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Abstract

In this paper, a (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada system is investigated in fluid mechanics via the symbolic computation. With the help of the Hirota method, we derive some singular soliton, shock-wave, breather-stripe soliton and hybrid solutions. Based on the finite difference method, we get some numerical one-soliton solutions. We graphically show the singular and shock-wave solutions, and observe that the singular one-soliton solutions are explosive and unstable, but the shock-wave solutions are nonsingular and stable. We observe that the breather-stripe soliton moves along the negative direction of the y axis, where y is a variable, and the amplitude and shape of the breather-stripe soliton remain invariant during the propagation. We graphically demonstrate the interaction among a **rogue** wave, a periodic wave and a pair of the stripe solitons: the **rogue** wave arises from the one stripe soliton; the rogue wave interacts with the periodic wave, the rogue wave splits into two waves and then the two waves merge into a wave; the **rogue** wave fuses with the other stripe soliton. We graphically present the numerical one-soliton solutions which agree with the analytic one-soliton solutions.

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Keywords: Fluid mechanics; (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada system; Soliton solutions; Shock-wave solutions, Breather-stripe soliton solutions; Hybrid solutions; Numerical simulations

1. Introduction

Fluid mechanics has been applied in a variety of disciplines such as meteorology, geophysics, biomedical engineering, oceanography and astrophysics [1–4]. In order to gain insight into certain fluid mechanical problems, researchers have focused their attention on some solutions for some nonlinear evolution equations, such as the soliton, breather-wave, periodic-wave and **rogue**-wave solutions [5–29].

Refs. [30–42] have considered the following (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada system in fluid mechanics:

$$\begin{cases} 36u_t + u_{xxxxx} + 15(uu_{xx})_x + 45u^2u_x - 5u_{xy} - 15uu_y - 15u_xv - 5v_y = 0, \\ u_y = v_x, \end{cases} \quad (1)$$

where u and v are both the differentiable functions with respect to the variables x , y and t , and the subscripts indicate the partial derivatives. Darboux transformations and N -soliton solutions for System (1) have been derived via the Darboux matrix method, where N is a positive integer [30]. Soliton, rational, triangular periodic, Jacobi and Weierstrass doubly periodic solutions for System (1) have been obtained via the algebraic method [31]. Various soliton, breather and periodic solutions for System (1) have been derived via the long wave limit method [32]. Via the Hirota-Riemann function method [33], quasi-periodic solutions for System (1) have been derived. Symmetry reductions and group-invariant solutions for System (1) have been obtained via the Lou's direct method [34]. Lump, mixed rogue wave-stripe soliton, mixed lump-stripe soliton for System (1) have been obtained via the Hirota bilinear method [35]. The higher-order breather, lump and hybrid solutions for System (1) have been obtained via the long wave limit method [36]. The periodic soliton solutions for System (1) have been derived via the Hirota bilinear method [37]. Ref. [38] has derived non-traveling wave solutions for System (1) via the Lie group analysis and exp-function method. Hybrid solutions among lumps, breathers and solitons for System (1) have been obtained via the long wave limit method [39]. Lump solutions have been obtained via the direct method [40]. Symmetry group theorem, some analytic solutions and infinite conservation laws for System (1) have been derived via the improved CK's method [41]. Hybrid solutions comprising the lumps and solitons have been obtained [42].

Under the transformations,

$$u = 2(\ln f)_{xx}, \quad v = 2(\ln f)_{xy}, \quad (2)$$

System (1) has been converted into the bilinear form as [39]

$$(5D_x^3D_y + 5D_y^2 - D_x^6 - 36D_xD_t)f \cdot f = 0, \quad (3)$$

where f is a differentiable function about x , y and t , and D is the Hirota bilinear differentiable

operator, which is defined as [43]

$$D_x^{\iota_1} D_y^{\iota_2} D_t^{\iota_3} (F \cdot G) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^{\iota_1} \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'} \right)^{\iota_2} \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^{\iota_3} F(x, y, t) G(x', y', t') \Big|_{x'=x, y'=y, t'=t},$$

with $F(x, y, t)$ being a differentiable function with respect to x, y and t , $G(x', y', t')$ being a differentiable function about the formal variables x', y' and t' , and ι_1, ι_2 and ι_3 being three non-negative integers.

However, to our knowledge, singular N -soliton solutions, shock-wave solutions, breather-stripe soliton solutions, hybrid solutions among a rogue wave, a periodic wave and a pair of the stripe solitons, and numerical solutions for System (1) have not been investigated. In Section 2, we will construct some singular N -soliton and shock-wave solutions for System (1). In Section 3, breather-stripe soliton solutions for System (1) will be established. In Section 4, we will discuss the hybrid solutions among a rogue wave, a periodic wave and a pair of the solitons for System (1). In Section 5, we will obtain certain numerical one-soliton solutions for System (1). In Section 6, conclusions will be drawn.

2. Singular soliton and shock-wave solutions for System (1)

2.1. Singular soliton solutions for System (1)

In this part, we will construct some singular N -soliton solutions for System (1) via the Hirota bilinear method [43]. We use the perturbation method to expand f with respect to a formal parameter ϵ as

$$f = 1 + \epsilon f_1 + \epsilon^2 f_2 + \epsilon^3 f_3 + \cdots + \epsilon^N f_N, \quad (4)$$

where f_i 's ($i = 1, 2, \dots, N$) are the real functions of x, y and t . Substituting Expression (4) into Bilinear Form (3) and equating the coefficients of the same power of ϵ to zero, we obtain some soliton solutions of System (1). Thus, when $\epsilon = -1$, singular N -soliton solutions can be written as

$$u = 2(\ln f)_{xx}, \quad v = 2(\ln f)_{xy}, \quad (5)$$

$$f = 1 - \sum_{i=1}^N e^{\eta_i} + \sum_{i < j}^N M_{ij} e^{\eta_i + \eta_j} - \sum_{i < j < \ell}^N M_{ij} M_{i\ell} M_{j\ell} e^{\eta_i + \eta_j + \eta_\ell} + \cdots + (-1)^N \left(\prod_{i < j} M_{ij} \right) e^{\sum_{i=1}^N \eta_i},$$

where

$$\eta_i = \kappa_i x + \rho_i y + \omega_i t + \eta_{i0}, \quad \omega_i = \frac{5\kappa_i^3 \rho_i - \kappa_i^6 + 5\rho_i^2}{36\kappa_i},$$

$$M_{ij} = -\frac{(\kappa_i - \kappa_j)^6 - 5(\kappa_i - \kappa_j)^3(\rho_i - \rho_j) - 5(\rho_i - \rho_j)^2 + 36(\kappa_i - \kappa_j)(\omega_i - \omega_j)}{(\kappa_i + \kappa_j)^6 - 5(\kappa_i + \kappa_j)^3(\rho_i + \rho_j) - 5(\rho_i + \rho_j)^2 + 36(\kappa_i + \kappa_j)(\omega_i + \omega_j)},$$

κ_i , ρ_i , ω_i and η_{i0} are the real constants and $i < j < \ell$ ($\ell = 3, 4, \dots, N$).

Assuming $N=1$ in Solutions (5), singular one-soliton solutions can be derived as

$$\begin{aligned} u &= 2(\ln f)_{xx} = 2[\ln(1 - e^\eta)]_{xx} = -\frac{1}{2}\kappa_1^2 \text{csch}^2\left(\frac{\eta_1}{2}\right), \\ v &= 2(\ln f)_{xy} = 2[\ln(1 - e^\eta)]_{xy} = -\frac{1}{2}\kappa_1\rho_1 \text{csch}^2\left(\frac{\eta_1}{2}\right), \end{aligned} \quad (6)$$

with $\eta_1 = \kappa_1 x + \rho_1 y + \frac{5\kappa_1^3\rho_1 - \kappa_1^6 + 5\rho_1^2}{36\kappa_1}t + \eta_{10}$.

The physical structure of Solutions (6) is shown in Figs. 1, which shows that Solutions (6) are explosive and unstable.

Figs. 1. Singular one soliton via Solutions (6) with $\kappa_1 = \rho_1 = 2$ and $\eta_{10} = 0$.

2.2. Shock-wave solutions for System (1)

According to Ref. [44], to calculate some shock-wave solutions, we suppose that

$$u = M \tanh^\lambda \zeta, \quad (7)$$

where $\zeta = ax + by + ct$, M is the wave amplitude, a , b , c and λ are all the real constants.

Substituting Expression (7) into u_{xxxxx} and $u^2 u_x$, we derive that

$$\begin{aligned} u_{xxxxx} &= a^5 \lambda M \left[(\lambda^4 - 10\lambda^3 + 35\lambda^2 - 50\lambda + 24) \tanh^{\lambda-5} \zeta - 5(\lambda^4 - 6\lambda^3 + 15\lambda^2 - 18\lambda + 8) \tanh^{\lambda-3} \zeta \right. \\ &\quad + 2(5\lambda^4 - 10\lambda^3 + 25\lambda^2 - 20\lambda + 8) \tanh^{\lambda-1} \zeta - 2(5\lambda^4 + 10\lambda^3 + 25\lambda^2 + 20\lambda + 8) \tanh^{\lambda+1} \zeta \\ &\quad \left. + 5(\lambda^4 + 6\lambda^3 + 15\lambda^2 + 18\lambda + 8) \tanh^{\lambda+3} \zeta - (\lambda^4 + 10\lambda^3 + 35\lambda^2 + 50\lambda + 24) \tanh^{\lambda+5} \zeta \right], \\ u^2 u_x &= a \lambda M^3 (\tanh^{3\lambda-1} \zeta - \tanh^{3\lambda+1} \zeta). \end{aligned} \quad (8)$$

From Expressions (8), making $\lambda+5 = 3\lambda+1$, we get $\lambda = 2$. Then, substituting $u = M \tanh^2 \zeta$ and $v = -\frac{bM \operatorname{sech}^2 \zeta}{a}$ into System (1), we obtain that

$$\begin{aligned} &45a(8a^4 + 6a^2M + M^2) \tanh^7 \zeta - 15(56a^5 + 34a^3M + 4a^2b + 3aM^2 + 2bM) \tanh^5 \zeta - \left[\frac{5b^2}{a} - 616a^5 \right. \\ &\quad \left. - 45M(6a^3 + b) - 100a^2b - 36c \right] \tanh^3 \zeta - \left(136a^5 + 30a^3M + 40a^2b - \frac{5b^2}{a} + 15bM + 36c \right) \tanh \zeta = 0, \end{aligned}$$

which gives us $M = -2a^2$ and $c = \frac{5b^2 - 76a^6 - 10a^3b}{36a}$. Therefore, we obtain the shock-wave solutions for System (1) as

$$u = -2a^2 \tanh^2 \left(ax + by + \frac{5b^2 - 76a^6 - 10a^3b}{36a} t \right), \quad v = 2ab \operatorname{sech}^2 \left(ax + by + \frac{5b^2 - 76a^6 - 10a^3b}{36a} t \right). \quad (9)$$

Figs. 2 show the propagation of the shock wave. We observe that the amplitude and the shape of the shock wave keep unchanged during the propagation. Therefore, Solutions (9) are the nonsingular and stable solutions.

Figs. 2. Shock wave via Solutions (9) with $a = -b = -1$.

3. Breather-stripe soliton solutions for System (1)

To construct the breather-stripe soliton solutions for System (1), inspired by Ref. [45], we assume that

$$f = s_1 \cosh \xi_1 + s_2 \cos \xi_2 + s_3 e^{\xi_3} + m_1, \quad (10)$$

where $\xi_\varrho = k_\varrho x + r_\varrho y + p_\varrho t + q_\varrho$ ($\varrho = 1, 2, 3$), and k_ϱ 's, r_ϱ 's, p_ϱ 's, q_ϱ 's, s_ϱ 's and m_1 are the real constants. Substituting Expression (10) into Bilinear Form (3), we derive that

$$p_1 = \frac{k_1^5}{4}, \quad p_2 = \frac{k_2^5}{4}, \quad p_3 = \frac{k_3^5}{4}, \quad r_1 = k_1^3, \quad r_2 = -k_2^3, \quad r_3 = k_3^3. \quad (11)$$

According to Expressions (10), (11) and Transformations (2), we get the breather-stripe soliton solutions for System (1) as

$$u = \frac{2(s_1 k_1^2 \cosh \xi_1 - s_2 k_2^2 \cos \xi_2 + s_3 k_3^2 e^{\xi_3})}{s_1 \cosh \xi_1 + s_2 \cos \xi_2 + s_3 e^{\xi_3} + m_1} - \frac{2(s_1 k_1 \sinh \xi_1 - s_2 k_2 \sin \xi_2 + s_3 k_3 e^{\xi_3})^2}{(s_1 \cosh \xi_1 + s_2 \cos \xi_2 + s_3 e^{\xi_3} + m_1)^2}, \quad (12a)$$

$$v = \frac{2(s_1 k_1 r_1 \cosh \xi_1 - s_2 k_2 r_2 \cos \xi_2 + s_3 k_3 r_3 e^{\xi_3})}{s_1 \cosh \xi_1 + s_2 \cos \xi_2 + s_3 e^{\xi_3} + m_1} - \frac{2(s_1 k_1 \sinh \xi_1 - s_2 k_2 \sin \xi_2 + s_3 k_3 e^{\xi_3})(s_1 r_1 \sinh \xi_1 - s_2 r_2 \sin \xi_2 + s_3 r_3 e^{\xi_3})}{(s_1 \cosh \xi_1 + s_2 \cos \xi_2 + s_3 e^{\xi_3} + m_1)^2}. \quad (12b)$$

Figs. 3. Breather-stripe soliton via Solutions (12) with $s_1 = k_3 = m_1 = 1$, $q_1 = q_3 = s_3 = -q_2 = 2$,

$k_1 = \frac{\sqrt{3}}{3}$ and $k_2 = \frac{9}{10}$

Figs. 3 display the propagation of the breather-stripe soliton. We observe that the breather-stripe soliton moves along the negative direction of the y axis. Amplitude and shape of the breather-stripe soliton remain invariant during the propagation.

4. Hybrid solutions among a rogue wave, a periodic wave and a pair of the stripe solitons for System (1)

In order to research the hybrid solutions among a rogue wave, a periodic wave and a pair of the stripe solitons for System (1), motivated by Ref. [46], we assume that

$$f = s_4 \xi_4^2 + s_5 \xi_5^2 + s_6 \cosh \xi_6 + s_7 \cos \xi_7 + m_2, \quad (13)$$

where $\xi_\sigma = k_\sigma x + r_\sigma y + p_\sigma t + q_\sigma$ ($\sigma = 4, \dots, 7$), and k_σ 's, r_σ 's, p_σ 's, q_σ 's, s_σ 's and m_2 are the real constants. Substituting Expression (13) into Bilinear Form (3), we obtain that

$$\begin{aligned} k_4 &= \frac{r_4}{3k_5 k_7^2} \sqrt{\frac{2k_7^2 (s_6^2 - s_7^2) - 4k_5^2 s_5 m_2}{s_5 m_2}}, \quad p_4 = -\frac{5k_7^2 r_4}{24k_5} \sqrt{\frac{k_7^2 (s_6^2 - s_7^2) - 2k_5^2 s_5 m_2}{2s_5 m_2}}, \\ r_5 &= -\frac{3k_7^2}{2} \sqrt{\frac{k_7^2 (s_6^2 - s_7^2) - 2k_5^2 s_5 m_2}{2s_5 m_2}}, \quad p_5 = -\frac{5k_5 k_7^4}{16}, \quad p_6 = -p_7 = \frac{k_7^5}{16}, \quad s_4 = \frac{9k_5^2 k_7^4 s_5}{4r_4^2}, \\ r_6 &= r_7 = \frac{k_7^3}{2}, \quad k_6 = -k_7, \quad k_5 k_7 r_4 \neq 0, \quad s_5 m_2 [k_7^2 (s_6^2 - s_7^2) - 2k_5^2 s_5 m_2] > 0. \end{aligned} \quad (14)$$

Based on Expressions (13), (14) and Transformations (2), we obtain the hybrid solutions among a rogue wave, a periodic wave and a pair of the stripe solitons for System (1) as

$$u = \frac{2(2k_4^2 s_4 + 2k_5^2 s_5 + k_6^2 s_6 \cosh \xi_6 - k_7^2 s_7 \cos \xi_7)}{s_4 \xi_4^2 + s_5 \xi_5^2 + s_6 \cosh \xi_6 + s_7 \cos \xi_7 + m_2} \quad (15a)$$

$$-2 \frac{(2k_4 s_4 \xi_4 + 2k_5 s_5 \xi_5 + k_6 s_6 \sinh \xi_6 - k_7 s_7 \sin \xi_7)^2}{(s_4 \xi_4^2 + s_5 \xi_5^2 + s_6 \cosh \xi_6 + s_7 \cos \xi_7 + m_2)^2},$$

$$v = \frac{2(2k_4 s_4 r_4 + 2k_5 s_5 r_5 + k_6 s_6 r_6 \cosh \xi_6 - k_7 s_7 r_7 \cos \xi_7)}{s_4 \xi_4^2 + s_5 \xi_5^2 + s_6 \cosh \xi_6 + s_7 \cos \xi_7 + m_2} \quad (15b)$$

$$- \frac{2(2k_4 s_4 \xi_4 + 2k_5 s_5 \xi_5 + k_6 s_6 \sinh \xi_6 - k_7 s_7 \sin \xi_7)(2s_4 r_4 \xi_4 + 2s_5 r_5 \xi_5 + s_6 r_6 \sinh \xi_6 - s_7 r_7 \sin \xi_7)}{(s_4 \xi_4^2 + s_5 \xi_5^2 + s_6 \cosh \xi_6 + s_7 \cos \xi_7 + m_2)^2}.$$

Figs. 4 show the interaction among a rogue wave, a periodic wave and a pair of the stripe solitons via Solutions (15). In Figs. 4(a_1) and (b_1), we observe that there is a periodic wave between a pair of the stripe solitons. **As t goes on, the rogue wave arises from the one stripe soliton and interacts with the periodic wave. The rogue wave splits into two waves, then the two waves merge into one wave, and finally the rogue wave fuses with the other stripe soliton, as seen in Figs. 4(a_2)-(a_5) and (b_2)-(b_5).**

Figs. 4. Interaction among a rogue wave, a periodic wave and a pair of the stripe solitons via Solutions (15) with $m_2 = r_4 = q_5 = q_6 = q_7 = k_7 = 1$, $q_4 = s_5 = 2$, $s_7 = 3$, $k_5 = \frac{1}{2}$ and $s_6 = \sqrt{10}$.

5. Numerical simulations for System (1)

In the section, for simplicity, we consider the dimensional reduction $\partial_x = \partial_y$, then System (1) is reduced to

$$36w_t + w_{xxxxx} + 15(ww_{xx})_x + 45w^2w_x - 5w_{xxx} - 30ww_x - 5w_x = 0, \quad (16)$$

with w being the differentiable functions about x and t . Next, we will construct the numerical one-soliton solutions for Eq. (16) via a finite difference method [47].

Choosing a finite interval $\Omega = [L_1, L_2]$, which is large enough, Eq. (16) with the vanishing **boundary condition** $\lim_{|x| \rightarrow +\infty} |w| = 0$ can be approximated by

$$\begin{aligned} 36w_t + w_{xxxxx} + 15(ww_{xx})_x + 45w^2w_x - 5w_{xxx} \\ - 30ww_x - 5w_x = 0, \quad L_1 < x < L_2, t > 0, \\ w(x, t = 0) = w_0(x), \quad x \in \Omega, \\ w(L_1, t) = w(L_2, t) = 0, \quad t \geq 0. \end{aligned} \quad (17)$$

According to Ref. [47], let $h = \frac{L_2 - L_1}{J}$ and τ be the x -direction step size and t -direction step size, respectively, then we get the mesh points $x_j = L_1 + jh$ ($j = 0, 1, \dots, J$) and $t_n = n\tau$, with J as a positive integer and n as a non-negative positive integer. For simplicity, we introduce some notations as follows [47]:

$$\begin{aligned} w_j^{n+\frac{1}{2}} &= \frac{w_j^{n+1} + w_j^n}{2}, \quad \delta_t w_j^{n+\frac{1}{2}} = \frac{w_j^{n+1} - w_j^n}{\tau}, \quad \delta_x w_j^n = \frac{w_{j+1}^n - w_{j-1}^n}{2h}, \\ \delta_x^2 w_j^n &= \frac{w_{j-1}^n + w_{j+1}^n - 2w_j^n}{h^2}, \quad \delta_x^3 w_j^n = \frac{-w_{j-2}^n + 2w_{j-1}^n - 2w_{j+1}^n + w_{j+2}^n}{2h^3}, \\ \delta_x^5 w_j^n &= \frac{-w_{j-3}^n + 4w_{j-2}^n - 5w_{j-1}^n + 5w_{j+1}^n - 4w_{j+2}^n + w_{j+3}^n}{2h^5}, \end{aligned} \quad (18)$$

where w_j^n denotes the approximate value of $w(x_j, t_n)$. Eq. (16) at the point $(x_j, t_{n+\frac{1}{2}})$ can be written as

$$\begin{aligned} 36 \frac{\partial w}{\partial t} \left(x_j, t_{n+\frac{1}{2}} \right) + \frac{\partial^5 w}{\partial x^5} \left(x_j, t_{n+\frac{1}{2}} \right) + 15 \frac{\partial w}{\partial x} \left(x_j, t_{n+\frac{1}{2}} \right) \frac{\partial^2 w}{\partial x^2} \left(x_j, t_{n+\frac{1}{2}} \right) \\ + 15w \left(x_j, t_{n+\frac{1}{2}} \right) \frac{\partial^3 w}{\partial x^3} \left(x_j, t_{n+\frac{1}{2}} \right) + 45w^2 \left(x_j, t_{n+\frac{1}{2}} \right) \frac{\partial w}{\partial x} \left(x_j, t_{n+\frac{1}{2}} \right) \\ - 5 \frac{\partial^3 w}{\partial x^3} \left(x_j, t_{n+\frac{1}{2}} \right) - 30w \left(x_j, t_{n+\frac{1}{2}} \right) \frac{\partial w}{\partial x} \left(x_j, t_{n+\frac{1}{2}} \right) - 5 \frac{\partial w}{\partial x} \left(x_j, t_{n+\frac{1}{2}} \right) = 0. \end{aligned} \quad (19)$$

Based on the Taylor expansion, we have

$$\begin{aligned} \frac{\partial w}{\partial t} \left(x_j, t_{n+\frac{1}{2}} \right) &= \delta_t w_j^{n+\frac{1}{2}} + O(\tau^2), \quad \frac{\partial w}{\partial x} \left(x_j, t_{n+\frac{1}{2}} \right) = \delta_x w_j^{n+\frac{1}{2}} + O(h^2), \\ \frac{\partial^2 w}{\partial x^2} \left(x_j, t_{n+\frac{1}{2}} \right) &= \delta_x^2 w_j^{n+\frac{1}{2}} + O(h^2), \quad \frac{\partial^3 w}{\partial x^3} \left(x_j, t_{n+\frac{1}{2}} \right) = \delta_x^3 w_j^{n+\frac{1}{2}} + O(h^2), \\ \frac{\partial^5 w}{\partial x^5} \left(x_j, t_{n+\frac{1}{2}} \right) &= \delta_x^5 w_j^{n+\frac{1}{2}} + O(h^2). \end{aligned} \quad (20)$$

Substituting Expressions (20) into Eq. (19), we derive that

$$36\delta_t w_j^{n+\frac{1}{2}} + \delta_x^5 w_j^{n+\frac{1}{2}} + \frac{15}{2}\delta_x w_j^{n+\frac{1}{2}} (\delta_x^2 w_j^{n+1} + \delta_x^2 w_j^n) + \frac{15}{2}w_j^{n+\frac{1}{2}} (\delta_x^3 w_j^{n+1} + \delta_x^3 w_j^n) + \frac{45}{2}(w_j^{n+\frac{1}{2}})^2 (\delta_x w_j^{n+1} + \delta_x w_j^n) - 5\delta_x^3 w_j^{n+\frac{1}{2}} - 15w_j^{n+\frac{1}{2}} (\delta_x w_j^{n+1} + \delta_x w_j^n) - 5\delta_x w_j^{n+\frac{1}{2}} = R_j^{n+\frac{1}{2}}, \quad (21)$$

where $R_j^{n+\frac{1}{2}} = O(h^2 + \tau^2)$. Omitting $R_j^{n+\frac{1}{2}}$, we obtain that

$$36\delta_t w_j^{n+\frac{1}{2}} + \delta_x^5 w_j^{n+\frac{1}{2}} + \frac{15}{2}\delta_x w_j^{n+\frac{1}{2}} (\delta_x^2 w_j^{n+1} + \delta_x^2 w_j^n) + \frac{15}{2}w_j^{n+\frac{1}{2}} (\delta_x^3 w_j^{n+1} + \delta_x^3 w_j^n) + \frac{45}{2}(w_j^{n+\frac{1}{2}})^2 (\delta_x w_j^{n+1} + \delta_x w_j^n) - 5\delta_x^3 w_j^{n+\frac{1}{2}} - 15w_j^{n+\frac{1}{2}} (\delta_x w_j^{n+1} + \delta_x w_j^n) - 5\delta_x w_j^{n+\frac{1}{2}} = 0. \quad (22)$$

Eq. (22) is collapsed to obtain that

$$w_j^{n+1} + \frac{\tau A^{n+1}}{144h^5} + \frac{5\tau GB^{n+1}}{96h^3} + \frac{\tau(15H-10)C^{n+1}}{288h^3} + \frac{\tau(45H^2-60H-20)D^{n+1}}{576h} = w_j^n - \frac{\tau A^n}{144h^5} - \frac{5\tau GB^n}{96h^3} - \frac{\tau(15H-10)C^n}{288h^3} - \frac{\tau(45H^2-60H-20)D^n}{576h}. \quad (23)$$

By the iterative method [47], the iterative scheme for Finite Difference Scheme (23) can be written as

$$w_j^{n+1,l+1} + \frac{\tau A^{n+1,l+1}}{144h^5} + \frac{5\tau \tilde{G}B^{n+1,l+1}}{96h^3} + \frac{\tau(15\tilde{H}-10)C^{n+1,l+1}}{288h^3} + \frac{\tau(45\tilde{H}^2-60\tilde{H}-20)D^{n+1,l+1}}{576h} = w_j^n - \frac{\tau A^n}{144h^5} - \frac{5\tau \tilde{G}B^n}{96h^3} - \frac{\tau(15\tilde{H}-10)C^n}{288h^3} - \frac{\tau(45\tilde{H}^2-60\tilde{H}-20)D^n}{576h}, \quad (24)$$

where

$$\begin{aligned} A^\gamma &= -w_{j-3}^\gamma + 4w_{j-2}^\gamma - 5w_{j-1}^\gamma + 5w_{j+1}^\gamma - 4w_{j+2}^\gamma + w_{j+3}^\gamma, \quad B^\gamma = w_{j-1}^\gamma + w_{j+1}^\gamma - 2w_j^\gamma, \\ C^\gamma &= -w_{j-2}^\gamma + 2w_{j-1}^\gamma - 2w_{j+1}^\gamma + w_{j+2}^\gamma, \quad D^\gamma = w_{j+1}^\gamma - w_{j-1}^\gamma, \quad H = w_j^{n+1} + w_j^n, \quad \tilde{H} = w_j^{n+1,l+1} + w_j^n, \\ G &= -w_{j-1}^{n+1} - w_{j-1}^n + w_{j+1}^{n+1} + w_{j+1}^n, \quad \tilde{G} = -w_{j-1}^{n+1,l+1} - w_{j-1}^n + w_{j+1}^{n+1,l+1} + w_{j+1}^n, \\ \gamma &= n \text{ or } n+1 \text{ or } n+1, l+1, \quad w_j^{n+1,l=0} = w_j^n, \quad \lim_{l \rightarrow +\infty} w_j^{n+1,l+1} = w_j^{n+1}, \end{aligned}$$

and $l = 0, 1, 2, \dots$ denotes the iteration time.

From t_n to t_{n+1} , the iteration stops when $|w_j^{n+1,l+1} - w_j^{n+1,l}| < \varepsilon$, with ε being a given error bond.

We let the error between the analytic solutions $w(x_j, t_n)$ and the numerical solutions w_j^n as

$$\|E(h, \tau)\|_\infty = \max_{\substack{0 \leq j \leq J \\ n \geq 0}} |w(x_j, t_n) - w_j^n|, \quad (25)$$

and define

$$\text{Rate} = \log_2 \left[\frac{\|E(h, \tau)\|_\infty}{\|E(h/2, \tau/2)\|_\infty} \right], \quad (26)$$

as an approximation of the rate of convergence when both h and τ are sufficiently small. As such, if the scheme is the second-order accurate in x direction and the second-order accurate in t direction, the obtained Rate should be 2.

We choose the analytic one-soliton solution for Eq. (16),

$$w = \frac{1}{2} \text{sech}^2 \left[\frac{1}{2} \left(x + \frac{1}{4}t \right) \right]. \quad (27)$$

Then, we simulate the one-soliton evolution by choosing Solution (27) at $t = 0$ as the initial data. In our simulations, we take the interval $\Omega = [-30, 30]$, $h = 0.2$, $\tau = 0.05$ and error bond $\varepsilon = 10^{-8}$ for the iterative computation of Scheme (24).

Fig. 5(a) shows the propagation of the one soliton via Solution (27). Fig. 5(b) displays the numerical one-soliton solution. Fig. 5(c) depicts a comparison of the analytical and numerical one-soliton solutions with t .

Figs. 5. (a) Analytical solution of the one soliton via Solution (27); (b) Numerical solution of the one soliton via Eq. (24); (c) Comparison of the analytical one-soliton solution and numerical one-soliton solution .

Table 1: Errors and convergence rates under different steps at $t=1$

(h, τ)	$\ E(h, \tau)\ _\infty$	Rate
$(\frac{1}{2}, \frac{1}{500})$	3.75727×10^{-3}	—
$(\frac{1}{4}, \frac{1}{1000})$	9.79221×10^{-4}	1.93998
$(\frac{1}{8}, \frac{1}{2000})$	2.49984×10^{-4}	1.96980
$(\frac{1}{16}, \frac{1}{4000})$	6.21383×10^{-5}	2.00828

Table 1 shows the errors and the convergence rates of the analytic and numerical one-soliton solutions under different h and τ at $t = 1$. From Table 1, we find that the errors between the

numerical and analytic one-soliton solutions are small and the convergence rates are close to 2, which is the result as expected.

6. Conclusions

In this paper, symbolic computation has been conducted on a (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada system in fluid mechanics, i.e., System (1). Via the Hirota method, Singular N -Soliton Solutions (5), Shock-Wave Solutions (9), Breather-Stripe Soliton Solutions (12) and Hybrid Solutions (15) have been derived. Via Finite Difference Scheme (23), we have simulated the one-soliton propagation for Eq. (16).

Figs. 1 and 2 have shown the propagations of the singular one soliton and the shock wave, respectively. We have observed that the singular one-soliton solutions are explosive and unstable, but the shock-wave solutions are nonsingular and stable.

Figs. 3 have displayed the propagation of the breather-stripe soliton. We have observed that the breather-stripe soliton moves along the negative direction of the y axis. We have found that the amplitude and shape of the breather-stripe soliton remain invariant during the propagation, as shown in Figs. 3.

Based on Solutions (15), we have investigated the interaction among a rogue wave, a periodic wave and a pair of the stripe solitons, as shown in Figs. 4. It has been seen that there is a periodic wave between a pair of the stripe solitons, as shown in Fig. 4(a_1) and (b_1). As t goes on, we have observed that the rogue wave arises from the one stripe soliton and interacts with the periodic wave, next the rogue wave splits into two waves, then the two waves merge into one wave, and finally the rogue wave fuses with the other stripe soliton, as seen in Figs. 4(a_2)-(a_5) and (b_2)-(b_5).

Figs. 5(a) and (b) have displayed the analytic one-soliton and numerical one-soliton solutions, respectively. Fig. 5(c) has depicted a comparison of the analytic and numerical solutions with t , which indicates that the numerical one-soliton solution which agree with the analytic one-soliton solution. Table 1 has shown the errors and the convergence rates of the analytic and numerical one-soliton solutions under different h and τ at $t = 1$. From Table 1, we have found that the errors between the numerical and analytic one-soliton solutions are small and the convergence rates are close to 2, which is the result as expected.

Hirota method has been used to find the analytic solutions for the NLEEs. Advantage of the Hirota method over the others is that it is more convenient to produce results for certain NLEEs. We expect that the results of this paper will be helpful to the propagation of waves in fluid mechanics.

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Data Availability Some or all data, models or code generated or used during the study are available from the corresponding author by request.

Compliance with ethical standards Research does not involve Human Participants and/or Animals.

Conflict of interest The authors declare that they have no conflict of interest.

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