

# Chaotic model for COVID-19 growth-factor

Thiago de Salazar e Fernandes (✉ [thiago.fernandes@ufpe.br](mailto:thiago.fernandes@ufpe.br))

Universidade Federal de Pernambuco <https://orcid.org/0000-0002-1053-1943>

---

## Short Report

**Keywords:** Fractal analysis, deterministic chaos, detrended fluctuation analysis, Coronavirus, COVID-19

**Posted Date:** April 22nd, 2020

**DOI:** <https://doi.org/10.21203/rs.3.rs-23661/v1>

**License:**  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

---

**Version of Record:** A version of this preprint was published at Research on Biomedical Engineering on July 26th, 2020. See the published version at <https://doi.org/10.1007/s42600-020-00077-5>.

# Abstract

The new COVID-19 Pandemic caused by SARS-CoV-2 initiated in the world a largest quarantine, due to its exponential capacity of the virus in spreading from human contact. In the present work, it was evaluated the dynamics of such spreading by the indicator of growth-factor, and applied to it the space phase of the time series, the detrended fluctuation analysis (DFA) of the series, and the fractal dimension of the space phase. It was possible to notice a strange attractor in the space phase of the growth-factor, indicating that the process is chaotic deterministic. The value of the alpha coefficient by DFA showed to be less than 0.5, characteristic of long-range memory of the series, in which large events precedes small events. The fractal dimension of the phase space was a fractal number, between 1 and 2, another indicator that the exponential growth-rate of the virus spreading among humans is fractal. These results, even with small number of data, is pointing that the spread of COVID-19 is fractal.

## Introduction

Some processes in nature can be described as being non-linear, that means that just small changes in the starting point can lead to enormous consequences. This is the case of the new infections by SARS-CoV-2, and the emergence of COVID-19 in the world. Just small changes in the genome of the first virus, SARS-CoV (2002), resulted in a more severe spread and human infection of SARS-CoV-2, leading to the Pandemic of COVID-19.

Recently, many countries in the world started the suppression of the virus by quarantine, with social distance of healthy individuals and isolation of elder or those with clinical symptoms such as fever. Schools, works, shopping centers were shut down. Work done by Imperial College showed that if the governments does not act in suppression, but only in mitigation, the number of deaths would reach the thousands in some countries, and millions in the world (Ferguson NM et al. 2020).

Understanding exponential growth is difficult for human minds that evolved in a world in which daily tasks need only the linearity of events. It is hard to figure out that when we reach 30 steps in an exponential manner, we do not reach something linear to that such as 30 "something", but we are already in the scale of millions. Therefore, for leading to coronavirus new infection, it is necessary to act very early for not allowing that SARS reach a great number of people leading to the collapse of the health system of the city, or of the entire country.

But if the understanding of exponential growth is already difficult to imagine for human mind, imagine the understanding of chaotic behavior of many biological systems and Epidemics. Chaos here does not mean the complete disorder of the system as the media is commonly associating this word with, but when a system is determined by iteration of logistic equations resulting in unpredictable results (Mandelbrot, 1983; Liebovitch 1998; Stam 2005; Kunicki et al. 2009). This means that even being very similar to random processes, or with an erratic behavior, deterministic chaos present self-similarity well

known as resulting from fractal processes, and this means that there is a long-range correlation also known as the memory of the system (Peng et al. 1994; Stanley et al. 1996; Havlin et al. 1999; Wang et al. 1995; Peng et al. 1995). Therefore, chaos is the result of complex systems, when components of the system are correlated and competing for surviving (such as rabbits and wolves, or fruit flies in a flask, sharks and shrimp, and also virus and humans).

In this work, it was investigated the chaotic or fractal behavior of the growth-factor of COVID-19 spread, in order to bring light to better understand the attractor of this disease, and how we can change it over time.

## Methods

The data of growth-factor of COVID-19 cases in the world that is calculated by every day's new cases divided by new cases on the previous day (from January 24<sup>th</sup> to April 9<sup>th</sup>), was obtained from the website:

<https://www.worldometers.info/coronavirus/coronavirus-cases/#cases-growth-factor>.

A growth-factor higher than 1 indicate an increase of cases. Between 0 and 1 it is a sign of decline, and constantly above 1 is a sign of exponential growth.

This series was then submitted to the Detrended fluctuation analysis (DFA). DFA was calculated as the algorithm of Peng et al. (1994; 1995), using the software GNU Octave (<https://www.gnu.org/software/octave/>). The series was integrated and divided into boxes of equal length ( $n$ ). In each box of length  $n$ , a least-square line was fitted to the data representing the trend in that box. The  $y$  coordinate of the straight line segments was denoted by  $yn(k)$ . Computing DFA was performed according to equation 1.

[Please see the supplementary files section to view the equation.] (1)

This computation was repeated over all the time scales (box sizes) to provide a relationship between the average root-mean fluctuation function  $F(n)$  and the box size  $n$ .  $F(n)$  would increase with box size  $n$ . A power-law relationship on a double log plot indicates the presence of scaling. The fluctuations could be characterized by a scaling exponent  $\alpha$ , the slope of the line relating  $\log F(n)$  to  $\log n$ :  $F(n) \sim n^\alpha$ , in order to provide a more accurate estimate of  $F(n)$ . If  $\alpha = 0.5$ , the series was the result of a random event;  $\alpha > 0.5$  indicated the persistent long-range correlations. The other values were:  $\alpha = 1$  corresponded to  $1/f$  noise (very rough landscape); the  $\alpha \geq 1$  – correlations existed but ceased to be of a power-law form or a random walk-like fluctuation; and finally,  $\alpha = 1.5$  – Brown noise; the integration of white noise (very smooth landscape).

The phase space of the data was also generated by plotting in Microsoft Excel (2007), by plotting each value of growth-factor of day 'x' by the value in the previous day 'x-1', building a logistic map.

The image of the phase space was submitted to fractal analysis using box-counting method of the plugin "fracLac" of the software ImageJ™, available for free download (<https://imagej.nih.gov/ij/plugins/fraclac/fraclac.html>), and using Fractal Dimension Estimator (FDE), a software tool to measure the fractal dimension (FD) of a 2D image, available (<http://www.fractal-lab.org/Downloads/FDEstimator.html>). This method consists in the superposition of boxes in the image, where the lateral of the boxes is continuously reduced in size. In each of this reduction, it is counted the number of boxes necessary to cover pixels of the image.

After this, the fractal dimension (FD) is calculated by equation 2:

[Please see the supplementary files section to view the equation.] (2)

Where  $N_r$  is the number of boxes necessary to cover the image in each progressive reduction of the side of the box ( $r$ ). FD will be the slope of the regression line, generated by the logarithm of the quantity of boxes in function to the logarithm of the size of the boxes.

## Results

In Figure 1, it is shown the time series of growth-factor from January to April, of the growth-factor of COVID-19.

The detrended fluctuation analysis (DFA) of the time series resulted in an alpha coefficient ( $\alpha$ ) of 0.27957, and a dimension of 2.7204.

The data was reorganized in order to produce a phase space, in which the next point of growth-factor was plotted to the point in previous time (Figure 2), showing a strange attractor characteristic of a deterministic chaotic process.

The fractal dimension of the attractor obtained in the phase space was calculated by Fractal Dimension Estimator, and the value of FD was 1.33027 (Figure 3).

When the fractal dimension was estimated by imageJ software, the value of FD was 1.1623.

## Discussion

It is well known that an alpha coefficient ( $\alpha$ ) from DFA between 0 and 0.5 corresponds to a power-law correlation such that large and small values of the time series are more likely to alternate (Peng et al.

1994, 1995).

In terms of growth-factor of cases of COVID-19, this represents the dynamics day by day from January 24<sup>th</sup> to April 9<sup>th</sup> of new cases of COVID-19. Growth-factor above 1 is a sign of increase of cases, between 0 and 1 is a decline, and constantly 1 is an exponential growth of cases. The results show that the pattern is sometimes above 1, but sometimes not, with a chaotic pattern oscillating between exponential increase and decline of COVID-19.

In practical terms, it is clear that the spread is exponential and suppression is the best way for controlling the high number of infected people that can lead to the collapse of health system. But in real terms of the dynamics of the disease, the spread is much more complex, non-random among people, and there is a fractal pattern behind the dissemination of the virus, probably related to the human social contacts, contact among member of the family or the community. The dynamics of social contact is not such as a Brownian movement of molecules in a test tube, but a much more complex system because members of a community and society is more likely to involve a certain number of possible knots of relationships, such as colleagues of work, relatives, friends, social behaviors and habits.

The fractal dimension of the phase space also indicated the fractality of this dynamical process, reinforcing the presence of self-similarity or long-range memory in this time series (Mandelbrot, 1983; Liebovitch 1998; Stam 2005; Kunicki et al. 2009). The practical consequence of these results is that the small changes in the initial conditions can bring great changes in the dynamics of the spread of this disease. In other words, social distance can bring the spread of this disease to a small number. But also in opposite side of the same strong argument, if nothing is done, the Epidemic can reach catastrophic consequences.

One should take attention to the peak in Figure 1, what seems a break of the attractor pattern, but actually it can be inferred that this peak may be repeated in the future. That is where our attention has to be, in order to avoid this future by changing the pattern of the spread of this disease.

## **Conclusion**

The growth-factor of COVID-19 shows a fractal pattern. This means that the growth of cases is oscillating from large values to small values, as a characteristic of a chaotic behavior, and not just by a constantly exponential growth. Understanding the complex dynamics of COVID-19 spread is crucial for decision making, such as keeping social distance and finding new drugs and vaccines, and for a better prospective protection to this disease.

## **Declarations**

### **Acknowledgement**

The author would like to thank the Federal University of Pernambuco.

## Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

## References

Ferguson NM, Laydon D, Nedjat-Gilani G, Imai N, Ainslie K, Baguelin M, Bhatia S, Boonyasiri A, Zulma C, Cuomo-Dannenburg G, Dighe A, Dorigatti I, Fu H, Gaythorpe K, Green W, Hamlet A, Hinsley W, Okell LC, van Elsland S, Thompson H, Verity R, Volz E, Wang H, Wang Y, Walker PGT, Winskill P, Whittaker C, Donnelly CA, Riley S, Ghani AC. Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand. Imperial College COVID-19 Response System, 2020.

Havlin S, Buldyrev SV, Bunde A, Goldberger AL, Ivanov PCh, Peng CK, et al. Scaling in nature: from DNA through heartbeats to weather. *Phys A*. 1999; 273: 46-49.

Kunicki ACB, Oliveira AJ, Mendonça MBM, Barbosa CTF, Nogueira RA. Can the fractal dimension be applied for the early diagnosis of non-proliferative diabetic retinopathy? *Braz J Med Biol Res*. 2009; 42: 930-934.

Liebovitch LS. *Fractals and chaos simplified for life sciences*. New York: Oxford University Press; 1998.

Mandelbrot BB. *The Fractal Geometry of Nature*. 2nd. ed. New York: Ed. Freeman; 1983.

Peng CK, Buldyrev SV, Havlin S, Simons M, Stanley HE, Goldberger AL. Mosaic organization of DNA nucleotides. *Phys Rev E*. 1994; 49: 1685-1689.

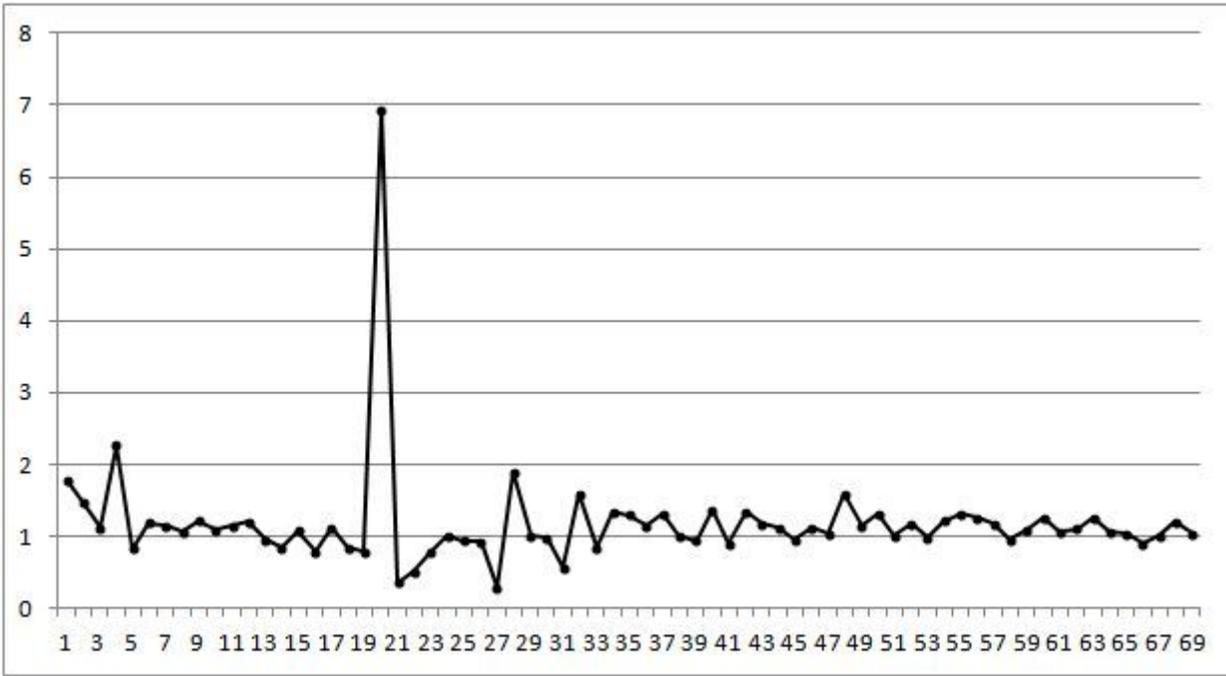
Peng CK, Havlin S, Stanley HE, Goldberger AL. Quantification of scaling exponents and crossover phenomena in non-stationary heartbeat time series. *Am Inst Phys*. 1995; 5: 82-87.

Stam CJ. Nonlinear dynamical analysis of EEG and MEG: Review of an emerging field. *Clin Neurophysiol*. 2005; 116: 2266-2301.

Stanley HE, Afanasyev V, Amaral LAN, Buldyrev SV, Goldberger AL, Havlin S, et al. Anomalous fluctuations in the dynamics of complex systems: from DNA and physiology econophysics. *Phys A*. 1996; 224: 302-321.

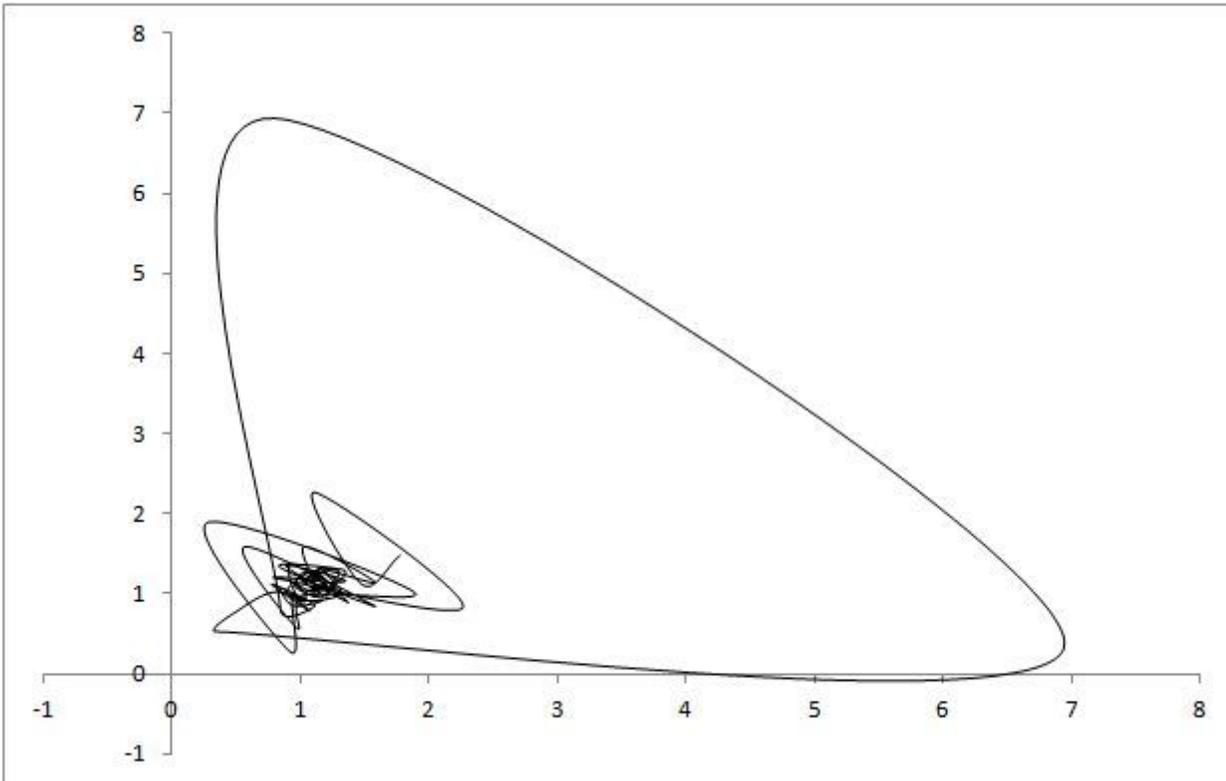
Wang SC, Li PC, Tseng HC. Long range correlation and possible electron conduction through DNA sequences. *Phys A*. 2008; 387: 5159-5168.

## Figures



**Figure 1**

Growth-factor of COVID-19 from January 24th to April 9th of 2020.



**Figure 2**

Strange attractor of COVID-19 cases from January 24th to April 9th.

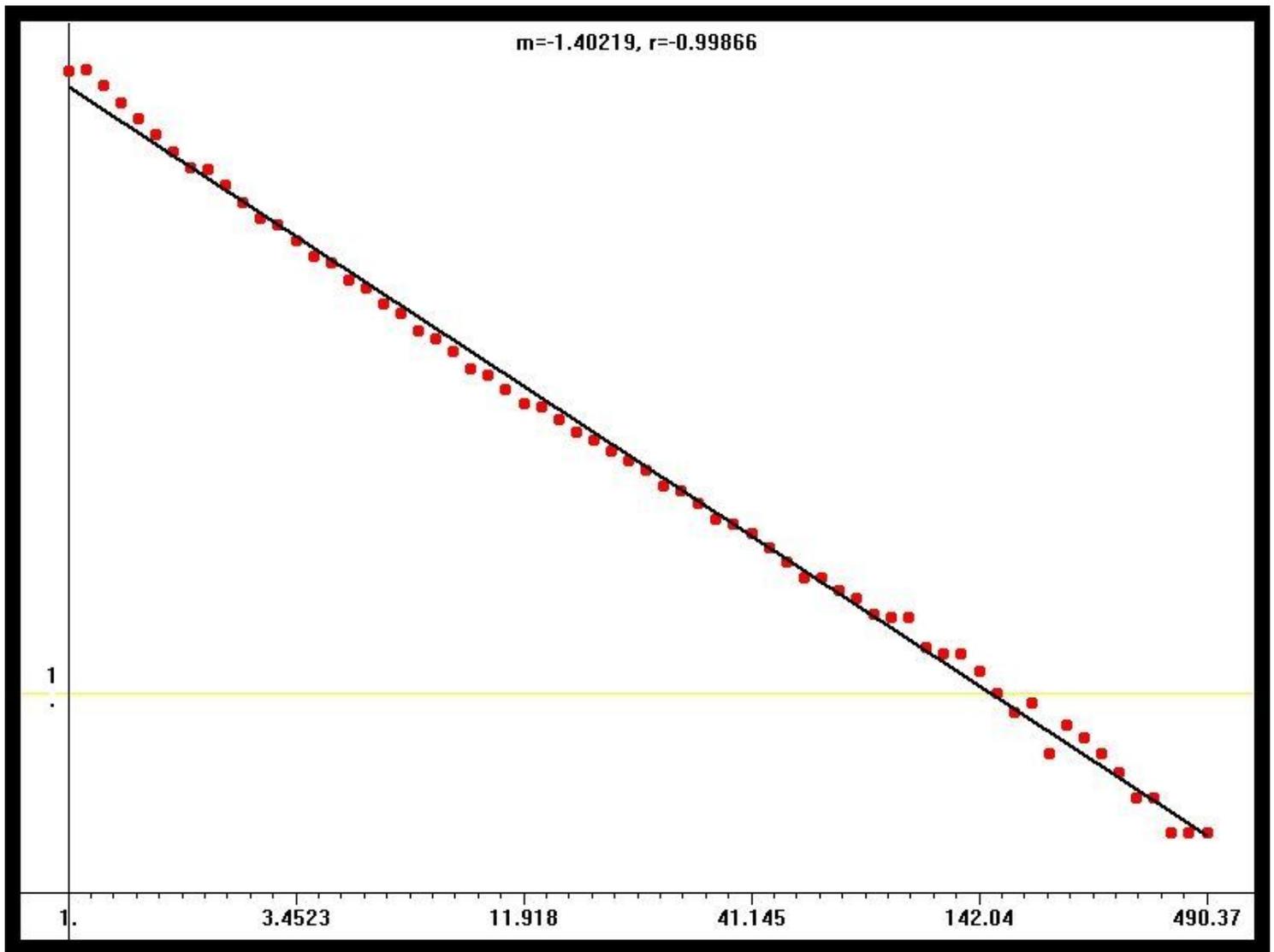


Figure 3

Fractal dimension of the strange attractor of COVID-19 growth-factor.

## Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [Equations.docx](#)