Jointly Optimized Design of Distributed RS Codes by Proper Selection in Relay

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Research Article

Keywords: Reed-Solomon codes, distributed coding schemes, information selection approach, joint decoding algorithms

Posted Date: June 2nd, 2021

DOI: https://doi.org/10.21203/rs.3.rs-239203/v1

Version of Record: A version of this preprint was published at Telecommunication Systems on August 16th, 2021. See the published version at https://doi.org/10.1007/s11235-021-00822-w.

Noname manuscript No.
(will be inserted by the editor)

Jointly Optimized Design of Distributed RS Codes by Proper Selection in Relay

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Received: date / Accepted: date

Abstract This paper proposes a distributed RS coding scheme which is comprised of two different Reed-Solomon (RS) codes over fast Rayleigh fading channel. Practically in any distributed coding scheme, an appropriate encoding strategy at the relay plays a vital role in achieving an optimized code at the destination. Therefore, the authors have proposed an efficient approach for proper selection of information at the relay based on subspace approach. Using this approach as the proper benchmark, another more practical selection approach with low complexity is also proposed. Monte Carlo simulations demonstrate that the distributed RS coding scheme under the two approaches can achieve nearly the same bit error rate (BER) performance. Furthermore, to jointly decode the source and relay codes at the destination, two different decoding algorithms named as naive and smart algorithms are proposed. The simulation results reveal that the advantage of smart algorithm as compared to naive one. The proposed distributed RS coding scheme with smart algorithm outperforms its non-cooperative scheme by a gain of 2.4-3.2 dB under identical conditions. Moreover, the proposed distributed RS coding scheme outperforms multiple existing distributed coding schemes, making it an excellent candidate for the future distributed coding wireless communications.

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1 Introduction

In wireless communication, the performance of the communication system is degraded due to signal fading. Diversity techniques [1, 21] are effective ways to combat the fading effects. In the wireless communication systems, the three basic technologies that provide diversity are time diversity, space diversity and frequency diversity. Multiple-input multiple-output (MIMO) is a breakthrough technology in wireless communications, where the transmitters and receivers are both equipped with multiple antennas. This technology can effectively combat the influence of wireless channel fading and has been recognized as a core technique for improving the spectral efficiency and channel capacity. However, owing to the limitation of their size, this useful technique cannot be directly applied to mobile equipment. In order to address this challenge, the idea of cooperative diversity [2, 3] technique came into being to realize the idea of MIMO in a distributed way. This technique means that the transmitters share each other's antennas to provide a virtual MIMO technique. Therefore, the idea of cooperative cooperation can be used to obtain similar spatial diversity gain as conventional MIMO technique.

A simplified cooperative communication system consists of three nodes named as source, relay and destination. In a cooperative communication system, a relay node repeats the received information from source node through either relaying or hard detection as discussed in [2]. Different protocols of cooperative communication have been proposed such as decoded-and-forward [24], compress-and-forward [25] and amplify-and-forward [26]. These protocols have been combined with channel codes to construct a more efficient way of cooperation known as coded-cooperative. Various coded-cooperative schemes have been presented in literature such as low density parity check (LDPC) codes [4, 5], turbo codes [6, 7], polar codes [8–10] and Reed-Muller (RM) codes [11, 12].

The fifth generation (5G) wireless systems are required to support the traffic originated by miscellaneous types of communication networks such as device-to device (D2D) and machine-to machine (M2M). Such type of communication may demand a short length message sequence rather than a long message sequence. In [19], the short length binary code RM code has been used in the coded-cooperative scheme and the scheme constructs a subcode at the destination which acquires a better bit-error-rate (BER) performance. Also, in [18, 20], polar codes have been applied in the code cooperation scheme. The idea of constructing subcode is also mentioned in the schemes of their paper. However, in the existing literature, the coded-cooperative scheme based on short length non-binary codes to generate a subcode has not been well investigated. As a well-known non-binary short length RS code [14, 16], it is a Maximum Distance Separable (MDS) code and performs well in correcting random burst errors. The author in [16] has proposed a coded-cooperative scheme based on RS code, however, the scheme does not include the idea of subcode. In this paper, a distributed RS coding scheme which jointly constructs a new code at the destination is proposed. In this proposed scheme, two different RS codes are applied to the source and relay nodes. The relay repeats the received information from the source node. Therefore, the code of the destination node which is jointly constructed by the source and relay nodes, is a subcode. Thus, a well designed code at the relay has a great influence on the coded cooperation scheme. Based on this fact, firstly an efficient approach is proposed as a benchmark for proper selection at the relay. Secondly, a low complexity information selection approach is proposed.

The remaining structure of this paper is organized as follows. In Section 2, a general design of distributed linear block coding scheme by subspace approach is introduced. Section 3 focuses on presenting the system model of distributed RS coding scheme and discussing the design of optimized subcode at the destination. Section 4 describes two design approaches for partial encoding at the relay. The joint RS decoding based on two decoding algorithms, naive and smart algorithms are established in Section 5. The numerical simulation results of the distributed RS coding scheme over fast Rayleigh fading channel are presented in Section 6. Finally, Section 7 concludes the paper.

2 General design of distributed linear block codes by subspace approach

A generalized distributed linear block coding scheme [15] is shown in Fig.1. It consists of three communication nodes named as the source S, the relay R, and the destination D. All these nodes transmit and receive signals through a single antenna. In the scheme, two linear block codes $C_1(N, K_1)$ and $C_2(N, K_2)$ are employed at the source and relay nodes respectively, where N is both the codeword length of C_1 and C_2, K_1 and K_2 ($K_1 < K_2$) are the code dimensions of C_1 and C_2, respectively. It takes two time slots to complete end-to-end transmission of a message sequence. During time-slot 1, the source encodes the message sequence $\mathbf{a}_1$ to the codeword $\mathbf{b}_1 \in C_1$ and broadcasts the coded symbol sequence $\mathbf{x}_S = [x_S^0, x_S^1, \cdots, x_S^{N-1}]$ to R and D nodes simultaneously, where $x_S^{i_1}$ denotes the M-QAM modulated symbol for $i_1 = 0, 1, \cdots, N-1$. At the i_1-th time instant, the modulated symbol $x_S^{i_1}$ is transmitted to the relay node through the Rayleigh fading channel. The received signals of the relay node $y_{S,R}^{i_1}$ and destination node $y_{S,D}^{i_1}$ can be written as

$$y_{S,R}^{i_1} = h_{S,R}^{i_1} x_S^{i_1} + n_{S,R}^{i_1} \tag{1}$$

$$y_{S,D}^{i_1} = h_{S,D}^{i_1} x_S^{i_1} + n_{S,D}^{i_1} \tag{2}$$

where $n_{S,R}^{i_1}$ is a complex Gaussian variable with zero-mean and $\sigma^2/2$ variance per dimension at the i_1-th time instant, where σ^2 is power spectral density (PSD) of noise. The $h_{S,R}^{i_1}$ is a complex Gaussian variable with zero-mean and 0.5-variance per dimension at the i_1-th time instant. The $n_{S,D}^{i_1}$ and $h_{S,D}^{i_1}$ are defined like $n_{S,R}^{i_1}$ and $h_{S,R}^{i_1}$. After transmitting N modulated symbols, the received signal sequence at the relay node is $\mathbf{y}_{S,R} = [y_{S,R}^0, y_{S,R}^1, \cdots, y_{S,R}^{N-1}]$ and the received signal sequence at the destination node is $\mathbf{y}_{S,D} = [y_{S,D}^0, y_{S,D}^1, \cdots, y_{S,D}^{N-1}]$.

During time-slot 2, the relay obtains $\mathbf{a}_1$ by correct decoding under the condition of an ideal source-relay channel. In the scheme, no additional message symbols are generated at the relay as all the message symbols are just transmitted from the source. Thus, these K_2

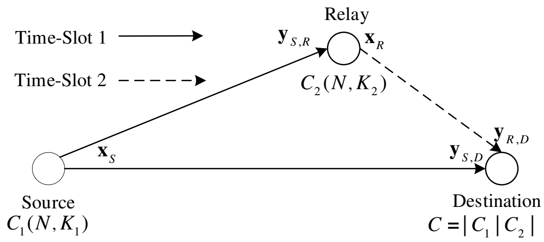

Fig. 1 General system model of distributed linear block codes

message symbols for $C_2(N, K_2)$ code are selected dependent on $\mathbf{a}_1$ and generated the codeword $\mathbf{b}_2 \in C_2$ at the relay. After modulation, the relay node transmits the signal $\mathbf{x}_R = [x_R^0, x_R^1, \cdots, x_R^{N-1}]$ to the destination node. At the i_2-th time instant, the received signal $y_{R,D}^{i_2}$ can be given as

$$y_{R,D}^{i_2} = h_{R,D}^{i_2} x_R^{i_2} + n_{R,D}^{i_2} \tag{3}$$

where $i_2 = 0, 1, \cdots, N-1$, $n_{R,D}^{i_2}$ and $h_{R,D}^{i_2}$ are defined like $n_{S,R}^{i_1}$ and $h_{S,R}^{i_1}$ in (1). After transmitting N modulated symbols, the received signal sequence at the destination node is $\mathbf{y}_{R,D} = [y_{R,D}^0, y_{R,D}^1, \cdots, y_{R,D}^{N-1}]$. At the destination, the overall received signal $\mathbf{y}$ is jointly constructed by the source and relay nodes, as shown below:

$$\mathbf{y} = |\mathbf{y}_{S,D}|\mathbf{y}_{R,D}| \tag{4}$$

where '|' represents the concatenation of the two received message sequences during the time-slot 1 and the time-slot 2. At the destination, the demodulated codeword $|\mathbf{b}_1|\mathbf{b}_2| \in |C_1|C_2|$ belongs to a new code C. If the relay selects K_2 additional message symbols independent of $\mathbf{a}_1$ to generate the codeword $\mathbf{b}_2^* \in C_2$. Through the joint construction, the destination gets the codeword $|\mathbf{b}_1|\mathbf{b}_2^*| \in \overline{C}$. The resultant code C is a subcode of $\overline{C}$, i.e., $C \subset \overline{C}$.

As mentioned earlier, few of non-binary codes are used in this model. The famous short length non-binary code RS code, which is a special type of MDS code, reaches the singleton bound and performs well in correcting random burst errors. Therefore, we employ RS code in this distributed coding scheme.

3 Optimized design for distributed RS codes

The system model of distributed RS coding scheme is shown in Fig.2. A complete distributed transmission of the source generated information sequence $\mathbf{m}_1$ requires two time slots.

During time-slot 1, the source maps the binary message sequence $\mathbf{m}_1$ to the non-binary M-ary symbol sequence $\mathbf{a}_1$ which are the symbols in GF(2^n), $n > 1$, where n is an integer. Then non-binary sequence $\mathbf{a}_1$ is encoded by systematic RS code $RS_1(N, K_1, d_1)$, where N is codeword length, K_1 is the code dimension and d_1 is the minimum distance of RS_1. Since RS code is MDS code, d_1 is exactly equal to $N - K_1 + 1$. Let α be a primitive element in GF(2^n). Then, the generator polynomial of RS code RS_1 contains $N - K_1$ consecutive roots which can be given as

$$\mathbf{g}_1(X) = (X - \alpha)(X - \alpha^2)\cdots(X - \alpha^{N-K_1}) \tag{5}$$

where $\alpha^{j_1} \in$GF(2^n), $1 \le j_1 \le N - K_1$. The codeword of RS_1 can be generated as: $\mathbf{b}_1(X) = \mathbf{g}_1(X)\mathbf{a}_1(X)$, where $\mathbf{a}_1(X) = a_0^{(1)} + a_1^{(1)}X + a_2^{(1)}X^2 + \cdots + a_{K_1-1}^{(1)}X^{K_1-1}$ is the polynomial of message sequence $\mathbf{a}_1$, $a_{p_1}^{(1)} \in$GF(2^n), $p_1 = 0, 1, \cdots, K_1 - 1$, $\mathbf{b}_1(X) = b_0^{(1)} + b_1^{(1)}X + \cdots + b_{N-1}^{(1)}X^{N-1}$ is the polynomial of codeword $\mathbf{b}_1$, $b_{i_1}^{(1)} \in$GF(2^n). The systematic codeword polynomial of $\mathbf{b}_1(X)$ can be given as: $\mathbf{b}_1(X) = X^{N-K_1}\mathbf{a}_1(X) + \mathbf{p}_1(X)$, where $\mathbf{p}_1(X) = p_0^{(1)} + p_1^{(1)}X + \cdots + p_{N-K_1-1}^{(1)}X^{N-K_1-1}$ is the parity check polynomial and can be computed using polynomial division such as: $\mathbf{p}_1(X) = X^{N-K_1}\mathbf{a}_1(X)/\mathbf{g}_1(X)$, $p_{l_1}^{(1)} \in$GF(2^n), $l_1 = 0, 1, \cdots, N - K_1 - 1$. Then, after M-QAM modulation, the source node further broadcast the sequence $\mathbf{x}_S$ to relay and destination nodes simultaneously. During time-slot 1, the relay and destination receive signals $\mathbf{y}_{S,R}$ and $\mathbf{y}_{S,D}$, respectively.

During time-slot 2, the relay demodulates the received symbol sequence $\mathbf{y}_{S,R}$ and gets the estimated sequence $\widetilde{\mathbf{b}}_1$. The sequence $\widetilde{\mathbf{b}}_1$ is then decoded by M-ary RS_1 decoder using Euclidean algorithm [13] for decoding and provides non-binary estimated message sequence $\hat{\mathbf{a}}_1$. The relay selects only K_2 symbols from message sequence $\hat{\mathbf{a}}_1$ as the relay message sequence $\mathbf{a}_2$. The information selection approaches at the relay will be discussed in Section 4. Then non-binary sequence $\mathbf{a}_2$ is encoded by systematic RS code $RS_2(N, K_2, d_2)$, where N is codeword length, K_2 is the code dimension and d_2 is the minimum distance of RS_2. The generator polynomial of RS code RS_2 contains $N - K_2$ consecutive roots which can be given as

$$\mathbf{g}_2(X) = (X - \alpha)(X - \alpha^2)\cdots(X - \alpha^{N-K_2}) \tag{6}$$

The codeword of RS_2 can be generated as: $\mathbf{b}_2(X) = \mathbf{g}_2(X)\mathbf{a}_2(X)$, where $\mathbf{a}_2(X) = a_0^{(2)} + a_1^{(2)}X + a_2^{(2)}X^2 + \cdots + a_{K_2-1}^{(2)}X^{K_2-1}$ is the polynomial of message sequence $\mathbf{a}_2$, $a_{p_2}^{(2)} \in$GF(2^n), $p_2 = 0, 1, \cdots, K_2 - 1$ and $\mathbf{b}_2(X) = b_0^{(2)} + b_1^{(2)}X + \cdots + b_{N-1}^{(2)}X^{N-1}$ is the polynomial of codeword $\mathbf{b}_2$, $b_{i_2}^{(2)} \in$GF(2^n). The systematic

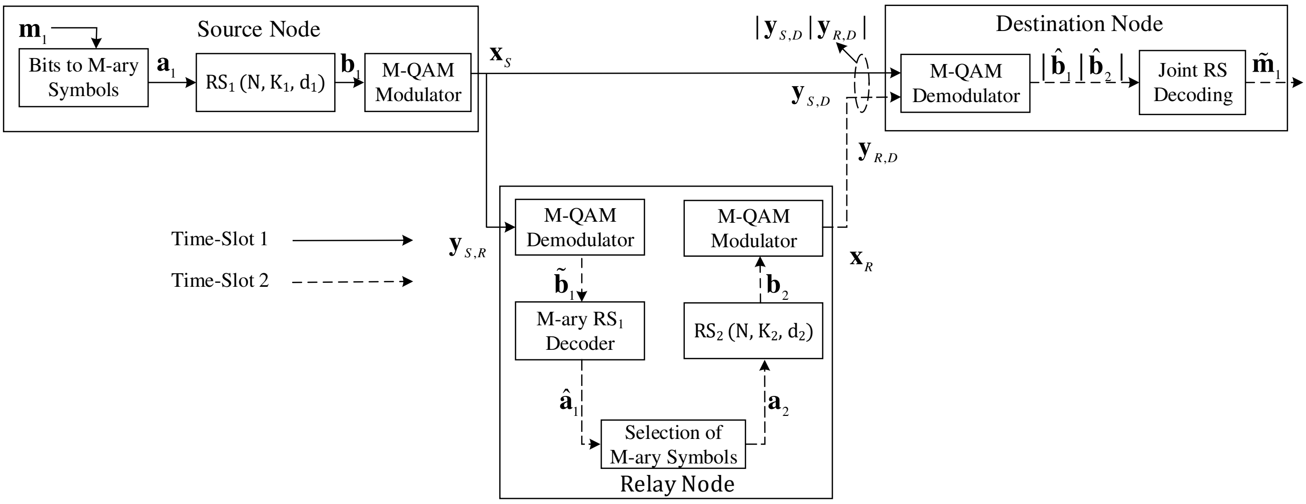

Fig. 2 Distributed RS coding scheme

codeword polynomial of $\mathbf{b}_2(X)$ can be given as: $\mathbf{b}_2(X) = X^{N-K_2}\mathbf{a}_2(X) + \mathbf{p}_2(X)$, where $\mathbf{p}_2(X) = p_0^{(2)} + p_1^{(2)}X + \cdots + p_{N-K_2-1}^{(2)}X^{N-K_2-1}$ is the parity check polynomial and can be computed using polynomial division such as: $\mathbf{p}_2(X) = X^{N-K_2}\mathbf{a}_2(X)/\mathbf{g}_2(X)$, $p_{l_2}^{(1)} \in$GF(2^n), $l_2 = 0, 1, \cdots, N-K_2-1$. The codeword $\mathbf{b}_2$ is modulated into $\mathbf{x}_R$ using M-QAM modulation. The relay transmits the modulated symbol sequence $\mathbf{x}_R$ to the destination. During time-slot 2, the destination receives signal $\mathbf{y}_{R,D}$.

At the destination, the signals transmitted from the source and relay are concatenated as $\mathbf{y}$ shown in (4). Owing to $K_2 > K_1$, the RS code RS_2 has better error correcting capability than RS_1 and the RS code RS_2 has more consecutive roots. These additional roots can be utilized by the joint RS decoder. Therefore, joint decoding can get performance gain from this construction. The two joint RS decoding algorithms are proposed i.e., naive algorithm and smart algorithm, which will be explained in section 5.

As mentioned in the previous section, the jointly constructed codeword at the destination belongs to the subcode C. The minimum Hamming distance of subcode C is d_3. We need to consider that large number of codewords with low weight d_3 might obtain at the destination owing to the worst scenario that the source generates a codeword with weight d_1 and the all-zero codeword is generated at the relay. Thus, the Hamming distance of the codeword at the destination is $d_3 = d_1$. To avoid the worst scenario described above, a new scheme is proposed to select a subcode C with as few as possible codewords of minimum weight d_3. If we meet the design standard, the destination node can obtain a subcode with a better weight distribution.

4 Design approaches for partial encoding at the relay

Two effective approaches for proper selection at the relay are proposed in this section. Before presenting design steps, some nomenclatures are explained below:

1) The first case is delineated as an event in which the Hamming weight at the source and relay are given as $wt(\mathbf{b}_1) = d_1$ and $wt(\mathbf{b}_2) = 0$, respectively. The codeword of minimum distance $d_3^{1st} = d_1$ is obtained at the destination.
2) The second case is delineated as an event in which $wt(\mathbf{b}_1) = d_1$, $wt(\mathbf{b}_2) = d_2$, and the codeword of distance $d_3^{2nd} = d_1 + d_2$ is obtained at the destination.
3) The third case is delineated as an event in which the codeword of Hamming distance d_3^{3rd} just greater than d_3^{2nd} is obtained at the destination.
4) Let v_1, v_2, v_3 indicate the number of occurrences of the first, second and third cases, respectively.
5) $h \to e$ indicates that the situation h leads to the situation e.
6) $|\theta|$ represents the cardinal number of any set θ.

4.1 Approach 1: Exhaustive search for all candidates

In this approach, an exhaustive search is performed for all candidates which are the message sequences with weight $0 < wt(\mathbf{a}_1) \le d_1$. All message sequences that can be encoded into codewords with weight d_1 are derived from these candidates. The specific steps of approach 1 are shown below:

Step 1: Determine $A = \{\mathbf{c}_k\}$ that is the set of all message sequences generating codewords with weight d_1 at the source, where $k = 1, 2, \cdots, |A|$.

Step 2: Determine $B = \{\boldsymbol{\lambda}_g\}$ that is the set of all selection patterns $\boldsymbol{\lambda}_g = [s_1, s_2, \cdots, s_{K_2}]$, where $s_i \in \{1, 2, \cdots, K_1\}(i = 1, 2, \cdots, K_2)$, $g = 1, 2, \cdots, L$ and L is defined as:

$$L = \binom{K_1}{K_2} = \frac{K_1!}{K_2!(K_1 - K_2)!} \tag{7}$$

Step 3: Determine v_1, $\forall \mathbf{c}_k \in A$ and $\forall \boldsymbol{\lambda}_g \in B$ by maintaining each unique combination $\boldsymbol{\lambda}_g$ fixed at the relay.

Step 4: Choose $\boldsymbol{\lambda}_g \to \min(v_1)$ and save them in the set H. If $|H| = 1$, move to step 9 else move to next step.

Step 5: Determine v_2, $\forall \mathbf{c}_k \in A$ and $\forall \boldsymbol{\lambda}_g \in H$ by maintaining each unique combination $\boldsymbol{\lambda}_g$ fixed at the relay.

Step 6: Choose $\boldsymbol{\lambda}_g \to \min(v_2)$ and save them in the set I. If $|I| = 1$, move to step 9 else move to next step.

Step 7: Determine v_3, $\forall \mathbf{c}_k \in A$ and $\forall \boldsymbol{\lambda}_g \in I$ by maintaining each unique combination $\boldsymbol{\lambda}_g$ fixed at the relay.

Step 8: Choose $\boldsymbol{\lambda}_g \to \min(v_3)$ and save them in the set J. If $|J| = 1$, move to step 9 else back to step 7 and increase the wt($\mathbf{b}_2$) by 1.

Step 9: The optimized selection pattern $\boldsymbol{\lambda}^{(1)} = \boldsymbol{\lambda}_g$ is choosed. End of approach 1.

The approach 1 is an effective way to select K_2 message symbols at the relay. However, for RS_1 code with larger N and K_1, the number of total candidates will become large. The complexity of determining the set A rises sharply. At the same time, the number of selection patterns will become huge. Therefore, this will bring more complexity to determine selection pattern $\boldsymbol{\lambda}^{(1)}$. Based on this fact, we propose another approach to deal with the increase in complexity.

4.2 Approach 2: Partial search for promising candidates

Approach 2 is applied to the selection procedure. At the source, each message sequence that can be encoded into a codeword with weight d_1 contains at least $\tau = K_1 - \min(K_1, d_1)$ zero symbols. The division strategy for dividing a codeword into two parts is designed. Scenario (a): the first part has one more symbol than the last part; Scenario (b): the last part has one more symbol than the first part. Therefore, the symmetric structure of the 11 message symbols is shown in Figure 4. According to this structure, we reasonably select part of all candidates as samples. This will effectively convert an exhaustive search into a partial search. The specific design steps of approach 2 are as follows:

Step 1: Determine the positions of τ zero symbols. Scenario (a): the w ($\lceil \tau/2 \rceil \le w \le \min(\lceil K_1/2 \rceil, \tau)$) zero symbols are randomly distributed in the first part, and the other $\tau - w$ zero symbols are uniquely assigned in the remaining part. Scenario (b): the w zero symbols are randomly distributed in the last part, and the other $\tau - w$ zero symbols are uniquely assigned in the remaining part, where $\lceil \cdot \rceil$ denotes ceil operation. Considering the two scenarios, the set $A_1 = \{\mathbf{c}_u\}$ is determined, which contains partial message sequences that generate the codewords with weight d_1, where $u = 1, 2, \cdots, |A_1|$.

Step 2: The relay selects K_2 message symbols out of K_1 recovered message symbols. Scenario (a): select μ ($\lceil K_2/2 \rceil \le \mu \le \min(\lceil K_1/2 \rceil, K_2)$) positions from the first part, and the other $K_2 - \mu$ positions are uniquely selected from the remaining part. Scenario (b): select μ positions from the last part, and the other $K_2 - \mu$ positions are uniquely selected from the remaining part. Considering the two scenarios, $B_1 = \{\boldsymbol{\lambda}_q\}$ is determined, which is a set of partial message selection patterns $\boldsymbol{\lambda}_q = [s_1, s_2, \cdots, s_{K_2}]$, where $q = 1, 2, \cdots, |B_1|$.

The subsequent steps of approach 2 refer to steps 3 to 9 in approach 1. Finally, the selection pattern $\boldsymbol{\lambda}^{(2)}$ is fixed at the relay, then K_2 message symbols selected by $\boldsymbol{\lambda}^{(2)}$ are re-encoded to get the codeword $\mathbf{b}_2$. End of approach 2.

To better understand the proposed approaches, an example is given below.

4.3 Design example for two approaches

Consider a distributed RS coding scheme, RS codes $RS_1(15, 11, 5)$ and $RS_2(15, 7, 9)$ at the source and relay are employed, respectively. The symbols of RS_1 and RS_2 are defined over GF(2^4) using the polynomial $X^4 + X + 1$. The selection procedures of $K_2 = 7$ out of $K_1 = 11$ message symbols under the proposed two approaches are shown as below.

Firstly, approach 1 is applied to the selection procedure:

Step 1: Determine $A = \{\mathbf{b}_u\}$ i.e., the set of all message sequences which can be encoded into the codewords with minimum Hamming distance $d_1 = 5$ at the source, where the cardinality of A is computed as $|A| = 45045$.

Step 2: Determine selection patterns $\boldsymbol{\lambda}_g = [s_1, s_2, \cdots, s_7]$ and save them in B, $L = 330$.

Step 3: Determine $\min(v_1) = 840$.

Step 4: $\boldsymbol{\lambda}_g \to \min(v_1) = 840$ are saved in H as illustrated in Table 1. Since $|H| = 4 > 1$, we proceed to step 5.

Step 5: Determine $\min(v_2) = 16635$.

Step 6: $\boldsymbol{\lambda}_g \to \min(v_2) = 16635$ is saved in the set I. Since $|I| = 1$, we proceed to step 9 and the optimized

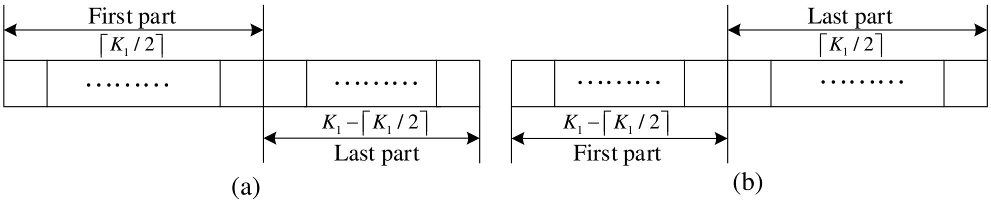

Fig. 3 The structures of the K_1 message symbols, scenario (a) more positions in the first part, scenario (b) more positions in the last part

Table 1 The selection patterns that result in $\min(v_1)$ and $\min(v_2)$ under Approach 1

Serial number	Selection pattern	v_1	v_2
1	[4,5,6,8,9,10,11]	840	17010
2	[4,5,7,8,9,10,11]	840	17280
3	[4,6,7,8,9,10,11]	840	17535
4	[5,6,7,8,9,10,11]	840	16635

Table 2 The selection patterns that result in $\min(v_1)$, $\min(v_2)$ and $\min(v_3)$ under Approach 2

Serial number	Selection pattern	v_1	v_2	v_3
1	[1,2,3,4,9,10,11]	360	10035	6615
2	[1,2,3,6,9,10,11]	360	10035	6540
3	[1,2,3,8,9,10,11]	360	10245	——

selection pattern is $\boldsymbol{\lambda}^{(1)} = \boldsymbol{\lambda}_g = [5, 6, 7, 8, 9, 10, 11]$. The approach 1 search is terminated.

Approach 2 is applied to the selection procedure. At the source, each message sequence that can generate the codeword with weight 5 contains at least 6 zero symbols. The symmetric structure of the 11 message symbols is shown in Figure 4.

Step 1: Determine the distribution of 6 zero symbols. Scenario (a): the $w(w = 3, 4, 5, 6)$ zero symbols are randomly distributed in the first part(6 positions), and the other $6 - w$ zero symbols are uniquely assigned in the last part(5 positions). Scenario (b): the w zero symbols are randomly distributed in the last part(6 positions), and the other $6 - w$ zero symbols are uniquely assigned in the first part(5 positions). The ways to distribute 6 zero symbols are $\beta_1 = 84$ and $|A_1| = 24075$.

Step 2: The relay selects 7 message symbols out of 11 recovered message symbols. Scenario (a): the $\mu(\mu = 4, 5, 6)$ positions are randomly selected from the first 6 positions, and the other $7 - \mu$ positions are uniquely taken from the remaining 5 positions. Scenario (b): the μ positions are randomly selected from the last 6 positions, and the other $7 - \mu$ positions are uniquely taken from the remaining 5 positions. The number of selection patterns is $|B_1| = 44$.

Step 3: Determine $\min(v_1) = 360$.

Step 4: $\boldsymbol{\lambda}_q \rightarrow \min(v_1) = 360$ are saved in H as illustrated in Table 2. Since $|H| = 3 > 1$, we proceed to step 5.

Step 5: Determine $\min(v_2) = 10035$.

Step 6: $\boldsymbol{\lambda}_q \rightarrow \min(v_2) = 10035$ are saved in I as illustrated in Table 2. Since $|I| = 2 > 1$, continue the next step 7.

Step 7: Determine $\min(v_3) = 6540$.

Step 8: $\boldsymbol{\lambda}_q \rightarrow \min(v_3) = 6540$ is saved in J. Since $|J| = 1$, we proceed to step 9 and the optimized selection pattern is $\boldsymbol{\lambda}^{(2)} = \boldsymbol{\lambda}_q = [1, 2, 3, 6, 9, 10, 11]$. The approach 2 search is terminated.

To compare the proposed approach 1 and approach 2, the above example with RS codes $RS_1(15, 11, 5)$ and $RS_2(15, 7, 9)$ is accordingly simulated. The BER performance is shown in Fig.5. Based on 16-QAM modulation, the ideal source to relay channel and joint RS decoding (will be discussed in section 5: smart algorithm) are used in the simulation. The simulated result reveals that the distributed RS coding scheme under the two approaches have almost identical BER performance at low to medium SNR. However, at high SNR, the distributed RS coding scheme using approach 1 outperforms the distributed RS coding scheme using approach 2 by a performance gain of around 0.23 dB at BER≈ 10^{-5}. The BER performance curves show that the proposed partial search approach 2 is valid. Therefore, we apply the proposed partial search approach 2 to the other two scenarios. In one scenario, the RS code $RS_1(31, 27, 5)$ is used at the source and RS code $RS_2(31, 17, 15)$ is employed at the relay. In the remaining one scenario, the two RS codes $RS_1(63, 59, 5)$ and $RS_2(63, 43, 21)$ are utilized at the source and relay, respectively. The detailed processes that obtain the selection patterns corresponding to the two scenarios are shown in Tables 3 and 4, respectively. Furthermore, Table 5 lists all three scenarios of distributed RS codes and selection patterns, which will be considered in Section 6 to generalize the proposed distributed RS coding scheme.

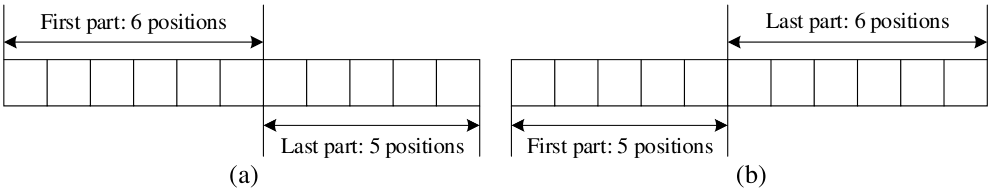

Fig. 4 The symmetric structures of the 11 message symbols, scenario (a) 6 positions in the first part, scenario (b) 6 positions in the last part

Table 3 The selection patterns that result in $\min(v_1)$, $\min(v_2)$ and $\min(v_3)$ under Approach 2 with $RS_1(31, 27, 5)$ and $RS_2(31, 17, 15)$

Serial number	Selection pattern	v_1	v_2	v_3
1	[2,3,4,6,8,10,11,12,15,16,20,21,22,23,25,26,27]	3515	505725	123550
2	[1,3,4,5,7,8,9,10,11,12,13,14,15,18,19,20,24]	3515	584250	———
3	[2,3,4,5,6,8,11,13,14,15,19,20,21,22,23,24,26]	3515	505725	132425

Table 4 The selection patterns that result in $\min(v_1)$and $\min(v_2)$ under Approach 2 with $RS_1(63, 59, 5)$ and $RS_2(63, 43, 21)$

Serial number	Selection pattern	v_1	v_2
1	[1,3,4,5,6,8,10,11,13,15,16,17,18,19,22,23,25, 26,27,28,30,31,33,34,35,36,37,38,40,42,43,44, 45,47,48,50,51,53,54,55,57,58,59]	155840	3457010
2	[1,2,4,5,8,9,10,12,14,15,16,17,18,19,20,24,26, 27,28,29,30,32,33,36,37,38,39,40,41,42,43,44, 47,48,49,50,52,53,54,55,56,58,59]	155840	3033535
3	[2,4,5,6,7,8,9,10,11,12,13,15,16,17,18,19,20,22, 23,25,26,27,28,29,31,33,34,35,36,37,38,39,40, 41,42,43,44,47,49,53,55,56,57]	155840	3902170
4	[2,5,6,7,8,9,10,11,12,13,15,16,17,18,19,20,21,22, 23,24,25,27,28,29,30,31,32,33,34,35,36,38,39, 41,42,44,45,46,47,50,51,53,55]	155840	4046635

4.4 Complexity analysis for the two searching approaches

Encoding one message sequence of length K_1 at the source requires $\xi_s^\times = K_1(N-K_1)$ multiplication operations and $\xi_s^+ = K_1(N-K_1)$ addition operations, hence, the number of total elementary operations is $\xi_s^1 = \xi_s^\times + \xi_s^+ = 2K_1(N-K_1)$. Then, for $|A|$ and $|A_1|$ message sequences, the number of elementary operations is $\xi_s^A = 2K_1|A|(N-K_1)$ and $\xi_s^{A_1} = 2K_1|A_1|(N-K_1)$, respectively. For $|B|$ and $|B_1|$ selection patterns, the number of all elementary operations is $\xi_r^B = 2K_2|A||B|(N-K_2)$ and $\xi_r^{B_1} = 2K_2|A_1||B_1|(N-K_2)$, respectively. Therefore, we can get the number of total elementary operations of approach 1 is $\xi^{(1)} = \xi_s^A + \xi_r^B = 2|A|[NK_1 - (K_1)^2 + K_2N|B| - (K_2)^2|B|]$ if approach 1 converges at step finding v_1. Similarly, when approach 2 converges at step finding v_1, the number of elementary operations of approach 2 is $\xi^{(2)} = \xi_s^{A_1} + \xi_r^{B_1} = 2|A_1|[NK_1 - (K_1)^2 + K_2N|B_1| - (K_2)^2|B_1|]$.

5 Joint Decoding for distributed RS codes

Joint decoding is one of the key features of coded cooperation system. In this section, two algorithms are proposed for the joint RS decoding, i.e., naive algorithm and smart algorithm.

5.1 Naive decoding algorithm

The specific steps of naive decoding algorithm are as follows:

Step 1: Decode the first part of sequence $|\hat{\mathbf{b}}_1|\hat{\mathbf{b}}_2|$ by RS_1 decoder and second part by RS_2 decoder to obtain the estimated non-binary message sequences $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_2$, respectively.

Step 2: Define a threshold ρ which is the SNR value of crossover point of the BER performance obtained by RS_1 and RS_2 over the fast Rayleigh fading channel.

Step 3: Send $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_2$ to the combiner, if SNR$\leq \rho$, the combiner output is $\tilde{\mathbf{a}}_1 = \hat{\mathbf{a}}_1$; SNR$> \rho$, the selected K_2 positions of $\hat{\mathbf{a}}_1$ are replaced by corresponding K_2 symbols in $\hat{\mathbf{a}}_2$, $\tilde{\mathbf{a}}_1 = \hat{\mathbf{a}}_1'$.

Table 5 Optimized selection patterns of corresponding distributed RS codes

Serial number	$RS_1(N, K_1)$	$RS_2(N, K_2)$	$\boldsymbol{\lambda}^{(1)}$	$\boldsymbol{\lambda}^{(2)}$
1	(15,11,5)	(15,7,9)	[5,6,7,8,9,10,11]	[1,2,3,6,9,10,11]
2	(31,27,5)	(31,17,15)	——	[2,3,4,6,8,10,11,12,15,16,20,21,22,23,25,26,27]
3	(63,59,5)	(63,43,21)	——	[1,2,4,5,8,9,10,12,14,15,16,17,18,19,20,24,26, 27,28,29,30,32,33,36,37,38,39,40,41,42,43,44, 47,48,49,50,52,53,54,55,56,58,59]

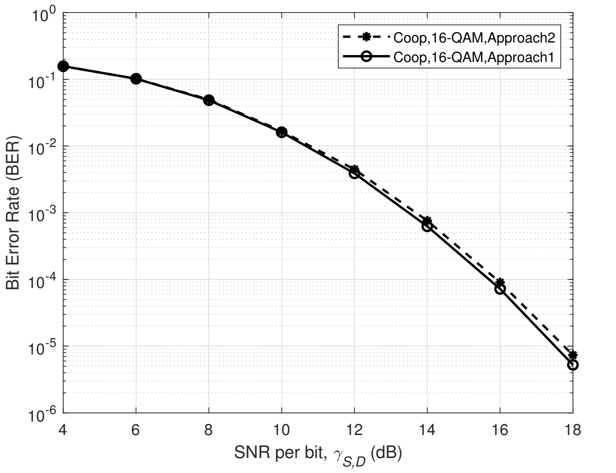

Fig. 5 BER performance comparison of distributed RS coding scheme over a fast Rayleigh fading channel with $RS_1(15, 11, 5)$ and $RS_2(15, 7, 9)$ under different information selection approaches

Step 4: Convert the combiner output $\tilde{\mathbf{a}}_1$ into binary sequence and get estimated binary message sequence $\tilde{\mathbf{m}}_1$.

The reason for defining a threshold: in channel coding, the BER performance of code with more error correcting capability is worse than the code with less error correcting capability for low SNR regime while it outperforms the less error correcting capability code over high SNR regime.

5.2 Smart decoding algorithm

The specific steps of smart decoding algorithm are as follows:

Step 1: Decode the second part of sequence $|\hat{\mathbf{b}}_1|\hat{\mathbf{b}}_2|$ by RS_2 decoder to obtain the systematic non-binary sequence $\hat{\mathbf{a}}_2$.

Step 2: The demodulated sequence $\hat{\mathbf{b}}_1$ is comprised of parity sequence $\hat{\mathbf{p}}_1$ and systematic message sequence $\hat{\mathbf{a}}_1$, these selected K_2 positions of $\hat{\mathbf{a}}_1$ are replaced with the K_2 symbols (all) of the estimated sequence $\hat{\mathbf{a}}_2$ in combiner block and get joint sequence $\overline{\mathbf{b}}_1$.

Step 3: Decode the combiner output sequence $\overline{\mathbf{b}}_1$ by RS_1 decoder and get the estimated non-binary sequence $\tilde{\mathbf{a}}_1$.

Step 4: Convert the estimated sequence $\tilde{\mathbf{a}}_1$ into binary message sequence $\tilde{\mathbf{m}}_1$.

Furthermore, a note for the step 2, the sequence $\overline{\mathbf{b}}_1$ is the joint source relay sequence and more reliable than $\hat{\mathbf{b}}_1$ due to the inclusion of $\hat{\mathbf{a}}_2$. This provides coding gain to the cooperative system.

6 Simulation results

For the simulations, three different scenarios are considered for the proposed distributed RS coding scheme. All scenarios are examined through fast Rayleigh fading channel. In the first scenario, $RS_1(15, 11, 5)$ and $RS_2(15, 7, 9)$ over GF(2^4) constructed based on the polynomial $X^4 + X + 1$ are used. The code rates for first scenario are $R_1 = 11/15$ and $R_2 = 7/15$ for source-to-destination and relay-to-destination links, respectively. For the second scenario, RS codes $RS_1(31, 27, 5)$ and $RS_2(31, 17, 15)$ over GF(2^5) which is constructed based on the polynomial $X^5 + X^2 + 1$ are employed in the scheme. In the second scenario, the code rates are $R_1 = 27/31$ and $R_2 = 17/31$, respectively. Moreover, the RS codes $RS_1(63, 59, 5)$ and $RS_2(63, 43, 21)$ over GF(2^6) which is constructed using polynomial $X^6 + X + 1$ are applied in the third scenario. Their code rates are $R_1 = 59/63$ and $R_2 = 43/63$, respectively. For the first scenario, 16-QAM modulation is employed. Besides, we have used 32-QAM modulation for the second scenario and 64-QAM for the third scenario. Supposing that the relay node has a 2 dB performance gain over source node, i.e., $\gamma_{R,D} = \gamma_{S,D} + 2$ dB, where $\gamma_{R,D}$ is the SNR per bit between the relay-destination link, $\gamma_{S,D}$ and $\gamma_{S,R}$ are SNR per bit among the source-destination and source-relay links, respectively. Furthermore, Euclidean algorithm is used for all scenarios.

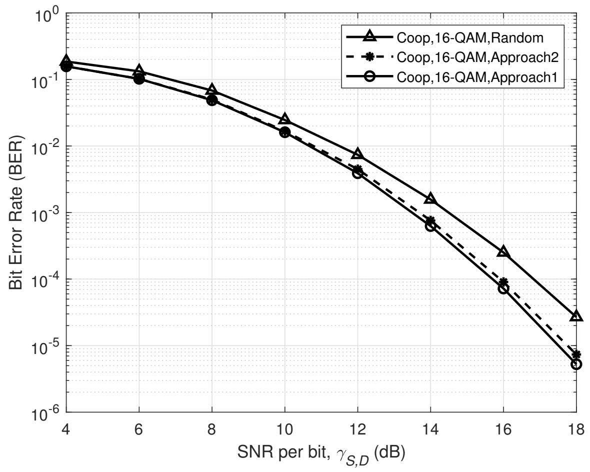

Fig. 6 BER performance comparison of distributed RS coding scheme with $RS_1(15, 11, 5)$ and $RS_2(15, 7, 9)$ under different information selection approaches.

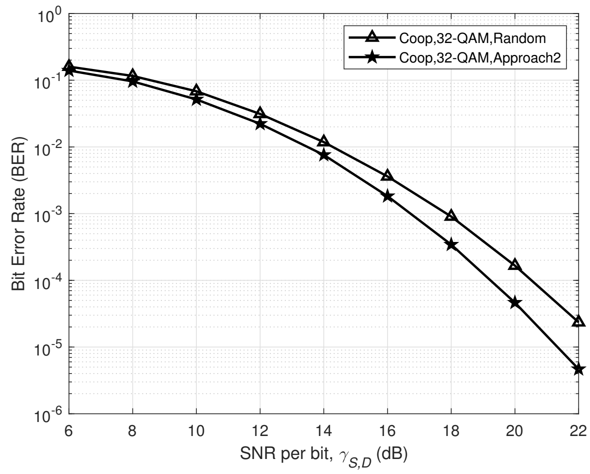

Fig. 7 BER performance comparison of distributed RS coding scheme with $RS_1(31, 27, 5)$ and $RS_2(31, 17, 15)$ under different information selection approaches.

6.1 Performance comparison of distributed RS coding scheme under different information selection approaches

Fig.6 demonstrates the BER performance comparison of distributed RS coding scheme under different selection approaches for the first scenario. The ideal source to relay channel ($\gamma_{S,R} = \infty$) and joint RS decoding (smart algorithm) are used. It is observed that the proposed scheme employing approach 1 and the proposed scheme employing approach 2 have almost the same performance at low to medium SNR. The BER performance of the scheme employing approach 1 improves slightly at high SNR. Whereas the proposed scheme employing random approach exhibits the worst performance and shows the importance of appropriate information selection at the relay. Similarly, Figs.7 and 8 show the BER performance for the second and third scenarios, respectively. The simulation results illustrate that the distributed RS coding scheme under approach 2 has a better BER performance than the proposed scheme employing random approach. These simulations clarify the effect of proper information selection at the relay and show the validity of approach 2.

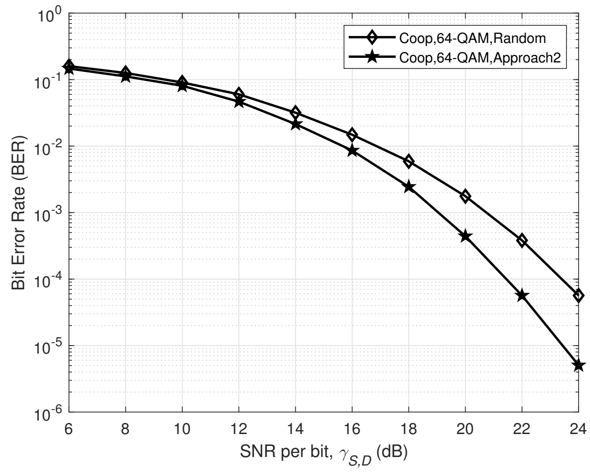

Fig. 8 BER performance comparison of distributed RS coding scheme with $RS_1(63, 59, 5)$ and $RS_2(63, 43, 21)$ under different information selection approaches.

6.2 Performance comparison of distributed RS coding scheme under naive and smart decoding algorithms

Figs.9, 10, 11 show the BER performance of distributed RS coding scheme with different joint decoding algorithms for the first, second and third scenarios, respectively. The source to relay channel is assumed to be ideal ($\gamma_{S,R} = \infty$) for all scenarios. Moreover, approach 2 is employed in the three scenarios. In Fig.9, the distributed RS coding scheme exploiting smart algorithm provides about 1.8 dB SNR over the distributed RS coding scheme exploiting naive algorithm ($\rho = 8$ dB) at BER$\approx 10^{-4}$. From Fig.10, it is observed that the distributed RS coding scheme utilizing smart algorithm outperforms the distributed RS coding scheme utilizing naive algorithm by a performance gain of 2.2 dB at BER$\approx 6 \times 10^{-5}$ under identical conditions. Similarly, Fig.10 shows that the distributed RS coding scheme with smart algorithm outperforms that of naive algorithm by about 2.4 dB gain at BER$\approx 8.6 \times 10^{-5}$.

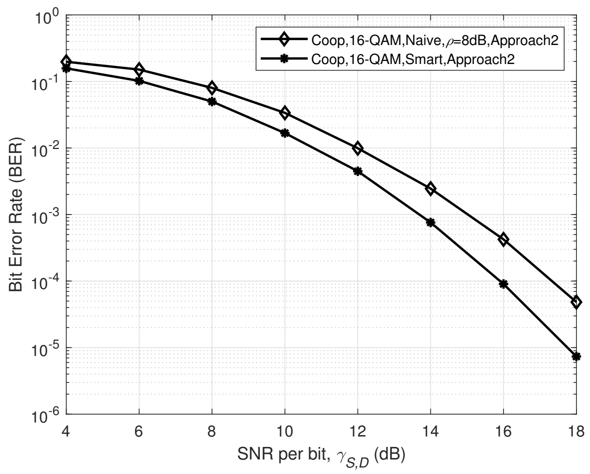

Fig. 9 BER performance comparison of distributed RS coding scheme with $RS_1(15, 11, 5)$ and $RS_2(15, 7, 9)$ employing joint RS decoding (naive and smart algorithms).

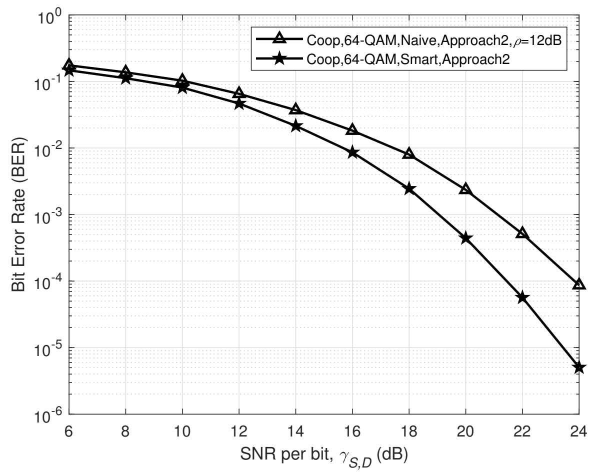

Fig. 11 BER performance comparison of distributed RS coding scheme with $RS_1(63, 59, 5)$ and $RS_2(63, 43, 21)$ employing joint RS decoding (naive and smart algorithms).

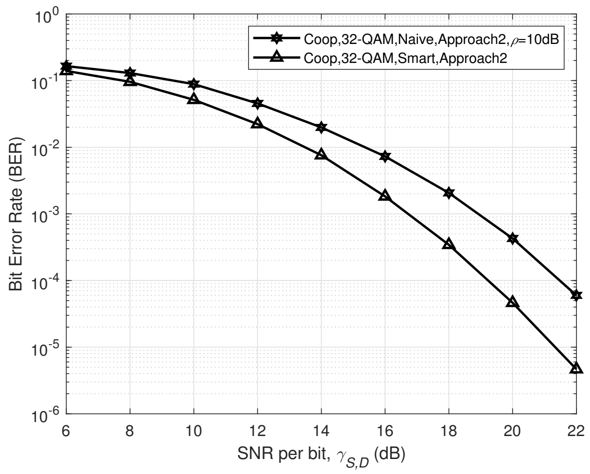

Fig. 10 BER performance comparison of distributed RS coding scheme with $RS_1(31, 27, 5)$ and $RS_2(31, 17, 15)$ employing joint RS decoding (naive and smart algorithms).

6.3 Performance comparison of non-ideal and ideal source to relay channels

Figs.12, 13 and 14 demonstrate the BER performance of distributed RS coding scheme and non-cooperative RS coding scheme for the first, second and third scenarios, respectively. The joint decoding (smart algorithm) and approach 2 are employed in all scenarios. For fair comparison, the distributed RS coding scheme and its non-cooperative scheme have the identical code rate at the destination. From Fig.12, it is noticed that the distributed RS coding scheme with ideal source to relay channel ($\gamma_{S,R} = \infty$) performs better than the corresponding non-cooperative RS coding scheme by a gain approximately 2.4 dB at BER$\approx 1.7 \times 10^{-4}$. This phenomenon indicates the influence of the relay node on path diversity. Moreover, it can be seen that the distributed RS coding scheme under $\gamma_{S,R} = 14$ dB has an approximative performance with that of distributed RS coding scheme under $\gamma_{S,R} = \infty$ at low SNR. However, at high SNR, the relay is useless. Therefore, the overall BER performance is significantly degraded, which results from the uncontrolled error propagation at the relay. The cyclic redundancy check (CRC) is an effective way to control this error propagation as mentioned in [17]. Also, the distributed RS coding scheme under $\gamma_{S,R} = \infty$ only gets about 0.15 dB gain as compared to the distributed RS coding scheme under $\gamma_{S,R} = 20$ dB at BER $\approx 9.5 \times 10^{-5}$. Similarly, as shown in Figs.13 and 14, it is observed that the distributed RS coding scheme with ideal source to relay channel ($\gamma_{S,R} = \infty$) outperforms the corresponding non-cooperative RS coding scheme. In Fig.13, the distributed RS coding scheme under $\gamma_{S,R} = 24$ dB approaches the performance of the distributed RS coding scheme under $\gamma_{S,R} = \infty$. The simulation results of Fig.13 reveal that the superiority of our proposed distributed RS coding scheme (with virtual 2×1 MIMO) over its non-cooperative scheme (with 2×1 MIMO). Furthermore, when the distributed RS coding scheme under $\gamma_{S,R} = 16$ dB, the error propagation is occurred at the relay. In Fig.14, the BER curve of the distributed RS coding scheme with $\gamma_{S,R} = 18$ dB becomes flat at medium SNR because there is no error control propagation employed at the relay. Moreover, at BER$\approx 5.6 \times 10^{-5}$, the distributed RS coding scheme under $\gamma_{S,R} = 28$ dB only lags behind about 0.22 dB gain as compared to the distributed RS coding scheme under $\gamma_{S,R} = \infty$.

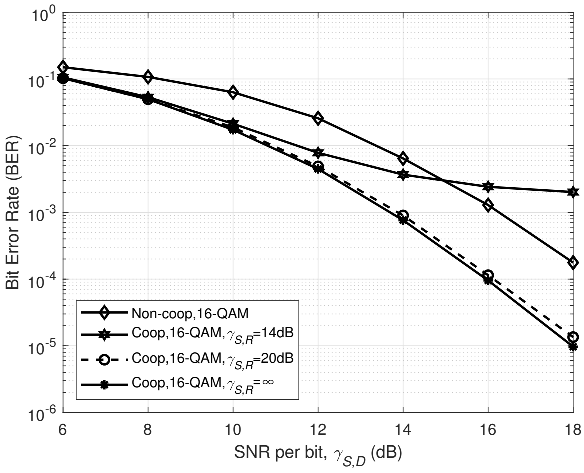

Fig. 12 BER performance of distributed RS coding scheme under non-ideal and ideal source to relay channels and non-cooperative RS coding scheme with $RS_1(15, 11, 5)$ and $RS_2(15, 7, 9)$.

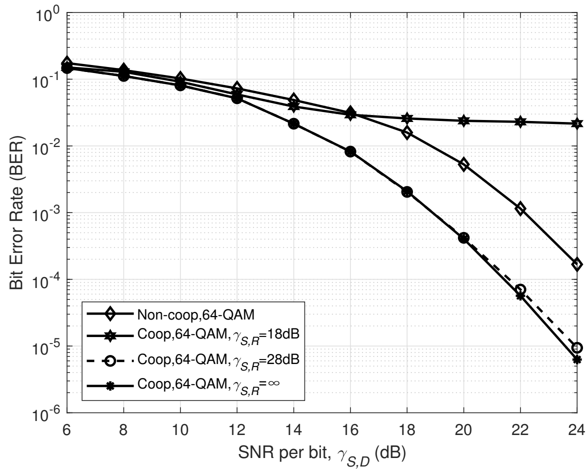

Fig. 14 BER performance of distributed RS coding scheme under non-ideal and ideal source to relay channels and non-cooperative RS coding scheme with $RS_1(63, 59, 5)$ and $RS_2(63, 43, 21)$.

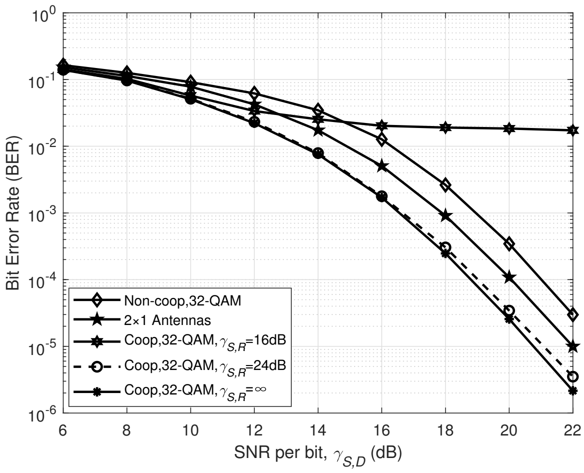

Fig. 13 BER performance of distributed RS coding scheme under non-ideal and ideal source to relay channels and non-cooperative RS coding scheme with $RS_1(31, 27, 5)$ and $RS_2(31, 17, 15)$.

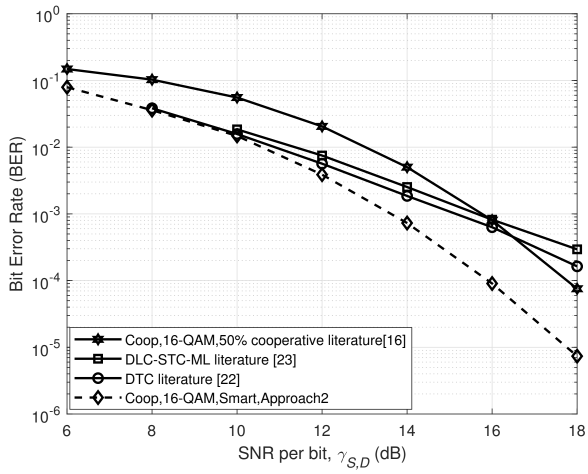

Fig. 15 BER performance comparison of distributed RS coding scheme and the existed schemes in [16], [22] and [23].

6.4 Performance comparison of the distributed RS coding scheme with the existed schemes

The BER performance comparison of the distributed RS coding scheme employing smart algorithm and approach 2 for the first scenario with the existed schemes in [16], [22] and [23] i.e., RS adaptive cooperation (50% cooperative), distributed turbo coding (DTC) and distributed linear convolutive space-time codes (DLC-STC) schemes is presented in Fig.15. The simulation results of Fig.15 demonstrate the superiority of our proposed distributed RS coding scheme over the existed adaptive scheme (50% cooperative) in [16], DTC scheme in [22] and DLC-STC-ML scheme in [23] through fast Rayleigh fading channel. For example, the distributed RS coding scheme obtains about 2.3dB, 2dB and 2.5dB gains at BER$\approx 7.6 \times 10^{-4}$ as compared to RS adaptive cooperation (50% cooperative), DTC and DLC-STC-ML schemes.

7 Conclusion

In this paper, we have proposed a distributed RS coding scheme for wireless communication which is comprised of two RS codes with different number of consecutive roots. In the scheme, two information selection approaches to achieve an optimized subcode at the destination are developed. Monte-Carlo simulations have

Table 6 BER Performance comparison of distributed RS coding scheme and the existed schemes in [16], [22] and [23]

SNR	Approach 2	50% cooperative	DTC	DLC-STC-ML
6	7.92×10^{-2}	1.48×10^{-1}	——	——
8	3.69×10^{-2}	1.03×10^{-1}	3.83×10^{-2}	——
10	1.47×10^{-2}	5.55×10^{-2}	1.58×10^{-2}	1.85×10^{-2}
12	3.87×10^{-3}	2.05×10^{-2}	5.66×10^{-3}	7.49×10^{-3}
14	7.29×10^{-4}	5.01×10^{-3}	1.86×10^{-3}	2.52×10^{-3}
16	9.05×10^{-5}	8.04×10^{-4}	6.31×10^{-4}	8.18×10^{-4}
18	7.36×10^{-6}	7.54×10^{-5}	1.63×10^{-4}	2.94×10^{-4}

demonstrated the superiority of the two selection approaches. Naive and smart are the two proposed algorithms to jointly decode at the destination. The various numerical results show that the distributed RS coding scheme clearly outperforms the RS non-cooperative scheme. Moreover, the distributed RS coding scheme using smart algorithm performs better than the distributed RS coding scheme using naive algorithm under the same conditions.

Acknowledgements This work was supported by National Natural Science Foundation of China under the contract No. 61771241.

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Figures

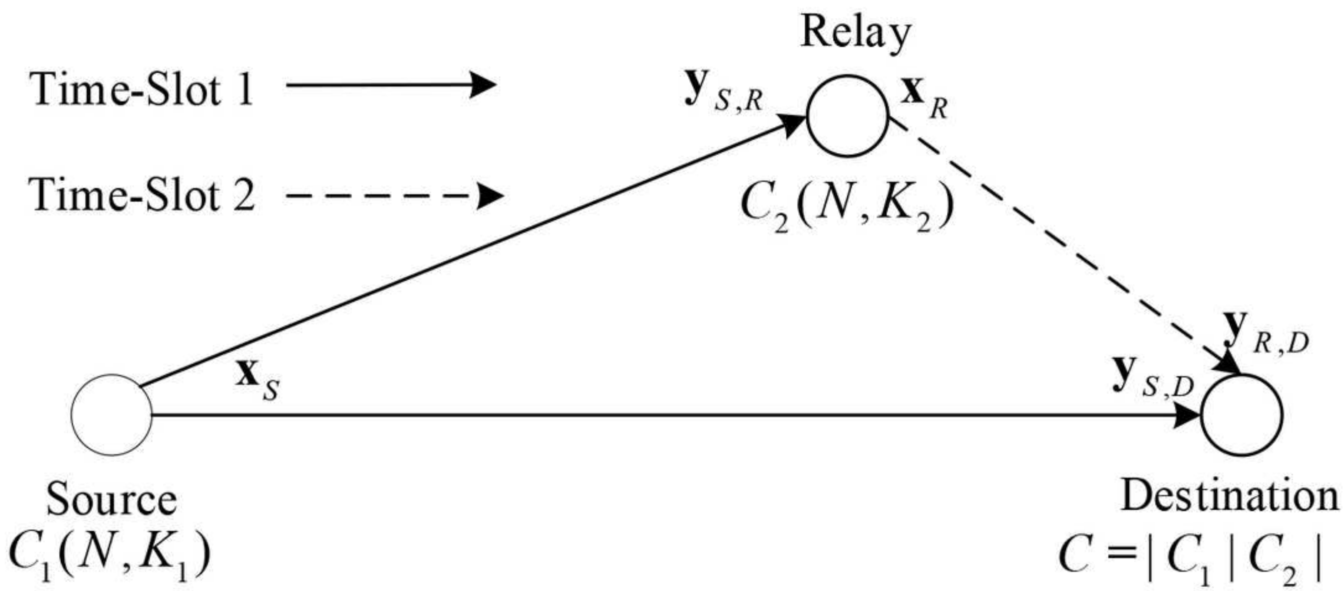

Figure 1

General system model of distributed linear block codes

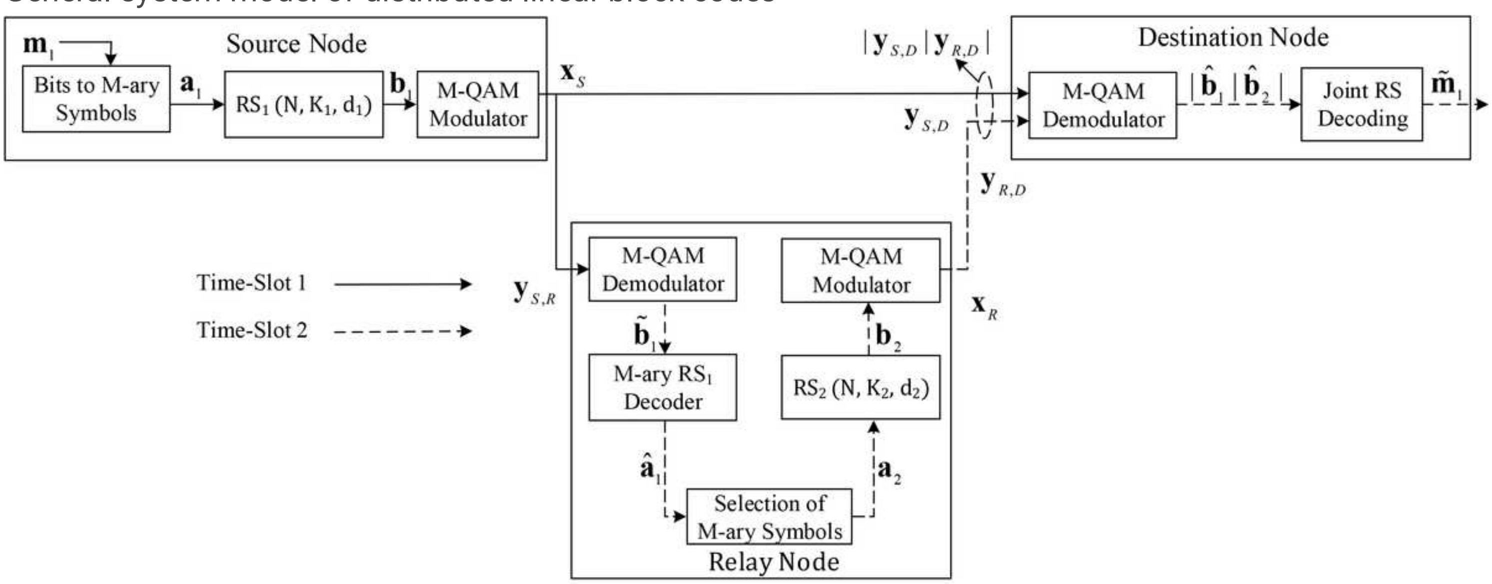

Figure 2

Distributed RS coding scheme

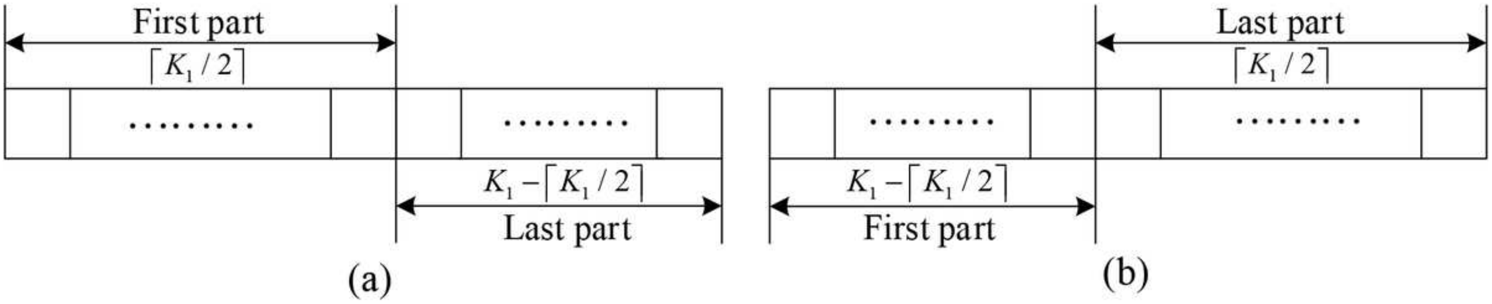

Figure 3

The structures of the K1 message symbols, scenario (a) more positions in the first part, scenario (b) more positions in the last part

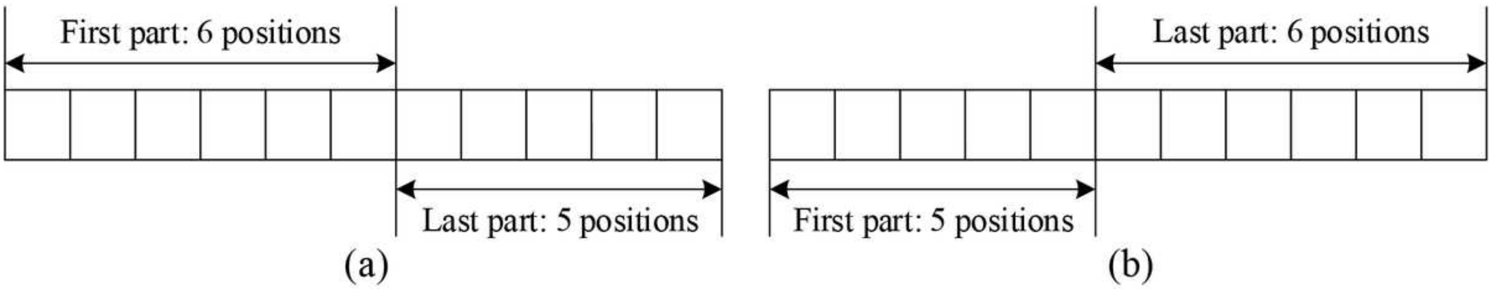

Figure 4

The symmetric structures of the 11 message symbols, scenario (a) 6 positions in the first part, scenario (b) 6 positions in the last part

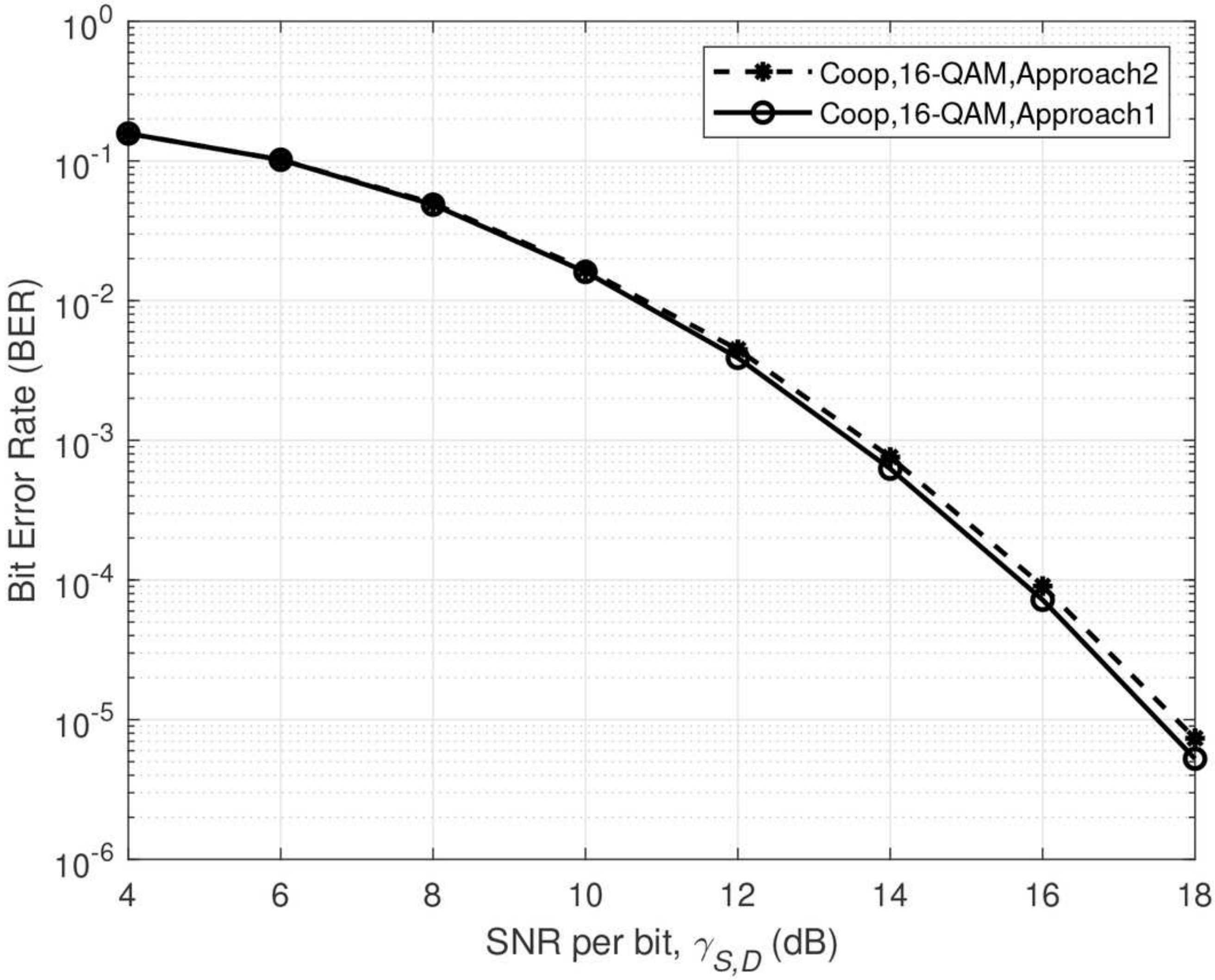

Figure 5

BER performance comparison of distributed RS coding scheme over a fast Rayleigh fading channel with RS1(15, 11, 5) and RS2(15, 7, 9) under different information selection approaches

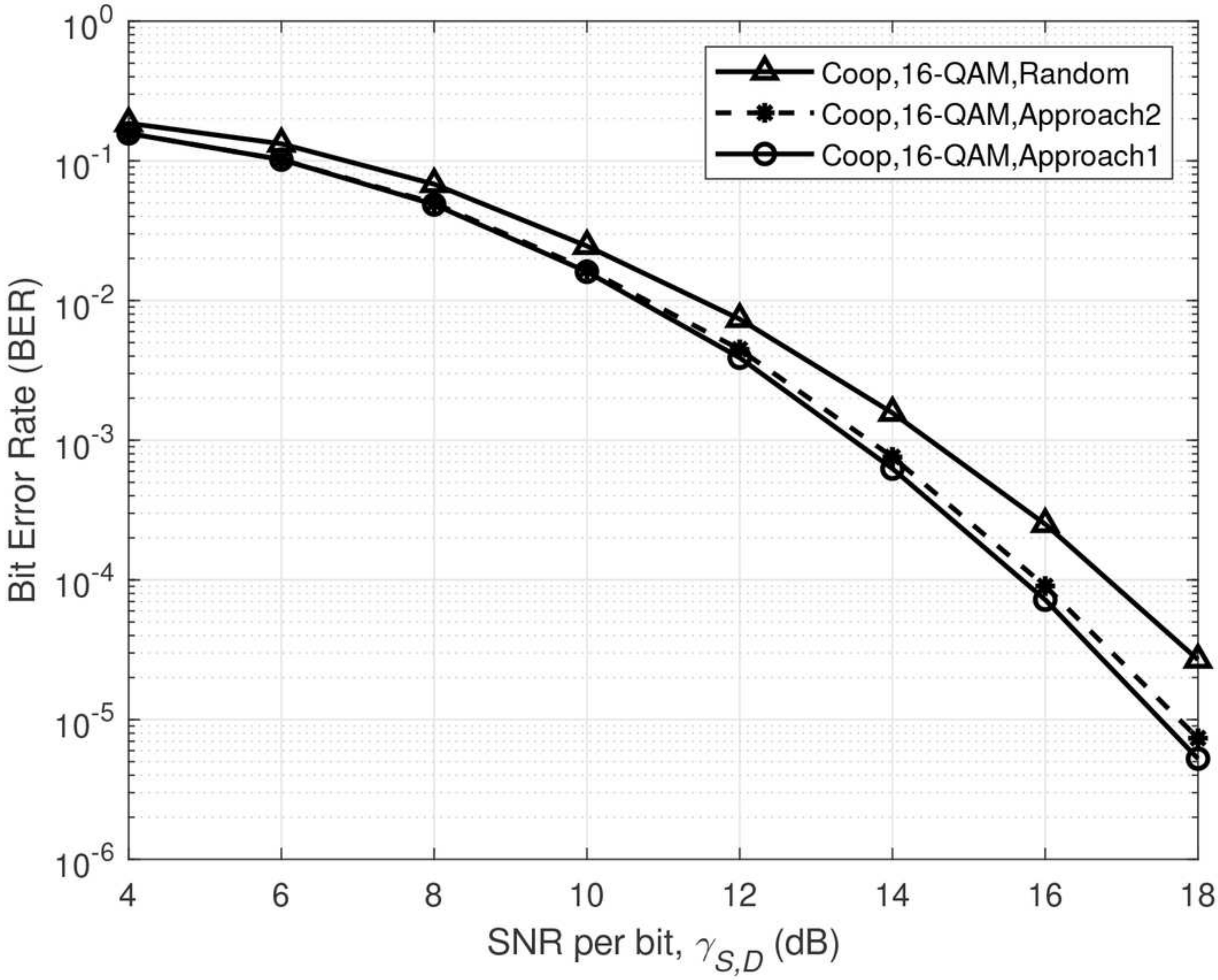

Figure 6

BER performance comparison of distributed RS coding scheme with RS1(15, 11, 5) and RS2(15, 7, 9) under different information selection approaches.

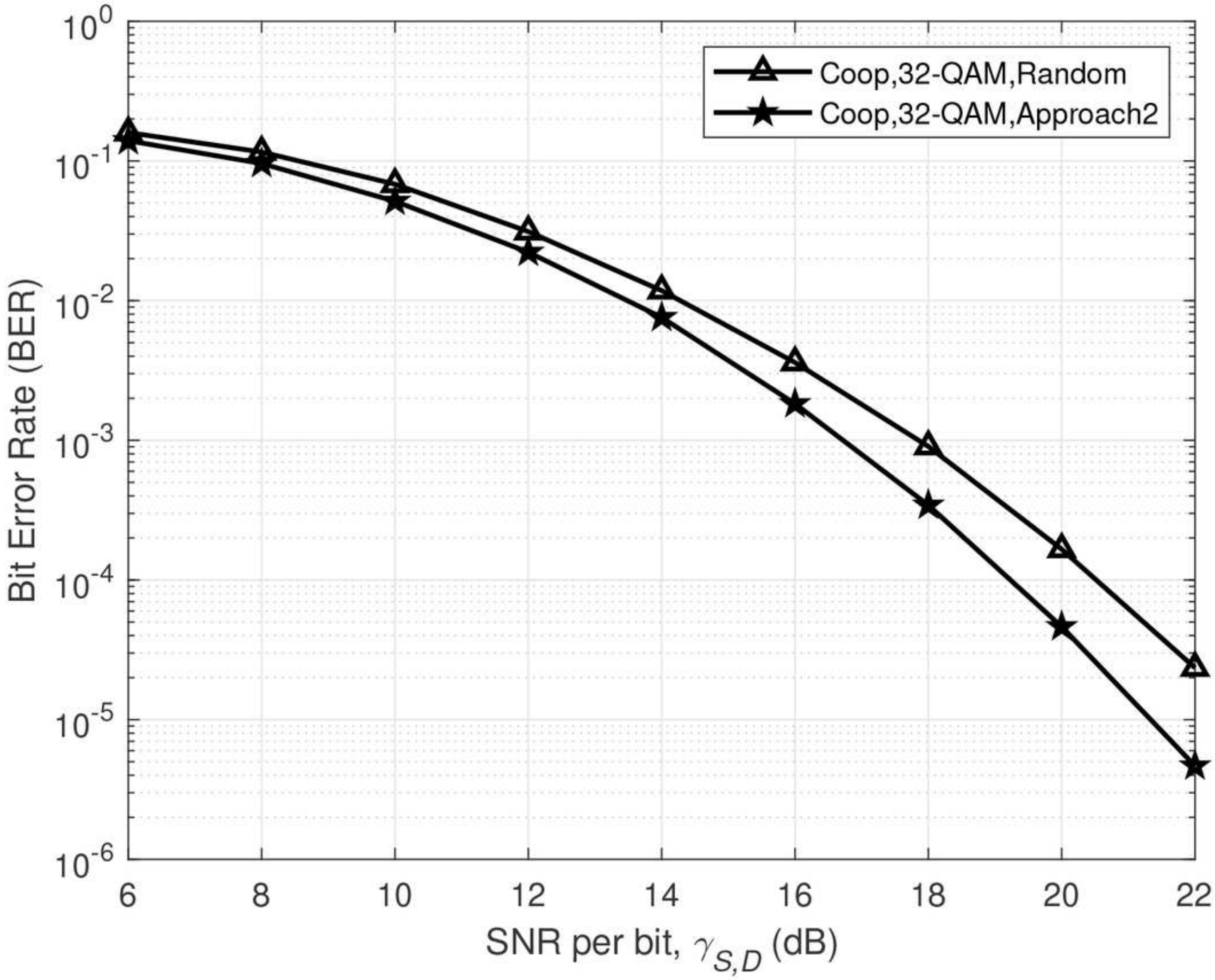

Figure 7

BER performance comparison of distributed RS coding scheme with RS1(31, 27, 5) and RS2(31, 17, 15) under different information selection approaches.

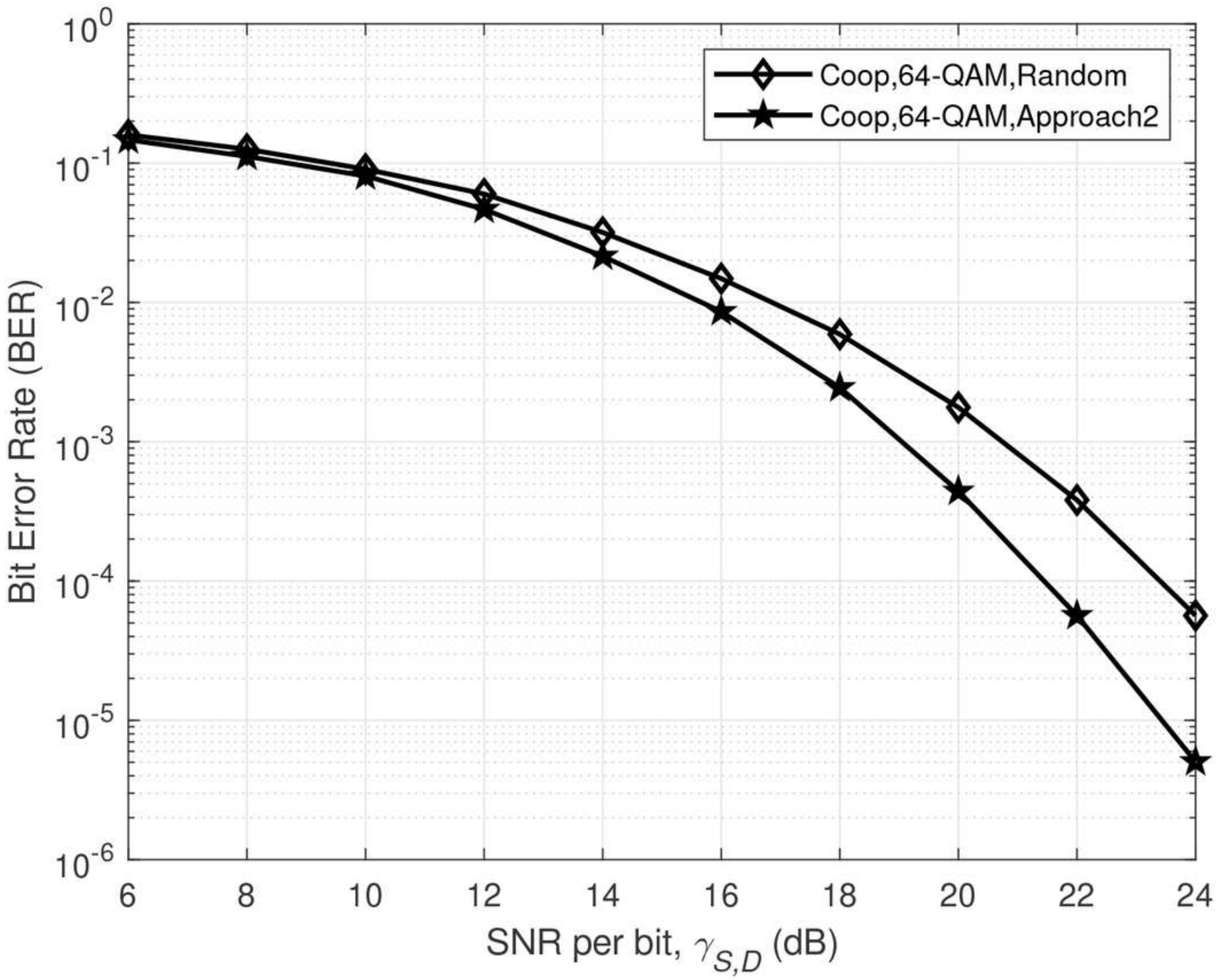

Figure 8

BER performance comparison of distributed RS coding scheme with RS1(63, 59, 5) and RS2(63, 43, 21) under different information selection approaches.

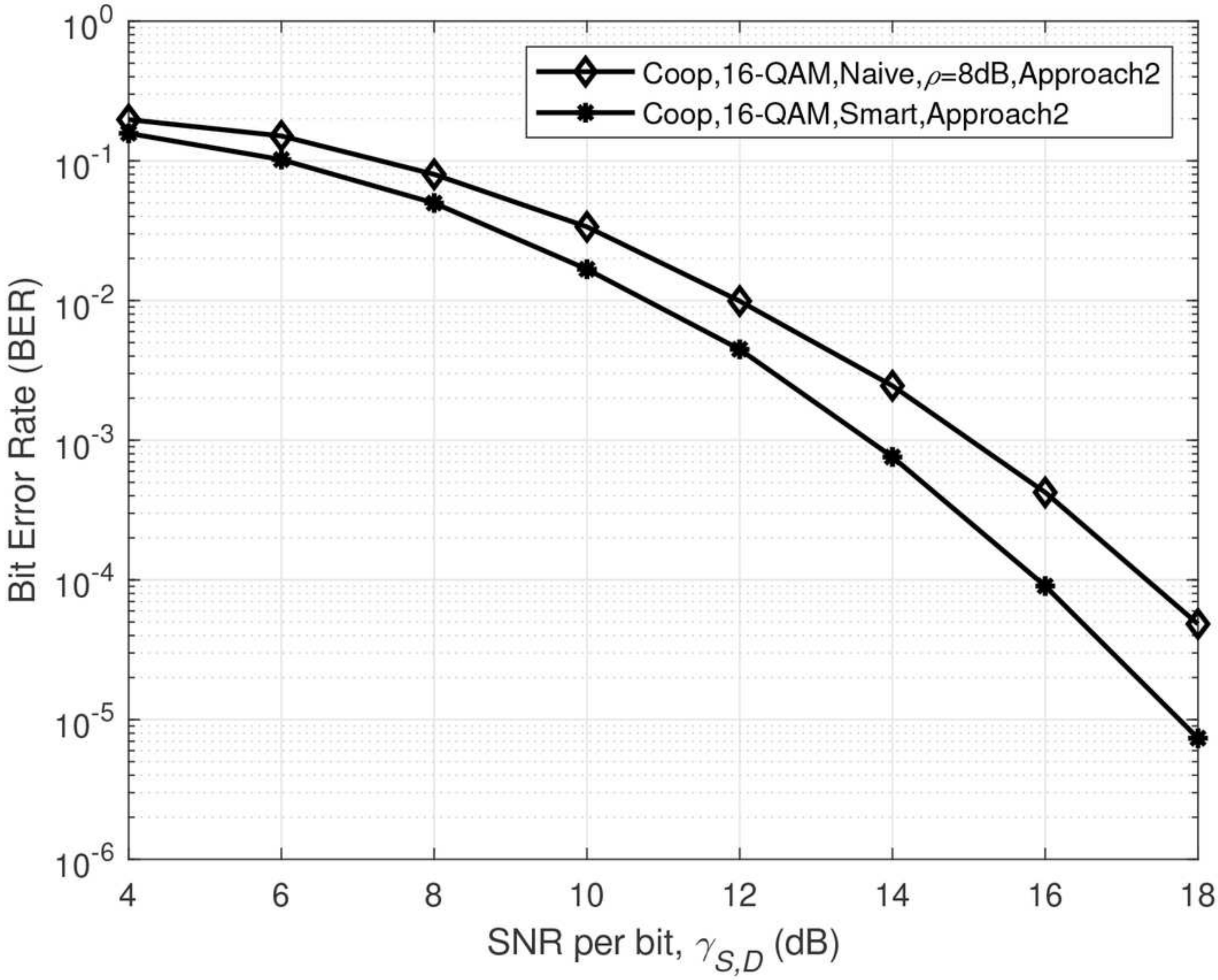

Figure 9

BER performance comparison of distributed RS coding scheme with RS1(15, 11, 5) and RS2(15, 7, 9) employing joint RS decoding (naive and smart algorithms).

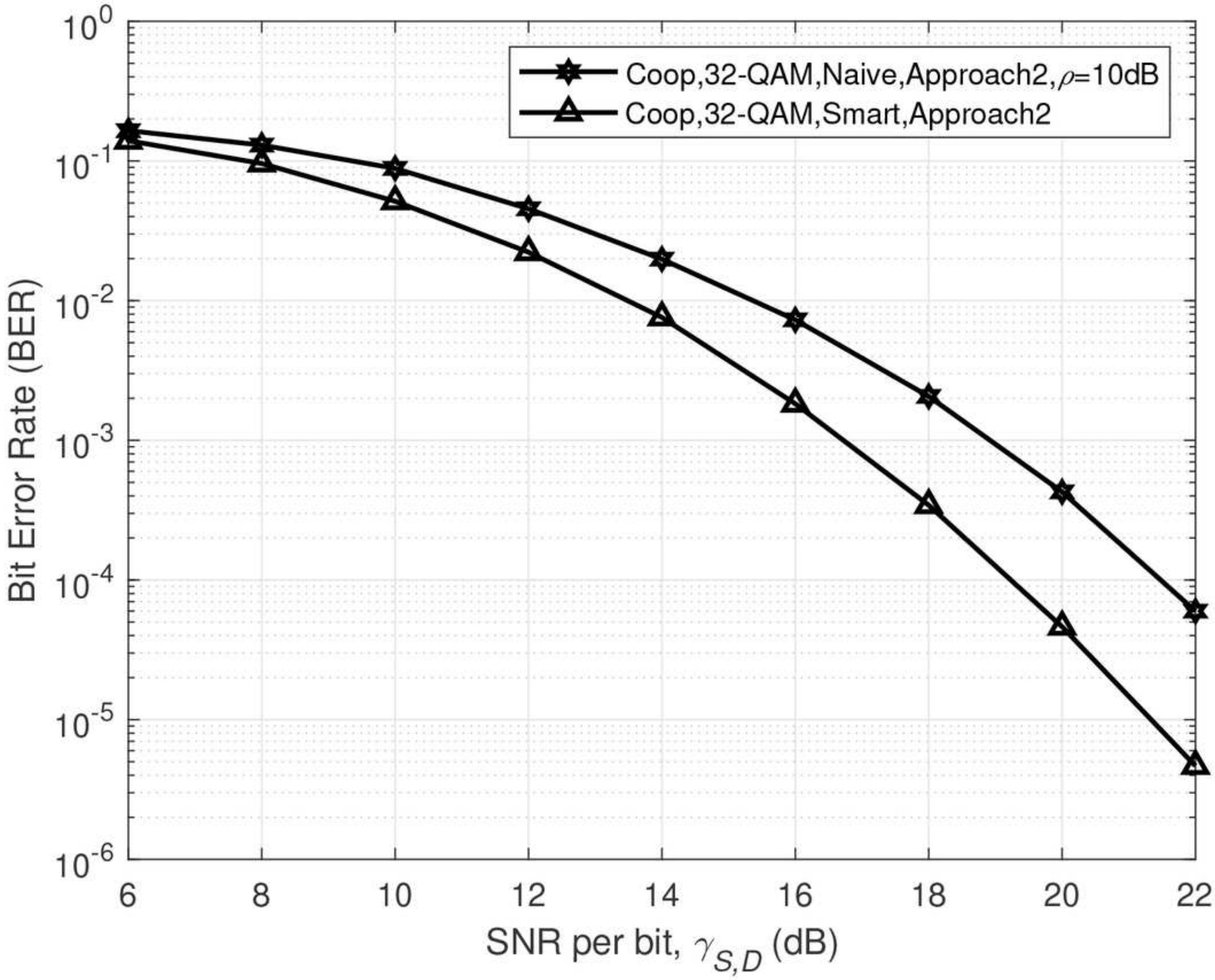

Figure 10

BER performance comparison of distributed RS coding scheme with RS1(63, 59, 5) and RS2(63, 43, 21) employing joint RS decoding (naive and smart algorithms).

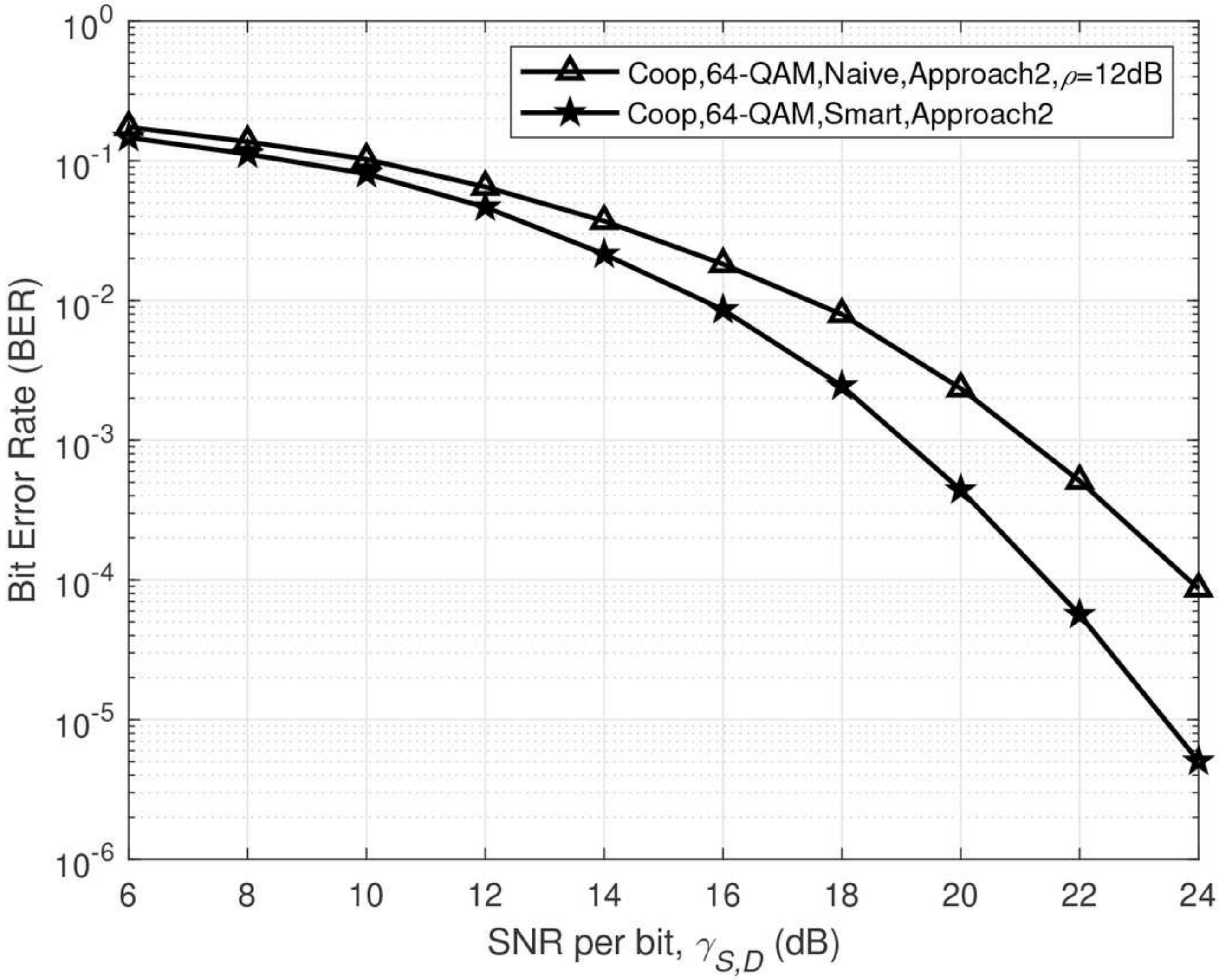

Figure 11

BER performance comparison of distributed RS coding scheme with RS1(31, 27, 5) and RS2(31, 17, 15) employing joint RS decoding (naive and smart algorithms).

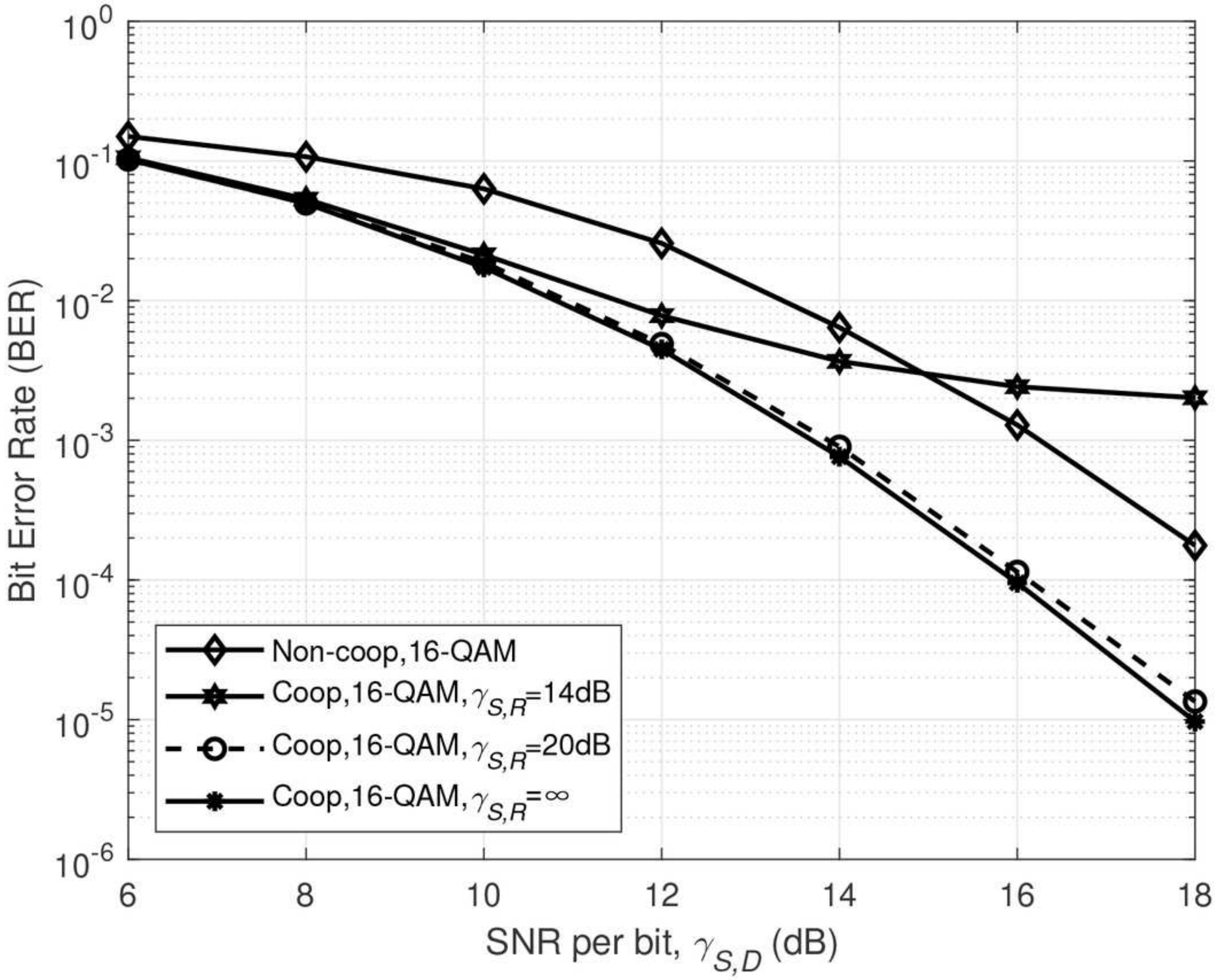

Figure 12

BER performance of distributed RS coding scheme under non-ideal and ideal source to relay channels and non-cooperative RS coding scheme with RS1(15, 11, 5) and RS2(15, 7, 9).

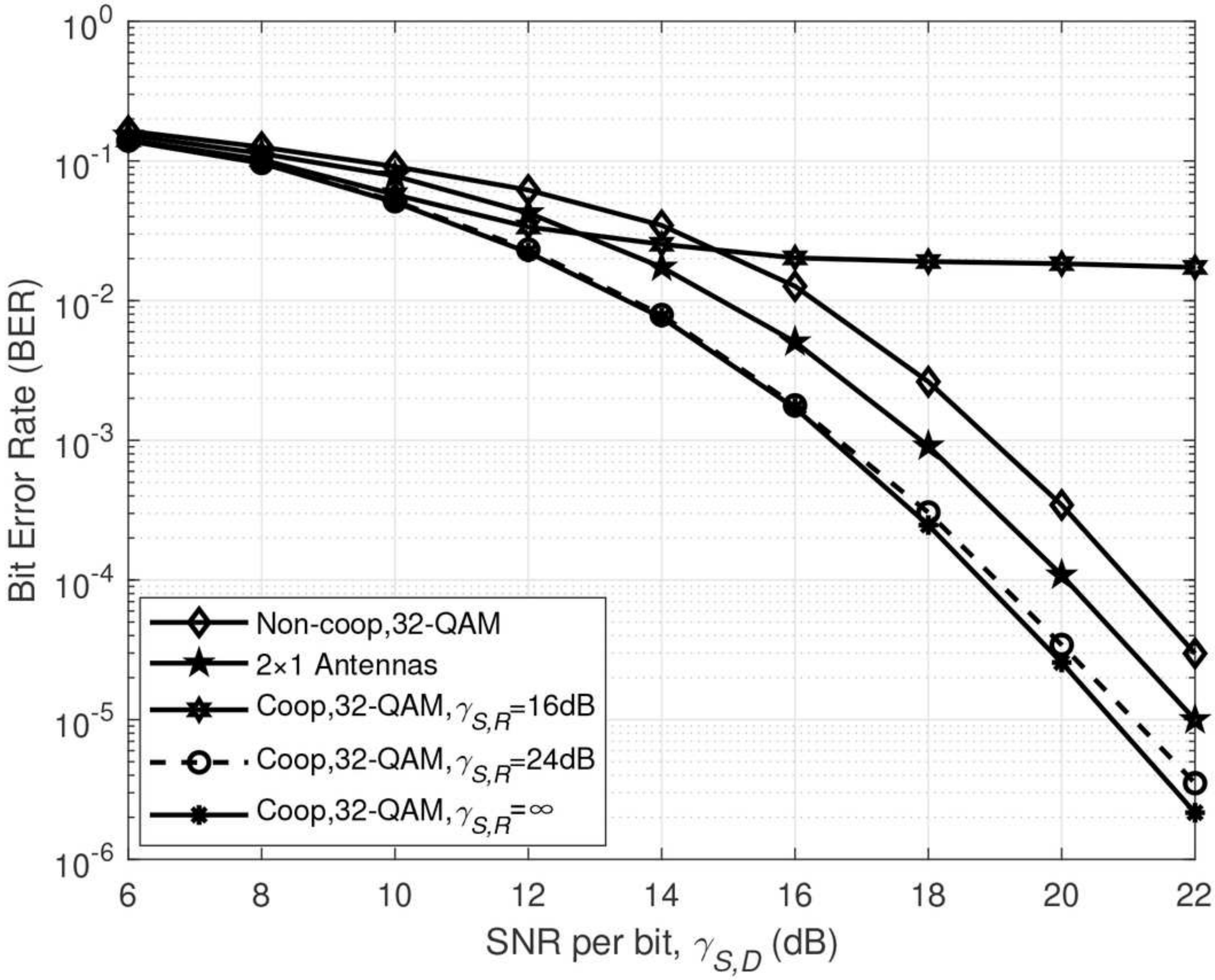

Figure 13

BER performance of distributed RS coding scheme under non-ideal and ideal source to relay channels and non-cooperative RS coding scheme with RS1(31, 27, 5) and RS2(31, 17, 15).

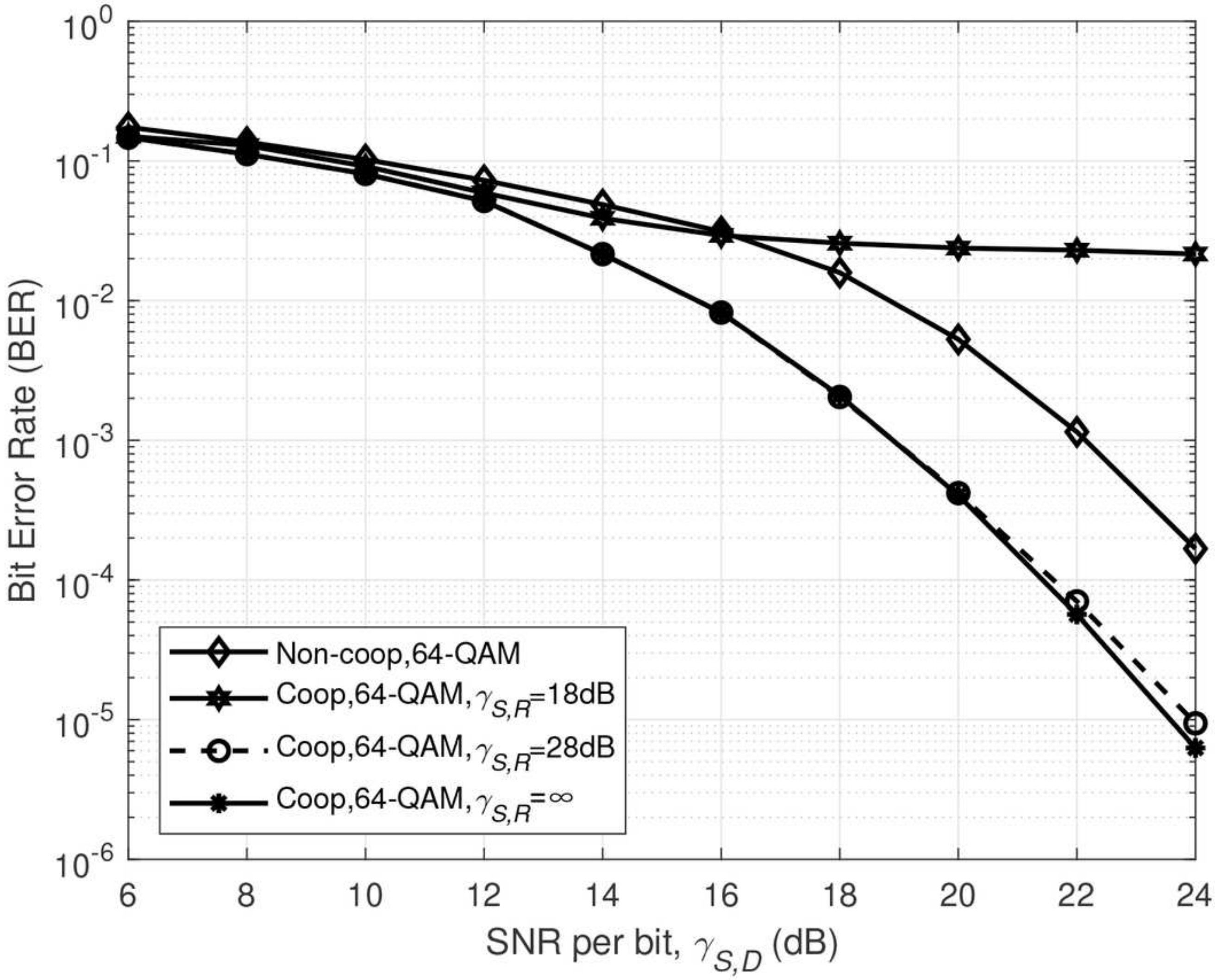

Figure 14

BER performance of distributed RS coding scheme under non-ideal and ideal source to relay channels and non-cooperative RS coding scheme with RS1(63, 59, 5) and RS2(63, 43, 21).

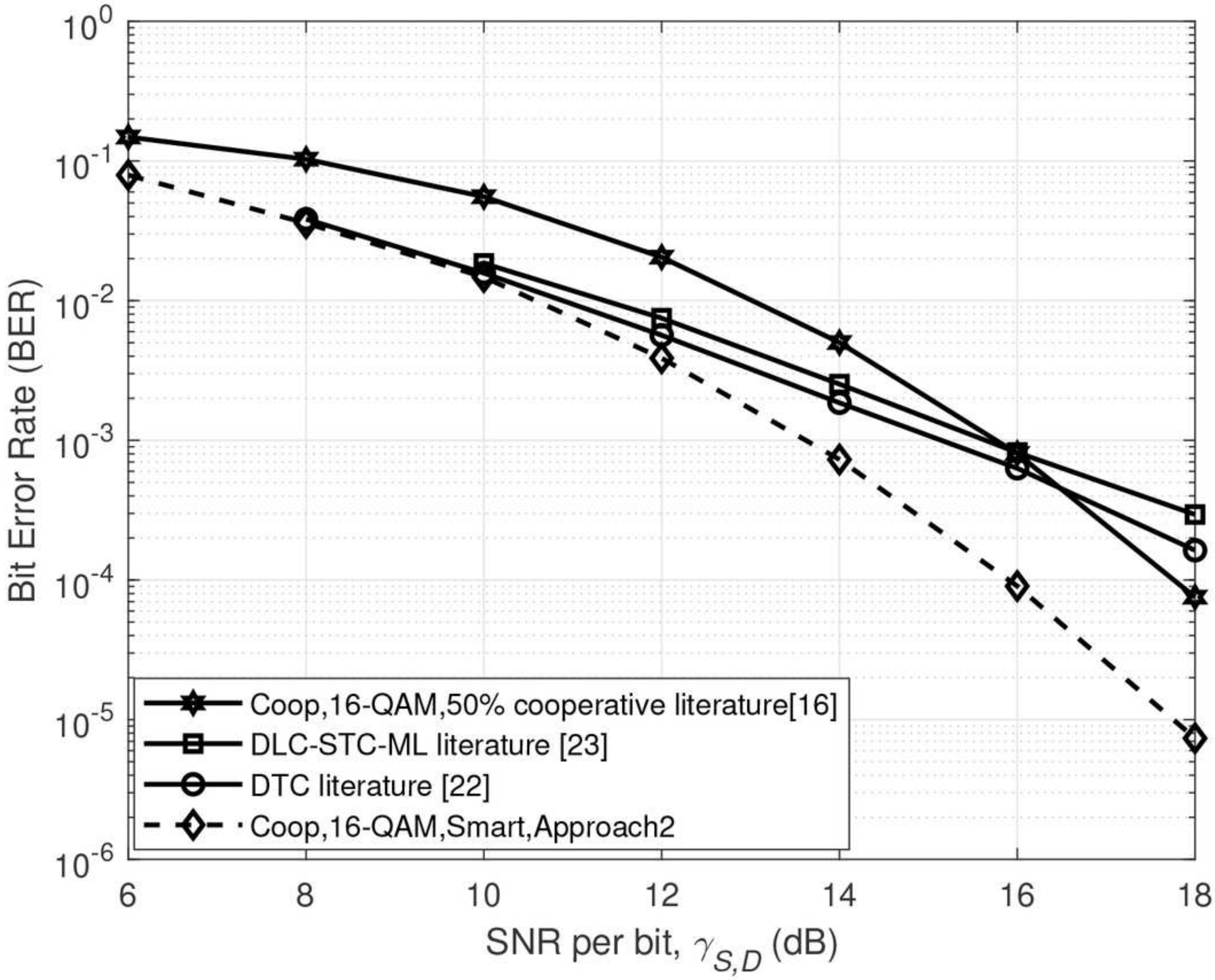

Figure 15

BER performance comparison of distributed RS coding scheme and the existed schemes in [16], [22] and [23].