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Research Article

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A new nonlinear mathematical model for forecasting the waves of the COVID-19 epidemic

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Abstract

Social distance and Hospitals' capacity are two very important factors in the COVID-19 pandemic, which usually change over time. Nevertheless, studies on forecasting this pandemic have not approximately considered these changes in their models. In this study, we accordingly provided a new nonlinear diffusion model by considering these changes for forecasting the COVID-19 outbreak in Iran. This new model generally indicated that it can well follow and predict the trends of the new cases, new recoveries and daily deaths relevant to Iran's COVID-19 epidemic. In this model, some outcomes also indicated that this model can detect some-things about the management style of the COVID-19 epidemic in Iran. Therefore, since this model could follow three waves of the COVID-19 outbreak in Iran, it can be used in mitigation strategies for the better management of the COVID-19 epidemic in Iran and even other world countries.

Keywords: COVID-19, Social distance, Hospitals' capacity, Diffusion model

1. Introduction

After identifying the COVID-19 in late December 2019 in Wuhan, Hubei Province, China and then the Middle Eastern and European countries [1], various mathematical models have been provided for COVID-19 disease transmission. These models are very important, because their outcomes can be considered in mitigation strategies for the better management of the COVID-19 pandemic in the future, if their parameters and outcomes can be compatible with reality.

In recent months, many research works have been reported on forecasting the COVID-19 diffusion in Iran and other world countries. For example, Ndairou and his colleagues [2] provided a compartment model for the COVID-19 transmission in Wuhan, China. These researchers generally stated that their model is suitable for the COVID-19 outbreak, which occurred in this city. Sarkar and his colleagues [3, 4] also proposed a classical SEIR (susceptible exposed infectious recovered)-type epidemiological model that predicts the dynamics of COVID-19 in 17 provinces of India and overall India. In a similar study, Samui and his colleagues [5] also employed a four-compartment SEIR model for the COVID-19 transmission in India with epidemic data up to April 30, 2020. Reports provided by Mandal, Chatterjee, Alaraj and their colleagues [6-8] about the corona-virus disease 2019 transmission, which was based on the SEIR models, are also other similar studies about the outbreak of this disease in India. Fortunately, these reports generally include significant results for predicting this disease in India, and it is remarkable that the reported results are also

suitable for predicting the COVID-19 disease in this country, because the COVID-19 outbreak in this country still contains one wave.

According to the SEIR models, Kuniya [9] also applied a compartmental model to the daily reported cases of COVID-19 in Japan from 15 January to 29 February and outcomes have been only suitable for forecasting the first wave of the disease in Japan. Similarly, the stochastic SEIR model to predict the spread of COVID-19 in Kuwait, which is reported by Almeshal and his colleagues [10], could not completely predict the procedure of the outbreak in this country. About forecasting the COVID-19 outbreak in Iran, there are also similar studies that provided mathematical models based on the daily reported cases of COVID-19 in the early outbreak of this disease after 20 February in Iran [11-14]. Nevertheless, although, the results of these models may be generally suitable for predicting the first wave of the COVID-19 outbreak in this country, these models cannot completely predict the procedure of the COVID-19 outbreak in Iran, which consists of three waves. The mathematical model provided by Ghanbari [15], which nearly followed the second wave of the COVID-19 outbreak in Iran, also seems that is only developed for forecasting the first and second waves, because this researcher has approximately provided no information about this issue that its model can predict later waves. In other words, although these researchers tried to provide the mathematical models for forecasting the COVID-19 outbreak in Iran and different world countries, they have forgotten that the parameters of their models such as social distance and Hospitals' capacity are dependent on the behavior of the humans, and these parameters actually change over time. Accordingly, in this study, we provided a new diffusion model for forecasting the

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COVID-19 outbreak in Iran, in which the changes of these parameters, i.e. social distance and Hospitals' capacity, are the engine of the proposed model.

The remainder of this paper is also organized as follows: Section 2 present the details of the new diffusion model proposed for forecasting the COVID-19 outbreak in different world countries. Section 3 provided the results of the proposed diffusion model for forecasting the COVID-19 outbreak in Iran. Finally, section 3 also presents a brief conclusion about the results obtained in this research.

2. The proposed diffusion model

In engineering, the spread of a pandemic similar to the COVID-19 is a diffusion problem [16], in which the concentration changes of compartments are dependent on force entered the compartments and the output rate of the compartments. Accordingly, as shown in Fig 1, we considered three compartments (new cases, new recoveries and daily deaths) in our diffusion model. Therefore, we wrote the diffusion equations for each of the three compartments as [16]:

$$\begin{cases} \frac{dC(t)}{dt} = F_C - K_C C(t) \\ \frac{dR(t)}{dt} = F_R - K_R R(t) \\ \frac{dD(t)}{dt} = F_D - K_D D(t) \end{cases} \quad (1)$$

where F_C , F_R and F_D are forces entered the compartments of the new cases, new recoveries and daily deaths, respectively. K_C , K_R and K_D are coefficients that set the output rate of these compartments. These forces and coefficients are generally are influenced by two fundamental factors, i.e. social distance and Hospitals' capacity. Since, these factors have different effects on three compartments of the new cases, new recoveries and daily deaths, we defined six variables (s_C , s_R , s_D , c_C , c_R and c_D) to model the effects of these two factors on the mentioned compartments. Also, since, these factors usually change between their best and worst states (0 to 100%), we can simply model the change of these six variables by using a first-order differential equation as follows:

$$\begin{cases} \frac{ds_i(t)}{dt} = \frac{a_i - b_i s_i(t)}{c_i} \\ \frac{dc_i(t)}{dt} = \frac{d_i - e_i c_i(t)}{f_i} \end{cases} \quad i = C, R, D \quad (2)$$

As explained, s_i and c_i are also variables that represent the status of the social distance and the Hospitals' capacity for each of the compartments (new cases, new recoveries and daily deaths). The multiplication of these two variables (Equation 3)

accordingly provides an overall status of the social distance and the Hospitals' capacity, which the force entered the compartments and the output rate of the compartments are directly dependent on it.

$$\phi_i(t) = s_i(t)c_i(t) \quad (3)$$

From Equations 1 and 3, therefore we can deduce that

$$\begin{cases} \frac{dC(t)}{dt} = \alpha \phi_C(t) - (1 - \phi_C(t))C(t) \\ \frac{dR(t)}{dt} = \beta \phi_R(t) - (1 - \phi_R(t))R(t) \\ \frac{dD(t)}{dt} = \theta \phi_D(t) - (1 - \phi_D(t))D(t) \end{cases} \quad (4)$$

Actually, we modeled the coefficients of K_C , K_R and K_D as $1 - \phi_i(t)$, because the value of these coefficients is reduced, when force entered compartments is increased. In other words, force entered a compartment increases the internal components of the compartment. Therefore, since, the mentioned coefficients are calculated from the ratio of the internal components of the compartment to the exited components of the compartment, the coefficients of K_C , K_R and K_D should be decreases, if force entered the compartment is increased. In the next section, we set parameters the proposed model (a_i , b_i , c_i , d_i , e_i , f_i , α , β and θ) for forecasting the COVID-19 outbreak in Iran.

Fig.1 . The proposed diffusion model

Table I. The values of the parameters in the system of equations 2 for forecasting the procedure of the COVID-19 outbreak in Iran within 335 days from 20 February 2019 to 26 December 2020

parameter	i	t (Day)														
		$t < 10$	$10 < t \leq 29$	$29 < t \leq 40$	$40 < t \leq 70$	$70 < t \leq 90$	$90 < t \leq 110$	$110 < t \leq 150$	$150 < t \leq 195$	$195 < t \leq 245$	$245 < t \leq 270$	$270 < t \leq 285$	$285 < t \leq 300$	$300 < t \leq 320$	$320 < t \leq 335$	$t > 335$
a	C	0.004	0.004	1	1	1	1	1	0	1	1	1	1	1	1	0
	R	0.001	0.001	1	1	1	1	1	0	1	1	1	1	1	1	0
	D	0.001	1	1	1	1	1	1	0	1	1	1	1	1	1	0
b	C	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
	R	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
	D	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
c	C	1	1	50	50	200	200	1000	60	140	20	100	100	100	100	500
	R	1	1	100	50	400	400	1000	60	300	100	15	5	5	5	90
	D	1	70	40	80	80	50	50	50	100	80	80	80	80	80	100
d	C	0	0	0	0	1	1	1	1	1	1	0	0	0	0	1
	R	0	0	0	0	1	1	1	1	1	1	0	0	0	0	1
	D	0	0	0	0	0	1	1	1	1	1	0	0	0	1	1
e	C	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
	R	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
	D	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f	C	1	1	25	15	230	230	2300	250	220	220	800	40	40	440	440
	R	1	1	1	15	230	230	2300	250	220	220	130	130	80	440	440
	D	1	50	30	30	30	200	200	180	180	180	45	45	45	180	180

3. Forecasting the COVID-19 outbreak in Iran by using the proposed diffusion model

As stated earlier, the social distance and the Hospitals' capacity change over time, because they are dependent on the behavior of the humans. Therefore, the parameters of a_i , b_i , c_i , d_i , e_i and f_i in the system of equations 2 change over time. Table I provides the values of these parameters within 335 days, which we determined based on the knowledge of first-order differential equations for forecasting the procedure of the COVID-19 outbreak in Iran from 20 February 2019 to 26 December 2020. In addition, since the parameters of α , β and θ are dependent on the maximum value of the new cases, new recoveries and daily deaths in the third wave of the COVID-19 outbreak in Iran, we rewrote the system of equations 4 as follows:

$$\begin{cases} \frac{dC(t)}{dt} = A\phi_C(t) - (1 - \phi_C(t))C(t) \\ \frac{dR(t)}{dt} = 0.93A\phi_R(t) - (1 - \phi_R(t))R(t) \\ \frac{dD(t)}{dt} = 0.04A\phi_D(t) - (1 - \phi_D(t))D(t) \end{cases} \quad (5)$$

where A is the maximum value of the new cases in the third wave of the COVID-19 outbreak in Iran ($A = 14000$).

Fig 2a and 2b show the outcomes of the proposed diffusion model (system of equations 5) for predicting the new cases, new recoveries and daily deaths within 365 from 20 February 2019 for Iran. As shown in this figure, this differential model could well model every three waves, which occurred in the real data of the COVID-19 outbreak in Iran. In the remaining two months of the solar year 2000 (i.e. January and February), this model has also forecasted the beginning of a new small wave for the COVID-19 outbreak in Iran. It is also interesting that the changes of the variables s_i and c_i , which is represented in Fig 2c, detected some-things about the management style of the COVID-19 epidemic in Iran. In other words, as shown in this figure, the variables s_i (s_C , s_R and s_D), which represent the status of the social distance, are reduced at the beginning of the first wave. This reduction is the reason for the first wave in the curves of the new cases, new recoveries and daily deaths. In the following, the variables c_i (c_C , c_R and c_D), which represent the status of the Hospitals' capacity, are incremented, which its outcome is the completion of the first wave. After this phase, while variables c_i are incremental, the variables s_i experiences a further reduction. The result of this reduction is the beginning of the second wave. After the completion of the second wave, which its reason is the increase in the variables s_i , the variables s_i are reduced

again in addition to decreasing the variables c_i due to the fatigue of hospital staff. Its result is also the third wave. To limit this big wave, the variables c_i is exponentially increased, which its reason is a mitigation

strategy applied by Iranian authorities. Therefore, as mentioned, the variables s_i and c_i , can detect some-things about the management style of the COVID-19 outbreak.

Fig.2 . The outcomes of the proposed diffusion model (system of equations 5) (a) For predicting the daily deaths within 365 from 20 February 2019, (b) For predicting the new cases and the new recoveries within 364 from 20 February 2019, (c) The changes of the variables s_i and c_i for predicting the new cases, new recoveries and daily deaths within 364 from 20 February 2019 for Iran

Fig.3 . Currently Infected according to equation 6

According to this model, as shown in Fig 1, the currently infected can be also estimated by the following equation:

$$I(t) = I(t - \Delta t) + C(t) - R(t) - D(t) \quad (6)$$

Where $I(t)$ is the currently Infected. Δt is also equal to one, because $I(t)$ is calculated in each day. Fig 3 shows the currently infected, which is predicted by the proposed model. As shown in this figure, this model could well follow the curve of the currently infected.

In the next section, we provided more information about the results obtained in this research.

4. Discussion and conclusion

In most world countries, the trend of the COVID-19 epidemic generally depends on social interactions and Hospitals' capacity, although other behavior of the people and government interventions can affect this trend. In other words, social distance and Hospitals' capacity are two fundamental factors in the spread of epidemics similar to the COVID-19 epidemic in a community. Accordingly, these two factors not only affect the infection rate, but also they can directly affect the rate of new recoveries and daily deaths.

In the diffusion model, which we proposed according to this viewpoint in this paper, these effects are clearly visible in variables s_i and c_i (Fig 2c). Actually, these variables, which are candidates from social distance and Hospitals' capacity and generally adjust the input and output rate of the model's compartments, are the motion engine of the proposed model.

It is remarkable that according to the estimated trend for these variables by the proposed model (Fig 2c), the social distance in the two stages has reduced for forecasting the trend of COVID-19 epidemic in Iran, which their outcome are also the first and third waves in

this epidemic. This reduction has been generally felt by the Iranian people and interventions have been also taken to fix them.

As explained earlier, for the second wave, especially in the trend of daily deaths, variables s_i is along with a further reduction, so that variables c_i could only fix this reduction with a delayed increase, which had been applied by the Iranian authorities. In other words, increasing the Hospitals' capacity could fix this problem (low social distance).

Therefore, these outcomes along with the model's outcomes (Fig 2a and 2b) for forecasting the new cases, new recoveries and daily deaths confirm that the model proposed in this study can be employed for predicting the COVID-19 outbreak in Iran in the next months, because this model has predicted that a new wave will likely occur in the coming months. Of course, as summarized in Table I, the parameters of a_i , b_i , c_i , d_i , e_i and f_i , especially the parameters of c_i and f_i , which are candidates for social distance and Hospitals' capacity, should be approximately set according to the condition of these factors (social distance and Hospitals' capacity). Therefore, a more comprehensive study on these parameters may improve the outcomes of the proposed model. In addition, there are other factors that affect the spread of the COVID-19 pandemic such as vaccination. Therefore, these factors can be investigated in future studies by adding a set of new compartments to the present model.

5. Conflict of Interest

The authors declare that they have no conflict of interest.

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Figures

Figure 1

The proposed diffusion model

Figure 2

The outcomes of the proposed diffusion model (system of equations 5) (a) For predicting the daily deaths within 365 from 20 February 2019, (b) For predicting the new cases and the new recoveries within 364 from 20 February 2019, (c) The changes of the variables s_i and c_i for predicting the new cases, new recoveries and daily deaths within 364 from 20 February 2019 for Iran

Figure 3

Currently Infected according to equation 6