

Fundamental optical linewidth of soliton microcombs

Fuchuan Lei

Photonics Laboratory, Chalmers University of Technology <https://orcid.org/0000-0003-2257-7057>

Zhichao Ye

Chalmers University of Technology

Attila Fülöp

Chalmers University of Technology

Victor Torres-Company ([✉ torresv@chalmers.se](mailto:torresv@chalmers.se))

Chalmers University of Technology <https://orcid.org/0000-0002-3504-2118>

Article

Keywords: Frequency Rulers, Precision Metrology, Broadband Phase Coherence, Optical Phase-noise Dynamics, Fundamental Linewidth

Posted Date: March 10th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-243867/v1>

License:  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Version of Record: A version of this preprint was published at Nature Communications on June 7th, 2022.
See the published version at <https://doi.org/10.1038/s41467-022-30726-5>.

Fundamental optical linewidth of soliton microcombs

Fuchuan Lei, Zhichao Ye, Attila Fülöp, and Victor Torres-Company*

Department of Microtechnology and Nanoscience,
Chalmers University of Technology SE-41296 Gothenburg, Sweden

(Dated: February 15, 2021)

Soliton microcombs provide a versatile platform for realizing fundamental studies and technological applications. To be utilized as frequency rulers for precision metrology, soliton microcombs must display broadband phase coherence, a parameter characterized by the optical phase or frequency noise of the comb lines and their corresponding optical linewidths. Here, we analyze the optical phase-noise dynamics in soliton microcombs and show that, because of the Raman self-frequency shift, the fundamental linewidth of some of the comb lines can, surprisingly, be narrower than the linewidth of the pump laser. This work elucidates information about the ultimate limits in phase coherence of soliton microcombs and illustrates a new strategy for the generation of spectrally coherent light on chip.

I. INTRODUCTION

The quantum-limited or fundamental linewidth of a continuous-wave single-mode laser is not zero due to the existence of spontaneous emission and the limited quality (Q) factor of the optical resonator cavity [1]. The study of the fundamental linewidth of optical coherent oscillators started with the seminal works of Schawlow and Townes [1], even before the invention of the laser. Noteworthy, the noise coming from spontaneous emission is white and, unlike technical noise sources, it cannot be effectively suppressed through a finite-bandwidth feedback control. Thus, the fundamental linewidth stands for the ultimate performance in temporal coherence of a coherent oscillator.

A frequency comb is a laser whose spectrum is composed of equidistant frequency components that are phase locked to a common frequency reference. The phase noise of the constituent optical lines sets a physical limit on the achievable time and frequency stability [2–4]. Significant efforts have been devoted to the systematic understanding of the linewidth of mode-locked lasers and frequency combs based on solid-state [5, 6], semiconductor [7] and fiber lasers [8, 9]. In 2007, a new type of frequency comb source (microcomb) was demonstrated. Microcombs harness the Kerr nonlinearity and large intensity buildup in a high-Q microresonator cavity [10]. Low-noise coherent states can be attained through the generation of dissipative solitons [11–13]. Unlike in conventional frequency combs based on mode-locked lasers, where the gain originates from stimulated emission in active gain media and the fundamental linewidth is dictated by spontaneous emission, the gain of soliton microcombs is based on resonantly enhanced continuous-wave-pumped parametric amplification, and the noise caused by spontaneous scattering is very weak. Another important difference is that in microcombs, the pump laser is coherently added to the comb spectrum, and therefore its noise is expected to be transferred equally to all comb lines. Indeed, earlier studies demonstrate that when the microcomb operates in a low-noise state, the comb lines inherit the linewidth of the pump [14, 15], with lines fur-

ther away from the pump degrading more due to thermorefractive noise in the cavity [16, 17].

In this work, we present both theoretical and experimental studies of the fundamental linewidth of soliton microcombs. We reveal that the interplay between the pump's frequency noise and soliton dynamics results in a linewidth distribution among comb lines that is consistent with the elastic-tape model [18], akin to what has been found previously with conventional mode-locked frequency combs [6, 7, 9]. An important difference in soliton microcombs is that the Raman self-frequency shift couples the repetition rate of the soliton with the pump frequency [19]. This results in a subset of comb lines becoming resilient to the frequency noise of the laser pump, and an encompassing decrease in fundamental linewidth. Our work stands for a universal understanding of the phase noise dynamics of soliton microcombs. Furthermore, the mechanism for the reduction of the fundamental linewidth provides a new strategy to generate ultra-low-phase-noise coherent optical oscillators on chip.

II. RESULTS

A. Elastic-tape model applied to soliton microcombs

The optical frequency of the m -th microcomb line ν_m is determined by two degrees of freedom, i.e. the frequency of the pump laser ν_p and the repetition rate of the soliton microcomb ν_{rep} ,

$$\nu_m = \nu_p + m\nu_{\text{rep}}, \quad (1)$$

with the comb line number, m , counted from the pump. The underlying assumption in the elastic-tape model [18] is that the noise sources will result in collective fluctuations of the comb lines. According to this, Eq. (1) indicates that the linewidth of the pump would be faithfully imprinted on all other comb lines if the repetition rate were fixed. However, in soliton microcombs, due to the existence of intrinsic intrapulse Raman scattering, the pump phase (frequency) noise will also affect the

repetition rate. This is because the repetition rate relies on the detuning between the cavity mode and the pump [20, 21]:

$$\nu_{\text{rep}} = \frac{1}{2\pi} [D_1 + \frac{D_2}{D_1} \Omega_{\text{Raman}} (\nu_c - \nu_p)], \quad (2)$$

where $D_1/2\pi$ is the cavity's free spectral range (FSR) at the pump mode, and $D_2 = -D_1^2 \beta_2 / \beta_1$ with β_1 and β_2 being the first- and second-order coefficients of the Taylor expansion of the mode propagation constant β [22]. The resonant frequency of the pump mode is ν_c , and Ω_{Raman} denotes the shift of the carrier frequency due to Raman scattering, i.e. the Raman self-frequency shift [19].

According to Eqs. (1) and (2), the frequency change

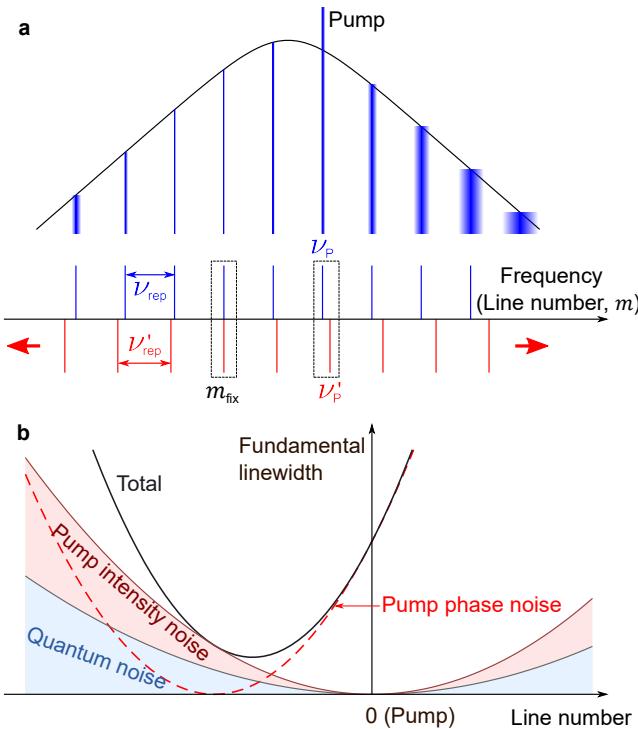


FIG. 1. Elastic-tape model applied to soliton microcombs. **a**, Illustration of the linewidth distribution in single-soliton microcombs affected by the Raman self-frequency shift. The pump laser has inherent frequency fluctuations. With respect to an arbitrary origin, variations of the pump frequency to the blue side result in an increase of the soliton repetition rate, and vice versa to the red side. The coupling between pump frequency and repetition rate thus results in the existence of a so-called "fixed point" in the comb spectrum, that is a comb line that is most resilient to the fluctuations of the pump's frequency noise. This translates into a linewidth distribution among comb lines whose minimum is found far from the pump frequency, **b**. In practice, the frequency linewidth of the lines is also affected by intensity-to-frequency noise conversion mediated by the Kerr nonlinearity, and the inherent quantum noise (vacuum fluctuations), which set a limit into the minimum attainable fundamental linewidth of the soliton microcomb.

of the m -th comb line $\delta\nu_m$ induced by that of the pump $\delta\nu_p$ can be written as

$$\delta\nu_m = \delta\nu_p (1 + m \frac{d\nu_{\text{rep}}}{d\nu_p}). \quad (3)$$

In the following, we will characterize the frequency noise of the comb lines by means of the power spectral density (PSD) of the frequency noise, $S_{\Delta\nu,m}(f)$, (sometimes called FM noise). The fundamental frequency noise PSD of the pump laser, $S_{\Delta\nu}^p(f)$, is characterized by a constant level and has a simple relation with its fundamental phase noise PSD $S_{\Delta\phi}^p$ as [23]

$$S_{\Delta\nu}^p(f) = f^2 S_{\Delta\phi}^p(f) = S_0. \quad (4)$$

When flicker noise and other technical sources are neglected, the pump features a Lorentzian lineshape, whose full-width at half maximum (FWHM) accounts for the fundamental linewidth, $\Delta\nu_p = \pi S_0$. From Eq. (3), the frequency noise PSD of the m -th comb line, $S_{\Delta\nu,m}(f)$, can be directly linked to the frequency noise PSD of the pump as

$$S_{\Delta\nu,m}(f) = S_{\Delta\nu}^p(f) (1 - \frac{m}{m_{\text{fix}}})^2. \quad (5)$$

where $m_{\text{fix}} = -(d\nu_{\text{rep}}/d\nu_p)^{-1}$ corresponds to the pump phase-noise fixed point [18]. Equation (5) indicates that the fundamental linewidth of the soliton microcomb follows a parabolic distribution with line number, with a minimum value reaching zero at the fixed point, see Fig. 1. For soliton microcombs affected by the Raman self-frequency shift, the fixed point appears to the red side of the pump because the repetition rate increases when the pump frequency increases. We anticipate that third-order dispersion or modal coupling could modify the location of the fixed point. For a realistic pump laser, where the frequency noise PSD contains both white and flicker noise, it is still possible to characterize the fundamental linewidth from the flat region at the bottom of the frequency noise PSD [24]. This is the procedure that we will follow in this work when assessing the fundamental linewidth of the comb lines. At the end of this section, we will clarify the differences between the *fundamental linewidth* and the *effective linewidth*.

In addition to phase noise, the intensity noise of the pump could also introduce frequency noise into the soliton microcomb because the repetition rate can be modified by the pump power via the Kerr and Raman effects. The frequency noise PSD induced by the relative intensity noise (RIN) of the pump can be written as [25]

$$S_{\Delta\nu,m}(f) = (\alpha P_{\text{in}})^2 S_{\text{RIN}}^p(f) m^2. \quad (6)$$

where $\alpha = d\nu_{\text{rep}}/dP_{\text{in}}$, and $S_{\text{RIN}}^p(f)$ is the PSD of the RIN. Note that the frequency dependence of the $S_{\text{RIN}}(f)$ may be dominated by flicker noise. However, in order to perform a quantitative analysis of the fundamental linewidth, here we use for simplicity the average value,

\bar{S}_{RIN} . As illustrated in Fig. 1b, the intensity noise introduces a parabolic distribution of the fundamental linewidth whose center is located at the pump mode.

We now proceed to include the frequency noise contribution induced by vacuum fluctuations (or quantum noise). In the studies of supercontinuum generation, quantum noise poses fundamental limitations in the achievable spectral coherence [26, 27]. In soliton microcombs, the quantum noise sets a fundamental-limited timing jitter [28–31]. The timing jitter PSD, $S_{tm}^Q(f)$, affects the optical frequency noise of the comb lines $S_{\Delta\nu,m}(f)$ through

$$S_{\Delta\nu,m}(f) = m^2 f^2 D_1^2 S_{tm}^Q(f). \quad (7)$$

With the fact $S_{tm}^Q(f) \propto 1/f^2$ if $f \ll 2\pi\nu_p/Q$ [28, 29], one can see that the quantum noise leads to a line-number-dependent white optical frequency noise. Since the effect arising from spontaneous Raman scattering is usually much weaker than the quantum noise [26], it is ignored here.

As the above three noise sources are independent from each other, their individual contribution to the fundamental linewidth can be added together. Thus, the fundamental linewidth of the m -th line of a single-soliton microcomb can be expressed as

$$\Delta\nu_m = \Delta\nu_p \left(1 - \frac{m}{m_{\text{fix}}}\right)^2 + m^2 (\Delta\nu_{\text{RIN}} + \Delta\nu_{tm}) \quad (8)$$

with $\Delta\nu_{\text{RIN}} = \pi(\alpha P_{\text{in}})^2 \bar{S}_{\text{RIN}}$ and $\Delta\nu_{tm} = \pi D_1^2 f^2 S_{tm}^Q(f)$. Because of the intensity noise of the pump and the quantum-noise-induced timing jitter, the comb line with the minimum fundamental linewidth (we term it here the *quiet mode*) is no longer the phase-noise fixed point, and its location appears closer to the pump but remains always to the longer-wavelength side. It is instructive to manipulate further Eq. (8) and calculate the reduction of the fundamental linewidth at the quiet mode, $\Delta\nu_{\min}$, relative to the pump's fundamental linewidth

$$\frac{\Delta\nu_{\min}}{\Delta\nu_p} = \frac{1}{1 + \frac{1}{m_{\text{fix}}^2} \frac{\Delta\nu_p}{\Delta\nu_{\text{RIN}} + \Delta\nu_{tm}}}. \quad (9)$$

This analysis indicates that the relative reduction is more efficient for pump lasers with larger linewidths, a feature that will be explored further in the following sections.

B. Optical phase noise dynamics and numerical simulations

To test the validity of the previous theoretical analysis and gain a better understanding, we conduct a series of numerical simulations based on the Ikeda map [32, 33]. The parameters of the simulations are chosen to match the characteristics of our silicon nitride (SiN) microresonators and are detailed in the Methods section. The simulations also allow us to study the influence of the individual contribution of the different noise sources on the

dynamics of the fundamental linewidth of soliton microcombs. We begin the analysis by considering the optical phase noise of the pump only, and the results are illustrated in Fig. 2a. In absence of Raman self-frequency shift, for a coherent dissipative soliton state, the phase noise is transduced equally among the comb lines, indicating a tight phase locking with the pump. However, when the Raman term is included, fluctuations in pump frequency are transduced into repetition rate changes. The repetition rate increases with detuning as a result of the Raman self-frequency shift, resulting in a fixed point to the red side of the pump characterized by zero frequency fluctuations in theory and validated by the simulations.

As anticipated in the previous section, both intensity noise and quantum noise prevent from attaining a zero fundamental frequency linewidth at the pump phase fixed point. This is analyzed in the numerical simulations presented in Figs. 2b and c. In both cases, the distribution of the fundamental linewidth is symmetric with respect to the pump and follows a parabolic profile, in agreement with the theoretical analysis presented in the previous section, cf. Eqs. (6) and (7). Note that the fundamental linewidth induced by quantum-noise timing jitter in our SiN microcomb is more than one order of magnitude higher than what has been previously reported for silica microcombs [29], mainly due to the larger nonlinear coefficient of SiN. Hence, the quantum noise sets a relatively high bound to the lowest achievable fundamental linewidth in this platform.

Figure 2d shows the simulation results when all noise contributions are added together. The fundamental linewidth based on the theoretical model (Eq. (8)) is plotted for comparison. The excellent match between the theoretical model and the simulations indicates that the elastic-tape model can account for the most salient features of the optical phase noise dynamics in soliton microcombs.

We end this section by analyzing the phase noise dynamics of the individual line components when all noise sources are included. Specifically, we compute the phase noise of individual frequency lines, see Fig. 2e. Clearly, the lines close to the quiet mode display reduced phase noise, with a standard deviation smaller than the inherent phase noise of the pump laser. Remarkably, the quiet mode stands out as a mirror symmetry point in the comb, whereby lines symmetrically located around it attain identical fundamental linewidth but anti-correlated phase noise. A similar behaviour has been observed before for electro-optic frequency combs, with the key difference that the fixed point corresponds there to the pump laser frequency [34, 35]. This observation can be further quantified with the aid of the Pearson's correlation coefficient:

$$C(m, n) = \frac{\text{cov}(\phi_m, \phi_n)}{\sigma_{\phi_m} \sigma_{\phi_n}}, \quad (10)$$

where ϕ_m denotes the sampled phase for the m -th comb

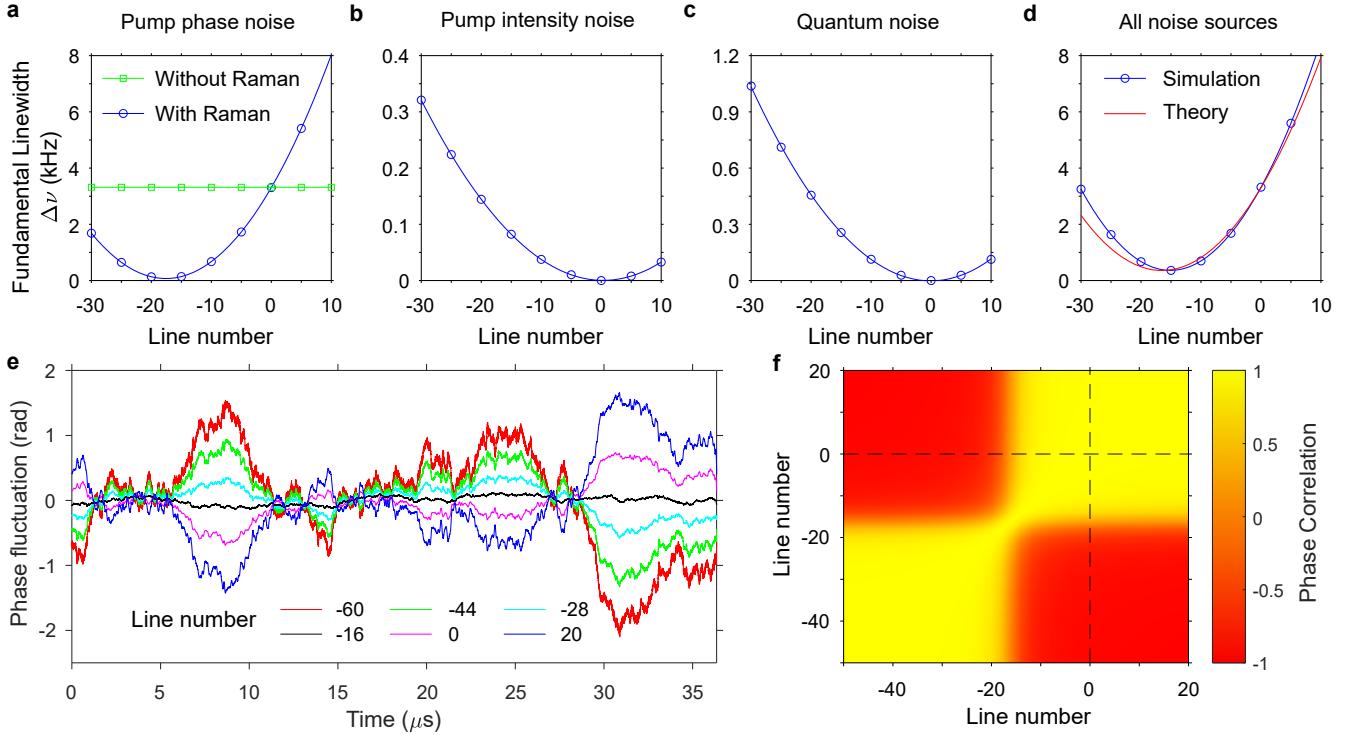


FIG. 2. Numerical study of the fundamental linewidth of soliton mirocombs. The fundamental linewidth of comb lines induced by pump phase noise, intensity noise, quantum noise and when all three noise sources are included, from **a** to **d**. The squares and circles denote the simulated data while the line stands for a quadratic fit. The theoretical result in **d** (red line) is obtained according to Eq. (8). **e**, Phase fluctuations of selected comb lines. **f**, Phase correlation among comb lines, computed according to Eq. (10).

line. cov is the covariance and σ_X is the standard deviation of X . The result is plotted in Fig. 2f. One can see that the phases of comb lines at the same side of $m = -16$ are highly correlated, and those on opposite sides anti-correlated, which can be explained with the elastic-tape model [6].

C. Experiments with SiN microcombs

We present experimental results of the distribution of the frequency noise and corresponding fundamental linewidth in a soliton microcomb implemented in a silicon nitride microresonator pumped by a narrow-linewidth external cavity tunable laser diode. The setup is shown in Fig. 3a. The FSR and the average intrinsic Q factor of the SiN microresonator are 227.5 GHz and 1.16×10^7 , respectively.

The pump power coupled into the bus waveguide is 120 mW. Other parameters and experimental details are described in the Methods. The soliton microcomb generation is enabled through fast thermo-optic tuning via an integrated metallic heater [36]. The optical spectrum of the generated single-soliton microcomb is shown in Fig. 3b. After attenuation of the pump line with a notch filter, the comb is amplified with either a C- or L-band

EDFA. Then each comb line is filtered out separately and amplified to ~ 20 mW. Subsequently, the phase and frequency noise of each comb line is measured through a self-heterodyne measurement technique implemented with the aid of a coherent receiver [23].

The measured fundamental linewidth for the comb lines from $m = -28$ to $m = 7$ are shown in Fig. 3b, which is obtained based on the average value at high offset frequencies (few MHz range). This region is chosen to avoid the contribution of flicker noise at low frequencies and the divergence at high frequencies caused by the white phase-noise component arising from both the optical amplifiers ASE noise and the thermal noise of the measurement system. The results indicate that a significant portion of the comb lines around -13 (equivalent 1580 nm wavelength) display a fundamental linewidth that is in fact smaller than that of the pump. Note that the location of the quiet mode is very close to the simulation in Fig. 2d. The line located at -13 achieves the smallest phase noise, with a fundamental linewidth of ~ 1 kHz, corresponding to more than threefold reduction of the pump's value. The measurement is higher than the predictions provided by Fig. 2d (~ 0.5 kHz). We believe the discrepancy is partly due to thermo-refractive noise in the microresonator, which has a non-negligible contribution in the frequency range used for computing

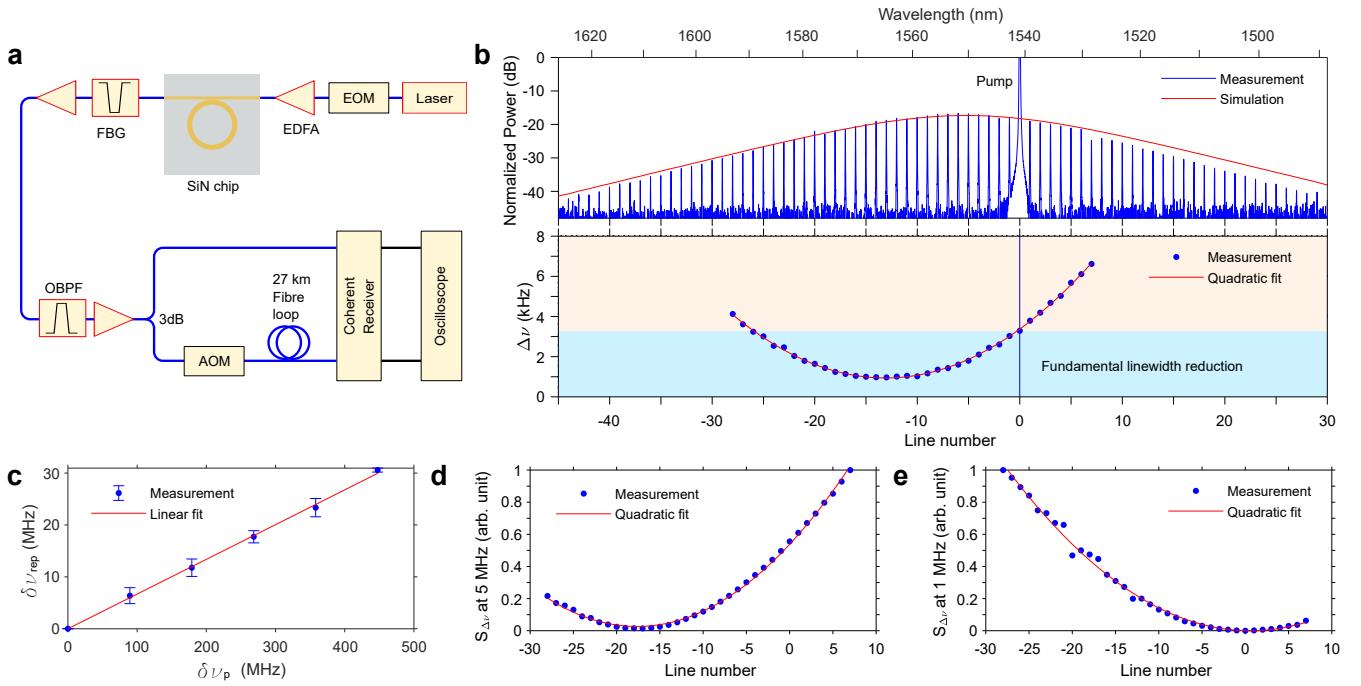


FIG. 3. Experimental study of the fundamental linewidth of silicon nitride soliton microcombs. **a** Setup for soliton microcombs linewidth measurement. EOM, (Phase or Intensity) Electro-optical modulator; EDFA, Erbium-doped fibre amplifier; FBG, Fibre Bragg grating for pump rejection; OBPF, Optical band-pass filter; AOM, Acousto-optic modulator. **b** Optical spectrum of the single-soliton microcomb and the measured fundamental linewidth of comb lines according to the mean value of $S_{\Delta\nu,m}(f)$ at high offset frequencies ranging 3 to 5 MHz. **c** Measured soliton microcomb repetition rate varying with pump frequency. The error bar stands for the standard deviation of three measurements. **d** and **e** The height of frequency noise PSD at 5 MHz and 1 MHz, originating from applying a 5 MHz phase or 1 MHz intensity modulation signal to the pump.

the fundamental linewidth. Other effects, not included in our model but that might contribute to the frequency noise dynamics include a frequency-dependent Q factor [37] or avoided mode crossings.

We continue with a quantitative analysis of the location of the fixed point by measuring the change of the soliton's repetition rate with pump frequency, cf Eq. (4). Specifically, the repetition rate of the soliton is measured by electro-optic downconversion [38] as the pump is set at different values, which are measured with a wavelength meter. The results of this measurement are presented in Fig. 3c. The slope of the variation is positive, explaining why the fixed point appears to the red side of the pump. Furthermore, the estimated fixed point $m_{\text{fix}} = -(dy_{\text{rep}}/dy_p)^{-1} = -15$ is very close to the measured point $m = -13$. However, this measurement provides limited insight into the phase-to-phase and intensity-to-phase transduction from the pump. Such measurements are further addressed in the following experiments. We modulated the pump laser with either a phase or intensity electro-optic modulator (EOM) at specific single radio frequencies before getting amplified by an EDFA. This introduces spikes in the $S_{\Delta\nu,m}(f)$ at the corresponding modulation frequencies, at levels about 3 orders higher than the noise background. The measurement results are presented in Figs. 3d and 3e, corresponding to the

phase and intensity modulation, respectively. The magnitude of the spike provides an indirect estimation of the influence of the pump's phase/amplitude noise on the fundamental linewidth of the corresponding comb line. One can see that while the intensity-to-phase noise transduction is symmetrically located around the pump, the phase-to-phase distribution attain its minimum at $m_{\text{fix}} = -17$. These results are in excellent agreement with the simulations presented in 2a and b.

D. Linewidth reduction and effective linewidth

So far, our analysis of the fundamental linewidth has ignored the effect of thermorefractive noise [39]. This noise source causes fluctuations in the location of the longitudinal modes and repetition rate of the soliton through the detuning parameter [16]. In microresonators with small cavity volumes, this effect has a dominant contribution [40, 41] at low offset frequencies in the frequency noise PSD of the comb lines [16]. To account for such non-white frequency noise, it is useful to calculate the effective linewidth of the comb lines $\Delta\nu_m^{\text{eff}}$ using the following definition [42]:

$$\int_{\Delta\nu_m^{\text{eff}}}^{\infty} \frac{S_{\Delta\nu,m}(f)}{f^2} df = \frac{1}{\pi}. \quad (11)$$

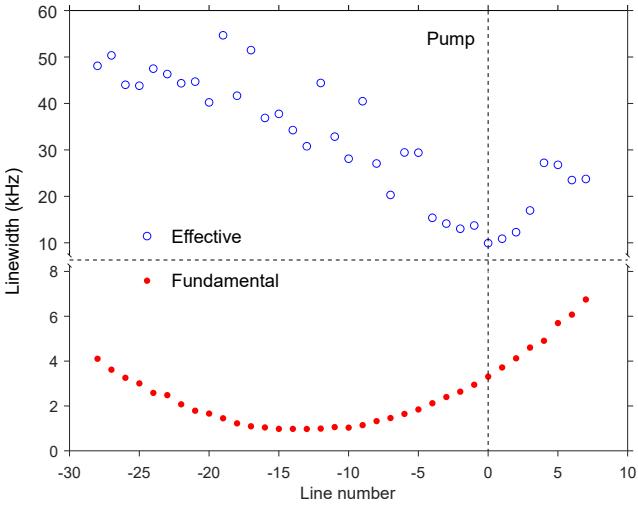


FIG. 4. Comparison of the effective linewidth and the fundamental linewidth. With partial inclusion of the low-frequency technique noise, the effective linewidth is much larger than the fundamental linewidth, and it is symmetrical with respect to pump.

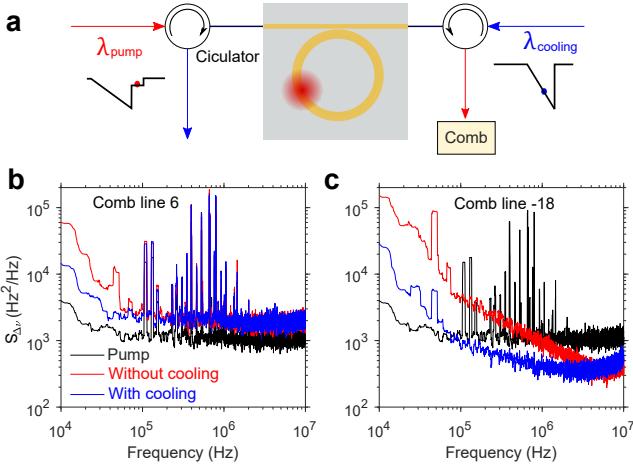


FIG. 5. Frequency noise reduction with laser cooling. **a** The schematic of the setup. An auxiliary laser (blue arrow) opposite to the pump (red arrow) is injected into the microresonator and blue-detuned to a cavity mode with the same family of pump mode. $S_{\Delta\nu,6}(f)$ and $S_{\Delta\nu,-18}(f)$ for without (red) and with (blue) laser cooling are shown in **b** and **c**, respectively. The black line denotes pump frequency noise PSD $S_{\Delta\nu}^P(f)$. The PSD at low-frequency region is reduced after cooling, but the flat bottom is maintained.

The results are displayed in Fig. 4, and compared to the results in Fig. 3b, reproduced again for convenience. Clearly, the non-white frequency noise region (or flicker noise) has a dominant contribution in the value of the effective linewidth. Note as well that the distribution is symmetrical with respect to the pump. This is expected, and in line with previous reports [17].

The contribution of thermo-refractive noise on the fre-

quency noise PSD of the comb lines can be effectively suppressed by e.g. laser cooling [16, 43], cavity dispersion engineering [44] or operation at cryogenic temperatures [45]. Here we show that the laser cooling technique is also compatible with the reduction of the fundamental linewidth described in the previous subsection. The laser cooling is performed with an auxiliary laser coupled to the microresonator from the opposite direction, as schematically depicted in Fig. 5a. The cooling laser has an on-chip power around 2 mW and is tuned into a resonance close to 1550 nm and kept blue detuned. The measured $S_{\Delta\nu,m}(f)$ for comb lines $m = 6$ and $m = -18$ are shown in Fig. 5b and 5c, respectively. It is clear that the low frequency noise up to a few MHz is reduced significantly while the plateau in the PSD that contributes to the fundamental linewidth remains nearly unchanged by the laser cooling. It is interesting to note that, the discrete spikes located between 0.1-2 MHz in pump frequency noise PSD cannot be observed in the comb line $m = -18$, indicating that the fundamental linewidth can be efficiently decreased in combination with laser cooling.

III. DISCUSSION

In summary, we have demonstrated that the elastic-tape model, widely used for describing the phase noise dynamics in mode-locked laser frequency combs, can also be used to analyze the fundamental optical linewidth in soliton microcombs. The coupling between soliton repetition rate and pump frequency, mediated by the Raman self-frequency shift, results in a broadband frequency region of phase-locked coherent oscillators whose fundamental linewidth is smaller than the fundamental linewidth of the laser pump itself. In practice, however, it is the thermorefractive noise of the cavity that contributes the most at low-offset frequencies in the frequency noise power spectral density, resulting in a dominant contribution in the effective linewidth of the comb lines. This effect can be notwithstanding damped with the aid of a cooling laser. We discover the existence of a fixed point in the soliton microcomb spectrum. Frequency lines symmetrically located around this point feature an identical fundamental linewidth, but the phase noise varies in an anti-correlated fashion.

This work unravels the role of the phase noise of the pump and its interplay with the cavity soliton dynamics, and elucidates the ultimate limits in phase coherence achievable in soliton microcombs. While here we focused on dissipative Kerr solitons in anomalous dispersion cavities, we anticipate that the location of the fixed point could be drastically different in other dissipative soliton states, such as multiple solitons [46], crystals [47, 48] or in novel cavity arrangements [49–52].

METHODS

Resonator characteristics and operating condition

The SiN microring resonator is fabricated via subtractive processing [53]. The radius of the microresonator used for experiment is 100 μm , and the corresponding FSR is about 227.5 GHz. The height and the width of the SiN waveguide are 750 nm and 2100 nm, respectively, which result in a group velocity dispersion coefficient of $\beta_2 = -80 \text{ ps}^2/\text{km}$ for mode TE₀₀. The average intrinsic Q -factor of the sample is 11.6 million and the total Q -factor is 8 million. Lensed fibres are used for coupling into and out of the on-chip SiN bus waveguide. The average coupling loss per facet is about 3.5 dB.

Theory and simulation

To apply the theoretical results corresponding to Eq. (8) in Fig. 2d, the parameters m_{fix} and α were calculated using their definitions, i.e., the derivative of repetition rate with respect to the pump's frequency and power. The repetition rate was attained by monitoring the speed of the soliton, whose temporal position t_p is calculated as [54]

$$t_p = \frac{\int t |A(t)|^2 dt}{\int |A(t)|^2 dt}. \quad (12)$$

The quantum-noise-induced timing jitter PSD $S_{tm}^Q(f)$ was obtained from simulations since there is no analytical result that includes the Raman self-frequency shift.

The simulation is performed with the Ikeda map [32]. By iteratively performing the coupling and propagation steps [55], this method allows introducing different noise sources each roundtrip. At each propagation step, the generalized nonlinear Schrödinger equation with (or without) inclusion of Raman scattering is solved [22, 56]:

$$\begin{aligned} \frac{\partial A}{\partial z} + \frac{1}{2}\alpha - i \sum_{n=1}^2 \frac{i^n \beta_n}{n!} \frac{\partial^n A}{\partial t^n} &= i\gamma A(z, t) \\ \times \int_0^\infty R(t') |A(z, t-t')|^2 dt'. \end{aligned} \quad (13)$$

Here $R(t) = (1 - f_R)\delta(t) + f_R h_R(t)$, and $h_R(t) = (\tau_1^{-2} + \tau_2^{-2})\tau_1 \exp(-t/\tau_2) \sin(t/\tau_1)$, and the parameters have the same meaning as [22]. In the simulation, the parameters $\tau_1 = 15 \text{ fs}$, $\tau_2 = 120 \text{ fs}$, $f_R = 0.027$ are used as they match the measured optical spectrum. The remaining parameters are directly measured or simulated and have the following values: $P_{\text{in}} = 0.1 \text{ W}$, $\gamma = 0.9 \text{ W}^{-1}\text{m}^{-1}$, $Q_{\text{ex}} = 3.15 \times 10^7$, $Q_{\text{in}} = 1.35 \times 10^7$, $\Delta\nu_p = 3.3 \text{ kHz}$, $\nu_c - \nu_p = 626 \text{ MHz}$ and $S_{\text{RIN}}^P = -127.5 \text{ dB/Hz}$. The time-dependent pump phase was generated through

$$\phi(t + \Delta t) = \phi(t) + \sqrt{2\pi\Delta\nu_p\Delta t} \times \eta \quad (14)$$

where $\Delta t = 2\pi/D_1$ is the roundtrip time and η stands for a normally distributed random number. The power fluctuation of the pump is determined by the RIN as

$$\delta P(t) = \sqrt{S_{\text{RIN}}^P \Delta t} \times \eta. \quad (15)$$

Next, frequency filters are used to generate a bandwidth-limited (0 to 10 MHz) white noise.

The quantum noise is treated via a semi-classical method described in [54, 57]. To be specific, we add a noise field coupled to the resonator and set its random amplitude $\delta E_{\text{in}}(t)$ with statistics [54]

$$\langle \delta E_{\text{in}}^*(t) \delta E_{\text{in}}(t + \tau) \rangle = \frac{h\nu_p}{2} \delta(\tau). \quad (16)$$

Here h is the Planck constant. The coupling coefficient is set to $\sqrt{2\pi\nu_p\Delta t/(Q_{\text{ex}} + Q_{\text{in}})}$, according to the fluctuation-dissipation theorem.

To extract the phase noise information from the optical modes, around 8 million roundtrips were simulated in order to have reliable statistics at low offset frequencies. The phases of selected modes (9 modes equally spaced from comb line $m = -30$ to $m = 10$) were recorded every 512 roundtrips. With the recorded phases, we computed $S_{\Delta\nu,m}(f)$ and the fundamental linewidth for each comb line according to their definitions.

Frequency noise measurement and effective linewidth

With a coherent receiver, the complex amplitude of the beat between the laser and its delayed self is measured. Using the setup shown in Fig. 3a, the phase difference at two times of the laser under test can be directly extracted:

$$\Delta\phi(t) = \phi(t) - \phi(t - T), \quad (17)$$

where T stands for the time delay caused by a fiber loop with length of 27 km, as shown in Fig. 3a. An acousto-optic modulator driven at 27 MHz is used to shift the measured signal out of baseband.

According to Fourier analysis, we have

$$|\mathcal{F}\Delta\phi(t)|^2 = |\mathcal{F}\phi(t)|^2 \times 4\sin^2\left(\frac{2\pi fT}{2}\right). \quad (18)$$

Therefore, the $S_{\Delta\nu}(f)$ can be calculated by:

$$S_{\Delta\nu}(f) = \frac{f^2 |\mathcal{F}\Delta\phi(t)|^2}{4\sin^2(\pi fT)}. \quad (19)$$

In practice, to avoid the divergence point of Eq. (19), one could do an average for Eq. (18). Noting $\langle \sin^2(x) \rangle = 1/2$, we reach an approximated expression for $S_{\Delta\nu}(f)$:

$$S_{\Delta\nu}(f) = \frac{1}{2} \langle f^2 |\mathcal{F}\Delta\phi(t)|^2 \rangle_T, \quad (20)$$

where $\langle \cdot \rangle$ stands for the average over a period T .

The effective linewidth was obtained according to Eq. (10), by fitting the measured $S_{\Delta\nu,m}(f)$ with a function of

$a + b/f + c/f^2 + d/f^3$. To emphasize the generality of the definition of the effective linewidth, the spikes (as shown in Fig. 5) in the power spectral density were filtered out.

* torresv@chalmers.se

- [1] A. L. Schawlow and C. H. Townes, Phys. Rev. **112**, 1940 (1958).
- [2] A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, Rev. Mod. Phys. **87**, 637 (2015).
- [3] S. A. Diddams, K. Vahala, and T. Udem, Science **369**, 267 (2020).
- [4] T. Fortier and E. Baumann, Commun. Phys. **2**, 153 (2019).
- [5] R. Schmeissner, J. Roslund, C. Fabre, and N. Treps, Phys. Rev. Lett. **113**, 263906 (2014).
- [6] A. Liehl, P. Sulzer, D. Fehrenbacher, T. Rybka, D. V. Seletskiy, and A. Leitenstorfer, Phys. Rev. Lett. **122**, 203902 (2019).
- [7] Y. Takushima, H. Sotobayashi, M. E. Grein, E. P. Ippen, and H. A. Haus, in *Active and Passive Optical Components for WDM Communications IV*, Vol. 5595 (International Society for Optics and Photonics, 2004) pp. 213–227.
- [8] N. R. Newbury and B. R. Washburn, IEEE J. Quantum Electron. **41**, 1388 (2005).
- [9] J. Kim and Y. Song, Adv. Opt. Photonics **8**, 465 (2016).
- [10] P. Del'Haye, A. Schliesser, O. Arcizet, T. Wilken, R. Holzwarth, and T. J. Kippenberg, Nature **450**, 1214 (2007).
- [11] F. Leo, S. Coen, P. Kockaert, S.-P. Gorza, P. Emplit, and M. Haelterman, Nat. Photonics **4**, 471 (2010).
- [12] T. Herr, V. Brasch, J. D. Jost, C. Y. Wang, N. M. Kondratiev, M. L. Gorodetsky, and T. J. Kippenberg, Nat. Photonics **8**, 145 (2014).
- [13] T. J. Kippenberg, A. L. Gaeta, M. Lipson, and M. L. Gorodetsky, Science **361** (2018).
- [14] P. Del'Haye, T. Herr, E. Gavartin, M. L. Gorodetsky, R. Holzwarth, and T. J. Kippenberg, Phys. Rev. Lett. **107**, 063901 (2011).
- [15] P. Liao, C. Bao, A. Kordts, M. Karpov, M. H. Pfeiffer, L. Zhang, A. Mohajerin-Ariaei, Y. Cao, A. Almaiman, M. Ziyadi, et al., Opt. Lett. **42**, 779 (2017).
- [16] T. E. Drake, J. R. Stone, T. C. Briles, and S. B. Papp, Nat. Photonics **14**, 480 (2020).
- [17] K. Nishimoto, K. Minoshima, T. Yasui, and N. Kuse, Opt. Express **28**, 19295 (2020).
- [18] H. R. Telle, B. Lipphardt, and J. Stenger, Appl. Phys. B **74**, 1 (2002).
- [19] M. Karpov, H. Guo, A. Kordts, V. Brasch, M. H. Pfeiffer, M. Zervas, M. Geiselmann, and T. J. Kippenberg, Phys. Rev. Lett. **116**, 103902 (2016).
- [20] X. Yi, Q.-F. Yang, X. Zhang, K. Y. Yang, X. Li, and K. Vahala, Nat. Commun. **8**, 14869 (2017).
- [21] C. Bao, Y. Xuan, C. Wang, J. A. Jaramillo-Villegas, D. E. Leaird, M. Qi, and A. M. Weiner, Opt. Lett. **42**, 759 (2017).
- [22] G. P. Agrawal, in *Nonlinear Science at the Dawn of the 21st Century* (Springer, 2000) pp. 195–211.
- [23] K. Kikuchi, Opt. Express **20**, 5291 (2012).
- [24] S. Gundavarapu, G. M. Brodnik, M. Puckett, T. Huff-
man, D. Bose, R. Behunin, J. Wu, T. Qiu, C. Pinho, N. Chauhan, et al., Nat. Photonics **13**, 60 (2019).
- [25] E. Lucas, P. Brochard, R. Bouchand, S. Schilt, T. Südmeyer, and T. J. Kippenberg, Nat. Commun. **11**, 374 (2020).
- [26] K. L. Corwin, N. R. Newbury, J. M. Dudley, S. Coen, S. A. Diddams, K. Weber, and R. Windeler, Phys. Rev. Lett. **90**, 113904 (2003).
- [27] J. M. Dudley, G. Genty, and S. Coen, Rev. Mod. Phys. **78**, 1135 (2006).
- [28] A. B. Matsko and L. Maleki, Opt. Express **21**, 28862 (2013).
- [29] C. Bao, M.-G. Suh, B. Shen, K. Şafak, A. Dai, H. Wang, L. Wu, Z. Yuan, Q.-F. Yang, A. B. Matsko, et al., Nat. Phys. **17**, 1 (2021), <https://doi.org/10.1038/s41567-020-01152-5>.
- [30] D. Jeong, D. Kwon, I. Jeon, I. H. Do, J. Kim, and H. Lee, Optica **7**, 1108 (2020).
- [31] K. Jia, X. Wang, D. Kwon, J. Wang, E. Tsao, H. Liu, X. Ni, J. Guo, M. Yang, X. Jiang, et al., Phys. Rev. Lett. **125**, 143902 (2020).
- [32] K. Ikeda, Opt. Commun. **30**, 257 (1979).
- [33] T. Hansson and S. Wabnitz, Nanophotonics **5**, 231 (2016).
- [34] L. Lundberg, M. Mazur, A. Mirani, B. Foo, J. Schröder, V. Torres-Company, M. Karlsson, and P. A. Andrekson, Nat. Commun. **11**, 201 (2020).
- [35] G. Brajato, L. Lundberg, V. Torres-Company, M. Karlsson, and D. Zibar, Opt. Express **28**, 13949 (2020).
- [36] C. Joshi, J. K. Jang, K. Luke, X. Ji, S. A. Miller, A. Klenner, Y. Okawachi, M. Lipson, and A. L. Gaeta, Opt. Lett. **41**, 2565 (2016).
- [37] A. B. Matsko and L. Maleki, J. Opt. Soc. Am. B **32**, 232 (2015).
- [38] P. Del'Haye, S. B. Papp, and S. A. Diddams, Phys. Rev. Lett. **109**, 263901 (2012).
- [39] G. Huang, E. Lucas, J. Liu, A. S. Raja, G. Lihachev, M. L. Gorodetsky, N. J. Engelsen, and T. J. Kippenberg, Phys. Rev. A **99**, 061801 (2019).
- [40] M. L. Gorodetsky and I. S. Grudinin, J. Opt. Soc. Am. B **21**, 697 (2004).
- [41] A. B. Matsko, A. A. Savchenkov, N. Yu, and L. Maleki, J. Opt. Soc. Am. B **24**, 1324 (2007).
- [42] D. R. Hjelme, A. R. Mickelson, and R. G. Beausoleil, IEEE J. Quantum Electron. **27**, 352 (1991).
- [43] X. Sun, R. Luo, X.-C. Zhang, and Q. Lin, Phys. Rev. A **95**, 023822 (2017).
- [44] J. R. Stone and S. B. Papp, Phys. Rev. Lett. **125**, 153901 (2020).
- [45] G. Moille, X. Lu, A. Rao, Q. Li, D. A. Westly, L. Ranzani, S. B. Papp, M. Soltani, and K. Srinivasan, Phys. Rev. Appl. **12**, 034057 (2019).
- [46] Y. Wang, F. Leo, J. Fatome, M. Erkintalo, S. G. Murdoch, and S. Coen, Optica **4**, 855 (2017).
- [47] D. C. Cole, E. S. Lamb, P. Del'Haye, S. A. Diddams, and S. B. Papp, Nat. Photonics **11**, 671 (2017).

- [48] M. Karpov, M. H. Pfeiffer, H. Guo, W. Weng, J. Liu, and T. J. Kippenberg, Nat. Phys. **15**, 1071 (2019).
- [49] X. Xue, Y. Xuan, P.-H. Wang, Y. Liu, D. E. Leaird, M. Qi, and A. M. Weiner, Laser & Photonics Rev. **9**, L23 (2015).
- [50] Ó. B. Helgason, F. R. Arteaga-Sierra, Z. Ye, K. Twayana, P. A. Andrekson, M. Karlsson, J. Schröder, and V. Torres-Company, Nat. Photonics (2021), <https://doi.org/10.1038/s41566-020-00757-9>.
- [51] A. Tikan, J. Riemsberger, K. Komagata, S. Hönl, M. Churaev, C. Skehan, H. Guo, R. N. Wang, J. Liu, P. Seidler, *et al.*, arXiv preprint arXiv:2005.06470 (2020).
- [52] B. Y. Kim, Y. Okawachi, J. K. Jang, M. Yu, X. Ji, Y. Zhao, C. Joshi, M. Lipson, and A. L. Gaeta, Opt. Lett. **44**, 4475 (2019).
- [53] Z. Ye, K. Twayana, P. A. Andrekson, *et al.*, Opt. Express **27**, 35719 (2019).
- [54] R. Paschotta, Appl. Phys. B **79**, 153 (2004).
- [55] M. Haelterman, S. Trillo, and S. Wabnitz, Opt. Commun. **91**, 401 (1992).
- [56] J. M. Dudley and J. R. Taylor, *Supercontinuum generation in optical fibers* (Cambridge University Press, 2010).
- [57] P. Drummond and J. F. Corney, J. Opt. Soc. Am. B **18**, 139 (2001).

ACKNOWLEDGMENTS

The devices demonstrated in this work were fabricated at Myfab Chalmers. We acknowledge funding support from the European Research Council (ERC, CoG GA 771410) and the Swedish Research Council (2016-03960, 2016-06077, 2020-00453).

AUTHOR CONTRIBUTIONS

F. L conducted the experiment with help from Z. Y.; F. L. provided the theoretical model and numerical simulations; Z. Y. designed and fabricated the SiN devices; A. F. developed the phase noise measurement system. All authors discussed the results. F. L and V. T.C. prepared the manuscript with input from all co-authors. V. T.C. supervised this project.

ADDITIONAL INFORMATION

Competing financial interests: The authors declare no competing financial interests.

Figures

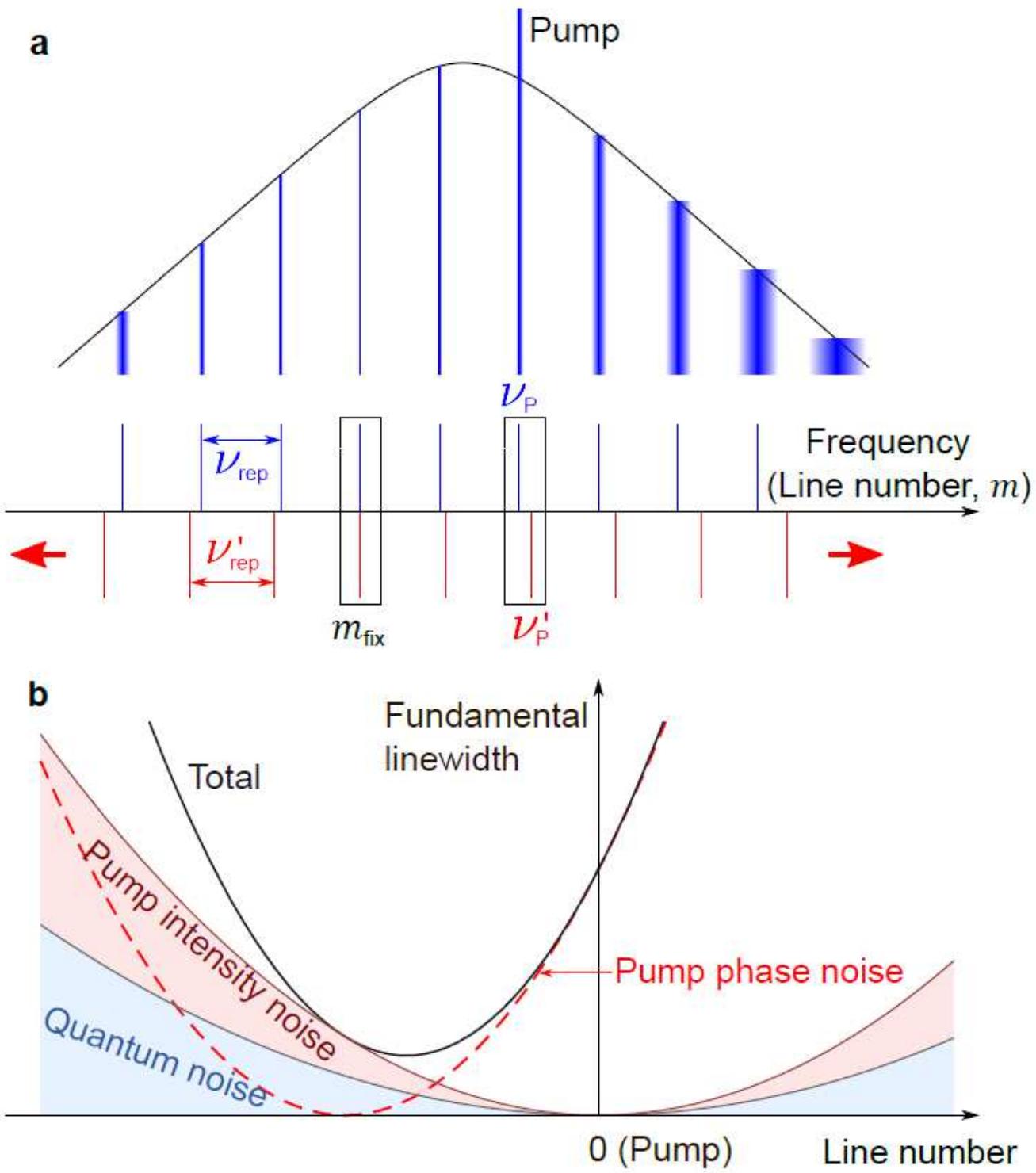


Figure 1

Elastic-tape model applied to soliton micro-combs. a, Illustration of the linewidth distribution in single-soliton microcombs affected by the Raman self-frequency shift. The pump laser has inherent frequency fluctuations. With respect to an arbitrary origin, variations of the pump frequency to the blue side result in

an increase of the soliton repetition rate, and vice versa to the red side. The coupling between pump frequency and repetition rate thus results in the existence of a so-called "fixed point" in the comb spectrum, that is a comb line that is most resilient to the fluctuations of the pump's frequency noise. This translates into a linewidth distribution among comb lines whose minimum is found far from the pump frequency, b. In practice, the frequency linewidth of the lines is also affected by intensity-to-frequency noise conversion mediated by the Kerr nonlinearity, and the inherent quantum noise (vacuum fluctuations), which set a limit into the minimum attainable fundamental linewidth of the soliton microcomb.

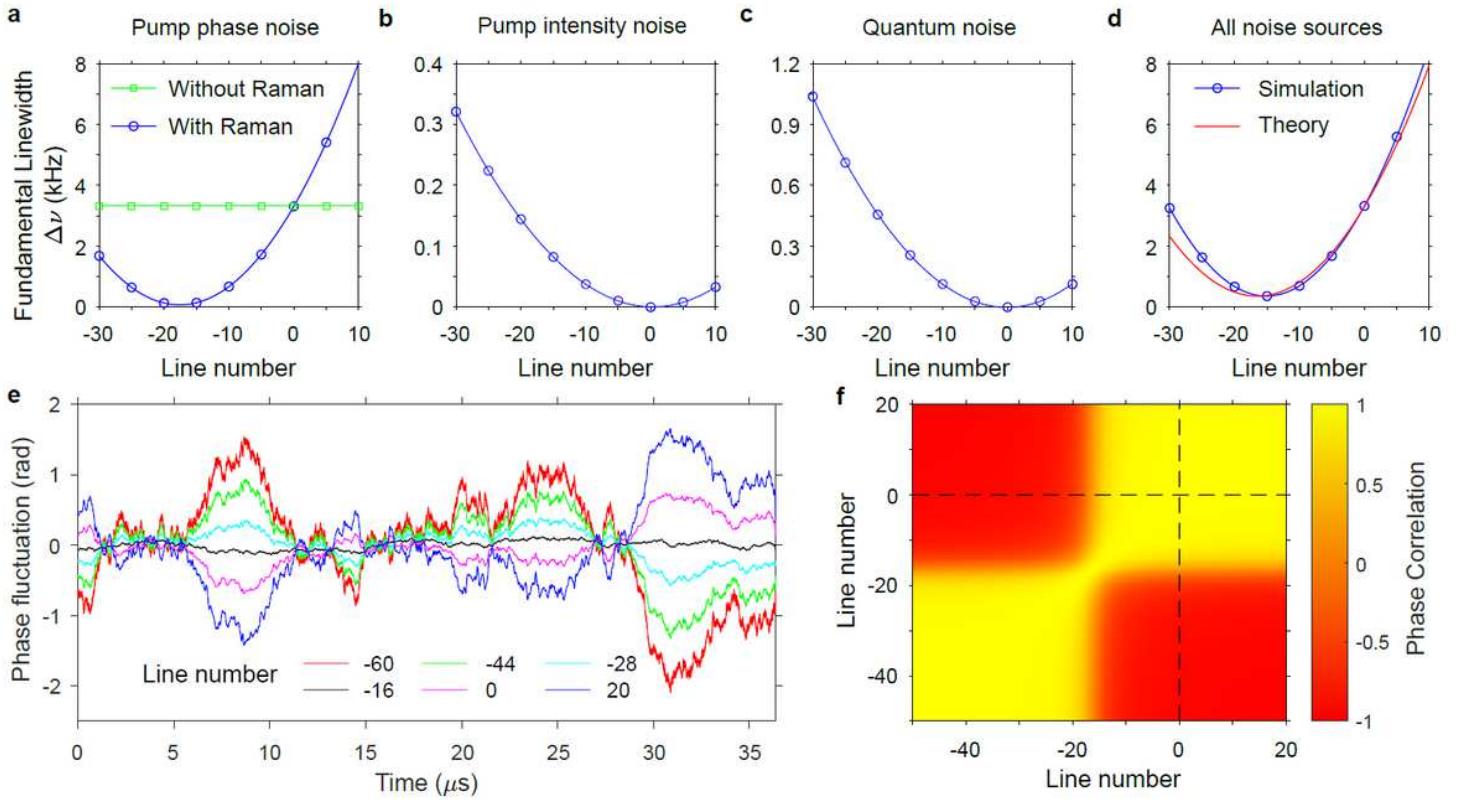


Figure 2

Numerical study of the fundamental linewidth of soliton microcombs. The fundamental linewidth of comb lines induced by pump phase noise, intensity noise, quantum noise and when all three noise sources are included, from a to d. The squares and circles denote the simulated data while the line stands for a quadratic fit. The theoretical result in d (red line) is obtained according to Eq. (8). e, Phase fluctuations of selected comb lines. f, Phase correlation among comb lines, computed according to Eq. (10).

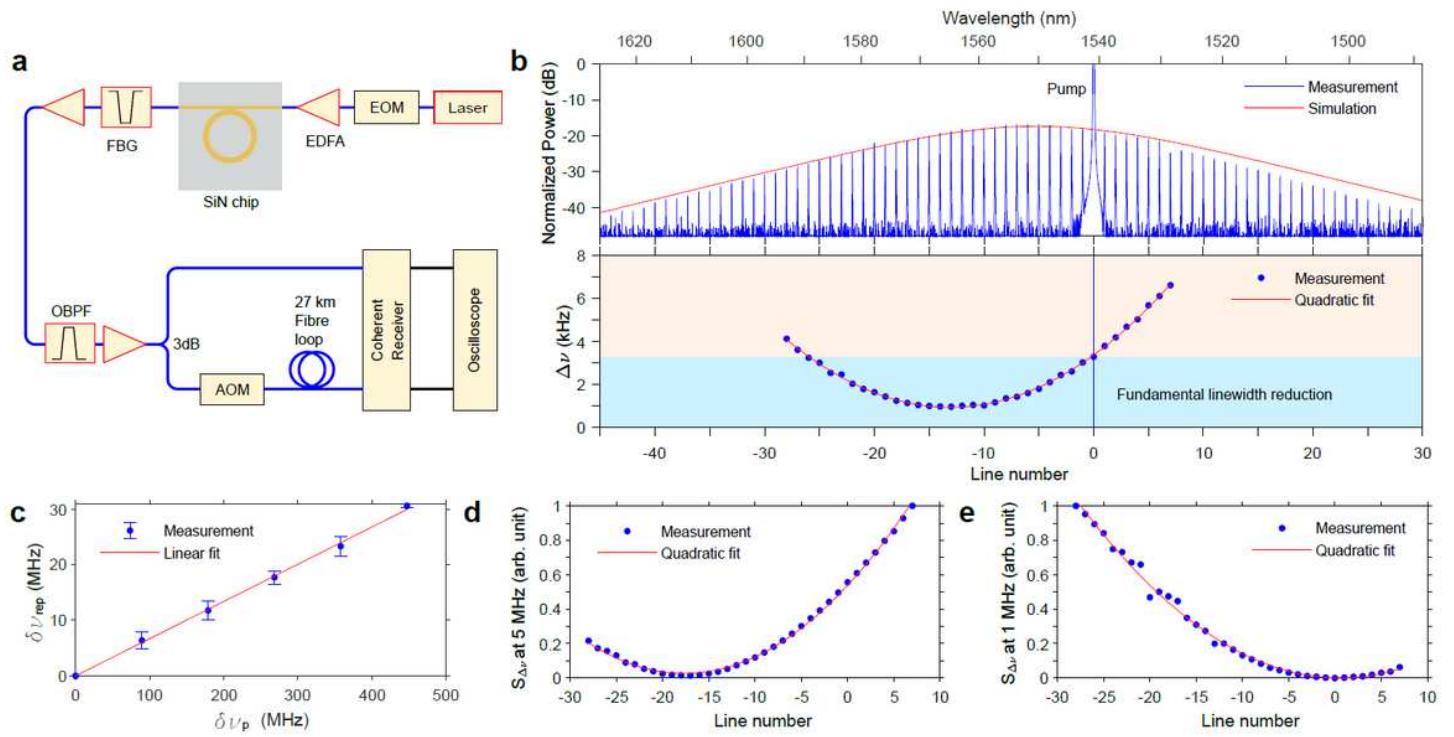


Figure 3

Experimental study of the fundamental linewidth of silicon nitride soliton microcombs. a Setup for soliton microcombs linewidth measurement. EOM, (Phase or Intensity) Electro-optical modulator; EDFA, Erbium-doped fibre amplifier; FBG, Fibre Bragg grating for pump rejection; OBPF, Optical band-pass filter; AOM, Acousto-optic modulator. b Optical spectrum of the single-soliton microcomb and the measured fundamental linewidth of comb lines according to the mean value of $\langle \Delta\nu \rangle$ (\pm) at high offset frequencies ranging 3 to 5 MHz. c Measured soliton microcomb repetition rate varying with pump frequency. The error bar stands for the standard deviation of three measurements. d and e The height of frequency noise PSD at 5 MHz and 1 MHz, originating from applying a 5 MHz phase or 1 MHz intensity modulation signal to the pump.

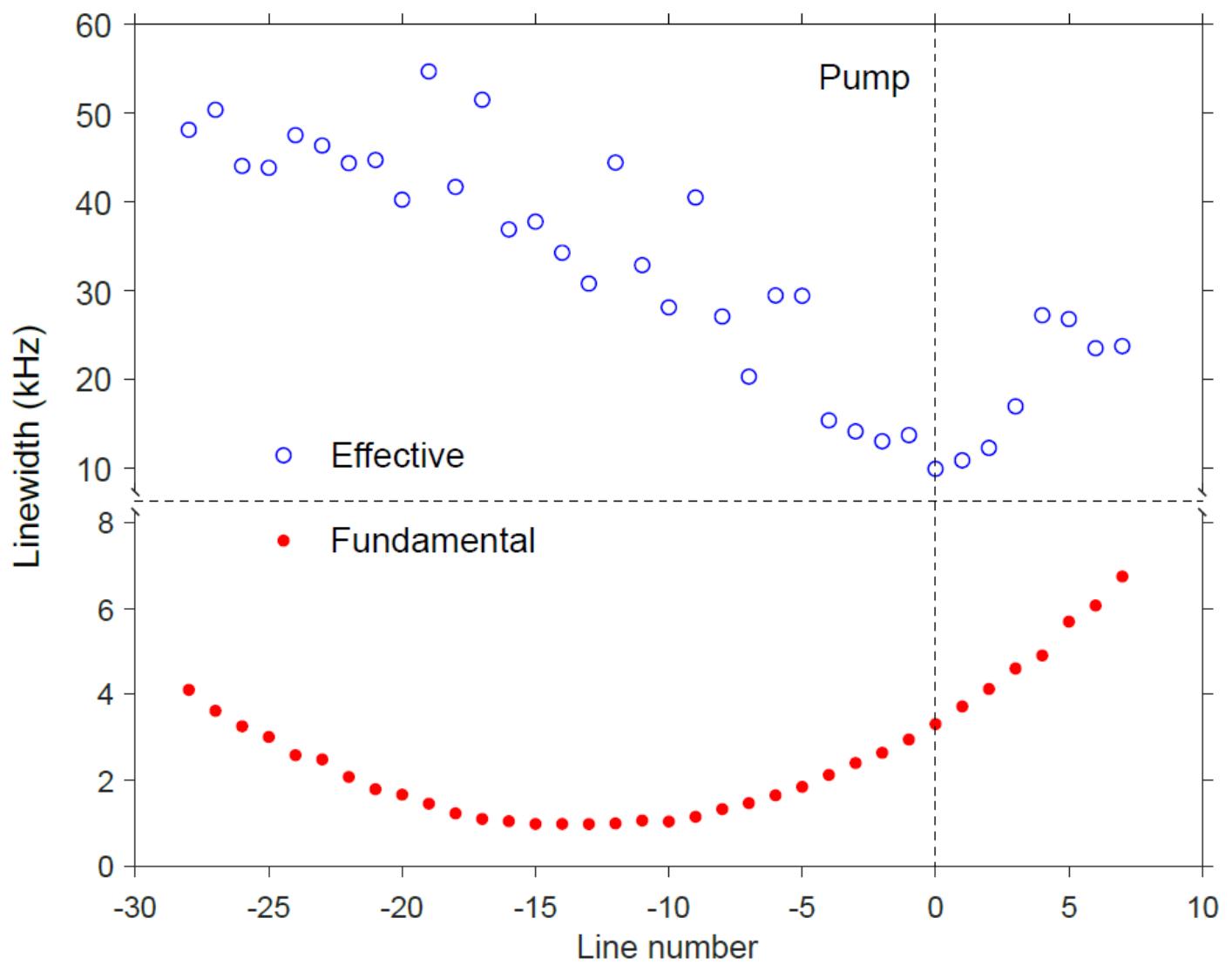


Figure 4

Comparison of the effective linewidth and the fundamental linewidth. With partial inclusion of the low-frequency technique noise, the effective linewidth is much larger than the fundamental linewidth, and it is symmetrical with respect to pump.

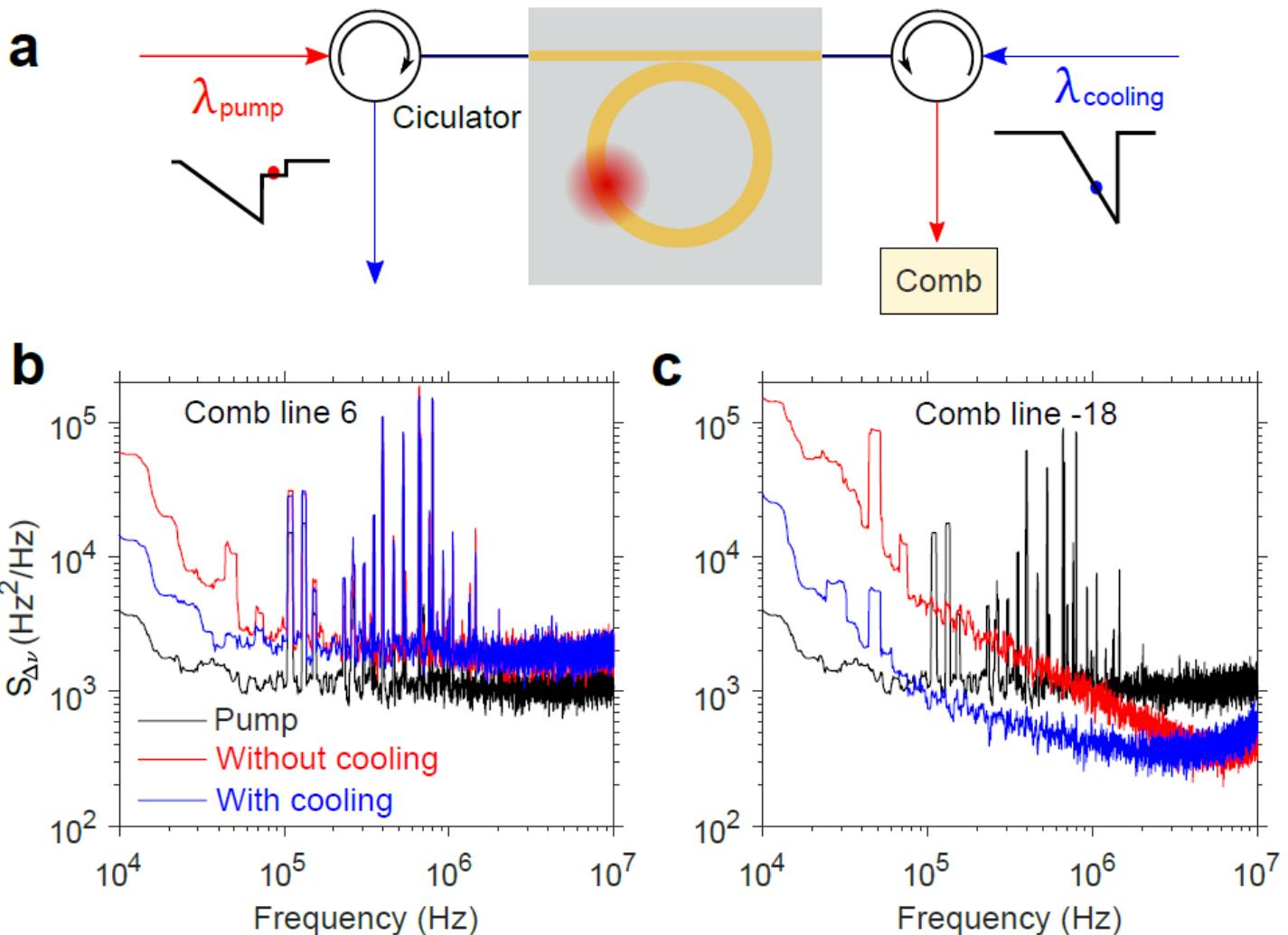


Figure 5

Frequency noise reduction with laser cooling. a The schematic of the setup. An auxiliary laser (blue arrow) opposite to the pump (red arrow) is injected into the microresonator and blue-detuned to a cavity mode with the same family of pump mode. $\Delta\omega_6$ () and $\Delta\omega_{-18}$ () for without (red) and with (blue) laser cooling are shown in b and c, respectively. The black line denotes pump frequency noise PSD $S_{\Delta\nu}$ (). The PSD at low-frequency region is reduced after cooling, but the flat bottom is maintained.