Probabilistic Modelling and Identifying Fluctuations in Annual Extreme Heatwave Regimes of Karachi

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Probabilistic Modelling and Identifying Fluctuations in Annual Extreme Heatwave Regimes of Karachi

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ABSTRACT

Climatic warming in the global mean has significantly increased the probability of occurrence of heat extremes on time scales ranging from months to seasons. As extreme heat events are most likely to become intense and frequent over the next decades, thus it's important to examine these events to mitigate its negative impacts on public health and society. This study focuses on Karachi heat extremes over the last 23 years. The power spectral analyses of Karachi heat extremes records have been carried out by considering two indices: Heat Index (HI) and Effective Temperature Index (TEE), which are also found to be significantly correlated. The result indicates a regular cyclic pattern of 4.5 years which is estimated to face a heat index of more than 73.63°C, associated with the El Niño–Southern Oscillation (ENSO). Other peaks are observed at 2.8 and 2.2 years with the expected value of the Karachi heat index of 70.53°C and 68.71°C, respectively. The probabilistic approach is also used to predict the future heatwave events of Karachi. Generalized Extreme Value (GEV) distribution is found to be the best-fitted probability distribution for the extreme heatwave events on the basis of goodness of fit test. Furthermore, the estimation of the return period of the heatwave event reveals that Karachi will be facing a maximum heat index of 84.37°C or more in the coming 33 years, which suggests an urgent need for mitigation strategies in Karachi to overcome the effects of extreme heatwave events.

Keywords

Spectral Analysis, El Niño Southern Oscillation, Quasi-Biennial Oscillation

1. INTRODUCTION

Extreme heat events have gained increasing attention since past few years because of the related effects on the increased mortality risk (IPCC, 2012). Densely populated areas are most susceptible to synergistic effects of extreme heat and humidity. These conditions may produce lethal effects and exert significant influence on the human-habitable atmosphere (Pal and Eltahir, 2015). The average temperature of a human body is 37°C. Although, the body temperature can sustain at less than 35°C for releasing heat (Sherwood and Huber, 2010). The human skin can not regulate the temperature below 35 and may contribute to develop troubles like cardiovascular disorders, hyperthermia and heat strokes.

The characterization and prediction of heat intensity, its varying frequency and duration of heat waves can be estimated in a proficient way through the combination of both statistical and physical approaches with large atmospheric driving forces in extreme value analyses (EVA). Extreme value theory (EVT) offers a statistical framework that is useful for the modeling of extreme events. It is one of the well-established techniques in the climate literature and has been extensively used for modelling of extreme meteorological and hydrological events (Goto-Maeda et al., 2008; Waylen et al., 2012; Keellings and Waylen, 2014). Furthermore, most of the naturally occurring phenomena, specifically extreme climatic events are described through the frequency-functions which rely on fluctuations of the event. Variations in precipitation and temperature for different areas have also been extensively studied in this regard (Hasanean, 2001; Morala et al., 2003; Tošić and Unkašević, 2005). The intraseasonal estimation of extreme events is beneficial in predicting future energy production and consumption (Roulston et al., 2003; Taylor and Buizza, 2003), water resources plans (Sankarasubramanian et al., 2009), and the insurance or financial markets (Zeng, 2000; Jewson and Caballero, 2003).

The metropolitan areas have become true hot spots of climate change because of the multiple environmental stresses, including perplexing phenomena of urban heat island, growing population and air pollution. Karachi is the largest city in Pakistan which is more susceptible to extreme events. Previous study indicate that Karachi heat events are linked with the phenomenon of UHI (Rizvi et al., 2019) and intensified due to rapid rate of urbanization, deforestation, industrialization and changing in LULC pattern (Arshad et al., 2020). Moreover, the built-up areas of Karachi were increased by 40% from 2009 to 2017, consequently the intensity of UHI was also increased (Rizvi

et al., 2020a). The regions of the maximum intensity of surface UHI nearly correspond with dense urban areas in Karachi, which covers the area of 201.28 km^2 (Rizvi et al., 2020b). The heatwave event, occurred in 2015 was one of the most severe climate extremes to hit Sindh province which caused more than 800 deaths in Karachi between 17 to 24 June 2015. An extreme event occurring across the province can increase the mortality rate and potential economic damages. Therefore, for enacting the mitigation strategies, a precise estimation of the risk in the region is needed. This study focuses on the spectral analysis to assess the regular cyclic patterns of heat waves and examines the extreme heat events with its future variability in Karachi using Generalized Extreme Value and Gumbel Maximum distribution to estimate the long return periods and levels via the best-fitted model.

2. MATERIAL AND METHODS

2.1 Data

In this study, different climate indices are utilized for the multivariate analysis of heat extremes. All the indices are accessible for the entire study period, 1997 - 2019. The Nino 3.4 (hereafter, N3.4) index is generally used to describe ENSO state (Kenyon and Hegerl, 2008). It represents Sea Surface Temperature (SST) anomalies in the N3.4 region ($5^{\circ}\text{N} - 5^{\circ}\text{S}$; $170^{\circ}\text{W} - 120^{\circ}\text{W}$), computed from the SST data taken from the Hadley Centre Sea Ice and Sea Surface Temperature (HadISST) data set, available at (<https://www.esrl.noaa.gov/psd/data/correlation/nina34.data>). The index has positive value during El Niño (warming phase of ENSO) and it shows negative value during La Niña (cooling phase of ENSO). Further details about N3.4 index can be obtained from Trenberth and Stepaniak (2001). The Quasi-Biennial Oscillation (QBO) index is also used in this study, which is identified as Singapore winds having the spatial extent of 60°N to 60°S ; -73°E to 90°W , available at NOAA Climate Prediction Center (<https://www.cpc.ncep.noaa.gov/data/indices/>). The daily observations of maximum temperature, relative humidity, and wind speed are taken from Weather Underground Organization (WUO) to calculate the Heat Index (HI) and Effective Temperature Index (TEE) of Karachi over a period of 23 years, during the summer season (AMJJ).

2.2 Heat Index and Effective Temperature Index

To estimate the heatwave intensity, various indices are used in environmental health research to analyze the combined effects of different climatic factors. In present study two indices of extremely hot climate are investigated: (i) Heat Index (HI) which is also known as Apparent Temperature (ii) Effective Temperature Index (TEE). Thus, the annual extreme values of HI are calculated through the succeeding regression equation which was achieved by using multiple regression analysis (Rothfusz, 1990)

$$HI = -42.379 + 2.049T + 10.143(RH) - 0.2247(T)(RH) - 6.83 \times 10^{-3}T^2 - 5.48 \times 10^{-2}(RH)^2 + 1.22 \times 10^{-3}(T)^2(RH) + 8.5 \times 10^{-4}(T)(RH)^2 - 1.99 \times 10^{-6}T^2(RH)^2 \quad (1)$$

where, T represents the temperature in degree Fahrenheit (°F) and RH shows relative humidity in percent. HI represents the heat index indicated as an apparent temperature in degree Fahrenheit (°F). In order to examine the influence of wind speed on Karachi Heat extreme, the annual extreme values of Effective Temperature Index (TEE) are also computed as (Gregorcuk, 1968)

$$TEE = 37 - \frac{37 - t}{0.68 - 0.0014(RH) - \frac{1}{1.76 + 1.4v^{0.75}}} - 0.29t(1 - 0.01RH) \quad (2)$$

where, v is the wind speed in m/s, t represents temperature in °C, and RH shows Relative Humidity. The rating scales of TEE varies with the latitude of the region, such as; for UK, 14.4 to 20.6°; for Yakovenko region of Russian Federation, 16.7 to 21.8°; for U.S.A., 18-22°, and for tropical regions it varies from 23.3 to 29.4° (Teodoreanu and Bunescu, 2007).

Figure 1 Timeseries of an annual extreme Heat Index (HI) and Effective Temperature Index (TEE) of Karachi for the last 23 years (1997-2019)

2.3 Spectral Analysis

Statistical spectral analysis has been traditionally used to detect regular cyclical patterns of the time series. In this study, Fourier-based spectral analysis has been chosen for the assessment to recover the periodicities in the Karachi heat extreme which has been used in previous climatological studies (Negrón-Juárez and Liu, 2001; Aldriana and Djamila, 2008; Iqbal et al., 2011; Hasanean et al., 2013). The annual HI records are transformed through a finite Fourier transform that decomposes it in different frequency waves. The power spectrum of the considered time series of heat extreme is computed via Autocorrelation spectral analysis (ASA), which is the Fourier transform of the autocorrelation function of HI time series (Blackman and Tukey, 1959). As the autocorrelation function possesses even symmetry, consequently the terms having sine function in Fourier series have become zero. As a result, the power spectrum (B_i) of the natural phenomenon becomes simplified, containing only real cosine terms

$$B_i = \frac{r_o}{n} + \frac{2}{n} \sum_{L=1}^{n-1} r_L \cos\left(\frac{360}{2n} iL\right) + (-1)^i \frac{r_n}{n}, \quad i \in \{1, 2, 3, \dots, n\} \quad (3)$$

where r_L and n represents the autocorrelation function of lag L and sample size, respectively (Sprott, 2003). The above spectrum function does not provide a reliable estimate until it will be smoothened by using lag features in a frequency domain. The latter estimates (F_i) can be computed by using the smoothing technique called Hamming Method with a five-term weighted average (Chatfield, 1989)

$$F_i = 0.0357B_{i-2} + 0.2411B_{i-1} + 0.4464B_i + 0.2411B_{i+1} + 0.0357B_{i+2} \quad (4)$$

2.4 Extreme Value Distribution

The time series of extreme events usually analyzed through extreme value theory (EVT), particularly, in the fields of meteorology, hydrology and in the prediction of financial time series (Rusticucci and Tencer, 2008; Iqbal and Ali, 2012; Mandal and Choudary, 2015; Du et al., 2019). This theory provides the statistical framework, that can estimate the occurrence probability of an extreme event with their corresponding return period in risk analysis (Wilks, 2006). In this study, the daily heat index is denoted by $\{X_1, X_2, \dots, X_n\}$ the conventional model for extremes is achieved

by observing the behavior of $M = \max\{X_1, X_2, \dots, X_n\}$ where $n = 23$, and M apparently represents to the annual maxima. The generalized extreme value distribution (GEV) was formulated in the EVT framework whose probability density function (PDF) is given by

$$f(x) = \frac{1}{\beta} (1 + Z)^{-1-\frac{1}{k}} e^{-(1+Z)^{-\frac{1}{k}}} ; \quad 1 + Z > 0 \quad (5)$$

$$\text{where} \quad Z = \frac{k(x-\xi)}{\beta}$$

where k, β and ξ are the shape, scale and location parameters respectively. By integrating the above equation, the following CDF function $F(x)$ can be obtained (Coles, 2001).

$$F(x) = e^{-(1+Z)^{-\frac{1}{k}}} \quad (6)$$

For the purpose of the explicit formulation of the quantile function, equation (6) can be inverted which gives

$$X = F^{-1}(p) = \xi + \frac{\beta}{k} \{[-\ln(P)]^{-k} - 1\} \quad (7)$$

where $P = F(x)$, represents cumulative probability. The L-moment or Maximum Likelihood Method (MLM) is applied to estimate the parameters of GEV distribution (Hosking, 1990; Stedinger et al., 1993). The method of L-moment is ideal for a small sample size (Hosking, 1990), but the MLM can be simply modified to include covariate effects, or other factors, such as the possibility of some distribution estimates to show a trend caused by climate change (Katz et al., 2005; Zhang et al., 2005). However, both methods showed similar results for large and moderate sample data. The GEV distribution consists of three cases based on the shape parameter. The type-I extreme value distribution is a special case having ($k = 0$) which is also called Gumbel distribution. Gumbel primarily focused on EVT application to engineering challenges, particularly in the modeling of climatological phenomena like annual extreme flood flows. The distribution function of Gumbel Maximum is given as

$$f(x) = \frac{1}{\beta} e^{\{-e^{-z}-z\}} \quad (8)$$

$$\text{where} \quad z = \frac{(x-\xi)}{\beta}$$

where ξ and β are location and scale parameters, respectively. The shape of this distribution is independent of its parameters. The Gumbel distribution probabilities can be found by its cumulative distribution function (CDF)

$$F(x) = e^{-(e^{-x})} \quad (9)$$

To obtain the quantile function, the inverse of CDF of Gumbel distribution is given by

$$F^{-1}(X) = \xi - \beta \ln\{-\ln(P)\} \quad (10)$$

2.5 Testing of the probability distribution model

The Goodness of Fit tests measures the compatibility between a theoretical PDF and a random sample. Thus, the results of these tests reveal how well a certain distribution fits the data set. The goodness of fit test is employed to test the null hypothesis, H_0 : the annual maximum daily heat index follow the particular distribution and the corresponding alternative hypothesis is, H_A : the annual maximum daily heat index does not follow the particular distribution. It is also highlighted that the test statistics of different goodness-of-fit test can produce a varied ranking position among the pdfs (Zhou et al., 2010; Chang, 2011). Though the Kolmogorov-Smirnov (K-S) test is extensively applied (Lo brano et al., 2011; Vivekanandan et al., 2012; Whan et al., 2015), so the distribution is selected on the basis of the (K-S) test at 5% level of significance. The methodology of the K-S test is as follows:

2.5.1 Kolmogorov-Smirnov (K-S) Test

The supreme class of Empirical distribution function (EDF) statistics is a K-S test statistic, originated on the maximum vertical difference among the empirical and theoretical distributions (Conover, 1999). Comparison of the CDF of an assumed theoretical distribution $F_X(x)$ and the empirical cumulative frequency $S_n(x)$ is a fundamental objective of this test. The K-S test statistic (D_n) is the maximum difference between $F_X(x)$ and $S_n(x)$. For 'n' samples, the data set is arranged in ascending order, $X_1 < X_2 < \dots < X_n$ and the statistic of (K-S) test is evaluated for each ordered value

$$\begin{aligned}
S_n(x) &= 0; \quad \text{if } X < X_1 \\
&= \frac{k}{n}; \quad \text{if } X_k \leq X < X_{k+1} \\
&= 1; \quad \text{if } X > X_n \\
D_n &= \max |F_X(x) - S_n(x)| \\
P(D_n \leq D_n^\alpha) &= 1 - \alpha
\end{aligned} \tag{11}$$

where D_n^α , α and k represent critical value, significance level and the ranking order of the data, respectively.

3. RESULTS AND DISCUSSION

3.1 Power Spectrum and Coherency

The power spectral analysis has been applied by the Fourier transform method to the annual time series of the Karachi heat index for the period of 1997-2019. The HI series is converted from real space (time-domain) to Fourier space (frequency domain). The Fourier transform provides a 1-1 correspondence into the signals at particular times and examines how specific frequency contributes to the signal. Hence, it would be possible to determine the signal statistics in Fourier space instead of real space. Firstly, the time series of HI is linearly detrended, and then the frequency spectrum is estimated by smoothing a periodogram. As the smooth periodogram can identify the specific frequencies with high spectral densities that have a significant contribution in the cyclic pattern of series. The outcomes of the FFT spectral analysis are depicted in Figure 2 which shows that the annual heat index series of Karachi for 23 years have the dominant peaks around 4.5, 2.8, and 2.2 years, and annual TEE series have the dominant peaks at 4.5 and 2.8 years, which reveals the regular cyclic behavior of heat events.

Figure 2 Spectral density plot of Karachi annual (a) HI and (b) TEE for 1997 - 2019 with periodicity. Dashed lines indicate a 95% confidence interval

The time series of HI and TEE has a significant correlation of 0.86 at 0.01 level and both series showed an almost similar return period. The 2.8-year middle frequency wave and 2.2-year short wave show some association with the quasi-biennial oscillations (Lamb, 1972; Iqbal et al., 2011). There would be multiple time scales in the trend and interannual variability of the monsoon-atmosphere-ocean systems; such as a quasi (2-year) cycle linked with Tropical Biennial Oscillation (TBO), along with (4-6) year cycle related with ENSO. Thus, the other peak at 4.5 years seems to be related to ENSO. The El Niño southern oscillation (ENSO) is recognized as a nexus of climate change throughout the world (Glantz et al., 1991; Allan; 2000; Van et al., 2005; Stevenson et al., 2017). Studies also reveal that oceanic forces have dominant effects on the extreme heatwave events of the 20th century (De and Mukhopadhyay, 1998; Della-Marta et al., 2007; Jenamani, 2012). In order to determine the association between the climate and ENSO phenomena, different circulation indices have been developed. El Niño phase is linked with an increased chance of extreme heat events while the La Niña phase is associated with a reduced chance of extreme heat events (White et al., 2014).

In order to assess the association between the spectral peak frequencies of Karachi heat extreme and the large-scale atmospheric system, spectral coherence analysis has been performed. The coherence function reveals whether or not variation in a particular series responds to a similar kind of variability in the other series. It is the correlation among different variables in the frequency domain and its value is normalized between 0 to 1. For a small number of samples, i.e., $n = 23$, the significant coherence and coherence-squared at 95% confidence interval are 0.35 and 0.12, respectively (Thompson, 1979). Figure 3 shows a lack of correlation among HI and QBO, for the frequencies, lie between 0.26 to 0.37. On the other hand, high coherence is observed at 0.46. The significant squared coherence is observed among the frequencies of heat index series and N3.4 frequencies except 0.26 to 0.29 (Figure 4). The obtained results related to the impact of ENSO on Karachi heatwaves are in line with the previous studies focused on the occurrence of heatwaves in any specific region (De and Mukhopadhyay, 1998; Parker et al., 2014). Perkins et al., (2015) also claimed that ENSO-strength provides a good prediction for the expected heatwave days when the N3.4 index increases positively, consequently, the number of heatwave days increases.

Figure 3 Coherence-squared function between annual HI series and QBO

Figure 4 Coherence-squared function between annual HI series and N3.4

3.2 Probability analysis of annual maximum daily Heat Index

The annual HI data of Karachi is utilized for Extreme value analysis, as both indexes have a significant correlation. A threshold value of the Heat index is 40.5°C as mentioned by NOAA (National Oceanic and Atmospheric Administration) corresponds to danger and extreme danger conditions. During the study period, the highest annual value of HI is 84.37°C, observed in 2010. However, there exist some other relative maxima of HI such as 81.63, 78.70, 76.39°C. To predict the future heatwave events using extreme value analysis, a univariate extreme value model is required that focuses on heat index observations. Statistical goodness of fit techniques and graphical representation of PDFs are capable to reveal about the consistency of the fitted-distributions with a certain data set. (Stedinger et al., 1993). This section comprises the comparison of results for the performance of each probability distribution to obtain a best-fitted probability distribution that could efficiently provide the estimate of HW in Karachi. The GEV is suggested as an appropriate distribution for the representation of extreme events because of its theoretical basis for maxima analysis and the procedure of estimating the model's parameter is quite simple. However, the Gumbel model has also been used to fit the annual maximum values of different indices including temperature (Kharin and Zwiers, 2000; Hong et al., 2013).

Figure 5 presents the distribution function of the GEV and Gumbel Maximum distributions, fitted to the extreme HW events to visually assess the level of Goodness-of-fitting for both distributions. It is observed from the PDF plot that both distribution models fit on the HI data quite well. Furthermore, the K-S test has been used to examine the goodness of fit. The test statistics (D) from the K-S goodness of fit test using GEV and Gumbel for extreme HW events are 0.086 and 0.087, respectively, which are smaller than the 95th percentile value of 0.274 (critical value) with 5% significance level. As the test statistic value in the goodness of fit techniques, specifies the difference among the set of observations and the fitted distributions, it is evident that the model having the lowest test statistic is a good fit. Hence, it is concluded on the basis of the K-S goodness of fit test that the GEV is the best-fitted probability distribution for the extreme HW events.

Figure 5 Probability Density Function of HI (°C) and fitted GEV and Gumbel Max Distributions

After the selection of best-fitted distribution, the probability-probability (P-P) plot has been plotted for the extreme annual heatwave of Karachi in Figure 6 to validate the results of the above goodness of fit test. It shows the relation between empirical and theoretical CDF values to identify how much a specific distribution agrees with HI data. If theoretical distribution fits precisely then the P-P plot will be roughly linear. The P-P plots are more advantageous than quantile-quantile (Q-Q) plots since the respective plotting positions are unbiased for the examined distributions (Soukissian, 2013). The result also reveals that the theoretical CDF values of GEV distribution have less deviation from the observed data points as compared to Gumbel Maximum distribution. Hence, it is concluded that the GEV distribution provides the best fit for the probabilistic modeling of heatwave events.

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Figure 6 P-P Plot of GEV and Gumbel Distribution of Karachi heatwave events

3.3 Estimates of the Return Period of Heatwave Events

The outcomes of extreme value analysis (EVA) can be considered as those quantiles which correspond to large cumulative probabilities, like an event that has an annual exceedance probability of 0.01. The direct assessment of these extreme quantiles is not possible unless n is a large number, however, a best-fitted extreme value distribution presents a practical approach for extrapolation of the probabilities that may be significantly larger than $1 - \frac{1}{n}$. The average return periods of such extreme probabilities are often expressed as (Wilks, 2011)

$$Return(R) = \frac{1}{\omega[1 - F(x)]} \quad (12)$$

Return periods $R(x)$ related to a quantile is considered to be the average period concerning the occurrences of extreme events of that extent or more. It is a function of the average sampling frequency w and the CDF of a specific distribution. For annual maximum observations $w = 1/year$, thus the event x having the cumulative probability $F(x) = 0.99$ will be the probability of exceedance $1 - F(x)$ at any certain time. The return period of this event would be 100 years and could be named as a 100-year event. The results reveal that the highest heat index of 84.37°C is observed on June 19, 2010, during the whole study period from 1997 to 2019. The return period is estimated to determine the number of years after which there will be a possibility of occurrence for the maximum heatwave in the future. Therefore, the fitting of the GEV distribution to the annual maximum daily heat index data of Karachi yielded the parameter $k = -0.11$, $\beta = 6.19$, and $\xi = 65.69$. To estimate the cumulative probability equation (6) is used

$$F(x) = P(x \leq 84.37) = e^{-(1+z)^{-\frac{1}{k}}}$$

$$F(x) = 0.97$$

Equation 12 yields;

$$R(x) = \frac{1}{1[1 - 0.97]} = 33 \text{ (years)}$$

The above calculation reveals that there is a chance for occurring heat index equals to 84.37°C or more in the coming 33 years. However, return levels are estimated against the different return periods by using equation (7). The regular cyclic pattern of 4.5 years for future heatwaves is estimated in section 3.1 through spectral analysis i.e; at 4.5 years, $p = 1/4.5 = 0.22$ and $F(x) = P = 1 - 0.22 = 0.78$, substituting these values in the inverse CDF of GEV Distribution yields $HI = 73.63^\circ\text{C}$, which shows that in the next 4.5 years, the expected value of Karachi heat index will be 73.63°C, with a 95% confidence interval [69.82,77.43]. Similarly, a small peak of 2.8 years, observed from the spectral analysis is projected to have a heat index of 70.53°C, with a 95% confidence interval [67.09,73.96]. Further calculations are summarized for different return levels against the different return periods with a 95% confidence interval in Table 1. Here, the sample size of data is quite small compared to the return levels. Anticipating up to 100-year return periods with only 23 years of data available unavoidably yields much lower confidence in the results as the return period increases.

Table 1 Return levels for different return periods with a 95% confidence interval

Years (T)	Probabilities $p = 1/T$	Cumulative Prob. $P = 1 - p$	Return Levels ($^\circ\text{C}$)	CI at 95% [lower, upper]
2	0.5	0.5	67.91	[64.76, 71.07]
5	0.2	0.8	74.25	[70.34, 78.16]
10	0.1	0.9	78.03	[73.04, 83.03]
20	0.05	0.95	81.38	[74.47, 88.29]
50	0.02	0.98	85.33	[74.76, 95.91]
100	0.01	0.99	88.04	[74.05, 102.02]

4. SUMMARY AND CONCLUSION

Large-scale atmospheric systems can greatly affect the local temperature and extreme heatwave events. The analysis in this chapter reveals that, besides anthropogenic heat, ENSO is also a significant aspect of varying HW activity. To estimate the periodicity of heatwave events, the HI data is subjected to power spectrum analysis and the results reveal the regular cyclic pattern of 4.5, 2.8, and 2.2 years seem to be affected by the well-known phenomenon of ENSO and QBO. As it has been observed that the periodicities of heat index series are linked with El Niño. This hypothesis is greatly strengthened by computing coherence between the annual HI series with QBO and an El Niño index which shows significant and strong coherence among them. Results indicate a prolonged association of these indices on Karachi heat extremes. The frequency of occurrence of extreme heatwave events in Karachi is also examined by comparing the performance of GEV and Gumbel maximum probability distributions to identify an appropriate model that could indicate extreme HW estimates, accurately. The performance of fitted distribution is evaluated by the statistical goodness of fit K-S test in which the selected distributions are ranked to identify the best-fitted distribution to the HI data of Karachi city. Among both probabilistic models, GEV is spotted as the best probability distribution to be fitted on the basis of (K-S) test. Furthermore, the estimation of the return period of the heatwave event reveals that Karachi will be facing a heat index of 78.03°C or more in the coming 10 years, while a 50-years return period is estimated to face more than 85.33°C heat index. This paper provides the future prediction of extreme heatwave events in Karachi to establish early warning alerts for residents and city planners to endorse smart sustainable strategies.

Declarations

The authors declare that they have no competing interests. The data and material used in this research are partly available in the body of the article; other data/material is also available to public and will be provided through writing to the corresponding author. No funding was received for conducting this study.

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Figures

Figure 1

Timeseries of an annual extreme Heat Index (HI) and Effective Temperature Index (TEE) of Karachi for the last 23 years (1997-2019)

Figure 2

Spectral density plot of Karachi annual (a) HI and (b) TEE for 1997 - 2019 with periodicity. Dashed lines indicate a 95% confidence interval

Figure 3
 Coherence-squared function between annual HI series and QBO

Figure 4
 Coherence-squared function between annual HI series and N3.4

Figure 5

Probability Density Function of HI (x) and fitted GEV and Gumbel Max Distributions

Figure 6

P-P Plot of GEV and Gumbel Distribution of Karachi heatwave events