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Workspace Analysis and Optimal Design of 3-PRR Planar Parallel Manipulators

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Abstract:

In the optimum design of parallel manipulators, workspace of the manipulator is of greater importance. The shape and area of the workspace are the main parameters under this. In this paper, a new geometrical approach is presented to determine the shape and size of the constant orientation workspace for the 3-PRR planar parallel manipulators. All possibilities of shapes of workspaces are determined with variation of different parameters. For each shape of workspace corresponding geometrical conditions are also put forth. Closed form area expression of workspace is derived by geometrical approach for each shape. Such closed form expression of area is not possible with non-dimensional approach. This becomes extremely useful during optimal design procedure. A *look-up table* is also presented seeing which the designer can choose geometrical conditions between different parameters which will ensure a void free workspace. A case study is presented wherein a user gives his required workspace area and an algorithm is presented which gives all possible combinations of geometrical parameters satisfying the workspace area requirement. Then based on various considerations including singularity analysis an optimal parallel manipulator is offered for the task which does not have any void within the workspace having least/nil singularities.

Keywords: Planar parallel manipulators, Workspace, Geometrical method, Closed form expression, 3-PRR parallel manipulators.

1.Introduction:

A parallel manipulator is a closed-loop kinematic mechanism whose end effector is linked to the base by several independent kinematic chains [1]. A parallel robot is defined as a robot which is made up of an end-effector with n degrees of freedom, and a fixed base, linked together by at least two independent kinematic chains. Actuation takes place through n simple actuators. Parallel robots for which the number of chains is strictly restricted to the number of DOF of the end effector are called fully parallel manipulators [2]. There are two main cases in the analysis of fully parallel manipulators. They are: planar manipulators and spatial manipulators. In the present paper we consider 3-PRR planar manipulator. The closed kinematic chain has potential applications where maneuverability and requirement of workspace is low but dynamic loading is severe and high speed and precision motions are of paramount importance [3]. The applications of parallel manipulators include force-torque sensor, machining center, aircraft simulator, earthquake simulator, pointing device, mechanism design, path planning, conformational trajectory planning of proteins [4] and in many other cases [5] etc. The 3-PRR planar parallel manipulator, due to its inherent stiffness and accuracy, and less inertia of moving links is a suitable mechanism for high-speed and high-accuracy robotic applications as a planar positioning and orientation device [6, 7, 8].

Workspace determination of parallel manipulators has been described in many papers. In [9], a complete method for determination of workspace boundary is described on general structure manipulators by using a branch and prune technique. Study of singular configurations of a three DOF planar parallel mechanisms with three identical legs is described in [10]. Dynamics and vibration control of a 3-PRR parallel manipulator with three flexible links is analyzed by generating dynamic equations [11]. It also studies the buckling effect on links and developing an active vibration control strategy. A methodology was described in [12] to obtain sensitivity coefficients of the pose of the moving platform of the

manipulator to variation in geometric parameters and actuated variables. A methodology to enlarge the workspace of parallel manipulators by using non-singular transitions has been described in [13]. Dynamics of a 3-PRR manipulator has been explained in many papers as in [14].

In case of 3DOF planar parallel manipulators, it is extremely difficult to determine the workspace area with the help of non-dimensional parameters especially when the shape of workspace is very complex as shown in [15-20]. Especially in case of 3-PRR planar parallel manipulator, the workspace shape becomes very complicated with combinations of straight lines, circles and arcs and consequently determination of area also becomes very difficult. Sometimes, researchers adopt to approximate algebraic or numerical method to determine the workspace area in the absence of such closed form expression of workspace area [21]. One disadvantage of such approach is that it would be computationally tasking and involves error.

A typical optimum design cycle for a parallel manipulator involves following steps: 1) specification of workspace area by the user and other specifications; 2) determination of possible parallel manipulators (combination of link lengths) satisfying this area and other constraints; 3) determination of singularity points in each possible parallel manipulator; 4) select the one with least singularity points.

Singularity analysis of parallel manipulators in general and specifically that of 3-PRR planar parallel manipulator has been addressed by many researchers [22-28]. Workspace analysis of this manipulator has also been addressed [8, 10, 29], however shapes of the workspace with variation in the value of the parameters has not been reported. Also closed form expression of workspace area of 3-PRR planar parallel manipulators has not been reported so far. Such closed form expression of workspace area becomes very useful for parallel manipulators because they are mostly proposed for high accuracy applications.

In this paper, effect of all possible variation of link lengths on the shape and area of workspace (constant orientation) of a 3-PRR planar parallel manipulators is studied and specific mathematical conditions for each workspace shape is determined. For each shape of the workspace, workspace area is also calculated in a geometrical way in closed form expression. Such geometrical expressions for determination of workspace area are very helpful for optimal design of such manipulators. A flow chart is presented to determine optimal parallel manipulator(s) for a given workspace area which free of singularities. A case study is given to verify this.

Figure 1: Line diagram of a 3-PRR planar parallel manipulator

2. Kinematics of a 3-PRR manipulator:

As shown in Fig. 1, each kinematic chain of such a manipulator is of PRR type. The triangular shaped platform $C_1C_2C_3$ is to be oriented by the manipulator. The triangle $A_1A_2A_3$ forms the base.

Generally, the workspace is determined on different parameters as follows.

l =maximum length that each slider can move

r = radius of the revolute link

b = side length of the base triangle

a = distance from the centroid of the platform to its vertex of the mobile platform

Determination of constant orientation workspace is shown in a step by step method in Fig. 2. Figure 2(a) shows the line diagram of the same manipulator as in Fig. 1 and Fig.2(b) shows the individual workspace of each PRR chain i.e. WS_1 is the workspace of 1st serial chain (A1-slider-C2); WS_2 is the workspace of 2nd serial chain (A2-slider-C3) and WS_3 is the workspace of 3rd serial chain (A3-slider-C1). Then, these workspace are moved in a direction determined by the orientation of the mobile platform by a distance 'a' as shown in Fig. 2(c). Consequently, the intersection of the three workspaces make up the total workspace of the manipulator as shown in Fig. 2(d). This procedure is followed in all the cases and please note that in all the cases, the location of the coordinate system is at A1 i.e. at the left vertex of the base triangle. Also it is to be observed that to determine the actual area of the workspace and have a comparison amongst them, all the figures are to be drawn to a fixed scale which is not the case here; however the numerical value of area of workspace can be determined from the algebraic expression given for area for each workspace.

Figure 2: Generation of constant orientation workspace for 3-PRR manipulator

Each parameter has its own effect on the shape and, hence, area of workspace. The variation of parameter is classified as follows:

Case 1: $l=b$

Case 2: $l < b$

Case 3: $l > b$

Among all the three cases, $l=b$ is the simple one because two parameters are taking only one value. The third case of $l > b$ is physically impractical to fabricate; however this case is also investigated for academic interest and, as it would be found out, there exists definite workspace even when $l > b$. In general, for every set of links on the PRR, maximum workspace is drawn and void area is subtracted from it. The intersection of workspaces of each link gives rise to total workspace of the manipulator. This is done for each and every individual case. The intersection area is determined by the integration of intersecting areas.

In all the subsequent figures, workspace is generated without showing the step by step procedure and in each case, coordinate system is chosen at left vertex of the base platform.

NOTE :

1. The colors green, pink and blue represent the workspace for each PRR chain
2. The maroon color represents the workspace of the manipulator
3. The empty space represents the void

CASE 1: When $l=b$

In this case, workspace of triangle shape starts to form when the lines are moved by a distance greater than $\frac{l}{2\sqrt{3}} - r$ i.e; ($a > \frac{l}{2\sqrt{3}} - r$) and beyond $\frac{l}{2\sqrt{3}} + r$ i.e., ($a > \frac{l}{2\sqrt{3}} + r$) there is no more workspace, thus in this case ($l = b$) workspace is formed only when

$$\frac{l}{2\sqrt{3}} - r < a < \frac{l}{2\sqrt{3}} + r$$

Three types of workspaces are formed before “ a ” reaches $\frac{l}{2\sqrt{3}} + r$

CASE 1.1: A triangle with its vertex downwards as shown in Fig.3.

Figure 3: Workspace with vertex down ($a > \frac{l}{2\sqrt{3}} - r$)

$$\text{Area} = \frac{\sqrt{3}}{4} * BC^2 = 3\sqrt{3}(a+r)^2 + \frac{\sqrt{3}}{4}l^2 - 3l(a+r) \quad (1)$$

CASE 1.2: A triangle with its vertex upwards when $\frac{l}{2\sqrt{3}} + \frac{r}{3} < a < \frac{l}{2\sqrt{3}} + r$ as shown in Fig.4

Figure 4: Workspace with vertex upwards ($a < \frac{l}{2\sqrt{3}} - r$)

$$\text{Area} = \frac{\sqrt{3}}{4} * BC^2 = 3\sqrt{3}(a-r)^2 + l^2 - 3l(a-r). \quad (2)$$

CASE 1.3: In between when $\frac{l}{2\sqrt{3}} - \frac{r}{3} < a < \frac{l}{2\sqrt{3}} + r/3$, a hexagonal workspace is formed as shown in Fig.5.

Figure 5: Hexagonal workspace

Different points are obtained by solving equations of different lines and the total area is obtained by determining the area of two trapeziums.

$$\text{Area} = 6al + 2lr - 4\sqrt{3}ar - \frac{\sqrt{3}}{2}l^2 - 6\sqrt{3}a^2 + \frac{2r^2}{\sqrt{3}} \dots \dots \dots (3)$$

CASE 2 ($l < b$):

All the workspace shapes under this case are shown in the following table-1.

Table 1: Workspace shapes for $l < b$

	$r=a$	$r<a$	$r>a$
FOR $b=r$	 $l < b = r = a$	 $l < b = r < a$	 $l < b = r > a, l > a$
	Workspace doesn't exists.	Workspace doesn't exist	 $l < b = r > a, l = a$

	Workspace doesn't exist	Workspace doesn't exist	$I\langle b, b=r \rangle a, I \rangle a$
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	$r=a$	$r<a$	$r>a$
FOR $b>r$	$I\langle b \rangle r=a, I=r$	$I\langle b \rangle r<a, I\langle r, b \rangle a$	$I\langle b \rangle r>a, a=I\langle r$
	$I\langle b \rangle r=a, I>r$	$I\langle b \rangle r<a, I=r, b>a$	$I\langle b \rangle r>a, I\langle a\langle r$
	$I\langle b \rangle r=a, I\langle r$	$I\langle b \rangle r<a, I=r, b=a$	$I\langle b \rangle r>a, a\langle I\langle r$

	Workspace doesn't exist	$l \langle b \rangle r \langle a, l \rangle r, b \langle a$	$l \langle b \rangle r \rangle a, a \langle r \langle l$
	Workspace doesn't exist	$l \langle b \rangle r \langle a, l \langle r, b \rangle a$	$l \langle b \rangle r \rangle a, a \langle l \rangle r$
	Workspace doesn't exist	$l \langle b \rangle r \langle a, l \langle r, b \rangle a$	Workspace doesn't exist
	Workspace doesn't exist	$l \langle b \rangle r \langle a, l \rangle r, b \rangle a$	Workspace doesn't exist
	Workspace doesn't exist	$l \langle b \rangle r \langle a, l \rangle r, b \rangle a$	Workspace doesn't exist

	Workspace doesn't exist		Workspace doesn't exist
	$r=a$	$r<a$	$r>a$
FOR $b<r$		Workspace doesn't exist
	Workspace doesn't exist	Workspace doesn't exist
	Workspace doesn't exist	Workspace doesn't exist
	Workspace doesn't exist	Workspace doesn't exist

Case 3 ($l>b$)

Although case $l>b$ is physically not preferable, for academic interest this case is presented below.

Table 2: Workspace shapes for case $l > b$

	$r=a$	$r<a$	$r>a$
FOR $b=r$	$l>b=r=a$	$l>b=r<a, l>a$	$l>b=r>a$
	Workspace doesn't exist	$l>b=r<a, l=a$	Workspace doesn't exist
	Workspace doesn't exist	$l>b=r<a, l>a$	Workspace doesn't exist
	$r=a$	$r<a$	$r>a$
FOR $b>r$	$l>b>r=a$	$l>b>r<a, l=a>b$	$l>b>r>a$

	Workspace doesn't exist	$l > b > r < a, a > l > b$	Workspace doesn't exist
	Workspace doesn't exist	$l > b > r < a, l > b > a$	Workspace doesn't exist
	Workspace doesn't exist	$l > b > r < a, l > b = a$	Workspace doesn't exist
	Workspace doesn't exist	$l > b > r < a, l > a > b$	Workspace doesn't exist
	$r = a$	$r < a$	$r > a$

FOR $b < r$	$l > b < r = a, l > r$	$l > b < r < a, r < a < l$	$l > b < r > a, l = r, b = a$
	$l > b < r = a, l = r$	$l > b < r < a, r < l = a$	$l > b < r > a, l = r, b = a$
	$l > b < r = a, l < r$	$l > b < r < a, l < r < a$	$l > b < r > a, l > r, b = a$
	Workspace doesn't exist	$l > b < r < a, r < l < a$	$l > b < r > a, l > r, b > a$

	Workspace doesn't exist	$l > b < r < a, l = r < a$	$l > b < r > a, l > r, b < a$
	Workspace doesn't exist	Workspace doesn't exist	$l > b < r > a, l = r, b > a$
	Workspace doesn't exist	Workspace doesn't exist	$l > b < r > a, l < r, b = a$
	Workspace doesn't exist	Workspace doesn't exist	$l > b < r > a, l < r, b > a$

	Workspace doesn't exist	Workspace doesn't exist
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3. Determination of closed form expression of workspace area by geometrical method:

From section 2, it can be inferred that the general shape of the workspace obtained is as shown in Fig. 6.

Figure 6: General shape of the workspace

The points P1, P2..., P11 are the points that form the vertices of the workspace. They are obtained by solving the below given combinations using the standard formulae for circle-circle intersection or circle-line intersection:

$$\begin{aligned}
 P1: C1 \cap L1 & \quad P7: C6 \cap L5 \\
 P2: C1 \cap C4 & \quad P8: C5 \cap L5 \\
 P3: C4 \cap Xaxis & \quad P9: C5 \cap Xaxis \\
 P4: C4 \cap L2 & \quad P10: C2 \cap C5 \\
 P5: C3 \cap L3 & \quad P11: C2 \cap L1 \\
 P6: C3 \cap C6 &
 \end{aligned}$$

The total area is separated into four parts as shown by red, green, blue and yellow colors above. The area of the red colored part is given by

$$\Delta 1 = \int_{Py3}^{Py2} C4(\text{intermsof } x = f(y))dy + \int_{Py2}^{Py1} C1(\text{intermsof } x = f(y))dy \dots \dots \dots (4)$$

The area of the green colored part is given by:

$$\Delta 2 = \text{area of rectangle} = l(r + a) \dots \dots \dots (5)$$

The area of blue colored part is given by:

$$\Delta 3 = \int_{Px11}^{Px10} C2(\text{intermsof } y = f(x))dx - \int_{Px9}^{Px10} C5(\text{intermsof } y = f(x))dx \dots \dots \dots (6)$$

The area of the yellow colored part is given by $\Delta 4 = \int_{P_{x3}}^{P_{x4}} C4(\text{interms of } y = f(x))dx + \int_{P_{x4}}^{P_{x5}} L3(\text{interms of } y = f(x))dx + \int_{P_{x5}}^{P_{x6}} C3(\text{interms of } y = f(x))dx + \int_{P_{x7}}^{P_{x8}} L5(\text{interms of } y = f(x))dx + \int_{P_{x8}}^{P_{x9}} C5(\text{interms of } y = f(x))dx + \int_{P_{x6}}^{P_{x7}} C6(\text{interms of } y = f(x))dx \dots\dots\dots(7)$

Total area is given by: $\Delta = \Delta 1 + \Delta 2 + \Delta 3 + \Delta 4 \dots\dots\dots(8)$

Now, observing the shapes of the workspace as described in section 2, it can be classified into following categories.

a) Workspace having no void within it

Under this category comes the following cases:

Table 3: Workspace shapes having no voids within it

b) Workspace shapes having voids in it, but the voids do not intersect with each other

This is as shown in Fig.7:

Figure 7: Shape of the workspace having non-intersecting voids

The area of void caused for each serial chain is same for each of the chains. The void area is given by:

$$\Delta' = 3[\pi(r - a)^2 + l(r - a)] \dots\dots\dots(9)$$

So the workspace area can be determined by subtracting the area as given in Eqn. (9) from that of Eqn. (8).

c) The voids are intersecting and that is of the following shape-Type 1 (Fig. 8):

Figure 8: Intercepted shape of the voids-type 1

The following workspace shapes come under this condition.

Table 4: Workspace shape having voids of type 1

$ r>a, a>b> $	$ r>a, b>a> $	$ r>a, b> >a$

$ r<a, l=r, b=a$	$ r<a, l=r, b<a$	$ r<a, l<r, b=a$	$ r<a, l<r, b<a$

The area of above shape is given by:

$$\begin{aligned}
\Delta'' = & \int_{Py2'}^{Py1'} C1(\text{intermsof } x = f(y))dy + \int_{Px1'}^{Px7'} L1'(\text{intermsof } y = f(x))dx \\
& + \int_{Px7'}^{Px6'} C2'(\text{intermsof } y = f(x))dx \\
& + \int_{Px3'}^{Px4'} C6'(\text{intermsof } y = f(x))dx + \int_{Py3'}^{Py2'} C6'(\text{intermsof } y = f(x))dx \\
& + \int_{Px4'}^{Px5'} L5'(\text{intermsof } y = f(x))dx
\end{aligned}$$

Now, the total void area is given by:

$$\Delta' = 3[\pi(r - a)^2 + l(r - a)] - 3\Delta'' \quad (10)$$

d) The voids are intersecting and that is of the following shape-Type 2 (Fig. 9)

The intersected shape is as shown in Fig.9

Figure 9: Intersected shape of the voids-type 2

The area of above shape in Fig.9 is given by:

$$\Delta'' = \int_{Px1'}^{Px2'} C4' - L3' dx$$

Total void area is given by:

$$\Delta' = 3[\pi(r - a)^2 + l(r - a)] - 3\Delta'' \quad (11)$$

The cases under this category are shown in table 5:

Table 5: Sub cases with workspace having the intersected shape of voids as in Fig.9

e) The voids are intersecting and that is of the following shape-Type 3 (Fig. 10)

Figure 10: Intercepted shape of the voids-type 3

The area of this shape is given by:

$$\Delta'' = \int_{P_{x1}'}^{P_{x6}'} L1' dx + \int_{P_{x6}'}^{P_{x5}'} L4' dx + \int_{P_{x5}'}^{P_{x4}'} C3' dx + \int_{P_{x4}'}^{P_{x3}'} C3' dx + \int_{P_{x3}'}^{P_{x2}'} L3' dx - \int_{P_{x1}'}^{P_{x2}'} L3' dx \quad (12)$$

The cases under this category are shown in table 6:

Table 6: Sub cases with workspace having the intersected shape of voids as in Fig.10

f) The voids are intersecting and that is of the following shape-Type 4 (Fig. 11)

Figure 11: Intercepted shape of the voids-type 4

The area of this shape is given by:

$$\Delta'' = \int_{Px1'}^{Px7'} L1' dx + \int_{Px7'}^{Px6'} L4' dx + \int_{Px6'}^{Px5'} C3' dx + \int_{Px5'}^{Px4'} C3' dx + \int_{Px3'}^{Px4'} L3' dx + \int_{Px2'}^{Px1'} C4' dx - \int_{Px2'}^{Px3'} L3' dx \quad (13)$$

The cases under this category are:

Table 7: Sub cases with workspace having the intersected shape of voids as in Fig.11

$l > b < r > a, l = r, b > a$	$l > b < r > a, l < r, b = a$
$l > b < r > a, l < r, b > a$	$l > b < r > a, l < r, b < a$	$l > b = r < a, l > a$

g) The voids are intersecting and that is of the following shape-Type 5 (Fig. 12)

Figure 12: Intercepted shape of the voids-type 5

The area of this shape is given by:

$$\Delta'' = \int_{Px2'}^{Px1'} C1' dx + \int_{Px1'}^{Px6'} L1' dx - \int_{Px5'}^{Px6'} L5' dx - \int_{Px4'}^{Px5'} C6' dx + \int_{Px3'}^{Px4'} C6' dx - \int_{Px2'}^{Px3'} C6' dx \quad (14)$$

The case under this category is shown in Fig.13:

Figure 13: The workspace and the void under case 5

By subtracting the void area from the general workspace area (Eqn. 8) for a particular case, actual workspace area for that respective case is obtained.

h) Void area exceeding the workspace area

Remaining cases in which the void area is exceeding the workspace area as shown in Fig.14.

Figure 14: Workspace having void area larger than the workspace area

The workspace area can be divided into three symmetric parts, each part as shown in Fig.15.

Figure 15: One of the three symmetric parts of the workspace as shown in Fig. 14

$$\Delta = 3 \left[\int_{Px2}^{Px3} C3 dx + \int_{Px3}^{Px4} C6 dx - \int_{Px4}^{Px5} C3' dx - \int_{Px5}^{Px6} C2' dx - l(r - a) - \int_{Px2}^{Px1} C1' dx \right] \quad (15)$$

The cases under this category are shown in table 8 and 9:

Table 8: Sub cases with void areas exceeding workspace area.

Table 9: Sub cases with void area exceeding workspace area.

4. Parameter Optimization

Normally, user comes with a requirement of workspace area and the designer has to give an optimized parallel manipulator which has no/least number of singularities. In section 3, geometrical and closed form expressions of all kinds of shapes of workspace areas are determined and based on that individual parameters can be optimized.

However, it can be decided that workspace shapes in which there is void at the center of the workspace are not much preferable compared to those where there is no void. Considering this, the possible conditions of the parameters, as can be determined from Table 1 and 2 are as follows:

Table 10: Conditions of parameters for no void area within the workspace

- $l > b = r = a$
- $l > b > r = a$
- $l < b < r = a$
- $l > b < r = a$
- $l > b < r = a (l = r)$
- $l > b < r = a (l > r)$
- $l > b < r = a (l < r)$
- $l < b > r = a (l = r)$
- $l < b > r = a (l < r)$
- $l < b > r = a (l > r)$

For all other cases, there would be voids in the workspace, hence not preferable. This is the great advantage of the *look-up table* looking at which one can determine which geometrical conditions of the parameters can give the designer a workspace free of voids.

The workspace area in all the above cases is determined in the following way (please refer to Fig. 6).

$$\Delta = A1 + A2 + A3 + A4 \quad \dots\dots\dots(16)$$

where

1. $A1 =$

$$\int_{P2}^{P1} \sqrt{(r+a)^2 - (y - \frac{\sqrt{3}l}{2})^2} + (b - \frac{l}{2}) + \sqrt{(r+a)^2 + y^2} dy$$

2. $A2 = l(r+a)$

3. $A3 =$

$$\int_{P11}^{P10} \frac{(x-l)\sqrt{((r+a)^2 + (x-l)^2)}}{2} dx + \int_{P10}^{P9} \left[\frac{(x-b/2)\sqrt{(r+a)^2 - (x-b/2)^2}}{2} + \frac{(r+a)^2}{2} \sin^{-1}\left(\frac{x-b/2}{r+a}\right) + \frac{\sqrt{3}}{2} bx \right] dx$$

4. $A4 =$

$$\int_{P7}^{P8} \left[\frac{\sqrt{3}x^2}{2} - 2(r+a)x \right] dx + \int_{P4}^{P5} \left[\sqrt{3}b - 2(r+a)x - \frac{\sqrt{3}x^2}{2} \right] dx + \int_{P8}^{P9} \left[\sqrt{(r+a)^2 - (x-b/2)^2} + \sqrt{3}b/2 \right] dx + \int_{P6}^{P7} \left[\frac{\sqrt{3(b-l)}}{2} \right] x dx + \int_{P5}^{P6} \left[\sqrt{(r+a)^2 - (x-l)^2} \right] dx + \int_{P8}^{P9} \left[\sqrt{(r+a)^2 + (x-b/2)^2} + \frac{\sqrt{3}b}{2} \right] dx$$

The algorithm that is followed to determine the optimal parameters of the parallel manipulator suitable for a given workspace area is described in Fig.16.

Figure 16: Flowchart describing the algorithm which is followed to determine the optimal parallel manipulator

In this algorithm, the designer first gets the required workspace area from the user. Then he varies the parameters l , r , b and a within certain ranges (different ranges for different parameters). Then he checks which values of these parameters satisfy the conditions mentioned in Table 10. By this he ensures that for these values of the parameters there is no voids inside the workspace. Out of these sets of parameters, he eliminates those cases in which $l > b$ considering feasibility of fabricating such manipulators. Then from the remaining sets of the values, he applies the Eqn. 16 to determine the workspace and selects those values of the parameters for which the workspace area satisfies the required workspace area considering the factor of safety depending upon the type of application, or any other consideration as demanded by the user. These sets of values are the feasible values of the parallel manipulators which will give the required workspace area without any void in the workspace. Then he applies singularity conditions to each one of these parallel manipulators to determine the best parallel manipulator having least or no singularity points within the workspace.

5. Case Study

In the case study, the user wants to 3-PRR planar parallel manipulator having workspace area of 300 sq. units. The task is thus to give him a 3-PRR planar parallel manipulator having following features:

- The workspace area is 300 ± 10 square units (This 10 square units depends upon the user's requirement. If it is a precision application then we may give zero tolerance. The algorithm has the capability to work on such close tolerances)
- All the proposed parallel manipulators do not have any void within the workspace.
- The proposed optimal parallel manipulator(s) has zero or least number of singularities.
- The solution gives us the optimal length of l , r , b and a . They are respectively the maximum displacement of the slider at the base joint, the length of the intermediate link, the base length of the base platform assumed to be a triangle and the radius of the top platform.

Based on the flowchart shown in figure 14, the following sets of parameters satisfy the workspace area constraint and each one of them are having no void within the workspace. Table 11 shows this result.

Table 11: Optimal parameters matching with workspace area requirement and having no voids within

Sl No	l (max slider length)	b (base length of the triangular base platform)	r (link connecting to slider to the mobile platform)	a (radius of the mobile platform)	Workspace area
1	20	20	6	8	301
2	20	20	8	10	301
3	20	20	10	12	301
4	20	20	12	14	301
5	20	20	14	16	301
6	20	20	16	18	301
7	20	20	18	20	301
8	10	50	10	10	281
9	50	50	12	12	307

Out of the 9 shown parallel manipulators, serial numbers 3, 5, 7, 8, 9 are chosen for the singularity analysis. The rest are all very close to these dimensions. Singularity analysis for 3-PRR planar parallel manipulators are very much studied in the literature. So based on that singularity analysis is performed and the singularities are plotted in the following figure 17. The circles in the figure represent the forward singularities, the triangle represents the base platform with the three lines connecting the vertices to the centroid of the base platform.

(a): Singularities of manipulator of Sl. No. 3

(b): Singularities of manipulator of Sl. No. 5

(c): Singularities of manipulator of Sl. No. 7

(d): Singularities of manipulator of Sl. No. 8

(e): Singularities of manipulator of Sl. No. 9

Figure 17: Singularity distribution of selected manipulators from table 11 (Sl. No 3, 5, 7, 8 and 9)

Considering the number of singularities, except Sl. No. 3, 8 and 9 (Fig. 17 (a),(d),(e)), the other two parallel manipulators can be proposed to the user as suitable parallel manipulators for his application requiring 300 sq. units area. These two manipulators are shown in Fig. 18 (a) and (b) respectively. Out of them Sl. No. 5 (Fig.18(a)) contains two singularities and Sl. No. 7 (Fig.18(b)) contains 3 singularities only. Both these manipulators can be proposed to be optimal parallel manipulators to the user for his required workspace area.

(a)

(b)

Figure 18: The optimal 3-PRR parallel manipulators; (a) Solid model of the parallel manipulator of serial no. 5 (Table 11); (b) Solid model of the parallel manipulator of serial no. 7 (Table 11)

6. Conclusions

1. This paper presents a geometrical method of determination of the shape of the constant orientation workspace for any kind of variation of its parameters. A *look up table* is also proposed which shows all kinds of shapes of the workspaces for any variation in the parameters of 3-PRR planar parallel manipulator. For each shape of the workspace, geometrical conditions are also mentioned. This helps a lot in choosing geometrical parameters for a required shape of the

workspace; it also helps to visualize which geometrical relationship between the parameters results in voids inside the workspace. Thus the look-up table is of a great help to the designer for optimal design of such parallel manipulators.

2. For each shape of the workspace, closed form mathematical expressions are obtained to determine the area of the workspace. This helps a lot for optimal design; this also helps a lot for applications where precision is of more importance. This is more pronounced in case of parallel manipulators because parallel manipulators are preferred for precision applications.
3. A case study is presented wherein the user gives his requirement of workspace area. A flowchart is presented wherein it is shown, in a step by step method, how to get singularity free (or minimum singularity) optimal parallel manipulators satisfying the workspace area requirement of the user and having no voids within the workspace.
4. For the case study, initially 9 sets of parameters are chosen which satisfy the workspace area requirement and having no voids. Out of them best 5 are selected and they are tested for singularity analysis and out of them 2 parallel manipulators are chosen finally having minimum singularities, satisfying workspace area constraint and having no voids within the workspace. Thus the algorithm is successfully tested for a specific application.

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Authors' Contribution

This work was taken up in two stages. In the first stage three authors namely Mr. Sree Sailesh, Mr. Manoj Kumar Reddy and Mr. Teja Prakash K. worked on development of different kinds of workspaces for different relationships among the links. This forms the most of part of section 2 and 3. In the second part, optimization and simulation were done by Mr. M.R. Shivakumar, Mr. R. Srikrishna, and Mr. R. Sriram. This forms the most part of sections 4 and 5. Dr Anjan Kumar Dash was the person behind the concept generation, execution and manuscript writing.

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Figures

Figure 1

Line diagram of a 3-PRR planar parallel manipulator

Figure 2

Generation of constant orientation workspace for 3-PRR manipulator

Figure 3

Workspace with vertex down ($\theta > \pi / 2\sqrt{3} - \theta$)

Figure 4

Workspace with vertex upwards ($\theta < \pi / 2\sqrt{3} - \theta$)

Figure 5

Hexagonal workspace

Figure 6

General shape of the workspace

Figure 7

Shape of the workspace having non-intersecting voids

Figure 8

Intercepted shape of the voids-type 1

Figure 9

Intersected shape of the voids-type 2

Figure 10

Intercepted shape of the voids-type 3

Figure 11

Intercepted shape of the voids-type 4

Figure 12

Intercepted shape of the voids-type 5

Figure 13

The workspace and the void under case 5

Figure 14

Workspace having void area larger than the workspace area

Figure 15

One of the three symmetric parts of the workspace as shown in Fig. 14

Figure 16

Flowchart describing the algorithm which is followed to determine the optimal parallel manipulator

(a): Singularities of manipulator of Sl. No. 3

(b): Singularities of manipulator of Sl. No. 5

(c): Singularities of manipulator of Sl. No. 7

(d): Singularities of manipulator of Sl. No. 8

(e): Singularities of manipulator of Sl. No. 9

Figure 17

Singularity distribution of selected manipulators from table 11 (Sl. No 3, 5, 7, 8 and 9)

(a)

(b)

Figure 18

The optimal 3-PRR parallel manipulators; (a) Solid model of the parallel manipulator of serial no. 5 (Table 11); (b) Solid model of the parallel manipulator of serial no. 7 (Table 11)