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A method for identifying and predicting the geometric errors of a rotating axis

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Abstract To improve the machine tool accuracy, an integrated geometric error identification and prediction method is proposed to eliminate the positioning inaccuracy of tool ball for a double ball bar (DBB) caused by the rotary axis' geometric errors in a multi-axis machine tool. In traditional geometric errors identification model based on homogenous transformation matrices (HTM), the elements of position-dependent geometric errors(PDGEs) are defined in the local frames attached to the axial displacement, which is inconvenient to do redundance analysis. Thus, this paper proposed an integrated geometric error identification and prediction method to solve the uncertainty problem of the PDGEs of rotary axis. First, based on homogeneous transform matrix (HTM) and multi-body system (MBS) theory, The transfer matrix only considering the rotary axes is derived to determine the tool point position error model. Then a geometric errors identification of rotary axis is introduced by measuring the error increment in three directions. Meanwhile the geometric errors of C-axis are described as position-dependent truncated Fourier polynomials

caused by fitting discrete values. Thus, The geometric error identification is converted to the function coefficient. Finally, the proposed new prediction and identification model of PDGEs in the global frame are verified through simulation and experiments with double ball-bar tests.

Keywords Geometric error identification   Rotary axis   Double ball bar
Five-axis machine tool

1 Introduction

Five-axis CNC machine tools provide greater productivity, better flexibility, and less fixture time than three axis machining centers, because the cutting tool can approach the workpiece from any direction. However, the two rotary axes would bring in additional geometric errors subsequently, such as squareness between a rotary axis and a translational axis [1]. Therefore, an accurate and efficient measurement method of geometric errors is a key prerequisite for the improvement of five-axis machine tool accuracy [2]. If there is an effective method to improve or compensate for those deviations, the machining performance of multi-axis machines will be improved drastically.

In the past decades, some researches have been taken to measure and identify the errors that are inherent to a rotary axis of a multi-axis machine tool. Tsutsumi and Saito proposed a calibration method using the simultaneous four-axis control technique for five-axis machining centers with a tilting rotary table. The eight deviations were estimated by the

observation equations from the ball bar measured results [3]. Zargarbashi identified eight link errors of a five-axis machine through measuring the tool center position deviations at a number of five-axis poses by using a ball-bar sensor [4]. Zargarbashi and Mayer presented a method consisting of five double ball bar tests to evaluate five trunnion axis motion errors including axial, radial and tilt errors.

Some new instruments have also been designed and adopted for high efficiency indirect measurement of geometric errors, such as the R-test, Capball, 3D probe-ball or a touch trigger probe [5-9]. In recent years, geometric errors have been identified more conveniently and accurately based on new measuring instruments. However, It must be remembered that the measuring instruments themselves do not have the ability to identify geometric error elements, and their direct function is limited to measuring tool center position errors related to the workpiece. Therefore, the key factor to identify the geometric error is to correctly establish the mathematical model between the cutting tool processing point and the worktable processing point. At present, the commonly used error modeling methods are based on homogenous transformation matrix (HTM) [10-12] or screw theory [13-14]. A global description of rigid body motion is allowed with the screw theory, this is the advantage and difference from HTM method. However, the geometric errors defined in the Cartesian space must be converted to twists in the three-dimensional space via screw

theory, which makes the calculation of screw theory relatively intricate compared to HTM method [15]. In this paper, A geometric error identification and compensation for the rotating axis of a five-axis machine tool is proposed based on HTM method.

Due to the complexity of the machine tool structure, geometric errors are commonly classified as position dependent geometric errors (PDGEs) and position independent geometric errors (PIGEs). PDGEs are mainly caused by imperfections of components, such as the straightness errors of the guide ways [16], while PIGEs are mainly caused by the imperfect assembly of parts, such as joint misalignments, angular offset and rotary axes separation errors [17]. Over the past few decades, many researches have reported the geometric errors identification methods for translational axes, such as 9-line method, the 15-line method and 12-line method by using the laser interferometer. Sepcially, the 9-line method has been recognized as a common method to detect translational errors according to ISO 230-1 (2012) [18]. While the error identification of the rotar axis yhas not been unified. Zhang et al. proposed a novel DBB measuring method, in which only the C-axis rotated. Unfortunately, it could only evaluate five PDGEs of the C-axis [19]. Similarly, Lei et al. presented a particular circular test path, which caused the two rotary axes only to move simultaneously and kept the other three linear axes stationary [20]

However, some others may have such a capacity, require complex mathematical formulations and especially involve a complicated measurement process to identify the error parameters at present. Therefore, in this paper, a new method for error identification of rotary axis is proposed, which is effective and uncomplicated to improve the accuracy of error compensation. Firstly, the geometric error truncated Fourier expansion model was established, and the geometric error identification was transformed into Fourier coefficient identification. Then, a multi-body theory based rotary axis error parameter identification model was proposed by using double ball bar. Finally, experiments and simulations proved that the method was effective and accurate for improving the accuracy of error identification. The structure of the article is arranged in this way. In section 2, the position errors is built based on MBS theory and HTMs, meanwhile the values of predicted PDGEs is fitted by the trunc Fourier polynomials in the machine coordinate system. In section 3, a identification method of C-axis is introduced subsequently. Senction 4 verifies the feasibility and effectiveness of the proposed method dedected by the results of experiments and simulation. Finally, the paper is summarized.

2 Geometric errors model

2.1 Description of geometric errors

Geometric errors caused mainly by manufacturing and assembly defects are considered to be systematic errors. The geometric errors are usually categorized as position-independent geometric errors (PIGEs) and position-dependent geometric errors (PDGEs). PDGEs have a certain relationship to the machine positions, while PIGEs are constant values which are usually produced during the assembly process[21]. For the five-axis CNC machine tools, whose configurations are depicted in Fig. 1, there are three translation axis(X,Y,Z) and two rotary axis (B,C) which is TTTRR Machine tool structure. Related parameters of rotary axis geometric errors are described in Table1. In these error parameters, $\sigma_x(B)$, $\sigma_y(B)$, $\sigma_z(B)$, $\varepsilon_x(B)$, $\varepsilon_y(B)$, $\varepsilon_z(B)$, $\sigma_x(C)$, $\sigma_y(C)$, $\sigma_z(C)$, $\varepsilon_x(C)$, $\varepsilon_y(C)$, $\varepsilon_z(C)$ are position-dependent geometric errors and S_{xb} , S_{xc} , S_{zb} , S_{yc} are position-independent geometric errors. Generally speaking, there are 16 geometric errors in a five-axis machine tool, including 12 PIGEs and 4 PDGEs. For example, $\sigma_x(C)$ represent the displacement error of the C axis in the X direction and S_{xc} represent the square errors between X axis and C axis shown in Fig. 1 and Fig. 2 respectively.

According to Ref [13] and Ref [15], the position-independent geometric errors are caused by the deviation between the actual machine tool installation and the ideal installation. Thus, in this paper, these position-independent geometric errors are regarded as a fixed size impact to related Accuracy of shaft movement. However, the PDGs are described as errors

to the movement of the axis. Based on previous studies, these position-independent geometric errors can be described as a function, which is more convenient to study geometric errors and more accurate for error compensation. The position -dependent geometric errors is fitted as the truncated Fourier polynomial as shown in Eq.(1). The fitting method includes Fourier polynomial, simple polynomial, Legendre polynomial, chebyshev polynomial and so on. Compared to truncated Fourier polynomial, The random unpredictability of the error can be reflected, and the lower and upper limits of the position-dependent geometric satisfy the Dirichlet condition. Hence, the truncate Fourier polynomials is introduced to fit the PDGEs. In this paper, error identification of rotary axis is the focus of research and the position-dependent geometric errors of rotary axis can be fully expressed in Eq.(1)

$$f(\theta) = \sum_{i=1}^n A_i \sin \frac{\theta}{2i-1}$$

1

Where $f(\theta)$ and θ denote geometric errors function and angles of rotation respectively. A_i represent the coefficient.

Fig.1 Geometric error of the C-axis

Fig.2 position-independent geometric errors of C-axis

Fig.3 Topological structure of machine tool

Fig.4 Schematic diagram of Machine tool

2.2 position error model based on HTM

The key of position error modeling is to establish the relationship P_e of the Ideal processing point and actual processing point. In other words, The identification of the actual position vector P_a and the ideal position vector P_i is the basis of volumetric error. According to the multi-body theory, the

relationship between the axes of a five-axis machine tool can be regarded as the interconnection of each rigid body as shown in Fig. 3 (0-Lathe bed 1-X axis 2-Y axis 3-Z axis 4-B axis 5-C axis 6-tool 7-workbench). Therefore, HTM is used to explain the position relationship between each rigid body, which can calculate the actual position vector and the ideal position vector. Assuming that the length of a cutting tool is L, the initial position of the cutting point and the initial position of cutting tool can be defined as $P_{\text{initial}}=[0,0,-L,1]^T$ in the cutting tool system respectively. If there are no geometric errors, the ideal position of the cutting tool can be expressed as follows

$$P_i = (T_0^1)^{-1} T_1^2 T_2^3 T_3^4 R_4^5 R_5^6 P_{\text{initial}} \quad 2$$

When the geometric errors are considered, the actual position of the cutting tool can be expressed as

$$P_a = (E_0^1 T_0^1)^{-1} E_1^2 T_1^2 E_2^3 T_2^3 E_3^4 T_3^4 E_4^5 R_4^5 E_5^6 R_5^6 P_{\text{initial}} \quad 3$$

Where T_j^i and R_j^i represent ideal homogenous transformation matrix between adjacent bodies (i,j) of translation axis and rotary axis respectively.

$$T_j^i(x) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_j^i(y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_j^i(z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_j^i(b) = \begin{bmatrix} \cos(b) & 0 & \cos(b) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(b) & 0 & \sin(b) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_j^i(c) = \begin{bmatrix} \cos(c) & -\sin(c) & 0 & 0 \\ \sin(c) & \cos(c) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4

E_j^i reflects the error transformation matrix between adjacent bodies(i,j) in machine tool. Where, E_j^i is represented by the position error matrix ${}^pE_j^i$ and motion error matrix ${}^sE_j^i$ between adjacent bodies.

$$E_j^i = {}^pE_j^i * {}^sE_j^i = \begin{bmatrix} 1 & -S_j^i(yz) & S_j^i(xz) & 0 \\ S_j^i(yz) & 1 & -S_j^i(xy) & 0 \\ -S_j^i(xz) & S_j^i(xy) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\xi_j^i(c) & \xi_j^i(b) & \sigma_j^i(x) \\ \xi_j^i(c) & 1 & -\xi_j^i(a) & \sigma_j^i(y) \\ -\xi_j^i(b) & \xi_j^i(a) & 1 & \sigma_j^i(z) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5

Therefore, the position error P_e can be expressed as

$$P_e = P_a - P_i \quad 6$$

3 Experiment and identify geometric errors of rotary axis

The identification and experiment of rotating axis geometric error are introduced in this paper. In previous studies, the identification of translational axis geometric error has been clearly specified, but the identification of rotating axis error has not been clearly specified. It can be seen from Table 1 that the two rotating axes B and C have 12 PDGs totally, except the four PIGs of rotary detected in the appendix of iso1091-2. The geometric errors of rotary axis can be detected using renishaw XR20W double ball bar device. The remaining 10 PDGs can be derived from the

multi-body theory and the Houston transformation matrix which contains six the linear errors and four angle errors. In order to express more briefly, the five PDGEs of C-axis are identified by using DBB, which is taken as an example of rotary axis.

PDGs	B-axis	C-axis
Linear errors	$\partial_x(b), \partial_y(b), \partial_z(b)$	$\partial_x(c), \partial_y(c), \partial_z(c)$
Angle errors	$\varepsilon_x(b), \varepsilon_y(b), \varepsilon_z(b)$	$\varepsilon_x(c), \varepsilon_y(c), \varepsilon_z(c)$

Table 1 The PDGs of rotary axis

3.1 Measurement

This article selects the double pendulum milling planer head for testing machine shown in Fig. 4, with translational axis geometric error compensated, using double cue instrument and machine tool axis of rotation movement relationship, success is a small local branch geometric error of each axis of rotation.

The detection of the C axis is divided into three steps: c-x, c-y and c-z. The detection mode of the cue meter is detected in the X, Y and Z directions respectively. As shown in Fig.5, assuming P_c is the ideal origin of C axis coordinates, P'_c is the projection of the actual position of the far point of C axis coordinate system in the XZ plane, δx is the projection of the positioning error of C axis in the x direction, δ is the projection of the degree error of C axis in the XZ plane. The error of the Angle and position of the C axis will inevitably lead to the error of the position of different

height points of the C axis in the X direction. During the measurement, firstly ensure that the Angle of B and C is 0° at the initial stage. Then when the C axis is rotated, the length of the measuring rod changes in the X direction. Two measuring points (c_1, c_2) are selected, and finally the geometric error parameters of the C axis are deduced through the position relationship. The key of this experiment is to measure the values of delta L in the X,Y, and Z directions of c_1 and c_2 .

$$\Delta L_c = \begin{bmatrix} \Delta L_{cx} & \Delta L_{cy} & \Delta L_{cz} \end{bmatrix}^T$$

7

Fig.5 C-X measurement
mode error generation
mechanism

Fig.6 C-X-1mode

Fig.7 C-X-2 mode

The ideal point D_{CX} and actual point D'_{CX} can be calculated from the HTM above Section 2. And the PDGEs of C-axis in the X direction is shown in Fig. 5. Among them, ΔL_{cx1} and ΔL_{cx2} represent the error increment measured in the X direction of the two experiments which conducted in Fig.6 and Fig.7 respectively. Therefore, the error increment of the length (ΔL_{cx}) of the cue instrument in the X direction is expressed as

$$\Delta L_{cx} = D'_{cx} - D_{cx} \quad 8$$

$$D_{cx} = \begin{bmatrix} \cos(c) & -\sin(c) & 0 & 0 \\ \sin(c) & \cos(c) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -H_c \\ 1 \end{bmatrix} \quad 9$$

$$D'_{cx} = \begin{bmatrix} 1 & 0 & S_{xc} & 0 \\ 0 & 1 & -S_{yc} & 0 \\ -S_{xc} & S_{yc} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(c) & -\sin(c) & 0 & 0 \\ \sin(c) & \cos(c) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\xi_z(c) & \xi_y(c) & \sigma_x(c) \\ \xi_z(c) & 1 & -\xi_x(c) & \sigma_y(c) \\ -\xi_y(c) & \xi_x(c) & 1 & \sigma_z(c) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -H_c \\ 1 \end{bmatrix} \quad 10$$

If Eq.(11) and Eq.(12) are put into Eq(10), the result will be described as

$$\begin{bmatrix} \cos(c) & -\sin(c) & -H_{c1} \cos(c) & -H_{c1} \sin(c) \\ \cos(c) & -\sin(c) & -H_{c2} \cos(c) & -H_{c2} \sin(c) \end{bmatrix} \begin{bmatrix} \delta_x(c) \\ \delta_y(c) \\ \xi_y(c) \\ \xi_x(c) \end{bmatrix} = \begin{bmatrix} \Delta L_{cx1} + H_{c1} S_{xc} + H_{c1} \sin(c) S_{bz} \\ \Delta L_{cx2} + H_{c2} S_{xc} + H_{c2} \sin(c) S_{bz} \end{bmatrix} \quad 11$$

Here the equation has four unknowns and the rank of the coefficient matrix is 2. Hence, the error relationship needs to be confirmed in the c-y direction.

(2) C-Y measurement

It will not be detailed here, since the detection method of c-y is the same as that of c-x. By combining Eq.10 and Eq.15, the error parameters of $(\sigma_x(c), \sigma_y(c), \xi_x(c), \xi_y(c))$ concerning the position of the four terms of the c-axis. C-Y can be inversely obtained.

$$\Delta L_{cy} = D'_{cy} - D_{cy} \quad 12$$

$$\begin{bmatrix} \cos(c) & -\sin(c) & -H_{c1} \cos(c) & -H_{c1} \sin(c) \\ \cos(c) & -\sin(c) & -H_{c2} \cos(c) & -H_{c2} \sin(c) \\ \sin(c) & \cos(c) & -H_{c1} \sin(c) & H_{c1} \cos(c) \\ \sin(c) & \cos(c) & -H_{c2} \sin(c) & H_{c2} \cos(c) \end{bmatrix} \begin{bmatrix} \sigma_x(c) \\ \sigma_y(c) \\ \xi_y(c) \\ \xi_x(c) \end{bmatrix} = \begin{bmatrix} \Delta L_{cx1} + H_{c1} S_{xc} + H_{c1} \sin(c) S_{bz} \\ \Delta L_{cx2} + H_{c2} S_{xc} + H_{c2} \sin(c) S_{bz} \\ \Delta L_{cy1} - H_{c1} S_{xc} - H_{c1} \cos(c) S_{bz} \\ \Delta L_{cy2} - H_{c2} S_{xc} - H_{c2} \cos(c) S_{bz} \end{bmatrix} \quad 13$$

Where ΔL_{ij} (i=x,y,z, j=1,2) represents the length increment of the jth measured in the i direction. Based on the Eq.(13), the PDGEs parameters will be obtained.

$$\xi_x(c) = \frac{\cos(c)(\Delta L_{cy1} - \Delta L_{cy2}) + \sin(c)(\Delta L_{cx2} - \Delta L_{cx1})}{H_{c1} - H_{c2}} - \cos(c) S_{yc} - \sin(c) S_{xc} - S_{bz} \quad 14$$

$$\xi_y(c) = \frac{\cos(c)(\Delta L_{cx2} - \Delta L_{cx1}) + \sin(c)(\Delta L_{cy2} - \Delta L_{cy1})}{H_{c1} - H_{c2}} - \cos(c) S_{xc} + \sin(c) S_{yc} \quad 15$$

$$\sigma_y(c) = \frac{\sin(c)(H_{c2} \Delta L_{cx1} - H_{c1} \Delta L_{cx2}) + \cos(c)(H_{c1} \Delta L_{cy2} - H_{c2} \Delta L_{cy1})}{H_{c1} - H_{c2}} \quad 16$$

$$\sigma_x(c) = \frac{\cos(c)(H_{c1} \Delta L_{cx2} - H_{c2} \Delta L_{cx1}) + \sin(c)(H_{c1} \Delta L_{cy2} - H_{c2} \Delta L_{cy1})}{H_{c1} - H_{c2}} \quad 17$$

(3) C-Z measurement

In the c-z detection mode, the rod length transformation detected is the axial displacement of C-axis. So $\sigma_z(c)$ can be obtained as

$$\Delta L_{cz} = D'_{cz} - D_{cz} = \sigma_z(c) \quad 18$$

Thus, Based on obtained constant, the five PDGEs of C axis will be identified totally.

4 Results of calculation and simulation

In the third part, a method of geometric error identification of rotation axis is proposed, which only thinks about picking two points at random.

However, considering the factor that the error of the guide machine varies with the movement of the machine, a comprehensive identification method for the rotary axis is proposed. For the first stage, all 16 geometric errors were described as third-orders truncated Fourier polynomials, and the identification of the polynomial coefficients were based on the selection of measurement points through an observability index. For the second stage, based upon the identification results of stage one, an integrated geometric error identification model of rotating axis is proposed to improve the identification accuracy.

For the five-axis machine tool, as one rotary axis has five related position dependent geometric errors besides its rotary positioning error , there were 10 position-dependent geometric errors. The truncated Fourier polynomial coefficients of 10 geometric errors was represented by using the row vectors $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_{10}$ .the parameters in the vector $\mathbf{A}$ as expressed in Eq.(22)

$$\mathbf{A}=[\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_{10}] \quad 19$$

Taking C axis as an example, its five PDGEs are represented by the following,

$$A_i = [A_{i1}, A_{i2}, A_{i3}, A_{i4}, A_{i5}] \quad 20$$

$$\begin{aligned}
\partial_x(c) &= \sum_{i=1}^n A_{i1} \sin\left(\frac{\theta}{2i-1}\right) \\
\partial_y(c) &= \sum_{i=1}^n A_{i2} \sin\left(\frac{\theta}{2i-1}\right) \\
\partial_z(c) &= \sum_{i=1}^n A_{i3} \sin\left(\frac{\theta}{2i-1}\right) \\
\xi_x(c) &= \sum_{i=1}^n A_{i4} \sin\left(\frac{\theta}{2i-1}\right) \\
\xi_y(c) &= \sum_{i=1}^n A_{i5} \sin\left(\frac{\theta}{2i-1}\right)
\end{aligned} \tag{21}$$

Where $A_{ij}(j = 1, 2, \dots, 5)$ is the coefficient of the truncated Fourier polynomials describing the PDGEs. n denotes the orders of the polynomials, which are determined by examining the residual errors. The constant terms of truncated Fourier polynomials are not considered since the five PDGEs of C-axis are zero at home position (i.e., $\theta_k = 0$).

By transforming Eq. (24), the following equation can be obtained.

$$A^T \theta = B \tag{22}$$

Where

$$A = \begin{bmatrix} A_{11} & A_{21} & A_{31} & A_{41} & A_{51} \\ A_{12} & A_{22} & A_{32} & A_{42} & A_{52} \\ \dots & \dots & \dots & \dots & \dots \\ A_{1i} & A_{2i} & A_{3i} & A_{4i} & A_{5i} \\ A_{1n} & A_{2n} & A_{3n} & A_{4n} & A_{5n} \end{bmatrix}_{n \times 5} \tag{23}$$

$$\theta = \begin{bmatrix} \sin \theta_1 & \sin \theta_2 & \dots & \sin \theta_j & \sin \theta_k \\ \sin \frac{\theta_1}{3} & \sin \frac{\theta_2}{3} & \dots & \sin \frac{\theta_j}{3} & \sin \frac{\theta_k}{3} \\ \sin \frac{\theta_1}{5} & \sin \frac{\theta_2}{5} & \dots & \sin \frac{\theta_j}{5} & \sin \frac{\theta_k}{5} \\ \dots & \dots & \dots & \dots & \dots \\ \sin \frac{\theta_1}{2n+1} & \sin \frac{\theta_2}{2n+1} & \dots & \sin \frac{\theta_j}{2n+1} & \sin \frac{\theta_k}{2n+1} \end{bmatrix}_{n \times k} \tag{24}$$

$$\text{and} \quad B = \begin{bmatrix} \sigma_x(\theta_1) & \sigma_x(\theta_2) & \dots & \sigma_x(\theta_j) & \sigma_x(\theta_k) \\ \sigma_y(\theta_1) & \sigma_y(\theta_2) & \dots & \sigma_y(\theta_j) & \sigma_y(\theta_k) \\ \sigma_z(\theta_1) & \sigma_z(\theta_2) & \dots & \sigma_z(\theta_j) & \sigma_z(\theta_k) \\ \xi_x(\theta_1) & \xi_x(\theta_2) & \dots & \xi_x(\theta_j) & \xi_x(\theta_k) \\ \xi_y(\theta_1) & \xi_y(\theta_2) & \dots & \xi_y(\theta_j) & \xi_y(\theta_k) \end{bmatrix}_{5 \times k} \quad 25$$

If A is the geometric error of the coefficient matrix, the position error of the machine tool in the workpiece coordinate system can be obtained as Eq.

(26) :

$$A = (\theta\theta^T)^{-1} \theta B^T \quad 26$$

4.2 Evaluation of identification accuracy

In the proposed detection method, different parameter assignment methods in A need to be evaluated and verified to seek for the optimal parameter assignment method. Thus, a method for evaluating the identification accuracy obtained by parameters in A is required. There are 20 sets of points that are randomly selected, one cycle of rotation on the C axis. By comparing the position errors of the test points measured by the cue instrument with the position errors predicted by the recognition results, the recognition accuracy of the current geometric error description can be evaluated. Therefore, the residual sum of squares and SSE estimation model of point estimation is used to evaluate the geometric error identification accuracy of rotary axis.

The geometric error parameters of the machine tool and the moving parts have periodic legal action, but there are many uncertainties at the same time. Thus, it is necessary to establish an error model to carry out

regression analysis on the measurement error parameters. In order to obtain specific model parameters, the regression analysis is calculated by the least square method. When the input point is (x,y), the estimated value $\hat{c}$ of the regression coefficient c is solved to satisfy the residual of the regression model.

$$\min_c \|f(c, x) - y\|_2^2 = \sum_{i=1}^n [y_i - f(c, x_i)]^2 \quad 27$$

Where x_i and y_i represent the sample data of the i th group, respectively. n is the number of input data points.

The residual squares of the estimated points and SSE are used to evaluate the estimated residual of the model. The smaller the SSE, the more accurate the estimation, as shown in equation

$$SSE = \sum_{i=1}^n (y_i - f(\hat{c}, x_i))^2 \quad 28$$

Where $\hat{c}$ is the point estimation of the regression coefficient. The goodness of fit test was carried out with the determination coefficient R-square of the fitting equation. And if R-square gets closer to 1, the accuracy of the fitting model will be higher. When R-square=1, the regression curve passed through all sample points and the sample residual was 0.

$$R\text{-square} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad 29$$

4.3 Analysis of experimental results

In order to verify effectiveness of the proposed method, a TTTRR type five-axis planer milling machine tool was utilized to measure the geometric errors of C-axis as shown in Fig.8. The workspace of the translational axes to be measured was $x * y * z = 2500 \text{ mm} * 2000 \text{ mm} * 500 \text{ mm}$. The rotational ranges of the swivelling heads to be measured were $C [0^\circ, 360^\circ]$ and $B [0^\circ, 180^\circ]$. In order to more accurately measure the geometric error, the room should be about 20°C constant temperature.

In the simulations, the identification accuracy of different sets of measurement points was compared. The result of identified PDGEs were shown in Fig. 9, which are fitted by third orders truncated Fourier series. From these, it can be seen that for both the predicted set and the measured sets of points, the motion positions of C-axis covered the entire measurement workspace and ensured a valid identification accuracy obtained from these sets of measurement points. It is obvious that the predicted values gets close to the measured values by comparing with the measured results. Among them, the maximum values of residual errors are 3.4um, 2.8um, 1.2um, 0.48udeg and 0.52udeg, which can describe the deviation from predicted and measured values, respectively. In addition, the values of R-square were about fitting curve shown in Table.2 that the maximum value of R-square gets to 0.976. As a matter of fact, the identification method used in this paper possesses less term numbers, smaller fitting residuals, and higher fitting accuracy.

In order to further prove that the proposed method is feasibility, a experiment is conducted which results are shown in Fig. 10 and Fig. 11. Under different identification method, the result of the compensated position error goes distinct based on Eq.(6). Histogram 1 represents original position error of C-axis and histogram 2 and 3 represent the position error of C-axis compensated by traditional identification method and the proposed method respectively. In these figures, the position error is the value of the error when the C axis is rotated 90 degrees and the C axis is rotated 180 degrees. It is obvious that the error compensation based on the proposed identification method is more effective and feasible than the traditional identification method. In addition, due to the influence of location error $\varepsilon_z(c)$, thermal error and other error source factors, the compensation results will be affected which won't lead the value of errors to get down to zero. Hence, there is no doubt that the proposed method is effective for error identification.

PDGs	$\partial_x(c)$	$\partial_y(c)$	$\partial_z(c)$	$\varepsilon_x(c)$	$\varepsilon_y(c)$
R-square value	0.973	0.969	0.976	0.962	0.963

Table. 2 The R-square of five PDGEs

Fig.8. Geometric errors of C-axis identification

(a)

(b)

(c)

(d)

(e)

Fig.9. Identification the PDGs of C-axis

Fig.10. Position error at 90° angle of axis C

Fig.11. Position error at 180° angle of axis C

5 Conclusion

This paper has proposed a novel and efficient on-machine measurement methodology for identification and prediction of the five PDGEs of the rotary axes on a five axis CNC machine tool by using double ball bar. Error identification model was built based on the Multiple body system theory and HTMs when the errors of translational axes were compensated during the test by measuring length change of double ball bar in the three measure model. Besides, the five PDGEs of C-axis were optimized by the truncate Fourier of coefficient matrix to ensure the stability of identified results. Furthermore, verification and comparisons were carried out through simulations and experiments. Comparing the result, it is obvious that the proposed measurement method is effective and feasible when the position errors are compensated.

In addition to the above, the following areas need to be studied in the future.

(1) In order to further verify the advancement of this method, error identification tests using the fourth and more higher orders are conducted.

(2) In this paper, the position error modeling and identification method for geometric errors of C-axis have been studied without considering thermal error and the cutting force of the machine tool. Thus, more error source factors should be considered into error identification in the future.

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Authors' Contributions

JWF assisted in modifying the structure and content of this paper. PTW put forward the framework and content of the paper and wrote the manuscript; All authors read and approved the final manuscript.

Competing Interests

The authors declare that they have no competing interests.

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Figures

Figure 1

Geometric error of the C-axis

Figure 2

position-independent geometric errors of C-axis

Figure 3

Topological structure of machine tool

Figure 4

Schematic diagram of Machine tool

Figure 5

C-X measurement mode error generation mechanism

Figure 6

C-X-1mode

Figure 7

C-X-2 mode

Figure 8

Geometric erros of C-axis identification

(a)

(b)

(c)

(d)

(e)

Figure 9

Identification the PDGs of C-axis

Figure 10

Position error at 90° angle of axis C

Figure 11

Position error at 180° angle of axis C