Dominating Competition Graph and its Application in Social Network

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Abstract: Domination and competition have been there since the beginning of the Universe. Bigger stars dominate the smaller ones and draw them towards themselves. Domination and competition are such phenomenon which nothing in this universe can stay without. Stronger things and animals dominate the weaker ones. Again, when there are more than one strong dominant, there comes competition among them. To measure domination and competition together, we introduce here a domination competition graph, through which we identify the most competitive and dominating variables. By dominating competition number, we calculate which nodes (when we draw a graph corresponding perception) are more powerful and how much powerful. We have used forest biodiversity network as an example here. In the end, an application of domination competitions to e-commerce industry is illustrated.

Keywords: Domination, Competition, Graph, Degree centrality, out degree, In degree.

1. Introduction

Domination means control over somebody or something or the state in which they are so controlled. The weakest are constantly dominated by the stronger people. It is there from the very beginning of the world's creation. There is dominion, not just for people, but also for other animals. The graph theory research began in the middle of the 20th century. The term ‘domination’ and ‘domination number’ were first introduced in 1962 by O. Ore. Now, graph theory has been established in a different approach at the beginning of the 21st century.

Competition happens when two or more parties seek to achieve a common goal that cannot be torn apart. The losses of the other are where one's profit is. ‘Competition’ is competition between bodies, persons, economic and social organisations and so forth. Competition takes place in nature among living species that coexist in the same habitat. All animals compete for water, food, mates and other biological resources. People usually compete for food and other resources. More serious conflicts form in the search for riches when these requirements are fulfilled. Men compete for strength, status and fame in an environment that is static or repetitious. Competition in a graph theory was first introduced by Cohen in 1968. Later in 1995, Kim et. Al. introduced a p-competition graph of a digraph, and now Competition is not only limited to simple graph theory, but it has been developed in hyper graph, fuzzy graph theory, game theory etc.

During the last six decades, many mathematicians have worked in the fields of domination and competition in graph theory separately. But no one has so far worked ona paper focusing on both domination and competition as a combined measure. In this paper, we introduce a new idea “dominating competition graph”. From this, we try to give a clear concept on the domination-competition related subject by graph theory.

Authors	**Contributions**
o.ore, 1962 [6]	First introduced domination set and domination number
Cohen, 1968 [5]	First introduced competition digraph
Cockayne, Dawes, Hedetniemi, 1980 [18]	Total domination
Kim et al., 1995[4]	p-competition graph of a digraph
Haynes et al., 1998 [24]	Power domination set
Sonnatag and Teichert, 2004[25]	Competition hypergraph
Samanta and Pal, 2013 [16]	Fuzzy k-Competition graphs and p-Competition graphs
Chellali et al., 2016 [21]	Roman {2}-domination
Pramanik et al., 2017 [10]	Fuzzy ∅-tolerance competition graphs
Maity et al.,2021	Domination based on degree centrality
Ibrahim et al., (2022) [27]	Upper whole domination in a graph
Maity et al., this paper	**Dominating Competition Graph and its Application in Social Network**

Table 1: Author’s contribution

In the world's nature, some cases exist where domination and competition are seen together, for example, in bio-diversity, business purpose, etc., where domination and competition are related.

For example, we take some particular variables related to forest biodiversity. Here, in the following figure-1, Cheetah, Lion, Tiger, Hyena, Deer, Zebra, Buffalo are denoted by node 1,2,3,4,5,6,7 respectively and relation with them are shown in that figure.

For this case,

$$\deg^+\big(n(1)\big) = 1, \qquad \deg^+\big(n(2)\big) = 3, \qquad \deg^+\big(n(3)\big) = 2, \qquad \deg^+\big(n(4)\big) = 2$$

[where node i is denoted by $n(i)$]

So, from the dominating competition graph, we can say that Lion ($n(2)$) dominants all the animals and then Tiger ($n(3)$), Hyena ($n(4)$), Cheetah ($n(1)$) are powerful animal respectively.

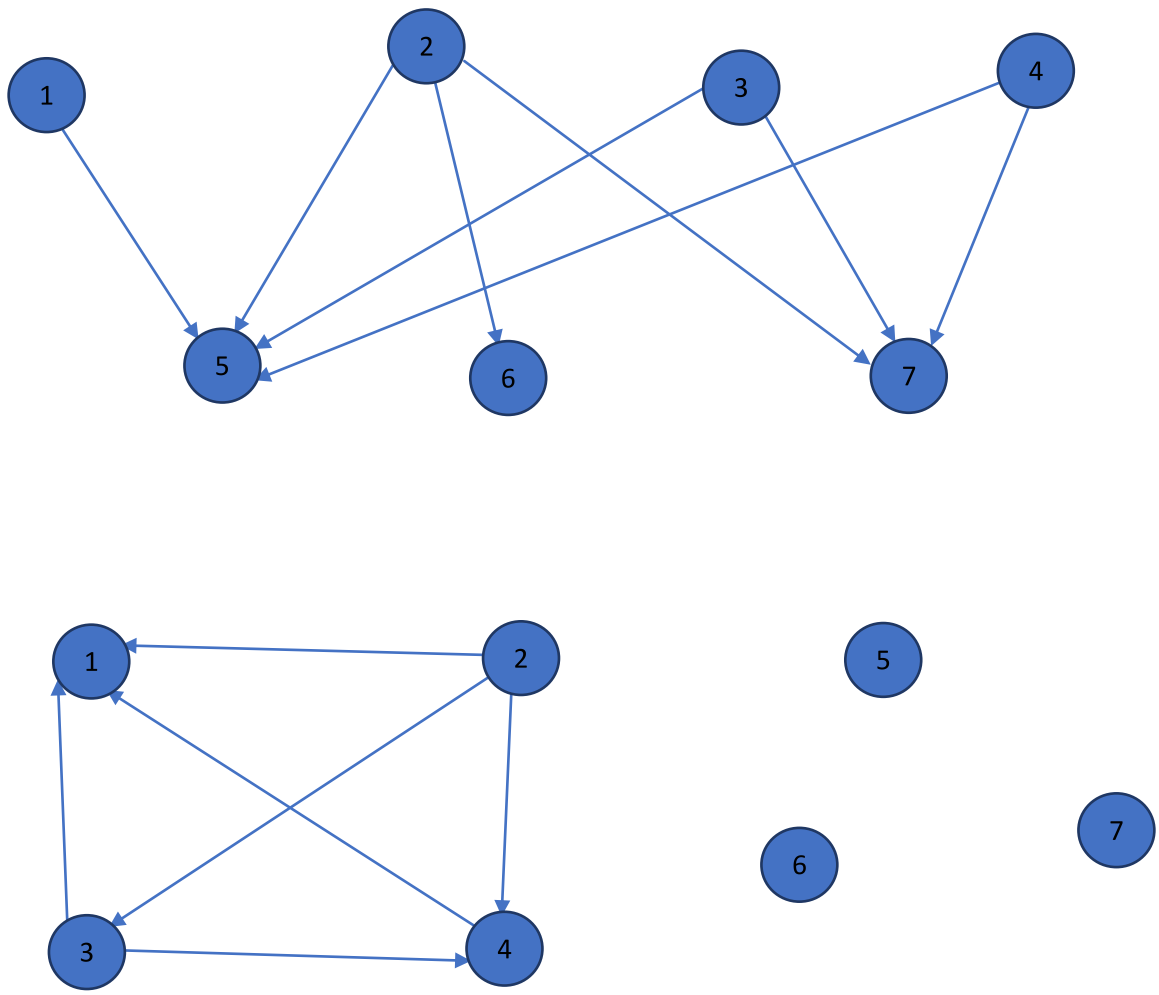

Figure 1: A directed graph and it’s dominating competition graph

2. Preliminaries:

Graph: A linear graph, or graph $G = (V, E)$, is made up of a set $V = \{v_1, v_2, v_3, \ldots\}$, vertices, points, or nodes of G and a set $E = \{e_1, e_2, e_3, \ldots\}$, edges, which are pairs of vertices that are not connected in any particular order.

Digraph: In graph theory, a directed graph (or digraph) is one type of graph in which the nodes are connected by directed lines, or "arcs."

Vertex's degree: The number of edges incident on a vertex v in a graph, is called the degree of the vertex v.

The indegree and outdegree of a vertex are the number of adjacent heads ends and tails ends, respectively, of that vertex.

Dominating set of a graph: A subset D of a vertex set V of a graph $G = (V, E)$, is called dominating set for G if $\forall\, x \notin D$ is adjacent to at least one $y \in D$ $(\forall\, x, y \in V)$.

Competition of a graph: Let $G = (V, E)$ be a digraph. A graph $G' = (V, E')$ is said to be a competition graph if $(\overrightarrow{x, y}), (\overrightarrow{z, y}) \in E$ implies $(x, z) \in E'$.

3. Dominating competition graph

3.1. Definition: Let $\vec{H} = (V, \vec{E})$ be a digraph. The dominating competition graph $\vec{H}'$ of a digraph $\vec{H}$ is also a digraph $\vec{H}' = (V, \vec{E}')$ which has same vertex set V and has a directed edge $(\overrightarrow{x, y})$ between two distinct vertices $x, y \in V$ if

(i) $\exists$ a vertex $s \in V$ and arcs $(\overrightarrow{x, s}), (\overrightarrow{y, s}) \in \vec{E}$

(ii) Vertex x is dominating to vertex y

(iii) $\deg^+(x) \geq \deg^+(y)$

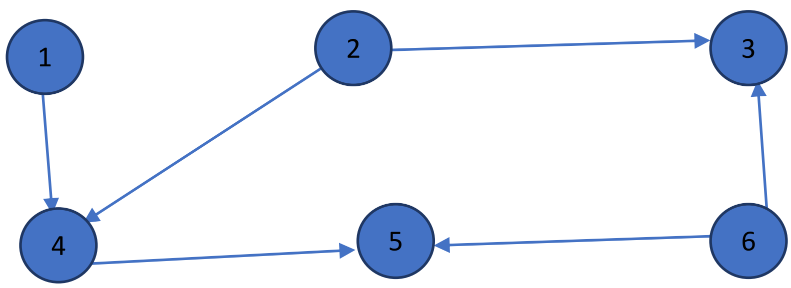

Figure 2: A digraph

From the above figure-2,

Here, {(1,4), (2,4)}, {(4,5), (6,5)}, {(2,3), (6,3)} exists and $\deg^+ n(1)$=1,$\deg^+ n(2) =$ 2,$\deg^+ n(4)$=1,$\deg^+ n(6)$=2

The dominating competition graph of the above digraph (Figure-1) is given bellow-

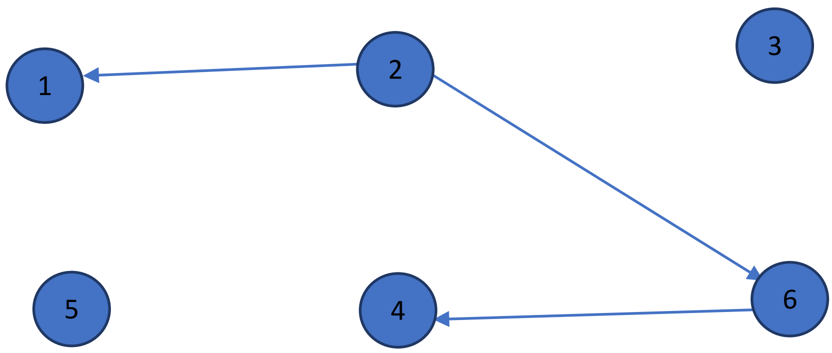

Figure 3: Dominating competition graph of figure-2

3.2.Algorithm:

Title: To find Domination competition graph from a digraph

Input: A digraph

Output: Domination competition graph

Step-1: At first, from the given graph, we consider it two set of vertices Let $\{V_i, i = 1,2,3, \dots, k\}$ are tail vertices and $\{V_j, j = 1,2,3, \dots, n-k\}$ are 'head' vertices.

Step-2: If any two vertices of 'tail' (let $V_i \& V_k$) are adjacent to same vertex (let V_j)(or vertices) of 'head', then between these two tail vertices, there exist an edge (V_i, V_k), where $degree^+(V_i) \geq degree^+(V_k)$ and so on.

Step-3: If any two vertices of 'tail' are not adjacent to same vertex (or vertices) of 'head', then between these two 'tail' vertices, there exist no edge.

Step-4: At last, all vertices of the given graph and forming edges construct a graph, called dominating competition graph.

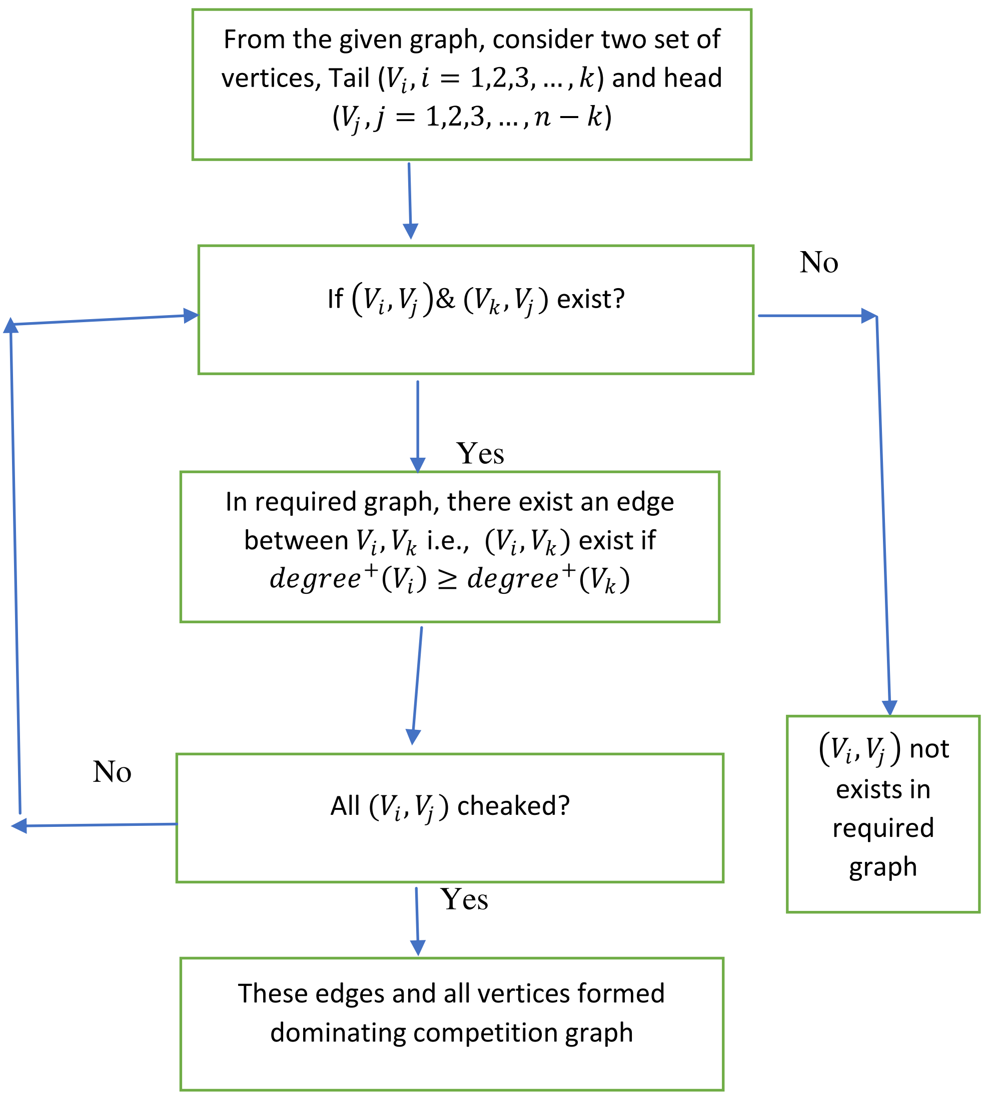

Figure 4: Flowchart on dominating competition graph

3.3. Remark: A dominating competition graph of any acyclic digraph has at least one isolated vertex.

3.4. Theorem: Every dominating competition graph is an acyclic digraph.

Proof: Let $\vec{G} = (V, \vec{E})$ be a digraph and whose dominating competition graph is $\overrightarrow{G'} = (V, \overrightarrow{E'})$.

If possible, let us assume that $\overrightarrow{G'}$ is a cyclic digraph and let $x, y, z \in V$, forms a cycle in $\overrightarrow{G'}$.

Then from the definition of domination competition graph, we can say that x is dominated y, y is dominated z and z is dominated x.

From the above, we can conclude that x is more centrality than y, y is more centrality than z and z is more centrality than x -it is not possible.

So, our assumption is wrong.

∴ Every dominating competition graph is an acyclic digraph.

3.5. Remarks: A dominating competition graph of a acyclic digraph contains the number of edge is equal to the successive distinct direction changes.

3.6. Remarks: Prime’s and Kruskal’s algorithm fail for dominating competition graph.

Prim’s algorithm is applicable when all vertices are connected. But in dominating competition graph, every node in this graph can’t be reached from any other node.

For Figure-1, no node is reachable from node 1. So, prime algorithm is not applicable.

Kruskal’s algorithm checked that if the produced spanning tree contains any cycles formed by the edges. But Kruskal’s algorithm can’t find cycles in a dominating competition graph because it incorrectly assumes that there is a cycle between the vertices and so ignores some edges.

For figure-1, dominating competition graph does not contain cycle. So, Kruskal’s algorithm fail.

3.7. Theorem:

Dominating competition graph of a complete digraph is a complete digraph.

Proof: If every pair of different points is connected by two unique edges, the graph is known as a Complete digraph, so, any one vertex is connected to other vertices.

For a digraph of n vertices, competition holds between any 2 or 3 or,…, or $n - 1$ vertices.

So, dominating competition graph of n vertices complete graph, there exists an edge between any two vertices. So, this forms a complete graph.

4. Domination based p-competition digraph:

4.1. Definition:

Let $\vec{H} = (V, \vec{E})$ be a digraph. The p-dominating competition graph$\vec{H}'$ of a digraph $\vec{H}$ is a dominating competition digraph$\vec{H}' = (V, \vec{E}')$ if there exist vertices s_i and arcs $(\overrightarrow{x, s_i}), (\overrightarrow{y, s_i}) \in E, \quad i \geq p, p \in \mathbb{Z}^+$

4.2. Theorem: For any p-dominating competition graph, $p \leq n - 2$, where $n$represents the total number of vertices.

Proof: Let $\vec{H} = (V, \vec{E})$ be an acyclic digraph of n vertices. Then$p$-dominating competition graph $\vec{H}'$ of a digraph $\vec{H}$ is a dominating competition digraph$\vec{H}' = (V, \vec{E}')$ if there exist vertices s_i and arcs $(\overrightarrow{x, s_i}), (\overrightarrow{y, s_i}) \in E, \quad i \geq p,$ p is a positive integer.

If possible, let us assume that, $p = n - 1$.

So, there exist vertices s_i and arcs $(\overrightarrow{x, s_i}), (\overrightarrow{y, s_i}) \in E,$ the least value of i is $n - 1$.

So, the total vertices of this graph are $n - 1 + 1 + 1 = n + 1$ (x and y are two another vertices)

So, here arise a contradiction.

So, it seems that we made a false assumption.

$\therefore$ For any p-dominating competition graph, $p \leq n - 2$, where n represents the total number of vertices.

4.3. Theorem: For p-dominating competition graph of a complete digraph with n vertices, $p = n - 2$

Proof: If every pair of different points is connected by two unique edges, the graph is known as a Complete digraph, so, for a n vertices complete digraph, any two vertices are connected to the most number of same vertices.

So, from the definition of p-dominating competition digraph of n vertices complete digraph, $p = n - 2$.

Example:

For, a complete digraph of 6 vertices,

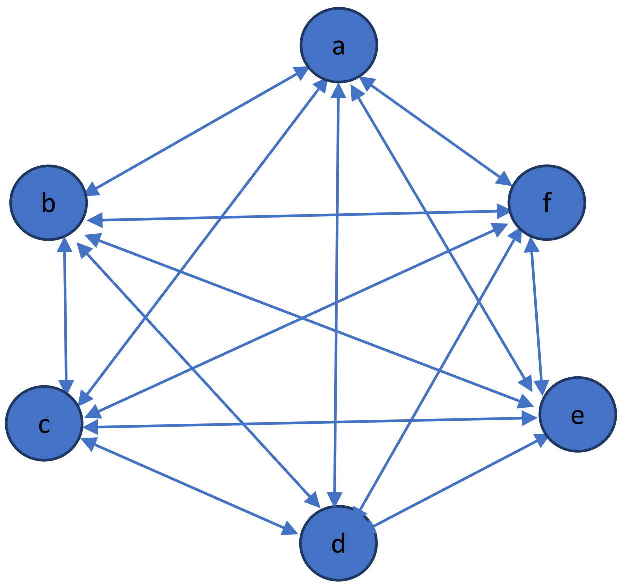

Figure 5: A Complete digraph

For this digraph, the p-dominating competition graph is-

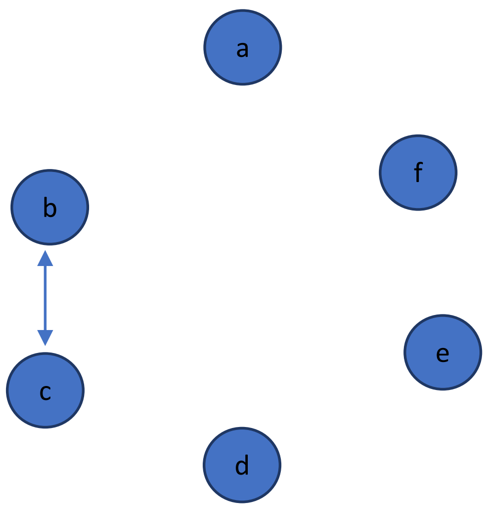

Figure6:p-dominating competition digraph of figure-5

For this digraph, $p = 6 - 2 = 4$

4.4. Remark: A p-dominating competition graph contains at least p isolated vertex.

5. Application:

The data set for this paper is collected from e-commerce websites and google. We choose 7 e-commerce platforms and taking data in 2022 on 29 July. We have selected 14 random segments, which are usually seen. Using the data of those 14 segments, a network is constructed in such a way that there should have a directed edge from e-commerce to segments if this segment is contained in this e-commerce platform. Then we tried to identify the dominating competition graph in the e-commerce platform and which is more superior and which dominates other employing our hypothesis. We identified the dominators and dominated from their competition in the e-commerce platform

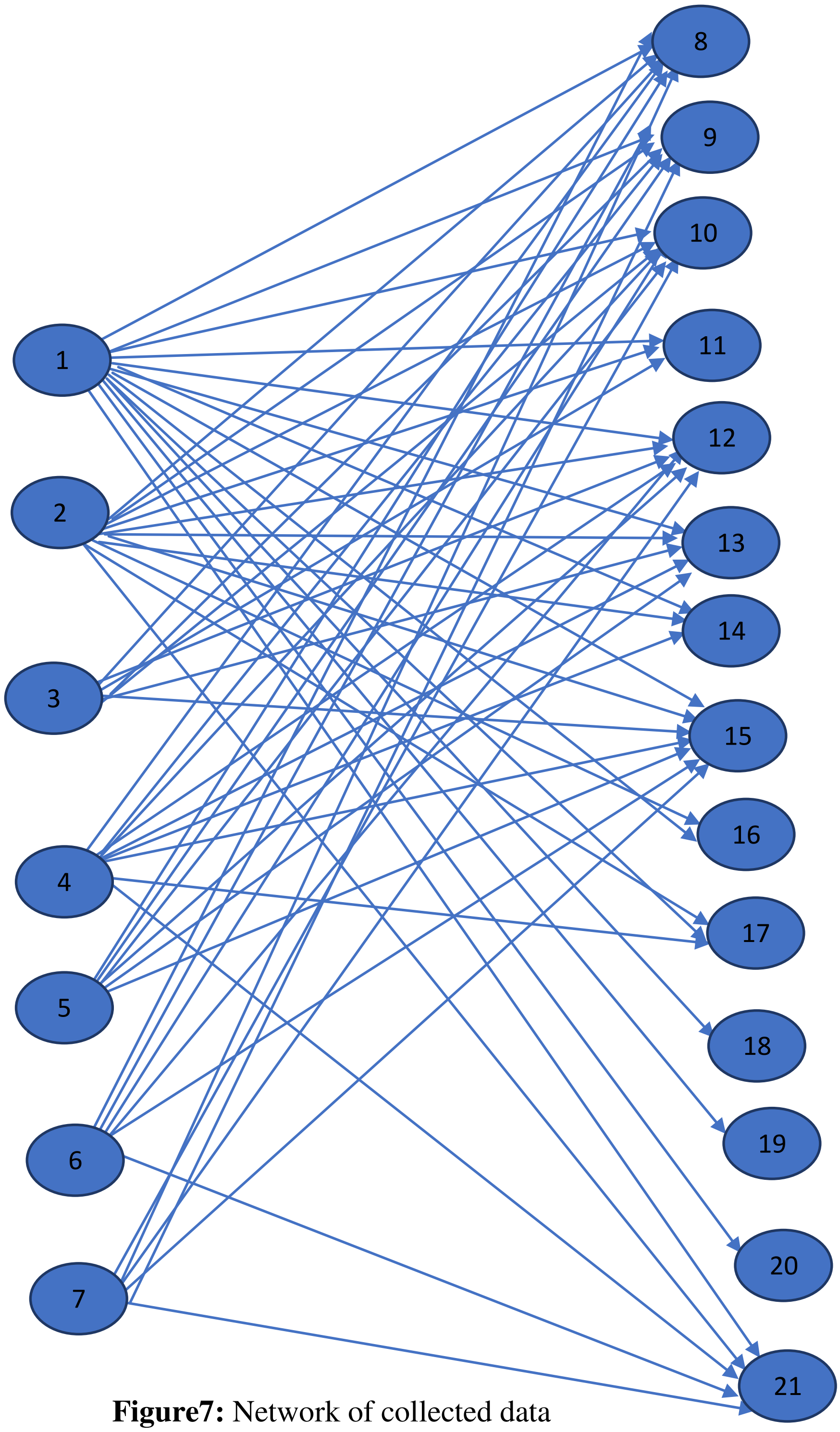

Figure7: Network of collected data

Nodes	Names
1	Amazon
2	Flipkart
3	Meesho
4	Snapdeals
5	Tatacliq
6	Myntra
7	Ajio
8	Apparel
9	Footwear & accessories
10	Beauty & health
11	Groceries
12	Jewellery
13	Electronics & gadgets
14	Computer accessories
15	Home & kitchen appliances
16	Furnitures
17	Books & stationaries
18	Recharges & bill payments
19	Media & entertainment
20	Payment wallet
21	Own brand products

Table 2: List of assumed nodes for the network

	Ecommerce platform							
Segment		Amazon	Flipkart	Meesho	snapdeals	Tatacliq	Myntra	Ajio
	Apparel	1	1	1	1	1	1	1
	Footwear & accessories	1	1	1	1	1	1	1
	Beauty & health	1	1	1	1	1	1	1
	Groceries	1	1	1	0	0	0	0
	Jewellery	1	1	1	1	1	1	1
	Electronics & gadgets	1	1	1	1	1	0	0
	Computer accessories	1	1	0	1	0	0	0
	Home & kitchen appliances	1	1	1	1	1	1	1
	Furnitures	1	1	0	0	0	0	0
	Books & stationaries	1	1	0	1	0	0	0
	Recharges & bill Payment	1	0	0	0	0	0	0
	Media & entertainment	1	0	0	0	0	0	0
	Payment wallet	1	0	0	0	0	0	0
	Own brand product	1	1	0	1	0	1	1

Table 3: Corresponding matrix of figure

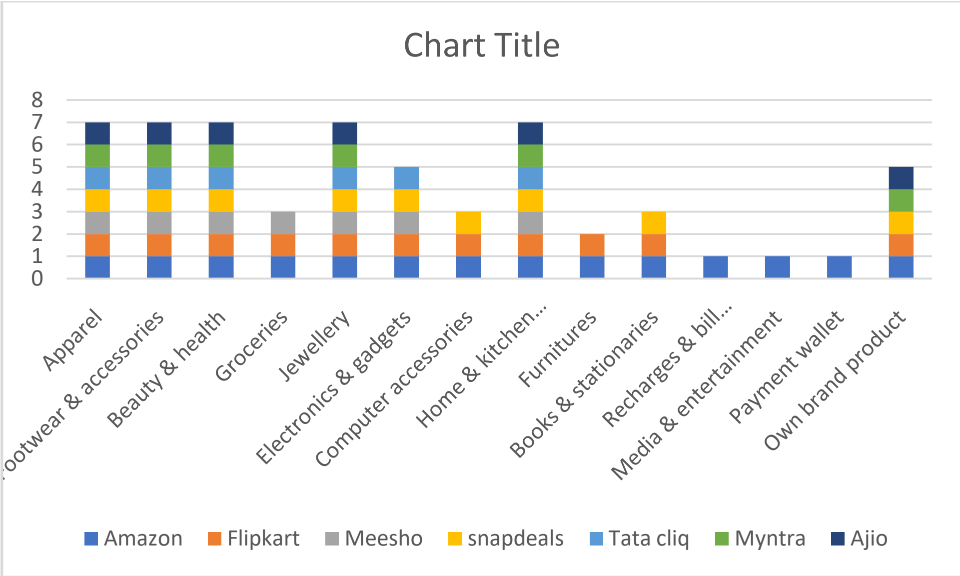

Figure8: Graphical representation of table-3

5.1. Result:

For the above graph (Figure-), tail vertices are {1,2,3,4,5,6,7} and head vertices are {8,9,10,11,12,13,14,15,16,17,18,19,20,21}.

Here {(1,8), (2,8), (3,8), …,(7,8)}, {(1,9), (2,9),(3,9), …,(7,9)}, {(1,10), (2,10), (3,10), … ,(7,10)}, {(1,11), (2,11), (3,11)}, {(1,12), (2,12), (3,12), …, (7,12)}, {(1,13), (2,13), …, (5,13)}, {(1,14), (2,14), (4,14)}, {(1,15), (2,15), …, (7,15)}, {(1,16), (2,16)}, {(1,17), (2,17), (4,17)}, {(1,21), (2,21), (4,21), (6,21), (7,21)}exists.

And $degree^{+}\big(n(1)\big) = 14$,$degree^{+}\big(n(2)\big) = 11$, $degree^{+}\big(n(3)\big) = 7$, $degree^{+}\big(n(4)\big) = 9$, $degree^{+}\big(n(5)\big) = 6$, $degree^{+}\big(n(6)\big) = 6$, $degree^{+}\big(n(7)\big) = 6$

∴The domination competition graph of the above digraph (Figure-7) is given bellow

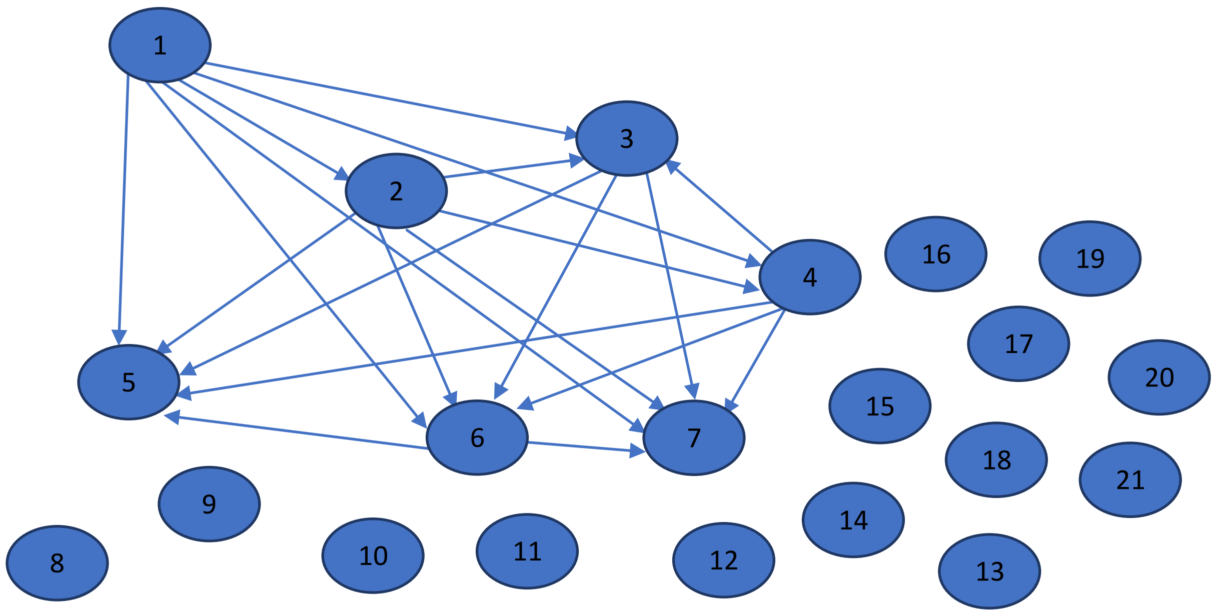

Figure8: Dominating competition graph of the figure-7

	Amazon	Flipkart	Meesho	snapdeals	Tata cliq	Myntra	Ajio
Amazon	**0**	**1**	**1**	**1**	**1**	**1**	**1**
Flipkart	**1**	**0**	**1**	**1**	**1**	**1**	**1**
Meesho	**1**	**1**	**0**	**1**	**1**	**1**	**1**
snapdeals	**1**	**1**	**1**	**0**	**1**	**1**	**1**
Tata cliq	**1**	**1**	**1**	**1**	**0**	**1**	**0**
Myntra	**1**	**1**	**1**	**1**	**1**	**0**	**1**
Ajio	**1**	**1**	**1**	**1**	**0**	**1**	**0**

Table 4: Corresponding matrix of figure-8

5.2.Observations:

1. Here domination and competition hold only among 7 e-commerce platform, in which Amazon is more dominator than the others.
2. This competition graph is a 2- dominating competition graph.

6. Conclusion:

Domination is omnipresent and where there is domination, there is competition. And a competitive environment helps us identify the superior players in any field. The dominating ones emerge as winners in the long run. The hypothesis of dominating competition graph can be applied to identify the dominators and the dominated in any competitive domain of life.

The hypothesis of dominating competition graph has some obvious limitations too. This method is difficult to be employed when the size of data and the quantity of nodes is very big. In those cases, it is very painstaking to detect exactly how much competition is there between which nodes and which ones are most dominating among those competitive nodes. In addition to that, it may not always be possible to accurately conclude which nodes are the most dominating. It can only give a random idea of the competitive entities and the majorly dominating ones putting us in the right path forward.

Talking about the future prospects of the hypothesis, the concept of dominating competition can be tried and tested upon various type of digraph. The idea of this dominating competition hypothesis can be used in business fields, professions, academics, global politics and so on.

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