Time-varying Formation Dynamics Modeling and Constrained Trajectory Optimization of Multi-quadrotor UAVs

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Time-varying formation dynamics modeling and constrained trajectory optimization of multi-quadrotor UAVs

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Abstract: The formation of multiple quadrotor UAVs with long navigation will encounter many flight constraints, so it is necessary to change the formation to avoid these constraints. In this paper, Voronoi graph theory is used to treat UAVs formation as a rigid body. The rigid body structure can be changed by changing the formation before different constraints, after passing constraints the rigid body structure remains the formation unchanged or form into the prescribed formation. The time-varying model is established to facilitate the use of optimization. Based on Gauss pseudospectral method (GPM), the path optimization of a single quadrotor UAV is carried out. The followed UAVs in formation are treated as constraints. The constraints of maximum turning radius and formation transformation time of other UAVs are considered in the optimization process. The minimum time required for formation transformation is optimized to solve the transformation optimization, and the performance index of trajectory optimization is to minimize the energy consumed by the leader quadrotor UAV within the specified time. Finally, the simulation proves the method proposed in this paper.

Key Words: time-varying, multiple quadrotor UAVs, constraints, optimization

1 Introduction

The cooperative path planning of multi-UAVs is the crucial technology of UAVs cooperative combat mission planning system [1-5]. The constraints of multi-UAVs collaborative path planning not only consider the physical performance and the mission requirements of a single UAV, but also the cooperative relationship among all UAVs is taken into account, which includes achieving the

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target constraint at the same time and the minimum safe distance constraint between multiple UAVs [6-8]. When the environment changes, the formation of multi-UAVs should be adjusted in real-time according to the environment to ensure the survival of UAVs [9, 10].

In the past, the research on the path planning of multiple UAVs mainly have focused on the assembling of multiple UAVs and the respective path planning of multiple UAVs, while the path planning of multiple UAVs maintaining formation has been studied less. Turpin et al. [11] considered the problem of controlling a group of Mavs to move rapidly in a 3D environment while maintaining a tight formation. The flight trajectory of a certain UAV was parameterized by using the coefficients of sequence polynomial function, and the flight trajectory is fully described according to these values. At the same time, in literature [12], each UAV independently plans its trajectory based on the local information of its neighboring robots, including the planned trajectory of its neighbors and the estimation of its state. In literature [13], aiming at multiple UAVs arrived at the target position from different positions, Shanmugavel et al. proposed a three-stage collaborative path planning method using Dubins paths with clothoid arcs when the constraints were met. Nikolos et al. [14] used an improved genetic algorithm to realize the 2-D trajectories planning of multiple UAVs. Besada-Portas et al. [15] adopted a centralized path planning algorithm based on the genetic algorithm to plan the path for multiple UAVs. Radmanesh et al. [16] researched the collision-free 3D trajectories for multi-UAVs in shared airspace, and proposed a decentralized method based on a Partial Differential Equation, in which, the dynamic characteristics of multiphase fluids in porous media are utilized, and the porosity is modeled as a function of collision risk. Chen et al. [17] solved the formation control path planning problem of multiple quadrotor UAVs by introducing virtual rate rigid body and virtual target point and updating the artificial potential field method based on additional control force.

At present, Gauss pseudospectral method (GPM) has many applications in trajectory optimization. To optimize the takeoff-mass for a two-stage-to-orbit RBCC-RKT launch vehicle, which is a crucial factor of integral performance, Zhang et al. [18] combined it with the trajectory optimization simultaneously by treating them as a nonlinear control problem, when the scopes of initial and terminal qualities as well as speeds are specified, the problem is solved by using the Gauss pseudospectral method. In literature [19], in order to optimize the trajectory of a general supersonic vehicle driven by a hydrocarbon fueled ramjet in the ascent phase, an optimization method based on GPM was proposed based on the characteristics of the propulsion system. In literature [20], in order

to solve the problem of autonomous landing trajectory optimization of mobile platform, with the minimum of time and energy consumption as the optimal goal, the GPM method was used to optimize the landing trajectory of a tilt rotor. However, there have been few literatures studied on the path optimization of variable formation of the quadrotor UAVs formation under the constraint of three-dimensional space.

To the best of our knowledge, there is no report about the conversion of the information of followed quadrotor UAVs(QUAVs) in formation into the constraints of the leader QUAV. Research on the information transformation of followed QUAVs in QUAVs formation into the constraints of leader QUAV can make the formation trajectory optimization problem stand-alone, simplify the complex multi-QUAVs formation trajectory optimization problem, and ensure the maintenance of multi-QUAVs formation and the timeliness of cooperation, and can optimize multiple performance indexes at the same time.

In this paper, we propose the path optimization problem of QUAVs formation with obstacles in three-dimensional space through kinematics analysis and numerical simulation. The QUAVs formation is regarded as a virtual rigid body, and the rigid body structure can be changed by changing the formation before different constraints. After pass one constraint, the QUAVs formation remains unchanged or changes to the specified formation. And the optimization model is established in this paper, and the optimization improvement of the end heading constraint and height constraint is proposed. The optimization model is transformed into a sequential quadratic programming problem by Gauss pseudospectral method.

The rest of this article is organized as follows. Section 2 introduces the model of a QUAV attitude system, and the dynamic model of multiple QUAVs is established to analyze the constraints and formation maintenance during its flight. Finally, the problems summary faced in the flight trajectory optimization model for QUAVs formation is given. In Section 3, principle of Gauss pseudospectral method is introduced and used to solve the problems in Section 2. In Section 4, simulation examples are given. Section 5 draws some conclusions.

2 Dynamics model of multiple QUAV

2.1 Dynamics model of a QUAV

87 A QUAUV is a kind of aircraft with four rotors, having two types of X-type distribution and cross-
 88 type distribution [21, 22]. In this paper, the X-type is used, and the schematic diagram of the QUAUV
 89 is shown in Fig. 1 [23, 24].

Fig. 1 Schematic view of the QUAUV system.

92 The quadrotor UAV can achieve various motions by changing the rotor speed. In the figure,
 93 $\{E\} = \{x_e, y_e, z_e\}$ is the north-east-down inertial coordinate system (NED). $\{B\} = \{x_b, y_b, z_b\}$ is the
 94 body coordinate system, the origin of which coincides with the mass center of the body, and the
 95 coordinate axis is fixedly connected with the QUAUV. The x-axis is the forward direction of the UAV.
 96 $F_i(t), i = 1, 2, 3, 4$ is the thrust produced by the four motors, respectively. $\{x(t), y(t), z(t)\}$ is the
 97 position of the QUAUV in the local NED frame $\{E\}$, and $\{\phi(t), \theta(t), \psi(t)\}$ is the Euler angle of the
 98 QUAUV.

99 To establish a relatively accurate mathematical model of QUAUV, the following assumptions are
 100 made:

- 101 (1) The small quadrotor UAV is regarded as a rigid body and is completely symmetrical;
- 102 (2) The origin of body coordinate O_b is completely coincident with the mass center of UAV;
- 103 (3) The propeller of UAV is a rigid-body without considering its structure and elastic
 104 deformation.

105 The nonlinear dynamic model of the QUAUV in $\{E\}$ is given as follows [25, 26]:

106

$$\begin{cases} \ddot{x} = -(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \frac{U_1}{m} + \frac{K_1 \dot{x}}{m}, \\ \ddot{y} = -(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) \frac{U_1}{m} + \frac{K_2 \dot{y}}{m}, \\ \ddot{z} = -(\cos \theta \cos \phi) \frac{U_1}{m} + g + \frac{K_3 \dot{z}}{m}, \\ \ddot{\phi} = \frac{1}{I_x} [lU_2 + \dot{\theta}\dot{\psi}(I_y - I_z) - K_4 \dot{\phi}], \\ \ddot{\theta} = \frac{1}{I_y} [lU_3 + \dot{\phi}\dot{\psi}(I_z - I_x) - K_5 \dot{\theta}], \\ \ddot{\psi} = \frac{1}{I_z} [U_4 + \dot{\theta}\dot{\phi}(I_x - I_y) - K_6 \dot{\psi}], \end{cases} \quad (1)$$

107 where, x, y, z are the positions of the QUAV; I_x, I_y, I_z are the rotational inertias of the QUAV
 108 respectively; because the mechanical structure of the QUAV is completely uniform, so $I_x = I_y$;
 109 ϕ, θ, ψ are the roll angle, pitch angle and yaw angle of the QUAV respectively; in the inertial frame
 110 of reference, $\dot{\phi}$ is roll angle velocity, $\dot{\theta}$ is pitching angle velocity, and $\dot{\psi}$ is yaw angle velocity; l
 111 is the length between the center of the aircraft and the rotor; U_1 is the vertical motion control
 112 quantity, U_2 is the rolling motion control quantity, U_3 is the pitch motion control quantity, U_4 is
 113 the yaw motion control quantity. K_1, K_2, K_3 are the coefficient of air resistance, K_4, K_5, K_6
 114 represent the coefficient of air resistance moment. In addition, $[v_x, v_y, v_z]$ and $[w_\phi, w_\theta, w_\psi]$ are
 115 defined as linear velocity components and angular velocity components around x, y, z coordinate
 116 in the NED coordinate system, where v_x is the forward velocity, v_y is the lateral velocity, v_z is
 117 the falling velocity, w_ϕ is the roll angular velocity, w_θ is the pitch angular velocity, and w_ψ is
 118 the yaw angle angular velocity.

119 The expression of the control quantity is expressed as follows: the control quantity directly
 120 controls the motor speed, and the actual implementation form is PWM wave to control the motor.

121

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} K_T(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ \frac{\sqrt{2}}{2} K_T(\Omega_1^2 - \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \\ \frac{\sqrt{2}}{2} K_T(\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2) \\ K_M(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{bmatrix}, \quad (2)$$

where, Ω_i ($i = 1, 2, 3, 4$) represents the rotation speed of each propeller, K_T represents the propeller lift coefficient, K_M represents the inverse torque coefficient of the propeller.

2.2 Description of flight problems of multiple QUAVs

The formation of QUAVs is shown in Fig. 2.

Fig. 2 Formation of QUAVs

The formation of QUAVs follow the leader (UAV1) which tracks the desired trajectory $x_1(t)$.

Each UAV is controlled to maintain a desired shape defined by shape vectors and denoted by $s_{i,j}$. In this paper, the formation is composed of N QUAVs. Due to turning or other situations when flying in formation, if multiple QUAVs keep the formation unchanged, their speeds may be inconsistent. Therefore, the following assumptions are defined in this paper:

(1) The speed of the leader UAV remains unchanged during flight. When it is not turning, all UAVs move at a constant speed. When it is flying in formation, the speed of each UAV is not consistent. It doesn't take into account the energy consumption of the turn.

(2) The UAVs can communicate reliably to prevent collisions among them.

(3) The UAVs formation can be changed according to the environmental constraints. After passing the obstacle, the formation recovers and continues to fly at the same speed and direction, or continues to fly with the formation unchanged.

(4) The formation of QUAVs is regarded as a virtual rigid body. Except for formation transformation, the formation of UAVs is regarded as a rigid body.

Virtual structure method: The basic idea of the virtual structure method is to regard the UAV formation system as a virtual structure of a rigid-body. Each UAV is considered as a fixed point on the virtual structure. When the UAVs' formation moves, the UAV can follow the corresponding fixed

145 point on the rigid body to achieve fixed formation flight.

146 In this paper, the consistency theory based on Voronoi graph theory [27-29] is used to define the
 147 formation keeping problem among the QUAVs. The formation system is transformed into a network
 148 topology. The state information of UAVs is consistent with the expected information through the
 149 consistency algorithm to realize the autonomous formation movement control of multiple UAVs.

150 According to the information communication relationship among the formation formed by N
 151 QUAVs, the flight formation is modeled as a directed graph G . The directed graph node v_i
 152 represents the i -th QUAV. The direction of information flow is represented by the direction of arrows
 153 in the directed graph G . The node v_1 represents the leader UAV, the remaining nodes represent
 154 the followed UAVs $v_i(t) = [x_i(t), y_i(t), z_i(t), \psi_i(t), \theta_i(t), \phi_i(t)]^T$. In the leader-followed formation
 155 mode, the leader position is the equilibrium point of the formation consistency algorithm. The
 156 expected position deviation between the leader UAV and followed UAVs is defined as the relative
 157 position deviation. The leader UAV sends its state to the followed UAVs. The topology of information
 158 among the followed UAVs can be any directed graph. The shape vectors are 6×1 vectors relating
 159 positions of pairs of UAVs. For UAV i and j :

$$160 \quad v_{i,j}(t) = v_j(t) - v_i(t) = \begin{bmatrix} x_j(t) - x_i(t) \\ y_j(t) - y_i(t) \\ z_j(t) - z_i(t) \\ \psi_j(t) - \psi_i(t) \\ \theta_j(t) - \theta_i(t) \\ \phi_j(t) - \phi_i(t) \end{bmatrix}. \quad (3)$$

161 These will be used to describe the desired relative configuration for each pair of UAVs in Fig. 2.
 162 In the rest of the paper, we assume UAV1 is specified as the leader for the team and the group motion,
 163 $g(t) = x_1$. The overall shape can be prescribed by a $(N-1) \times (N-1)$ vector-valued, skew-
 164 symmetric matrix [27].

165 Considering the change of the communication topology among the formation systems of the
 166 QUAVs with the time change, let $G = \{G_1, G_2, \dots, G_m\}$, $m \geq 1$ represents the possible formation set,
 167 and m represents the maximum number of possible formations.

168 **Definition 1** [30]: Time-varying formation can be represented by a set of functions

169 $h_i(t), i = 1, 2, \dots, N$, for a given formation $h_i(t)$ and a certain trajectory $c(t)$, if the formation
 170 system of UAVs satisfies in any given initial situation

171
$$\lim_{t \rightarrow \infty} (\xi_i(t) - h_i(t) - c(t)) = 0, i = 1, 2, \dots, N. \quad (4)$$

172 then it is said that the UAVs achieve formation $h_i(t)$. $c(t)$ is called the center track of formation.

173 where, $\xi_i(t) = [x_i(t), v_i(t)]^T$, here $x_i(t)$ and $v_i(t)$ respectively represent the position and velocity
 174 in the three-dimensional space of the i -th UAV.

175 When a UAV formation encounters an obstacle, it sometimes needs to change its formation to
 176 get through the obstacle. Formation transformation in the flight of UAVs formation can be regarded
 177 as reassembly of UAVs, namely formation reconstruction. At this time, the UAVs formation is
 178 regrouped from formation A into new formation B, as shown in Fig.3. In formation transformation,
 179 minimum energy consumption or time is required [31]. In this paper, the minimum time is taken as
 180 the required target of formation transformation.

181
 182 **Fig.3** Formation transformation

183 QUAVs have the advantages of hovering and maneuvering in all directions, so formation change
 184 can be made by methods of formation rotation, formation reduction or new formation.

185 (1) Rotating

186 To keep the formation rigid, the formation can be rotated at the same angle around a centerline
 187 or a center point to keep the relative position of each UAV unchanged. Supposed the position of the
 188 i -th UAV in the inertial coordinate system is (x_i, y_i, z_i) , the position of its rotation center in the

189 inertial coordinate system is (x_r, y_r, z_r) , and the rotation angles in three-dimensional space are α ,
 190 then the rotation path length of the i -th UAV is an arc, and the arc length is

$$191 \quad l_i = \alpha \left[(x_i - x_r)^2 + (y_i - y_r)^2 + (z_i - z_r)^2 \right]^{1/2}. \quad (5)$$

192 If the minimum rotation time t is required, only the minimum rotation time corresponding to
 193 the UAV with the maximum rotation length $l_{\max}$ is required, namely, let Eq.(5) be minimized.

$$194 \quad J_1 = t_{\min} = \min \left(\max \left(\frac{l_i}{v_i} \right) \right), i = 1, 2, \dots, N. \quad (6)$$

195 At this time, the rotation speed of other UAVs is

$$196 \quad v_i = \frac{l_i}{t_{\min}}, i = 1, 2, \dots, N. \quad (7)$$

197 If the UAV with the maximum rotation path is allowed to reach its maximum flight speed $v_{\max}$, it can
 198 also be obtained

$$199 \quad t_{\min} = \frac{l_{\max}}{v_{\max}}. \quad (8)$$

200 All UAVs can rotate synchronously.

201 After passing the obstacle, then the formation rotates back to the original formation at an angle
 202 of $-\alpha$ and continues to fly, or keep the formation unchanged.

203 (2) Formation shrinking

204 Formation of QUAVs draw close to the center of formation in proportion to shrink formation
 205 and pass obstacles, as shown in Fig. 4 [31].

206
207 **Fig. 4** Reduction diagram of formation

208 Formation shrinking can be regarded as a translational movement. In this paper, formation
 209 shrinking transformation is still carried out based on the requirement of minimum time. The distance
 210 of the UAV farthest from the center $f(x_f, y_f, z_f)$ and formation center (x_r, y_r, z_r) is

$$l_{\max} = \left[(x_f - x_r)^2 + (y_f - y_r)^2 + (z_f - z_r)^2 \right]^{1/2}. \quad (9)$$

If the formation shrinking time t is required to be the minimum, then only the transformation time corresponding to UAV f is required to be the minimum, namely

$$J_2 = t_{\min} = \min \left(\frac{l_f}{v_f} \right). \quad (10)$$

At this time, the rotation speed of other UAVs is

$$v_i = \frac{l_i}{t_{\min}}, i = 1, 2, \dots, N. \quad (11)$$

If the UAV is allowed to fly at its maximum speed $v_{\max}$, we can also get

$$t_{\min} = \frac{l_{\max}}{v_{\max}}. \quad (12)$$

That is, all UAVs can rotate synchronously.

After passing the obstacle, the formation recovers and continues to fly at the same speed and direction, or continues to fly with the formation unchanged.

(3) The new formation

When the UAVs meet some constraints and need to be transformed into a new formation, the UAV formation can be regarded as regrouping without obstacles. The minimum time is required as the performance index requirements. Here, if all UAVs are required to fly at a constant speed $v_{\max}$ with their maximum speed, the time t_i taken by each UAV is proportional to its distance, namely,

$$t_i = \frac{l_i}{v_{\max}}. \quad (13)$$

It can be based on the existing formation regrouping methods (such as literature [2, 6, 7, 31], etc.), taking the following as the objective function to solve the optimal value:

$$J_3 = t_{\min} = \min \{ \max t_i \}, i = 1, 2, \dots, N. \quad (14)$$

After $t_{\min}$ is obtained, the UAVs can fly at a constant speed $v_i = l_i / t_{\min}$ again to realize synchronous formation transformation.

2.3 The constraints of leader UAV

(1) Initial boundary condition constraint

The initial condition means that the UAV needs to meet the initial position and attitude constraints when at the initial point $t = 0$, namely:

$$X(t_0) = X_0, Y(t_0) = Y_0, Z(t_0) = Z_0, \theta(t_0) = \theta_0, \phi(t_0) = \phi_0, \psi(t_0) = \psi_0. \quad (15)$$

(2) Terminal condition constraint

The terminal condition refers to the terminal position and terminal attitude constraints of the state variable that need to be met when the UAV arrives at the target point at $t = t_f$, namely:

$$X(t_f) = X_f, Y(t_f) = Y_f, Z(t_f) = Z_f, \theta(t_f) = \theta_f, \phi(t_f) = \phi_f, \psi(t_f) = \psi_f. \quad (16)$$

(3) Minimum turning radius

Because the flight physics of the UAV determines the value of its turning radius, the limitation of the turning radius should be taken into account when the path planning is carried out; otherwise, the UAV cannot fly according to the set path. Besides, a larger turning radius prevents the UAVs from colliding with each other when they change formation. The minimum turning radius refers to the circular radius of the UAV when it moves in a circular motion with extreme acceleration at the horizontal plane. In general, this radius should meet the following requirements:

$$\frac{V_1^2}{R_1 \cdot g} \leq n_{1\max}, \quad (17)$$

where, V_1 is the speed of the leader UAV, R_1 is the turning radius of the leader UAV, g is the gravitational acceleration, and $n_{1\max}$ is the maximum normal acceleration of the leader UAV. We can obtain the minimum transition radius of the leader UAV flight, which shall meet the following requirements:

$$R_1 \geq \frac{V_1^2}{n_{1\max} \cdot g}. \quad (18)$$

(4) Control constraints

$$U_{i\min} \leq U_i \leq U_{i\max}, U_{i\max} > 0, U_{i\min} \leq 0, i = 1, 2, 3, 4. \quad (19)$$

(5) Flight altitude minimum constraint

In general, UAV will encounter high buildings, mountains or enemy fire range during flight. If the UAV flies too low, it will increase the probability of collision or be shot down. Therefore, the minimum height of the UAV flight should be set during path planning to ensure that all UAVs fly

261 above the safe height.

$$262 \quad z_1(t) \geq h_{\min}(t), h_{\min}(t) > 0, \quad (20)$$

263 where, $z_1(t)$ is the real-time flying altitude of the leader UAV in the formation, and $h_{\min}(t)$ is the
264 corresponding real-time minimum flying altitude requirement.

265 (6) Horizontal constraint

266 When flying in a certain height, it is also needed to avoid some obstacles in horizontal direction,
267 including mountains, islands, generally political no-fly zone, etc. It is needed to keep a safe distance
268 to the peripheral contour of the obstacle to prevent the UAV from colliding with obstacles due to
269 inertia or lack of mobility. Meanwhile, when the obstacle is a building, keeping a certain distance
270 from the obstacle can also reduce the probability of the UAV being detected by the enemy. In this
271 paper, at a certain flying altitude, the horizontal obstacles are represented by discs and the constraints
272 to be satisfied in the horizontal direction are as follows:

$$273 \quad (x_1(t) - x_j)^2 + (y_1(t) - y_j)^2 > R_j^2, j = 1, 2, \dots, m, \quad (21)$$

274 where, $(x_1(t), y_1(t))$ represents the real-time position of the leader UAV, (x_j, y_j) and R_j
275 represents the center and radius of the j no-fly zone disk respectively, and m represents the total
276 number of no-fly zones.

277 2.4 The constraints of followed UAVs

278 The leader UAV is only considered that path optimization of multi-QUAVs formation studied in
279 this paper, and the followed UAV exists as a constraint condition. Its conversion into constraints is
280 dealt with as follows:

281 (1) According to the position information of followed UAVs and leader UAV in formation, the
282 position of followed UAV in the inertial coordinate system can be expressed as follows:

$$283 \quad \begin{cases} x_i(t) = x_1(t) - x_{i1}, \\ y_i(t) = y_1(t) - y_{i1}, \\ z_i(t) = z_1(t) - z_{i1}, \end{cases} \quad (22)$$

284 where, $(x_i(t), y_i(t), z_i(t))$ is the real-time position of the i -th followed UAV,
285 $(x_1(t), y_1(t), z_1(t))$ is the real-time position of the leader UAV, and (x_{i1}, y_{i1}, z_{i1}) is the relative

position between the i -th followed and the leader UAV in the formation, as shown in Fig. 5.

Fig. 5 Relative position in formation between followed UAV and leader UAV

In this case, the followed UAV constraints can be transformed into constraints on the leader UAV.

(2) Flight altitude minimum

$$\begin{cases} z_i(t) = z_1(t) - z_{i1} \geq h_{\min}(t) \\ \Rightarrow z_1(t) \geq h_{\min}(t) + z_{i1}, \quad h_{\min}(t) > 0. \end{cases} \quad (23)$$

(3) Horizontal constraint

$$\begin{aligned} & (x_i(t) - x_j)^2 + (y_i(t) - y_j)^2 \\ &= (x_1(t) - x_{i1} - x_j)^2 + (y_1(t) - y_{i1} - y_j)^2 \\ &> R_j, \quad j = 1, 2, \dots, m. \end{aligned} \quad (24)$$

(4) Turning radius minimum

Like the leader UAV, the turning radius of the followed UAVs should also meet the requirements:

$$\frac{V_i^2}{R_i \cdot g} \leq n_{i\max}, \quad (25)$$

where, V_i is the speed of the i -th followed UAV, R_i is the turning radius of the i -th followed UAV,

g is the gravitational acceleration, and $n_{i\max}$ is the maximum normal acceleration of the i -th

followed UAV. For this purpose, we obtain a minimum flight transition radius $R_i = \frac{V_i^2}{n_{i\max} \cdot g}$ of the

i -th followed UAV.

In formation flight, the leader UAV and followed UAVs rotate around the same center, that is, fly around the same center of a circle. The speed has the following relationship:

303

$$\begin{aligned} \frac{V_i}{V_1} &= \frac{\frac{1}{2}\pi R_i^2}{\frac{1}{2}\pi R_1^2} = \frac{R_i^2}{R_1^2} \\ \Rightarrow V_i &= \frac{R_i^2}{R_1^2} V_1, \end{aligned} \quad (26)$$

304

According to Fig. 5,

305

$$R_i = \sqrt{x_i^2 + y_i^2 + z_i^2} = \sqrt{(x_1 - x_{i1})^2 + (y_1 - y_{i1})^2 + (z_1 - z_{i1})^2}, \quad (27)$$

306

We can get

307

$$\frac{V_i^2}{R_i \cdot g} = \frac{\left(\frac{R_i^2}{R_1^2} V_1\right)^2}{R_i \cdot g} = \frac{R_i^3}{R_1^4} V_1^2. \quad (28)$$

308

That is

309

$$\frac{\left(\sqrt{(x_1 - x_{i1})^2 + (y_1 - y_{i1})^2 + (z_1 - z_{i1})^2}\right)^3}{R_1^4} V_1^2 \leq n_{i\max}. \quad (29)$$

310

Thus, they can be converted to related the minimum turning radius, speed and position constraints of the leader UAV.

311

312

2.5 Improvement of the performance index function

313

(a) Energy consumption index for total flight distance

314

When formation transformation is not taken into account in formation flight, the minimum energy consumption performance index traditionally adopted is:

316

$$J_4 = \min \left(\sum_{i=1}^N \omega_i \right), \quad (30)$$

317

where, $\omega_i = \int_{t_0}^{t_f} \omega_i(t) dt$ denotes the total energy consumption of UAV_i, and $\omega_i(t)$ denotes the real-time energy consumption. For all UAV_i, since the concept of virtual rigid formation is adopted in this paper, and there is a cooperative time constraint, each UAV must reach the specified location at the same time t_f , so the speed and distance of all UAV_i are in direct proportion [31], namely

320

321

$$v_i = \frac{l_i}{t_f - t_0}. \quad (31)$$

322

If the impacts of different UAV_i on air density and windward area, as well as thermal effects, etc. are

323 ignored, the energy consumption generated by UAV_i is

324
$$\omega_i = F_i l_i, \quad (32)$$

325 where, $F_i = k_f v_i^2$ represents the air resistance during UAV_i flight, k_f is the air resistance
326 coefficient, and is a constant value. The same UAV is used in this paper, so k_f of all UAVs are of
327 the same value.

328 (b) Analysis on the influence of formation transformation

329 The formation transformation used in this paper takes the leader UAV as the central axis for
330 transformation. The leader UAV used still follows its trajectory during formation transformation. It's
331 the followed UAVs that make the formation change. Formation change or rotation can be ignored
332 relative to the total flight distance. The formation change can also be completed during the flight.
333 Therefore, the time solution of formation change can be completed online quickly, and the influence
334 on the overall flight optimization of the leader UAV can be ignored.

335 (c) The terminal demand

336 Generally speaking, if a UAV formation is required to fly in a straight line from a certain
337 horizontal direction at a certain altitude to the target point, in order to ensure the UAV's terminal flight
338 path is straight and reduce the control demand at the terminal stage, we introduce the following
339 performance index function:

340
$$J_5 = \min(Q(t)(\psi_1(t) - \psi_r)), \quad (33)$$

341
$$J_6 = \min(Q(t)(z_1(t) - h_r)), \quad (34)$$

342 where, $Q(t)$ is the weighting function. To ensure the requirements of the terminal flight, we finally
343 adopt the performance index function as follows:

344
$$J = a_1 J_4 + a_2 J_5 + a_3 J_6, \quad (35)$$

345 where, a_1, a_2, a_3 are the weighting coefficients, which can be selected according to the actual
346 requirements. In this paper, the energy consumption index of UAV takes up a significant proportion
347 and $a_1 = 0.8, a_2 = 0.1, a_3 = 0.1$.

348 The flight track diagram and control curve of the improved single QUAV are shown in Fig. 6, in
349 which different weighting functions are added, respectively [32]:

(1) $Q(t)=1$; (2) $Q(t)=t^2$; (3) $Q(t)=1/(t_f-t+0.5)$; (4) $Q(t)=1/(t_f-t+0.5)^2$.

In order to verify the influence of different weighting functions $Q(t)$, the following simulation is carried out: a single QUAUV takes off from $(0, 0, 2) m$ and lands on $(1000, 1000, 2) m$, and the end requires the UAV to fly from -90° direction to the target point and gradually land from a higher altitude. Fig. 6 is the x - y plane flight diagram, and Fig. 7 is the corresponding 3D flight diagram.

As can be seen from Fig. 6, the four weighting functions selected by us can ensure that the UAV will turn into a flat state in the latter part of flight. However, as can be seen from the 3D flight diagram in Fig. 7, when taking $Q(t)=1$, the altitude of UAV in the later period of flight will change a lot, and it cannot be guaranteed that the UAV will land from a higher position; when taking $Q(t)=t^2$ and $Q(t)=1/(t_f-t+0.5)^2$, the altitude of UAV in the later period of flight will be higher and the descent time will be longer; when taking $Q(t)=1/(t_f-t+0.5)$, the altitude of UAV in the latter period of flight will not be so high, and the variation of UAV flight is smooth. Therefore, considering the simulation results comprehensively, $Q(t)=1/(t_f-t+0.5)$ is selected as the time function.

Fig.6 x - y plane flight diagram

Fig.7 3D flight diagram

2.6 The problem summary

Based on the above analysis, we can conclude the following flight path optimization model for formation flight of QUAUVs:

For dynamic differential Eq. (1) for QUAUV, to find the control quantity to make the performance index Eq. (35) optimal, satisfying boundary value constraints Eq. (15) and Eq. (16), process constraints Eqs. (18-21), (23), (24), (29).

Now, the problem we faced are:

(1) The optimal solution is a curved form and an infinite dimensional problem.

(2) It is a dynamic equation in differential form.

(3) It is an objective function of integral form.

(4) There are constraints.

To solve these problems, we use Gauss pseudospectral method principle [33,34] to solve:

(1) The optimization problem of the curve is transformed into the optimization problem of several characteristic points on the curve: the infinite-dimensional problem is reduced to a finite dimensional-problem.

(2) The whole curve is fitted by the polynomial.

(3) Transform the differential dynamic equation into algebraic constraint by sampling function.

(4) The integral form of the objective function is transformed into an algebraic equation by using the Gauss quadrature formula.

In this way, the whole optimal control problem is put into the solving framework of nonlinear programming.

3 Principle of Gauss pseudospectral method

In Gauss pseudospectral method, control variables and state variables are dispersed first in a selected Gauss point series. Select the discrete points for the node to construct though Lagrange interpolation polynomial, and approximate state and control variables based on the polynomial, and then approximate the derivative of the state variable by taking the derivative of these global interpolation polynomials. The differential equation constraints can be converted into a set of algebraic equation constraints. For the integral term in the performance index, Gauss integral is used for discretization. The terminal state is obtained from the initial state and the integral of the right function, and Gauss integral is used for discretization calculation. After the above transformation, the original optimal control problem is transformed into a parametric optimization problem with a series of algebraic constraints, which is also called Nonlinear Programming problem (NLP) [35-37].

(1) The time-domain transformation

When Gauss pseudospectral method is adopted, the time interval $[t_0, t_f]$ needs to be

transformed into $[-1, 1]$, so the time t is converted as follows:

$$t \in [t_0, t_f] \rightarrow \tau \in [-1, 1], \tau = \frac{2t}{t_f - t_0} - \frac{t_f + t_0}{t_f - t_0}. \quad (36)$$

(2) Approximation of state variables and control variables

The Gauss pseudospectral method uses K order Legendre Gauss (LG) points (the roots of K order Legendre polynomials $P_K(\bar{\tau})$) and $\tau_0 = -1$ as the discrete node to construct $K+1$ order Lagrange interpolation polynomials, based on the Lagrange interpolation polynomial. The approximate expression of the state variable is constructed, namely,

$$x(\tau) \approx X(\tau) = \sum_{i=0}^K L_i(\tau) X(\tau_i), \quad (37)$$

where,

$$L_i(\tau) = \prod_{j=0, j \neq i}^K \frac{\tau - \tau_j}{\tau_i - \tau_j}, \quad (38)$$

$$P_K(\bar{\tau}) = \frac{1}{2^K K!} \frac{d^K}{d\bar{\tau}^K} \left[(\bar{\tau}^2 - 1)^K \right], K = 1, 2, \dots. \quad (39)$$

let the approximate state value at the node in Eq. (37) equal to the actual state value, and get

$$X_i = X(\tau_i) = x(\tau_i).$$

For the control variables, the same method is used to discretize, that is

$$u(\tau_k) = U(\tau_k), (k = 1, \dots, K), \quad (40)$$

namely,

$$u(\tau) \approx U(\tau) = \sum_{i=1}^K L_i(\tau) U(\tau_i). \quad (41)$$

(3) Transformation of dynamic differential equation constraints

Take the derivative of Eq. (37)

$$\dot{x}(\tau) \approx \dot{X}(\tau) = \sum_{i=0}^K D_{ki}(\tau_k) X(\tau_i), \quad (42)$$

where, the differential matrix $D \in R^{K \times (K+1)}$ can be determined offline, namely,

$$D_{ki} = \dot{L}_i(\tau_k) = \begin{cases} \frac{(1+\tau_k)\dot{P}_K(\tau_k) + P_K(\tau_k)}{(\tau_k - \tau_i)[(1+\tau_i)\dot{P}_K(\tau_i) + P_K(\tau_i)]} & i \neq k, \\ \frac{(1+\tau_i)\ddot{P}_K(\tau_i) + 2\dot{P}_K(\tau_i)}{2[(1+\tau_i)\dot{P}_K(\tau_i) + P_K(\tau_i)]} & i = k, \end{cases} \quad (43)$$

where, τ_k ($k=1, \dots, K$) is a point in the set $\kappa = \{\tau_1, \tau_2, \dots, \tau_K\}$, and $\bar{\tau}_i$ ($i=0, \dots, K$) belongs to the set $\kappa_0 = \{\bar{\tau}_0, \bar{\tau}_1, \dots, \bar{\tau}_K\}$. By substituting Eq. (43) into the dynamic equation, the algebraic constraint equation that the state variable should satisfy on the discrete node can be obtained:

$$\sum_{i=0}^K D_{ki} X(\tau_i) - \frac{t_f - t_0}{2} f(X(\tau_k), U(\tau_k), \tau_k; t_0, t_f) = 0, \quad (44)$$

where, $k = 1, \dots, K$.

(4) Processing of terminal state constraints

The terminal moment node is not included in the approximate expression of the state variable, but the state at the terminal moment should also satisfy the constraint of the dynamic equation:

$$x(\tau_f) = x(\tau_0) + \int_{-1}^1 f(x(\tau), u(\tau), \tau) d\tau. \quad (45)$$

Eq. (45) is approximated by Gauss integral formula. We get

$$X(\tau_f) = X(\tau_0) + \frac{t_f - t_0}{2} \sum \omega_k f(X(\tau_k), U(\tau_k), \tau; t_0, t_f), \quad (46)$$

where, $\omega_k = \int_{-1}^1 L_i(\bar{\tau}) d\tau$ is Gauss weight, τ is Legendre-Gauss point.

(5) Approximation of integral term in the performance index function

The integral term in the performance index function is approximated by the Gauss integral formula:

$$J = \Phi(X_0, t_0, X_f, t_f) + \frac{t_f - t_0}{2} \sum_{k=1}^2 \omega_k g(X_k, U_k, \tau_k; t_0, t_f), \quad (47)$$

where, $\Phi()$ is the no-integral term in the performance index function.

(6) General description of the Gauss pseudospectral method

Based on the above mathematical transformation, the principle of Gauss pseudospectral method to solve the optimal control problem can be described as follows: calculate state variables X_i , $i=0, \dots, K$ and control variables U_k , $k=1, \dots, K$ and t_0 , t_f on the discrete nodes, to make the minimum performance index Eq. (47), and satisfy the dynamics equation of algebraic constraint

Eq. (44), the terminal state constraint Eq. (46), and the boundary conditions of the original optimal control problem

$$\varphi(X_0, t_0, X_f, t_f) = 0, \quad (48)$$

and process constraints

$$C(X_k, U_k, \tau_k; t_0, t_f) \leq 0, \quad k = 1, \dots, K, \quad (49)$$

thus, the original optimal control problem is transformed into a nonlinear programming problem (NLP), namely:

$$\begin{aligned} \min \quad & F(y), \quad y \in R^M, \\ \text{s.t.} \quad & g_j(y) \geq 0, \quad j = 1, 2, \dots, p, \\ & h_j(y) = 0, \quad j = 1, 2, \dots, l. \end{aligned} \quad (50)$$

where, y is a designed variable, including control variable, state variable and two endpoint time.

At present, there are many effective methods to solve the nonlinear programming problem, among which the sequential quadratic programming algorithm (SQP) is widely used for its local superone-time convergence and global convergence, so the Gauss pseudospectral method uses SQP algorithm to deal with the transformed NLP problem.

4 The simulation verification

The simulation in this paper is aimed at the formation of six QUAUVs. The type of UAVs we used is the ZY F150 QUAUV. When taking off, the formation is formed by equilateral triangle, the formation keeping strategy is the leader-followed method. The relevant parameters of the QUAUVs used in the experiment are shown in Eq. (51) and Tab.1.

$$\left\{ \begin{aligned} &0 \leq x_i \leq 1500 \text{ m}, 0 \leq y_i \leq 1500 \text{ m}, 0 \leq z_i \leq 35 \text{ m}, \\ &|v_{xi}| \leq 5 \text{ m/s}, |v_{yi}| \leq 5 \text{ m/s}, |v_{zi}| \leq 2 \text{ m/s}, \\ &|\phi_i| \leq \pi, |\theta_i| \leq \pi/3, |\psi_i| \leq \pi/3, \\ &|p_i| \leq 32^\circ/\text{s}, |q_i| \leq 10^\circ/\text{s}, |r_i| \leq 5^\circ/\text{s}, \\ &0 \leq U_{1i} \leq 5.2 \text{ N}, \\ &|U_{2i}| \leq 0.0501 \text{ N} \cdot \text{m}, \\ &|U_{3i}| \leq 0.0501 \text{ N} \cdot \text{m}, \\ &|U_{4i}| \leq 0.0209 \text{ N} \cdot \text{m}. \end{aligned} \right. \quad (51)$$

Table 1 Simulation parameters of the ZY F150 QUAV

Symbol	Physical meaning	Value	Unit
m	Mass of the quadrotor	0.5	kg
l	Length between the center of the aircraft and the rotor	0.075	m
K_T	Lift coefficient	1.105e-5	$\text{N}/(\text{rad/s})^2$
K_M	Inverse torque coefficient	1.779e-7	$\text{N} \cdot \text{m}/(\text{rad/s})^2$
I_{xx}	Rotational inertia of the x-axis	0.0219	$\text{kg} \cdot \text{m}^2$
I_{yy}	Rotational inertia of the y-axis	0.0219	$\text{kg} \cdot \text{m}^2$
I_{zz}	Rotational inertia of the z-axis	0.0366	$\text{kg} \cdot \text{m}^2$
g	Gravitational acceleration	9.8	m/s^2
K_1, K_2, K_3	Coefficient of air resistance	0.073	$\text{N}/(\text{m/s})^2$
K_4, K_5, K_6	Coefficient of air resistance moment	0.0055	$\text{N} \cdot \text{m}/(\text{rad/s})^2$

465 The terrain constraint model adopts the ground height generated by Eq. (52) and the mountain
466 terrain data generated by Eq. (53) [38, 39].

$$467 \quad z_1(x, y) = \text{abs} \left(\sin(y + a) + b \cdot \sin(x) + c \cdot \cos \left(d \cdot \sqrt{x^2 + y^2} \right) + e \cdot \cos(y) + f \cdot \sin \left(g \cdot \sqrt{x^2 + y^2} \right) \right), \quad (52)$$

$$468 \quad z_2(x, y) = \sum_{i=1}^n h_i \exp \left[- \left(\frac{x - x_i}{x_{si}} \right)^2 - \left(\frac{y - y_i}{y_{si}} \right)^2 \right], \quad (53)$$

469 where, in Eq. (52), x and y are the point coordinates of the model projected on the horizontal plane,
470 z_1 is the ground surface elevation value corresponding to the horizontal surface point, a , b , c , d ,
471 e , f , g are the constant coefficients to control the rise and fall of the ground surface reference
472 terrain in the digital map model, function $\text{abs}()$ represents absolute function, that is, all terrain heights
473 are above sea level, as the reference terrain of the formation environment of the QUAVs. This paper
474 takes $x = y = [0 \ 0.001 \ 0.002 \dots 1500]$, $a = 10$, $b = 0.2$, $c = 0.1$, $d = 0.6$, $e = 1$, $f = 0.1$, $g = 0.1$.

475 In Eq. (53), z_2 is the elevation value given at the corresponding point (x, y) on the map, (x_i, y_i)
476 is the central coordinate of the i -th mountain, h_i is the terrain parameter to controls the height of the

477 mountain, x_{si} and y_{si} are the attenuation of the i -th mountain along the x -axis and y -axis
 478 respectively, which controls the slope, and n represents the total number of mountains. By setting the
 479 above parameters, different number of peaks with different heights and slopes can be achieved. This
 480 paper take $h_i = [55 \ 20 \ 40 \ 30 \ 48 \ 25 \ 31]$, $x_i = [500 \ 300 \ 300 \ 1300 \ 760 \quad 700 \ 1300]$,
 481 $y_i = [1100 \ 410 \ 800 \ 450 \ 750 \ 200 \ 1300]$, $x_s = [50 \ 40 \ 30 \ 86 \ 87 \ 60 \ 30]$, $y_s = [50 \ 30$
 482 $20 \ 79 \ 83 \ 90 \ 30]$, so there are seven peaks.

483 In the formation of UAVs with leader-followed strategy, the reference path of the leader UAV
 484 should be specified first, and the method proposed in this paper is used to generate the reference path
 485 of the leader UAV. The path generated by GPM is given in the form of nodes. In this paper, B-spline
 486 smoothing strategy is adopted to smooth the process and limit the maximum value.

487 **Simulation 1: Path planning of the formation of UAV when $0 \leq z_i \leq 35\text{m}$**

488 When we limit the maximum height that a quadcopter can fly to as its maximum height
 489 $z_i=35\text{ m}$, the task to be accomplished by the leader UAV is to start from the specified starting point
 490 $(x_0, y_0, z_0) = (2, 1, 3)\text{ m}$, with the initial velocity vector $(v_{x0}, v_{y0}, v_{z0}) = (2, 2, 0)\text{ m/s}$ and the initial
 491 attitude angle $(\psi_0, \theta_0, \phi_0) = (45^\circ, -1^\circ, 0^\circ)$. It passes through the non-flat area distributed over a
 492 number of mountains, and arrive at the specified location $(1200, 1000, 2)\text{ m}$ with the specified
 493 velocity vector $(v_{xf}, v_{yf}, v_{zf}) = (0, 0, 0)\text{ m/s}$ and attitude angle $(\psi_f, \theta_f, \phi_f) = (45^\circ, 0^\circ, 0^\circ)$ at the
 494 specified time $t = 275\text{ s}$, and gradually descend from the high altitude to the specified height 2 m .
 495 The take-off positions of the five followed UAVs are $(0, 0, 3)\text{ m}$, $(0, 2, 3)\text{ m}$, $(1, 0.5, 3)\text{ m}$,
 496 $(0, 0.5, 3)\text{ m}$, $(1, 1.5, 3)\text{ m}$, respectively. The take-off velocity vector, attitude angle, terminal
 497 velocity vector and attitude angle are the same as the leader UAVs. The simulation result is shown in
 498 Fig. 8 ~ Fig. 14.

499 Fig. 8 and Fig. 9 show the flight path from different visual angles of the formation of UAVs in
 500 the environment with multiple obstacles distributed, and it can be seen that it does not collide with
 501 any obstacles and can reach the specified end point from the starting point. As can be seen from Fig.

8, the flight path is smooth throughout the flight, and all of which meet the constraints; due to large maximum flying height of the ZY F150 QUAUV, the take-off point nearest obstructions can be directly overflow, and then keep flying at a certain height, don't need to turn to evade, which can prevent again to evade other obstacles; in the followed flight process, UAVs formation by changing the flight direction and height to consume the excess flight time. Also, we can see that, after crossing the first obstacle, the flight path of UAV formation is far from other obstacles, so UAVs don't need to transform formation. However, from the perspective of energy consumption, the landing process can save energy consumption while consuming extra time compared with the horizontal circling flight. At the end of flight, it can be seen that the formation of UAV gradually lands at the specified altitude of 2 m from the particular direction. Fig. 9 also shows the change of the heading angle of the leader UAV. The pitch angle and roll angle are showed in Fig. 10.

Fig. 8 3D view of the optimized path

Fig. 9 x-y plane of the optimized path

Fig. 10 The curves of pitch angle and roll angle of the leader UAV

Fig. 11 Formation of UAVs

From Fig. 10 we can see that the change of pitch angle and roll angle of the leader UAV is small

519 during the entire flight, this is also related to the characteristics of the QUAV, that to change the
 520 heading angle, ascent or descent of a QUAV does not need to change its pitch angle, and because the
 521 whole flight process is in a higher airspace, there is no need to roll to avoid obstacles or change
 522 formation, so the roll angle of the whole flight process is also small. The enlarged figure is shown in
 523 Fig.11, which shows the formation of QUAVs during the flying process (The dots represent the six
 524 QUAVs).

525 Fig. 12 and Fig. 13 show the curves of variation of control variables U_1 , U_2 , U_3 and U_4 ,
 526 respectively. It can be seen from Fig. 12 that U_1 is used to change the speed of the three spatial
 527 directions to realize the change of the trajectory of the leader UAV, so as to avoid obstacles or consume
 528 extra time for the leader UAV. It can be seen from Fig.13 that roll angle control quantity keeps a small
 529 value in the entire process of flight, which shows that the roll angle of the leader UAV almost does
 530 not changed during the flight; in the process of flight, the pitch angle control quantity is also small,
 531 although there are up and down, but due to the special structure of QUAV, the pitch angle does not
 532 needed to be great changed, which also accord with the results of simulation; but the change of yaw
 533 angle control quantity is bigger, because it need to change the flight direction of leader UAV.

534
535 Fig. 12 The curves of U_1 of the leader UAV

536 Fig. 13 The curves of U_2 , U_3 and U_4 of the leader UAV

536 Fig. 14 shows the curves of variation of linear velocity variables v_x , v_y and v_z respectively.
 537 Since we are using the NED coordinate system, the change of the Z axis direction is the opposite
 538 of the normal situation. It can be seen from Fig. 14 that when the vertical velocity v_z changes, the
 539 horizontal velocity can be changed through the change of v_x and v_y , so as to realize the UAV's
 540 obstacle avoidance or consume extra flight time in the corresponding x - y plane.

Fig. 14 The curves of v_x , v_y and v_z of the leader UAV

Simulation 2: Path planning of the formation of UAV when $0 \leq z_i \leq 15\text{m}$

When we limit the maximum height that the QUAV can fly to as $z_i=15\text{m}$, the task to be accomplished by the leader UAV is to start from the specified starting point $(x_0, y_0, z_0) = (2, 1, 3)\text{m}$, with the initial velocity vector $(v_{x0}, v_{y0}, v_{z0}) = (2, 2, 0)\text{m/s}$ and the initial attitude angle $(\psi_0, \theta_0, \phi_0) = (60^\circ, -18^\circ, 0^\circ)$, and arrive at the specified location $(1200, 1000, 2)\text{m}$ with the specified velocity vector $(v_{xf}, v_{yf}, v_{zf}) = (0, 0, 0)\text{m/s}$ and attitude angle $(\psi_f, \theta_f, \phi_f) = (-60^\circ, 0^\circ, 0^\circ)$ at the specified time $t = 350\text{s}$. The simulation result is shown in Fig. 15 ~ Fig. 20.

Fig. 15 3D view of the optimized path

Fig. 16 x-y plane of the optimized path

As can be seen from Fig. 15, when the maximum height that the QUAV can fly to is limited as $z_i=15\text{m}$, it can't fly over the mountain and has to avoid the need to detour. From the optimization path in x-y plane graph in Fig. 16, we can see that the UAV formation need to change flight course

555 between peaks, and ultimately to heading to the end point, which is significant different from
 556 Simulation-1. However, as we stipulate that the UAV formation should reach the target point at the
 557 specified time of 350 s, so the UAV formation needs to make some detours in the flight process to
 558 consume the extra flight time, which is also automatically considered in the optimization process.
 559 Of course, we can also consider the time optimization as the optimization goal. Fig. 16 also shows
 560 the change of the heading angle of the leader UAV. The pitch angle and roll angle are showed in
 561 Fig. 17.

562 From Fig. 17 we can see that the change of pitch angle of the leader UAV is also small during
 563 the whole flight, but because there are some turns to avoid obstacles or to change formations during
 564 the whole flight process, so the roll angle of the whole flight process is a little larger than the value
 565 in Simulation 1.

566 Fig. 18 shows the curves of variation of linear velocity variables v_x , v_y and v_z , respectively.
 567 It can be seen from Fig. 18 that as we reduce the maximum flying height of the UAV, the change of
 568 v_z in the Z-axis direction is also smaller than that in Simulation 1. However, due to many changes
 569 in the flight direction, the change of v_x and v_y in the horizontal direction is more, and the change
 570 of v_x and v_y is not consistent as that in Simulation 1.

571 Fig. 17 The curves of pitch angle and roll angle

572 Fig. 18 The curves of v_x , v_y and v_z of the leader UAV of

573 the leader UAV

574 Fig. 19 and Fig. 20 show the curves of variation of control variables U_1 , U_2 , U_3 and U_4 ,
 575 respectively. It can be seen from Fig. 19 and Fig. 20, relatively speaking, due to the long flight time
 576 of UAV formation, the value of U_1 is smaller than U_1 in Simulation-1, but the control quantity U_2 ,

577 U_3 and U_4 which are used to change the attitude are little bit bigger than U_2 , U_3 and U_4 in
 578 Simulation-1.

580 Fig. 19 The curves of U_1 of the leader UAV

Fig. 20 The curves of U_2 , U_3 and U_4 of the leader UAV

581 5 conclusion

582 In this paper, the formation problem of multi-QUAVs was studied. Firstly, the strategy of the
 583 leader-followed method was adopted, and the formation keeping problem between each QUAV was
 584 defined based on the consistency theory. The formation system was transformed into a network
 585 topology, and then the formation was treated as a time-varying virtual rigid body by using the virtual
 586 structure method. When a UAV formation encounters an obstacle, it sometimes needs to change its
 587 formation to get through the obstacle. Formation transformation in the flight of UAVs formation can
 588 be regarded as reassembly of UAVs, namely formation reconstruction. The methods of formation
 589 rotation, formation reduction or new formation were introduced in this paper. On this basis, firstly,
 590 some constraints of the leader QUAV were analyzed. Then, by analyzing the constraint information
 591 of the followed UAVs in the formation, such as the position of the followed UAVs in the formation
 592 and the minimum turning radius of the followed UAVs, the information of the followed UAVs in the
 593 formation was converted to constraints of the leader UAV, so that the formation problem can be
 594 isolated. After that, the requirement of performance index was analyzed, if a UAV formation is
 595 required to fly in a straight line from a certain horizontal direction at a certain altitude to the target
 596 point, so the terminal flight requirement was added, and the traditional performance index was
 597 improved. Then this paper gave the analysis of the formation path optimization of the QUAV,
 598 summarized the final problems of path optimization, namely infinite dimension, differential, integral,

599 constrained, etc, and proposed the Gauss pseudospectral method to transform the optimal control
600 problem of the path optimization into the solving framework of nonlinear programming. Finally, we
601 conducted two simulations for different maximum flight height. The simulation results showed the
602 effectiveness of the proposed method in this paper.

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607 **Conflicts of interest/Competing interests**

608 The authors declare that they have no conflict of interest.

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Figures

Figure 1

Schematic view of the QUAVER system.

Figure 2

Formation of QUAUVs

Figure 3

Formation transformation

Figure 4

Reduction diagram of formation

Figure 5

Relative position in formation between followed UAV and leader UAV

Figure 6

x-y plane flight diagram

Figure 7

3D flight diagram

Figure 8

3D view of the optimized path

Figure 9

x-y plane of the optimized path

Figure 10

The curves of pitch angle and roll angle of the leader UAV

Figure 11

Formation of UAVs

Figure 12

The curves of $U1$ of the leader UAV

Figure 13

The curves of U_2 , U_3 and U_4 of the leader UAV

Figure 14

The curves of v_x , v_y and v_z of the leader UAV

Figure 15

3D view of the optimized path

Figure 16

x-y plane of the optimized path

Figure 17

The curves of pitch angle and roll angle

Figure 18

The curves of v_x , v_y and v_z of the leader UAV of the leader UAV

Figure 19

The curves of $U1$ of the leader UAV

Figure 20

The curves of U_2 , U_3 and U_4 of the leader UAV