

Low Complexity Rotation Algorithm for Reducing PAPR in Partial Transmits Sequence

Thamer Alameri (✉ alamery.thamer@gmail.com)

Iraqi Police College <https://orcid.org/0000-0001-8521-9711>

Nabeel ali

University of Kufa

Mothana Attiah

Middle Technical University

Mohammed Saad Talib

University of Babylon

Jawad Mezaal

University of Baghdad

Research Article

Keywords: PAPR, PTS, IL-PT, Ad-PTS, PR-PTS

Posted Date: March 16th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-264221/v1>

License:  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

1 **Low Complexity Rotation Algorithm for Reducing PAPR in Partial Transmits**
2 **Sequence**

6 Thamer Alameri^{1,2}, Nabeel Salih Ali³, Mothana L. Attiah^{2,4}, Mohammed Saad Talib⁵, Jawad Kadhim Mezaal⁶

7 ¹Department of Computer Engineering, University of Technology, Baghdad, Iraq.

8 ²Police College of Iraq, Baghdad, Iraq.

9 ³Information Technology Research and Development Centre, University of Kufa, Iraq.

10 ⁴Department of Computer Engineering, Electrical Engineering Technical College, Middle Technical University,
11 Baghdad, Iraq

12 ⁵College of Administration and Economics, University of Babylon – Iraq

13 ⁶State Company for Oil Projects (SCOP), Baghdad, Iraq

16 **Corresponding Author:** Thamer Alameri **Email:** Alamery.thamer@gmail.com

17 **Abstract:** High peak to average power ratio (PAPR) is considered as a prime challenge in orthogonal frequency
18 division multiplexing. The partial transmits sequence (PTS) technique is one of the most effective methods for
19 restraining the PAPR pattern. This study proposes a novel approach for enhancing PAPR reduction performance
20 in a PTS by partitioning each subblock into two parts then exchanging the first sample with the last selection in
21 each part of the subblock to generate a new partitioning scheme. The proposed algorithm is analysed and applied
22 to typical traditional segmentation schemes, namely, the adjacent, interleaving and pseudo-random schemes.
23 Moreover, simulation is conducted with two scenarios in which the number of subcarriers is set to 128 and 256.
24 In both systems, the improved segmentation schemes demonstrate PAPR reduction performance that is superior
25 to that of the traditional strategies. Furthermore, the computational complexity level of the enhanced adjusted PTS
26 scheme is low compared with that of the conventional schemes.

28 **Keywords:** PAPR. PTS. IL-PT. Ad-PTS. PR-PTS.

30 1. **Introduction**

32 Recently, 5th generation (5G) networks have drawn increasing attention from industrial and academic
33 researchers owing to the rapid growth of communication markets [1]. Several applications have been introduced
34 in 5G networks to meet the new requirements, such as machine-to-machine, Internet of things and vehicle-to-
35 vehicle communications [2]. Partial Transmit Sequence (PTS) is one of the promising methods to mitigate PAPR
36 [3]. One of the most significant discussions on the partial transmit sequence (PTS) approach is that input data are
37 segmented into several subblocks. A set of phase coefficients rotates the subblock samples before combining them
38 once again to produce a group of picked signals. Lastly, the signal with a minimum peak to average power ratio
39 (PAPR) level is selected for transmission [4]. Recently, researchers have examined the effects of subblock
40 segmentation schemes on PAPR reduction levels. Specifically, Ibraheem et al. discussed the influence of varying

41 subblock lengths on the adjacent (Ad) partitioning scheme and found that fixed subblock lengths can realise PAPR
42 reduction performance better than varying subblock lengths [5]. Vidya presented traditional partitioning schemes
43 and compared their influence on PAPR reduction performance [6].

44 Moreover, the authors concluded that the pseudo-random (PR) method is the ideal scheme among the
45 partitioning methods. Researchers have developed new segmentation schemes. For example, Xiao suggested a
46 novel technique for enhancing PAPR reduction performance by copying PR subblocks in the time domain (TD)
47 to increase candidate signals [7]. Hong presented a hybrid scheme for minimising high PAPR patterns by merging
48 the PR-PTS and interleaving PTS (IL-PTS) scheme [8]. Meanwhile, Miao proposed a novel approach by
49 combining the IL and PR schemes with the Hadamard matrix [9]. Moreover, Ibraheem introduced a new algorithm
50 by combining the Ad-PTS and IL-PTS scheme to minimise PAPR values [5]. Jawhar attempted to acquaint new
51 partitioning algorithms to improve PAPR reduction capacity compared with conventional methods [4]. However,
52 a significant problem with the application of high PAPRs in OFDM systems involves incremental PAPR levels,
53 which make the system works in a nonlinear region of the power amplifier (PA), thereby, deteriorating the bit
54 error rate (BER) and signal spectrum effectiveness [10]. Nevertheless, most previous research avoids the PAPR
55 challenge. Several algorithms have been suggested as practical solutions rather than large dynamic-range PAs,
56 such as tone reservation, nonlinear companding transform [11], clipping and filtering [12], PTS [4], selective
57 level mapping [13], and active constellation extension [14]. Among the PAPR reduction techniques, the PTS,
58 which was suggested by Muller in 1997, is the most efficient reduction method [15]. The PTS algorithm can lessen
59 PAPR values effectively by increasing the computational complexity (CC) of a system. However, discussions on
60 the impact of segmentation schemes on PAPR are limited. The current study proposes three traditional
61 segmentation schemes that are enhanced through the application of a new algorithm, namely, the rotation
62 algorithm. The PAPR reduction capacity of the enhanced schemes is better than that of the conventional
63 techniques. However, the CC of the enhanced strategies records different levels compared with that of the
64 traditional schemes.

65 The rest of this paper is structured as follows: The PTS technique is explained in Section 2, and Section
66 3 analyses the PTS segmentation method. Section 4 introduces the proposed algorithm, and Section 5 discusses
67 the results. Finally, Section 6 presents the conclusion of the study.

68

69 **2. Partial Transmit Sequence (PTS) Technique**

70

71 **2.1 Peak to Average Power Ratio (PAPR) and PTS**

72 In an OFDM system, baseband subcarriers are often modulated using an Inverse Fast Fourier transform
 73 (IFFT) algorithm. Hence, the OFDM signal is expressed as (Kim, 2016) in [Eq. 1 \[16\]](#).

$$y(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} Y_k e^{j2\pi kn/N} , 0 \leq n \leq N - 1 \quad (1)$$

74
 75 Where N . represents, and the number of subcarriers Also, $\{Y_k, k = [0, 1, \dots, N - 1]\}$ is a baseband signal
 76 in a complex representation after mapping by one of the modulation methods, such as quadrature amplitude
 77 modulation (QAM). The OFDM signal in the TD is produced by superpositioning multiple sinusoidal functions.
 78 Thus, the instantaneous power of individual samples may be increased several times more than the mean intensity
 79 of the signal [\[17\]](#). Therefore, the envelope fluctuation of the OFDM signal drives the high PA to operate outside
 80 the linear region, thereby deteriorating the signal spectrum and BER performance of the system [\[18\]](#). Therefore,
 81 PAPR is adopted to compute the fluctuation of the OFDM signal in the TD. Thus, PAPR is defined as the highest
 82 instant signal power divided by the average power and expressed in decibel (dB) as in [Eq. 2](#).

83

$$PAPR = 10 \log_{10} \frac{\max|y(n)|^2}{E\{|y(n)|^2\}} \quad (2)$$

84
 85 Where the $E\{\cdot\}$ represents the expected value. The complementary cumulative distribution function
 86 (CCDF) is contributed to calculate the PAPR probability that exceeds a threshold value as in [Eq. \(3\) \[19\]](#).

$$P_r[PAPR(y(n)) \geq PAPR_0] = 1 - (1 - e^{-PAPR_0})^N \quad (3)$$

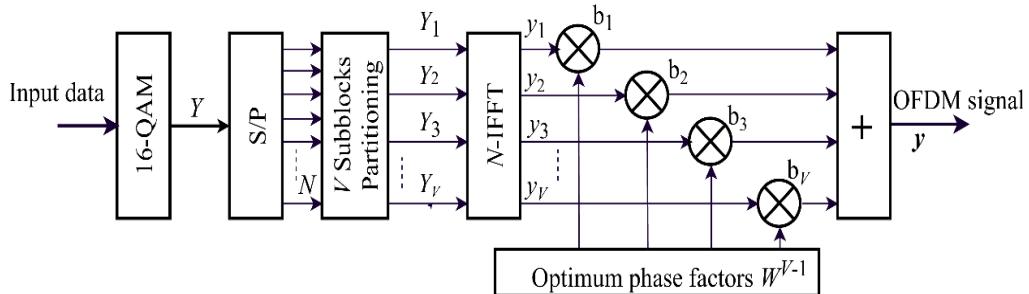
87 Where P_r represents the probability function, and $PAPR_0$ symbolises the threshold value. Besides, to
 88 increase the prissily of the PAPR computations, the oversampling factor (L) is generally used by setting $(L-1) N$
 89 zeros among the samples. Thus, the CCDF can be expressed as in [Eq. 4 \[19\]](#).

$$P_r[PAPR(y(n)) \geq PAPR_0] = 1 - (1 - e^{-PAPR_0})^{NL} \quad (4)$$

90 The PAPR problem should be resolved before the OFDM signal is transmitted. The PTS technique is
 91 among the methods that can diminish high PAPRs effectively. The mechanism of the PTS technique involves the
 92 partitioning of the input data symbol into V subblocks. Next, the N-IFFT block is utilised to modulate the N -
 93 subcarriers with the input data symbols. Subsequently, a group of phase coefficient vectors rotates the subcarriers

94 of each subblock to produce a set of candidate signals. Given the influence of the phase coefficients, the candidate
 95 signals will exhibit different PAPR value levels. Accordingly, the candidate signal that achieves a low PAPR is
 96 sent to the receiver side [20] as shown in [Figure 1](#).

97



98

99 [Figure. 1](#) C-PTS block diagram [21]

100

101 Figure 1 presents the simple scheme of the PTS approach, where 16-QAM maps the data symbol $Y_k =$
 102 $[Y_0, Y_1, \dots, Y_{N-1}]$, and the complex data symbol is divided into V subsets $\{Y_v, v=1, 2, \dots, V\}$, as shown in [Eq.](#)
 103 [\(5\)](#).

$$Y_k = \sum_{v=1}^V Y_v \quad (5)$$

104

105 Next, the partitioned subblocks of the OFDM signal are multiplied by the phase rotation coefficients
 106 according to [Eq. \(6\)](#) as follows:

107

$$Y_k = \sum_{v=1}^V b_v Y_v \quad (6)$$

108

109 Where $\{b_v, v = 1, 2, \dots, V\}$ is the phase coefficients. Afterwards, the IFFT operation is utilised to produce
 110 the OFDM signal, consequently; [Eq.7](#) expresses the output OFDM signal as:

$$y = \text{IFFT} \left\{ \sum_{v=1}^V b_v Y_v \right\} \quad (7)$$

111

112 Equation (8) depicts the OFDM output signal due to linear characteristic of the IFFT function as follows:

$$y = \sum_{v=1}^V b_v \text{IFFT}(Y_v) \quad (8)$$

113 In which,

$$y = \sum_{v=1}^V b_v y_v \quad (9)$$

114

115 The optimum phase coefficient that drives the OFDM signal to attain the minimum PAPR value can be
 116 obtained by the following expression in [Eq. \(10\)](#) [22].

117

$$\{b_1, b_2, \dots, b_V\} = \arg_{\{b_1, b_2, \dots, b_V\}} \left(\min \left(\max_{1 \leq n \leq N} \left\{ \left| \sum_{v=1}^V b_v Y_v \right| \right\} \right) \right) \quad (10)$$

118

119 Where $\arg(\cdot)$ is the term that achieves the minimum PAPR value by setting the phase coefficient factors,
 120 furthermore, the optimum phase coefficient is obtained after $(WV-1)$ attempts are performed, where W denotes
 121 the number of different phase coefficients, which is often limited to or for reducing the multiplication operations.
 122 Consequently, the PTS technique suffers from high computational complexity when determining the proper phase
 123 rotation coefficient. Furthermore, the receiver should recognise the index of the appropriate phase coefficient to
 124 recover the transmitted signal correctly. Hence, $(\log_2 WV - 1)$ bits should be sent to the receiver by the transmitter
 125 as side information, which requires a portion of the bandwidth and can lead to the reduction of system capacity
 126 [\[23\]](#).

127

128 3. Segmentation Schemes

129 In the PTS technique, several subblocks are extracted by dividing the input data to reduce the high
 130 correlation level among the samples [\[24\]](#). The partitioning input data sequence is the central part of the PTS
 131 technique; thus, the selection of a suitable partitioning scheme plays an essential role in mitigating the PAPR
 132 value. Three typical partitioning schemes are IL partitioning (IL-PTS), Ad partitioning (Ad-PTS) and PR
 133 partitioning (PR-PTS). The PAPR reduction gain of conventional schemes differs from one scheme to another.
 134 Furthermore, the partitioning operation must satisfy different conditions, such as subblocks must be equal in
 135 length, whereas the second concerning active subcarriers must not overlap. Ordinary partitioning schemes are
 136 summarised in the following subsections.

137 3.1 Interleaving Segmentation Scheme (IL-PTS)

138
139 Figure 2 depicts the IL-PTS algorithm, in which N/V subcarriers allocate equal V intervals within the
140 subblocks. Thus, $V-1$ of zeros inside every subblock separates the active subcarriers. The PAPR reduction capacity
141 of the IL-PTS is the worst among the three ordinary division schemes [24]. They are owing to the periodic pattern
142 of the subcarriers in the IL-PTS structure, which imposes a high correlation among the subcarriers.

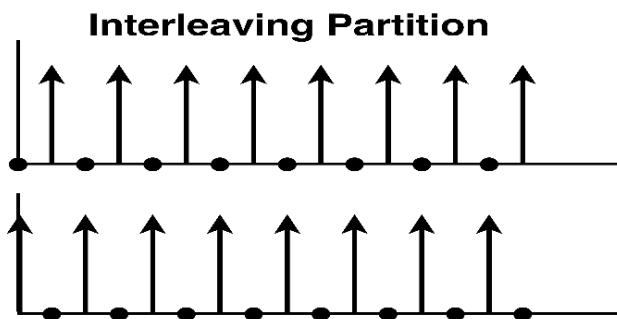


Figure 2. Interleaving segmentation scheme

145 By contrast, the CC of the IL-PTS method is reduced by applying the divide-and-conquer approach via
 146 Cooley–Tukey IFFT transformation. In every single subblock of the IL-PTS, the data vector is arranged column-
 147 wise, in which the number of subblocks equals the total number of rows V in the IL-PTS. Meanwhile, the number
 148 of subsets equals the number of columns R in one subblock, as shown in [Table 1](#). However, the signal obtained
 149 from every subblock is restored via row-wise mapping based on the divide-and-conquer approach.

Table 1 Subblock arrangement of the IL-PTS method

$v \setminus r$	1	2	...	R
1	Y_1^v	Y_{V+1}^v	...	$Y_{V(R-1)+1}^v$
2	Y_2^v	Y_{V+2}^v	...	$Y_{V(R-1)+2}^v$
...
V	Y_V^v	Y_{2V}^v	...	Y_{VR}^v

154 Tukey IFFT algorithm.

$$y_{(q,p)} = \frac{1}{N} \sum_{v=1}^V \left\{ W_N^{vq} \left[\sum_{r=1}^R Y_{v,r} W_R^{rq} \right] \right\} W_V^{vq} \quad (11)$$

156

157

158 Where $W_N^{vq} = e^{j2\pi vq/N}$, $W_R^{rq} = e^{j2\pi rq/R}$, and $W_V^{vq} = e^{j2\pi vq/V}$ are the twiddle factors, while p and q ,
 159 where $1 \leq p \leq V$ and $1 \leq q \leq R$ represent the elements of the p -th row and q -th column of $y(q,p)$, respectively.
 160 The active subcarriers are located only at the v row depending on the IL property of the IL-PTS method; therefore,
 161 R-IFFT is performed only at the v row, and the formula above can be simplified as [Eq. \(12\)](#).

162

$$y_{(q,p)} = \frac{1}{N} W_N^{v(Rp+q)} \sum_{r=1}^R Y_{v,r} W_R^{rq} \quad (12)$$

163

164 Thus, each subblock of the IL-PTS needs to compute only one R-IFFT and N -times complex
 165 multiplications. Therefore, the number of the addition ($C_{\text{add-IL}}$) and multiplication operations ($C_{\text{mult-IL}}$) of the
 166 IL-PTS is written as in [Eq. \(13 and 14\)](#) [23].

167

$$C_{\text{add-IL}} = V \left[\frac{N}{V} \log_2 \frac{N}{V} \right] \quad (13)$$

$$C_{\text{mult-IL}} = V \left[\frac{N}{2V} \log_2 \frac{N}{V} + N \right] \quad (14)$$

168 The CC of the IL-PTS is the lowest among the traditional schemes [24]. This result is because the IL-
 169 PTS method requires few IFFT stages to transform the subblocks in the TD depending on the Cooley–Tukey IFFT
 170 algorithm.

171

172 3.1.1. Pseudo-random Segmentation Scheme (PR-PTS)

173 The PR-PTS algorithm is the most convenient traditional segmentation scheme with PAPR reduction
 174 capacity [24]. In the PR-PTS, the subcarriers are allocated arbitrarily within the subblocks, with active subcarriers
 175 assigned only once inside each subblock, as shown in [Figure 3](#).

176
177
178
179
180
181
182

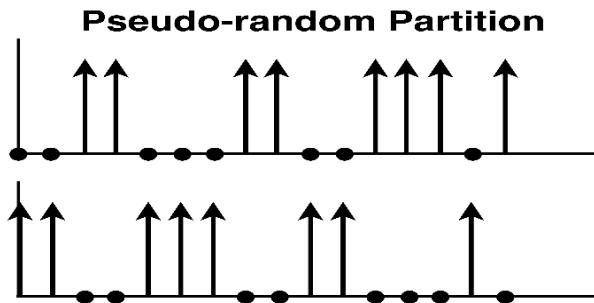


Figure. 3 Pseudo-random segmentation scheme

Figure 3 shows the PR-PTS scheme for two subblocks. The PR-PTS method can realise significant PAPR reduction, as it has low autocorrelation among the subcarriers inside the subblocks [25]. By contrast, high CC is considered as an inherent feature of the PR-PTS method, which can be assigned to the random distribution of the subcarriers. Thus, all the stages of the IFFT operation should be performed using the PR-PTS method when transforming the subblocks in the TD. The number of addition ($C_{\text{add-PR}}$) and multiplication operations ($C_{\text{mult-PR}}$) of the PR-PTS method is written in [Eqs. \(15\) and \(16\)](#) utilising the Cooley–Tukey IFFT algorithm as:

$$C_{\text{mult-PR}} = V \left[\frac{N}{2} \log_2 N \right] \quad (15)$$

$$C_{\text{add-PB}} = V[N \log_2 N] \quad (16)$$

189 3.1.2. Adjacent Segmentation Scheme (Ad-PTS)

190 The Ad-PTS algorithm involves a simple partitioning process, but its PAPR reduction performance is
191 lower than that of the PR-PTS scheme. N/V consecutive subcarriers are assigned sequentially inside the subblocks
192 of the Ad-PTS, and the rest positions of the subblocks are set to zeros, as shown in [Figure 4](#).

193

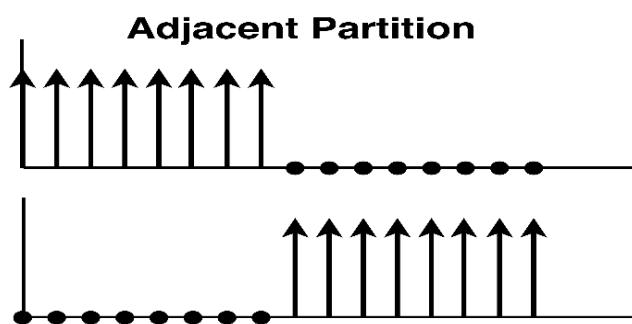


Figure. 4 Adjacent segmentation scheme

196 Accordingly, the CC of the Ad-PTS is similar to that of the PR-PTS [25] because the Ad-PTS performs
 197 all the IFFT stages to transform the subblocks in the TD. The number of addition ($C_{\text{add-Ad}}$) and multiplication
 198 operations ($C_{\text{mult-Ad}}$) of the Ad-PTS is expressed in [Eqs. \(17\) and \(18\)](#) by implementing the Cooley–Tukey IFFT
 199 algorithm as [\[23\]](#).

$$C_{\text{add-Ad}} = V[N \log_2 N] \quad (17)$$

$$C_{\text{mult-Ad}} = V\left[\frac{N}{2} \log_2 N\right] \quad (18)$$

200 **4. Proposing Algorithm (R-PTS)**

201 In this section, a new segmentation method is proposed to enhance PAPR reduction capacity. In OFDM
 202 systems, the input data symbol has a noticeable correlation with its elements; thus, the transmitted OFDM signal
 203 will include a high PAPR value [\[25\]](#). The proposing algorithm depends on the rotation of several subcarriers inside
 204 the subblocks arranged in a specific pattern to downgrade the correlation of the subcarriers. Hence, the PAPR
 205 value of the OFDM signal is reduced, accordingly. The procedure of the rotation PTS algorithm (R-PTS) begins
 206 when the input data sequence Y is divided into V subblocks similar to the partitioning operation of the traditional
 207 segmentation schemes, as given in [Eq. \(5\)](#), which is rewritten below for convenience.

209

$$Y = \sum_{v=1}^V Y_v \quad (19)$$

210

211 Where Y_v is the partitioned subblocks. Subsequently, each subblock is subdivided into two equal parts,
 212 that is, A_v and B_v ,

$$Y_v = [A_v, B_v] \quad (20)$$

213 In which,

$$A_v = [Y_{v,1}, Y_{v,2}, \dots, Y_{v,N/2}] \quad (21)$$

214

215

216

217

And,

$$B_v = [Y_{v,(N/2)+1}, Y_{v,(N/2)+2}, \dots, Y_{v,N/2}] \quad (22)$$

218 Next, the first element is exchanged with the last element in each part of the subblocks. Therefore, the
219 two parts of a subblock can be expressed as:

$$A'_v = [Y_{v,N/2}, Y_{v,2}, \dots, Y_{v,1}] \quad (23)$$

220

$$B'_v = [Y_{v,N}, Y_{v,(N/2)+2}, \dots, Y_{v,(N/2)+1}] \quad (24)$$

221 The rotation process is implemented for all the subblocks in the PTS matrix. Therefore, the new matrix
222 of the R-PTS method can be expressed as:

$$Y' = \sum_{v=1}^V Y'_v \quad (25)$$

223

224 In which,

$$Y'_v = [A'_v, B'_v] \quad (26)$$

225 Therefore, the final R-PTS matrix is,

226

$$Y' = \begin{bmatrix} Y_{1,N/2} & Y_{1,2} & \cdots & Y_{1,1} & Y_{1,N} & Y_{1,(N/2)+2} & \cdots & Y_{1,(N/2)+1} \\ Y_{2,N/2} & Y_{2,2} & \cdots & Y_{2,1} & Y_{2,N} & Y_{2,(N/2)+2} & \cdots & Y_{2,(N/2)+1} \\ \vdots & \vdots \\ Y_{v,N/2} & Y_{v,2} & \cdots & Y_{v,1} & Y_{v,N} & Y_{v,(N/2)+2} & \cdots & Y_{v,(N/2)+1} \end{bmatrix} \quad (27)$$

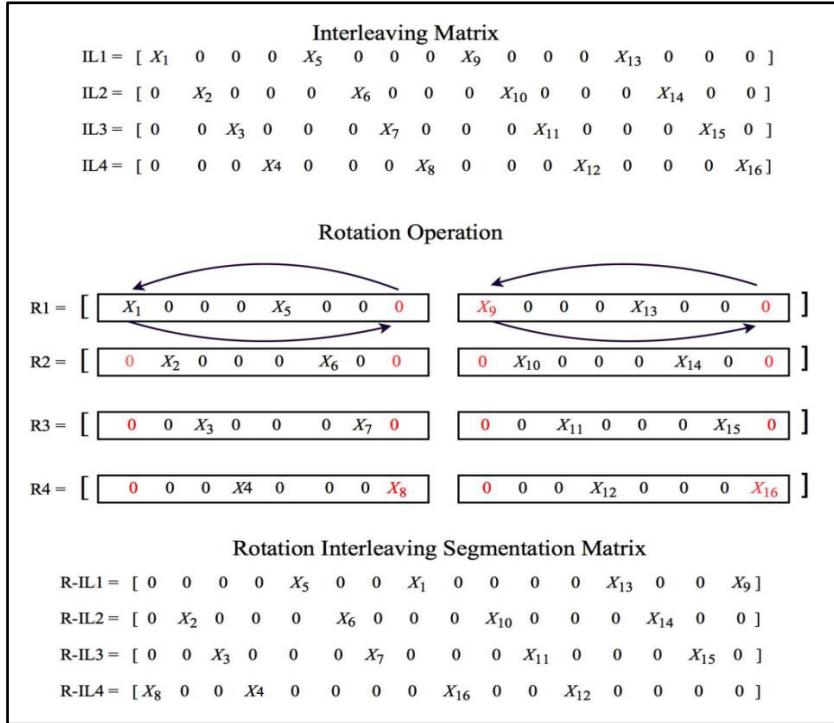
227 The R-PTS algorithm exchanges the terminal samples within the subblocks; thus, the correlation among
228 the samples will be lower than that in the original matrix. Accordingly, the PAPR reduction gain of the PTS
229 technique will be improved. In this study, the R-PTS algorithm is applied to the three conventional segmentation
230 schemes, as shown in the following subsections.

231

232 **4.1. Rotation IL-PTS Scheme**

233 As discussed in Section 2, the PAPR performance of the IL-PTS is the worst among the ordinary
 234 schemes. In this section, the R-PTS algorithm is applied to the IL-PTS to enhance the PAPR gain. The new
 235 algorithm is called the R-IL-PTS, as we can see in [Figure 5](#).

236



237

238 [Figure 5](#) R-IL-PTS segmentation scheme when $N = 16$ and $V = 4$

239 [Figure 5](#) demonstrates the procedure of the R-IL-PTS algorithm, in which the V subblocks are extracted
 240 from the input data symbol like the IL-PTS scheme, and every subblock is subdivided into two parts. Next, in
 241 each part, the first sample is exchanged with the last sample, and this process continues until the last subblock
 242 obtains the final R-IL-PTS matrix. Finally, the R-IL-PTS algorithm is applied to the PTS procedure to produce
 243 the optimum OFDM signal. The PAPR reduction capacity of the R-IL-PTS is preferable to that of the IL-PTS and
 244 better than that of the Ad-PTS method. However, the CC of the R-IL-PTS algorithm in [Figure 5](#) is calculated
 245 using the divide-and-conquer approach with the Cooley–Tukey algorithm as well as the subblock arrangement in
 246 Table 1, where half of the subblocks in the R-IL-PTS algorithm are needed to compute two R-IFFT functions and
 247 N -times complex multiplications. The rest of the subblocks need to compute only one R-IFFT function and N -
 248 times complex multiplications. Therefore, the number of complex additions ($C_{\text{add-R-IL}}$) and complex
 249 multiplications ($C_{\text{multi-R-IL}}$) of the IL-PTS can be expressed in [Eqs. \(28 and 29\)](#) as follows:

$$C_{\text{add-R-IL}} = \frac{3N}{2} \log_2 \left(\frac{N}{V} \right) + N \quad (28)$$

$$C_{\text{mult-R-IL}} = \frac{3N}{4} \log_2 \left(\frac{N}{V} \right) + \frac{N}{2} + NV \quad (29)$$

4.2. Rotation Ad-PTS Scheme

As mentioned previously, the Ad-PTS algorithm is deemed the most straightforward implementation among the segmentation methods. The PAPR reduction capability of the Ad-PTS method is better than that of the IL-PTS but worse than that of the PR-PTS. In this section, the R-PTS algorithm is utilised with the Ad-PTS (R-Ad-PTS) to enhance PAPR reduction performance as can be shown in **Figure 6**.

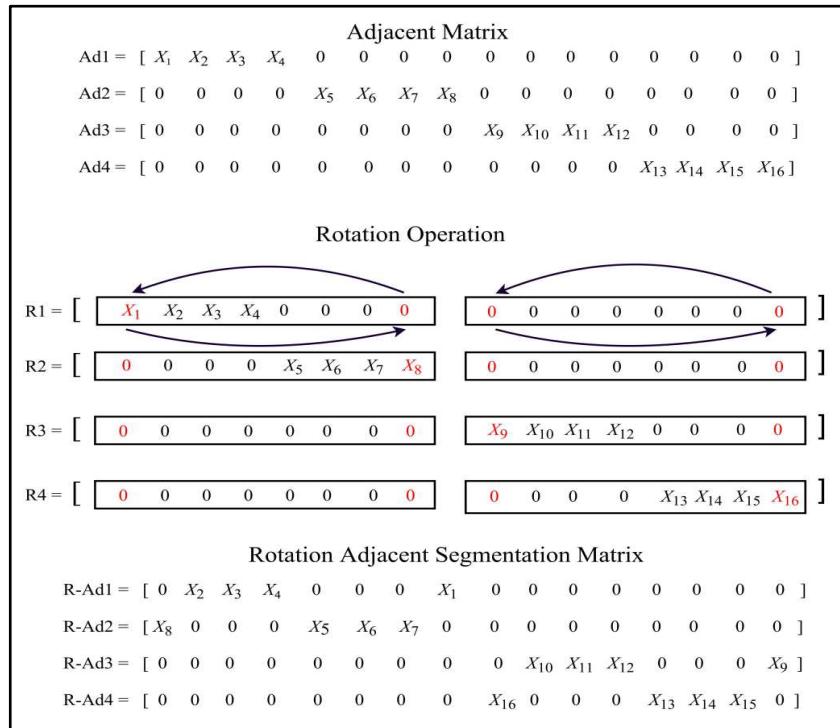


Figure 6 R-Ad-PTS segmentation method, $N = 16$ and $V=4$

The input data symbol in **Figure 6** is segmented into V subblocks, which is like the procedure discussed for the Ad-PTS algorithm, and each subblock of the adjacent matrix is separated into two parts. However, in the subblocks, the samples of each part are modified by exchanging the first sample with the last sample. Lastly, the R-Ad-PTS matrix performs the remaining procedure of the PTS algorithm to produce the optimum OFDM signal.

The PAPR reduction performance of the R-Ad-PTS corresponds to that of the Ad-PTS algorithm because the correlation among the samples of the R-Ad-PTS is less than that among the samples of the Ad-PTS. However, CC is obtained according to the divide-and-conquer approach with the Cooley–Tukey algorithm and the subblock arrangement in **Table 1**, where each subblock in the R-Ad-PTS needs to compute three R-IFFT functions and N -

266 times complex multiplications. Accordingly, the number of the addition and multiplication complexities of the R-
 267 Ad-PTS is obtained as **Eqs. (30 and 31)**.

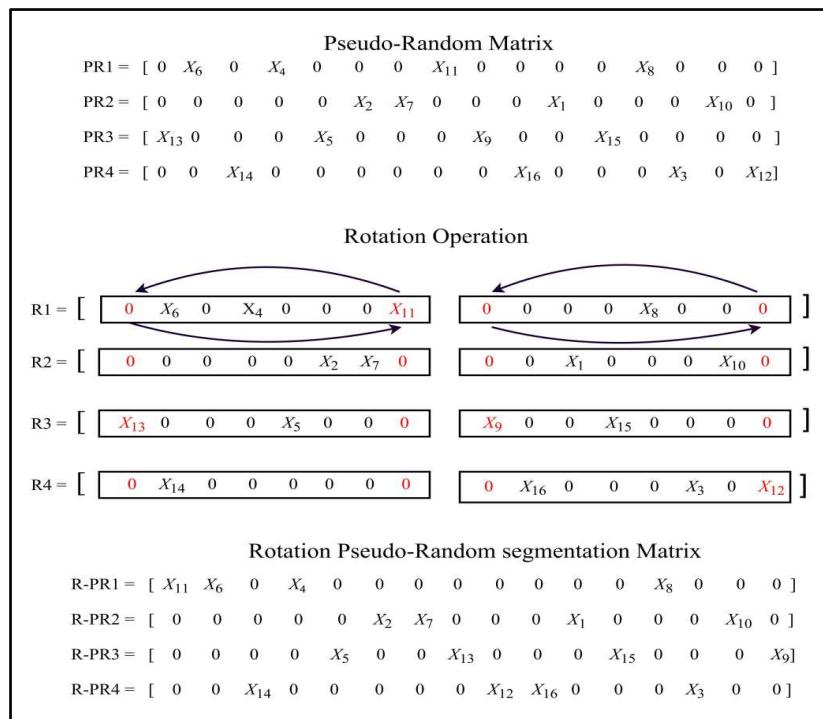
$$C_{\text{add-R-Ad}} = 3N \left[\log_2 \left(\frac{N}{V} \right) + \log_2 3 \right] \quad (30)$$

$$C_{\text{mult-R-Ad}} = \frac{3N}{2} \left[\log_2 \left(\frac{N}{V} \right) + \log_2 3 \right] + NV \quad (31)$$

268 **4.3. Rotation PR-PTS Scheme**

269 In this study, we classify the PR-PTS as the best partitioning scheme among the ordinary segmentation
 270 methods in terms of PAPR reduction efficiency. Sample exchange is applied to the R-PR-PTS to lessen the PAPR
 271 pattern. The procedure of the R-PR-PTS is the same as that of the Ad-PTS and IL-PTS, except the input data
 272 symbol is partitioned similar to the PR-PTS, as demonstrated in **Figure 7**. The PAPR-reducing capability of the
 273 R-PR-PTS method is similar to that of the PR-PTS algorithm, as illustrated in the next section. This similarity is
 274 because the random property of the PR-PTS matrix can achieve the lowest subcarrier correlation inside the
 275 subblocks. Therefore, the PAPR-reducing efficiency of the R-PR-PTS is uninfluenced by the rotation algorithm.
 276 By contrast, the CC of the R-PR-PTS algorithm is the same as that of the PR-PTS, as the subcarriers are allocated
 277 randomly inside each subblock. Thus, all the stages of the IFFT operation must be performed when transforming
 278 the subblocks in the TD.

279



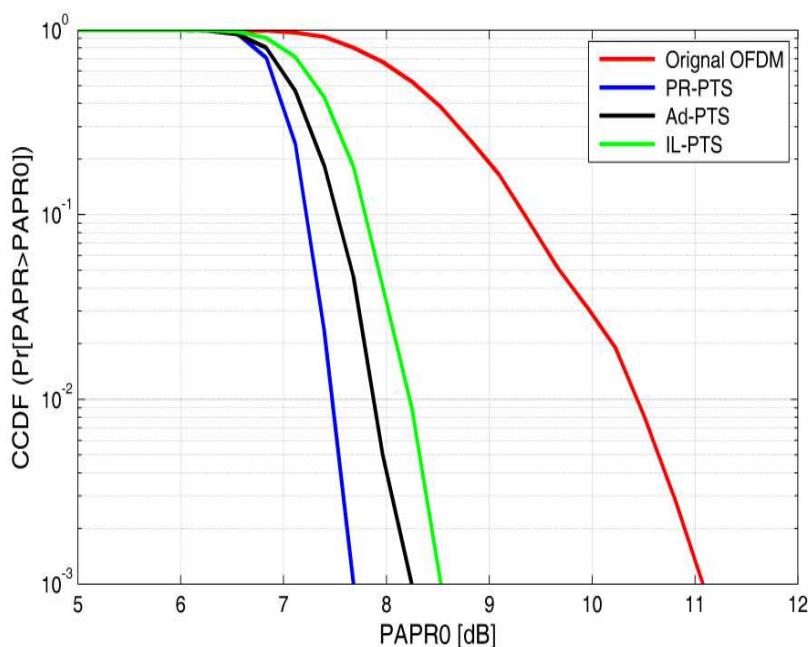
280

281 **Figure. 7** PR-PTS segmentation method, N = 16 and V = 4

282 **4. Results and Discussion**

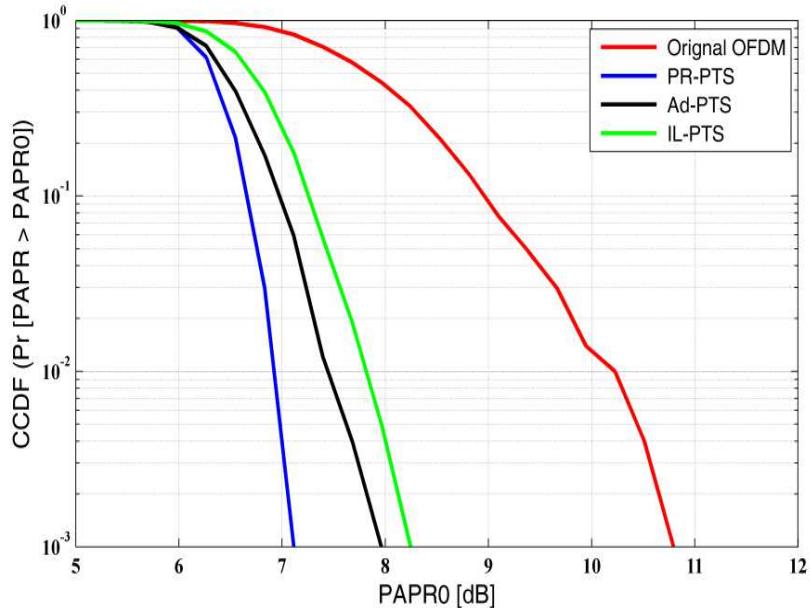
283 **4.1. Simulation Results Analysis**

284 The PAPR reduction performance of the standard and enhanced algorithms is analysed. In terms of the
285 parameters utilised in the simulation, the subblock number V is 4, the subcarrier numbers N are 128 and 256, the
286 oversampling factor L equals 4 and the number of the different phase coefficient vectors W is 4. The CCDF is
287 employed to evaluate the PAPR pattern of the input symbols, and 16-QAM is adopted for constellation mapping.
288 Firstly, simulation is conducted to evaluate the three ordinary segmentation schemes, as shown in [Figures 8 and](#)
289 [9](#). In case the number of subcarriers is 128, the PAPR value of the PR-PTS is 7.11 dB, that of the IL-PTS is 8.24
290 dB, and that of the original signal (without a reduction technique) is 10.79 dB, as shown in [Figure 8](#). Furthermore,
291 when N = 256, the PAPR capacity of the PR-PTS, Ad-PTS, IL-PTS and original OFDM is 7.68dB, 8.24 dB, 8.52
292 dB and 11.07 dB, respectively, as shown in [Figure 9](#). As a result, the PR-PTS algorithm is the best segmentation
293 scheme, whereas the Ad-PTS algorithm demonstrates a lower reduction performance compared with the PR-PTS.
294 By contrast, the IL-PTS algorithm is the worst segmentation scheme in terms of PAPR reduction efficiency.



295

296 [Figure. 8](#) Comparison of the three ordinary partitioning methods, N =128 and V= 4



297

298

Figure. 9 Comparison of the three ordinary partitioning methods, N = 256 and V=4

299

Figures 10 and 11 compare the R-IL-PTS algorithm with the Ad-PTS and IL-PTS ordinary segmentation

300

methods when N = 128 and 256. In the first scenario, when N is set to 128, the R-IL-PTS method outperforms the

301

original signal by 2.88 dB, the Ad-PTS by 0.28 dB and the IL-PTS by 0.56 dB, as depicted in **Figure 10**. However,

302

in the second scenario, when N = 256, the R-IL-PTS outperforms the original OFDM by 3.46 dB and the IL-PTS.

303

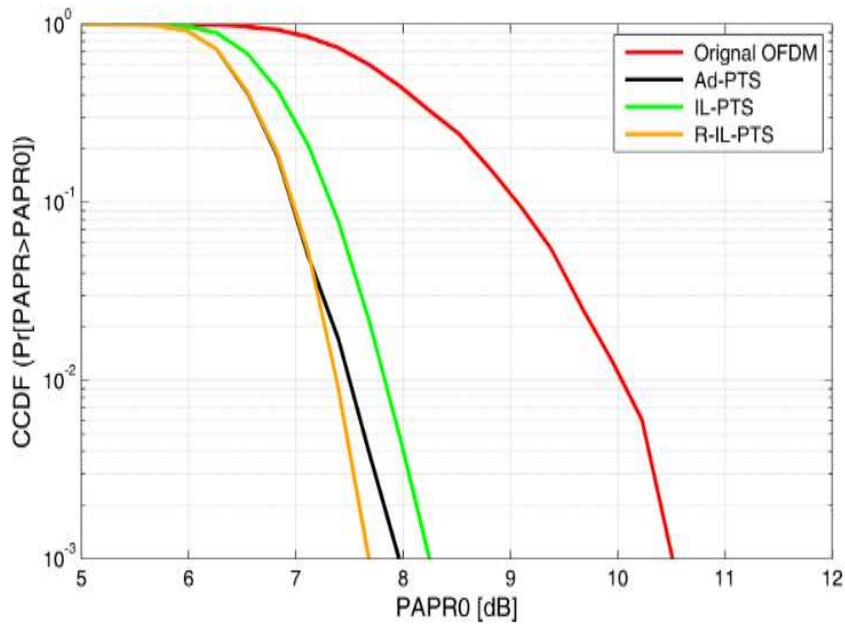
By 0.28 dB. Meanwhile, the R-IL-PTS and Ad-PTS achieve the same PAPR performance at 8.24 dB, as

304

demonstrated in **Figure 11**. Therefore, the PAPR reduction capacity of the proposed method, that is, the R-IL-

305

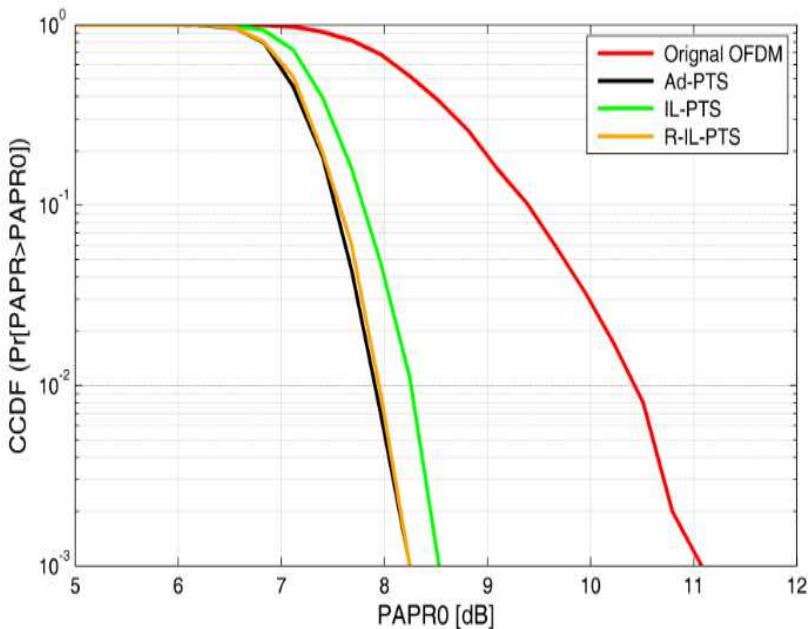
PTS, is better than that of the IL-PTS and the same as that of the Ad-PTS.



306

307

[Figure 10.](#) Comparison R-IL-PTS, IL-PTS and Ad-PTS, N = 128 and V= 4



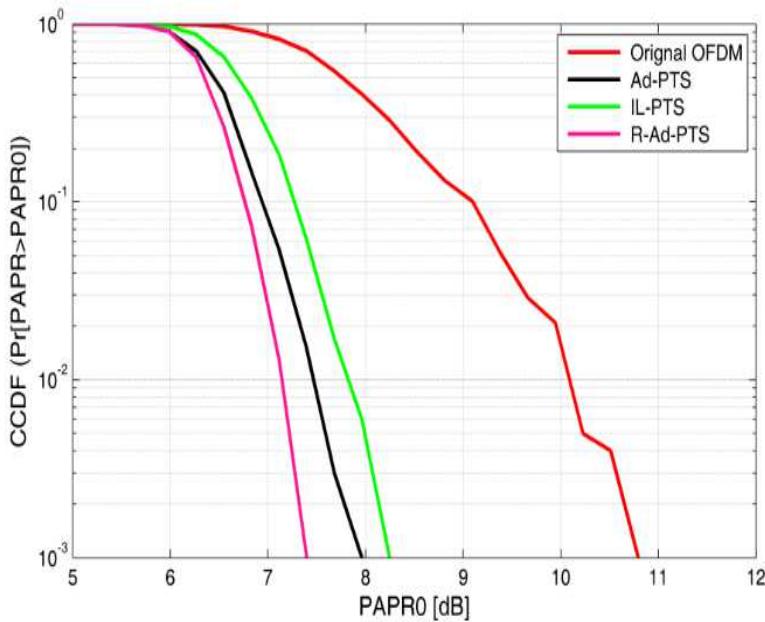
308

309

[Figure 11.](#) Comparison R-IL-PTS, IL-PTS and Ad-PTS, N = 256 and V= 4

310 In the simulation results shown in [Figures 12 and 13](#), the PAPR reduction efficiency of the R-Ad-PTS
 311 is compared with that of the Ad-PTS, IL-PTS and OFDM signal when N = 128 and 256. In [Figure 12](#), when N =
 312 128, the R-Ad-PTS algorithm outperforms the Ad-PTS, IL-PTS and original OFDM by 0.57 dB, 0.85 dB and 3.4
 313 dB, respectively. However, when N is set to 256, the R-Ad-PTS algorithm exceeds the Ad-PTS by 0.28 dB, the

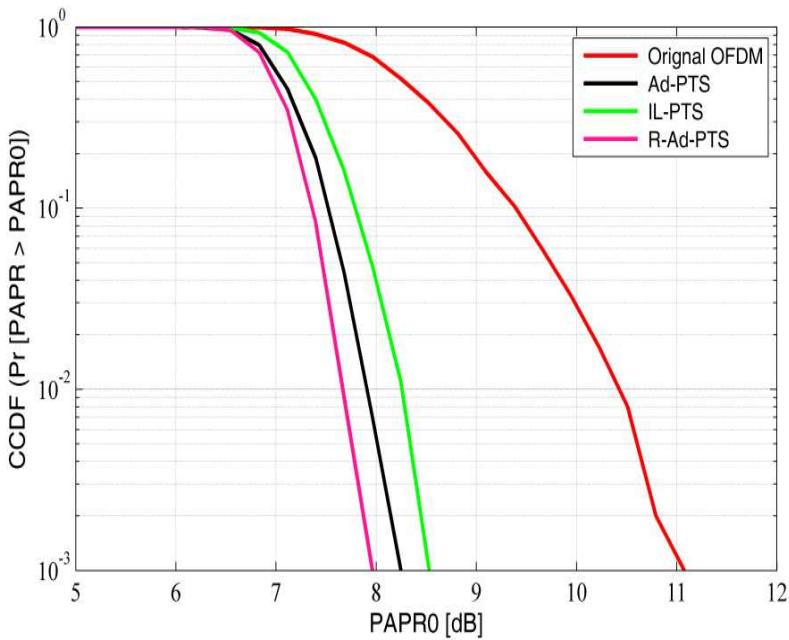
314 IL-PTS by 0.56 dB and the original OFDM by 3.74 dB, as shown in [Figure 13](#). Consequently, the enhanced R-
315 Ad-PTS algorithm can reduce PAPR values better than the Ad-PTS for any number of subcarriers.



316

317 [Figure 12](#). Comparison R-Ad-PTS, Ad-PTS and IL-PTS, $N = 128$ and $V = 4$

318



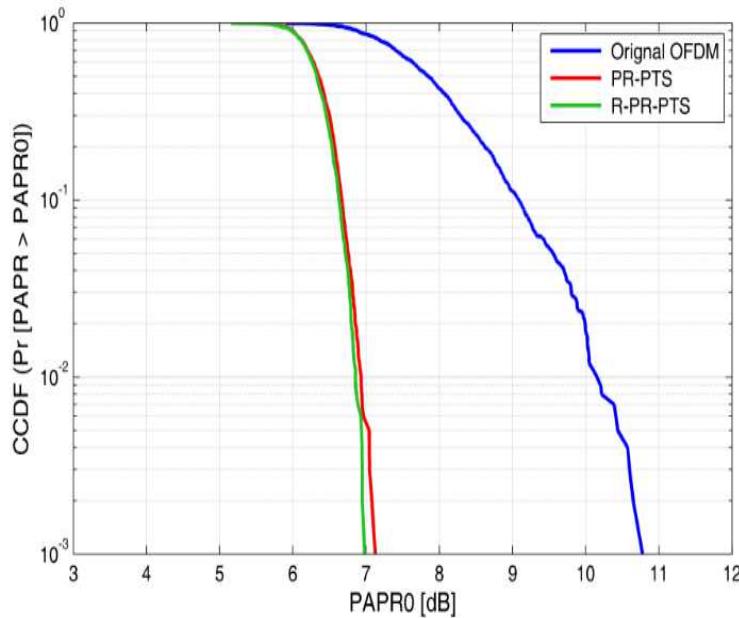
319

320 [Figure 13](#). Comparison R-Ad-PTS, Ad-PTS and IL-PTS, $N = 256$ and $V = 4$

321 Moreover, the simulation results in [Figure 14 and 15](#) illustrate the influence of the R-PR-PTS on PAPR
322 reduction capacity. When N is set to 128, the PAPR level of the R-PR-PTS is 6.95 dB, whereas that of the PR-

323 PTS and the ordinary signal is 7.12 dB and 10.78dB, as shown in [Figure 14](#). However, when N is set to 256, the
324 PAPR value of the R-PR-PTS reaches 7.6 dB, whereas that of the PR-PTS and original OFDM is 7.72dB and
325 10.91 dB, respectively, as presented in [Figure 15](#). Moreover, the proposed R-PR-PTS method has a slight
326 influence on PAPR reduction capacity compared with the PR-PTS, because the R-PR-PTS and P-PTS have nearly
327 the same effect on the correlation of the subcarriers within the subblocks.

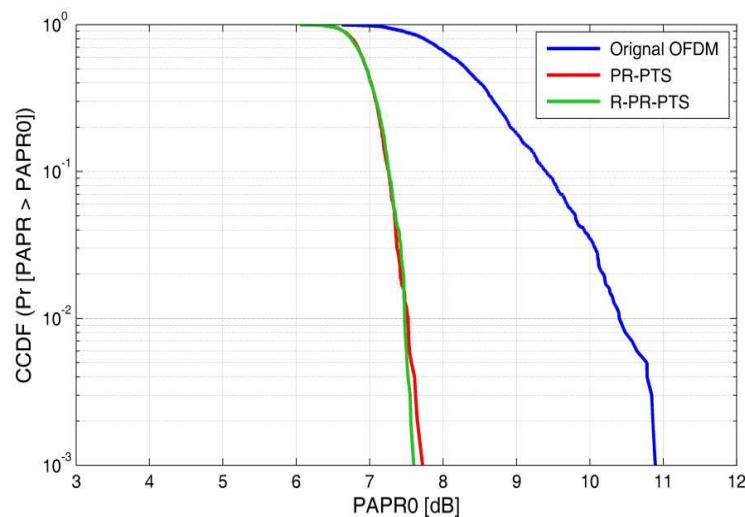
328



329

330

[Figure 14](#). Comparison R-PR-PTS and PR-PTS, N = 128 and V= 4



331

332

[Figure 15](#). Comparison R-PR-PTS and PR-PTS, N = 256 and V= 4

333 **4.4. Numerical Results Analysis**

334 From the perspective of mathematical calculations, the proposed method demonstrates different CC
 335 levels when the three traditional segmentation schemes are applied. **Tables 2 and 3** record the CC level of the
 336 ordinary and improved partitioning schemes, namely, the R-IL-PTS, R-Ad-PTS and R-PR-PTS, as well as the
 337 calculations of the CC reduction ratio (CCRR) of the different simulated schemes, where the CCRR between the
 338 proposed and corresponding traditional schemes can be expressed as [26] in **Eq. (32)** below.

$$\text{CCRR} = 1 - \frac{\text{Complexity of proposed scheme}}{\text{complexity of conventional scheme}} \times 100\% \quad (32)$$

339

340 **Table 2** CCRR of the traditional and the improved schemes when N= 128
 341 $N= 128, V= 4$

PTS method	C_{add}	C_{mult}	CCRR_{add}	$\text{CCRR}_{\text{mult}}$
IL-PTS	640	832		
R-IL-PTS	1088	1056	-41.17%	-21.21%
Ad-PTS	3584	1792		
R-Ad-PTS	2528.64	1776.32	29.44%	1.05%
PR-PTS	3584	1792		
R-PR-PTS	3584	1792	0%	0%

342 **Table 3** CCRR of the traditional and the improved schemes when N= 256
 343 $N= 256, V= 4$

PTS method	C_{add}	C_{mult}	CCRR_{add}	$\text{CCRR}_{\text{mult}}$
IL-PTS	1536	1792		
R-IL-PTS	2560	2304	-40%	-22.22%
Ad-PTS	8192	4096		
R-Ad-PTS	5825.28	3936.64	28.9%	3.9%
PR-PTS	8192	4096		
R-PR-PTS	8192	4096	0%	0%

344 In **Tables 2 and 3**, when the R-PTS algorithm is applied to the IL-PTS, the number of complex addition
 345 and multiplication operations is higher than that in the IL-PTS for both scenarios ($N = 128$ and 256). Therefore,
 346 when $N = 128$, improvement of PAPR performance when the proposed algorithm is applied to the IL-PTS is at the
 347 expense of degradation in the CCRRadd and CCRRmult, where CCRRadd increases by 41.17%, and CCRRmult

348 increases by 21.21. Similarly, when $N = 256$, the CCRRadd of the R-IL-PTS algorithm records degradation by
349 40%, whereas the CCRRmult records degradation by 22.22%. Furthermore, when $N = 128$, CCRRadd and
350 CCRRmult are increased to 29.44% and 1.05%, respectively, when the R-PTS algorithm is applied to the Ad-PTS
351 scheme. Likewise, when the number of subcarriers equals 256, CCRRadd and CCRRmult are increased by 28.9%
352 and 3.9%, respectively. Accordingly, the proposed method improves the Ad-PTS scheme in terms of PAPR
353 reduction performance and CC level. Finally, the CC level of the R-PR-PTS and PR-PTS schemes is the same for
354 both scenarios. This result can be referred to the fact that the random nature of these schemes makes the subblocks
355 perform all the IFFT stages when transforming the data in the TD.

356 **5. Conclusion**

357 A new algorithm for subblock segmentation in the PTS technique is proposed to restrain high PAPR
358 values in OFDM systems. The conducted algorithm relies on dividing every subblock of the PTS technique into
359 two parts and exchanging the first and last samples in each part to obtain a new segmentation scheme, which is
360 called the R-PTS. The presented algorithm is analysed and applied to the conventional segmentation schemes.
361 The results of the simulation indicate that the enhanced schemes, that is, R-IL-PTS and R-Ad-PTS, can accomplish
362 PAPR reduction better than the corresponding ordinary schemes, that is, IL-PTS and Ad-PTS. However, the
363 mathematical calculation model states that the CC level of the R-IL-PTS is higher than that of the IL-PTS, whereas
364 the number of the process complexity for the R-Ad-PTS algorithm is lower than that for the Ad-PTS. However,
365 the R-PR-PTS approach and PR-PTS scheme demonstrate the same PAPR reduction efficiency and CC level.
366 Consequently, the rotation algorithm can be utilised as an effective method to achieve a high PAPR pattern with
367 a low complexity level for the Ad-PTS scheme.

368

369 **REFERENCES**

- 370 1. Attiah, M. L., Isa, A. A. M., Zakaria, Z., Abdulhameed, M. K., Mohsen, M. K., & Ali, I. (2020). A survey
371 of mmWave user association mechanisms and spectrum sharing approaches: an overview, open issues and
372 challenges, future research trends. *Wireless Networks*, 26, 2487–2514. <https://doi.org/10.1007/s11276-019-01976-x>
- 374 2. Attiah, M. L., Isa, A. A. M., Zakaria, Z., Mohsen, M. K., Abdulhameed, M. K., & Dinar, A. M. (2020). Joint
375 QoE-based user association and efficient cell–carrier distribution for enabling fully hybrid spectrum sharing
376 approach in 5G mmWave cellular networks. *Wireless Networks*, 26, 2487–2514.
377 <https://doi.org/10.1007/s11276-019-02109-0>

- 378 3. Padarti, V. K., & Rao, N. V. (2020). Adaptive SOICAF Algorithm for PAPR Mitigation in OFDM Systems.
379 *Wireless Personal Communications*, 113(2), 927–943. <https://doi.org/10.1007/s11277-020-07260-y>
- 380 4. Jawhar, Y. A., Audah, L., Taher, M. A., Ramli, K. N., Shah, N. S. M., Musa, M., & Ahmed, M. S. (2019).
381 A Review of Partial Transmit Sequence for PAPR Reduction in the OFDM Systems. *IEEE Access*, 7, 18021–
382 18041. <https://doi.org/10.1109/ACCESS.2019.2894527>
- 383 5. Ibraheem, Z. T., Rahman, M. M., Yaakob, S. N., Razalli, M. S., & Kadhim, R. A. (2015). Effect of partition
384 length variability on the performance of adjacent partitioning PTS in papr reduction of OFDM systems.
385 *ISCAIE 2014 - 2014 IEEE Symposium on Computer Applications and Industrial Electronics*, 24–28.
386 <https://doi.org/10.1109/ISCAIE.2014.7010203>
- 387 6. Vidya, M., Vijayalakshmi, M., & Ramalingareddy, K. (2016). Performance enhancement of efficient
388 partitioning technique for PAPR reduction in MIMO-OFDM system using PTS. *2015 Conference on Power,
389 Control, Communication and Computational Technologies for Sustainable Growth, PCCCTSG 2015*, 247–
390 253. <https://doi.org/10.1109/PCCCTSG.2015.7503942>
- 391 7. Wen, Q., Xiao, Y., & Li, S. (2007). Improved PTS for PAPR Reduction in OFDM Systems. *ICCCAS 2007
392 - International Conference on Communications, Circuits and Systems 2007*, 280–283.
393 <https://doi.org/10.1109/ICCCAS.2007.6250053>
- 394 8. Hong, C., Qin, Q., & Chao, T. (2013). An PTS optimization algorithm for PAPR reduction of OFDM system.
395 *Proceedings - 2013 International Conference on Mechatronic Sciences, Electric Engineering and Computer,
396 MEC 2013*, 3775–3778. <https://doi.org/10.1109/MEC.2013.6885649>
- 397 9. Miao, L., & Sun, Z. (2013). PTS algorithm for PAPR suppression of WOFDM system. *Proceedings - 2013
398 International Conference on Mechatronic Sciences, Electric Engineering and Computer, MEC 2013*, (3),
399 1144–1147. <https://doi.org/10.1109/MEC.2013.6885236>
- 400 10. Taher, M. A., Radhi, H. S., & Jameil, A. K. (2020). Enhanced F-OFDM candidate for 5G applications.
401 *Journal of Ambient Intelligence and Humanized Computing*, (0123456789). [https://doi.org/10.1007/s12652-020-02046-3](https://doi.org/10.1007/s12652-
402 020-02046-3)
- 403 11. León, O., Hernández-Serrano, J., & Soriano, M. (2010). Securing cognitive radio networks. *International
404 Journal of Communication Systems*, 23(5), 633–652. <https://doi.org/10.1002/dac>
- 405 12. Taher, M. A., Mandeep, J. S., Ismail, M., Samad, S. A., & Islam, M. T. (2010). Reducing the power envelope
406 fluctuation of OFDM systems using side information supported amplitude clipping approach. *International
407 Journal of Circuit Theory and Applications*, 24(4), 425–435. <https://doi.org/10.1002/cta>

- 408 13. Taher, M. A., Mandeep, J. S., Ismail, M., Samad, S. A., & Islam, M. T. (2013). Sliding the SLM-technique
409 to reduce the non-linear distortion in OFDM systems. *Elektronika ir Elektrotechnika*, 19(5), 103–111.
410 <https://doi.org/10.5755/j01.eee.19.5.2075>
- 411 14. Jiang, T., & Wu, Y. (2008). An overview: Peak-to-average power ratio reduction techniques for OFDM
412 signals. *IEEE Transactions on Broadcasting*, 54(2), 257–268. <https://doi.org/10.1109/TBC.2008.915770>
- 413 15. Mueller, S. H., & Huber, J. B. (1997). Novel peak power reduction scheme for OFDM. *IEEE International
414 Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC*, 3, 1090–1094.
415 <https://doi.org/10.1109/pimrc.1997.627054>
- 416 16. Kim, K. H. (2016). On the shift value set of cyclic shifted sequences for PAPR reduction in OFDM systems.
417 *IEEE Transactions on Broadcasting*, 62(2), 496–500. <https://doi.org/10.1109/TBC.2016.2529292>
- 418 17. Mhatre, K., & Khot, U. P. (2015). Efficient Selective Mapping PAPR reduction technique. *Procedia
419 Computer Science*, 45, 620–627. <https://doi.org/10.1016/j.procs.2015.03.117>
- 420 18. Sundararajan, M., & Govindaswamy, U. (2014). Multicarrier spread spectrum modulation schemes and
421 efficient FFT algorithms for cognitive radio systems. *Electronics*, 3(3), 419–443.
422 <https://doi.org/10.3390/electronics3030419>
- 423 19. Lee, K. S., Cho, Y. J., Woo, J. Y., No, J. S., & Shin, D. J. (2016). Low-complexity PTS schemes using
424 OFDM signal rotation and pre-exclusion of phase rotating vectors. *IET Communications*, 10(5), 540–547.
425 <https://doi.org/10.1049/iet-com.2015.0192>
- 426 20. Joo, H. S., Kim, K. H., No, J. S., & Shin, D. J. (2017). New PTS Schemes for PAPR Reduction of OFDM
427 Signals Without Side Information. *IEEE Transactions on Broadcasting*, 63(3), 562–570.
428 <https://doi.org/10.1109/TBC.2017.2711141>
- 429 21. Al-Jawhar, Y. A., Ramli, K. N., Taher, M. A., Shah, N. S. M., Audah, L., & Ahmed, M. S. (2018). Zero-
430 padding techniques in OFDM systems. *International Journal on Electrical Engineering and Informatics*,
431 10(4), 704–725. <https://doi.org/10.15676/ijeei.2018.10.4.6>
- 432 22. Tokur Bozkurt, Y., & Taşpinar, N. (2017). Peak-to-Average Power Ratio Reduction in Lifting Based
433 Wavelet Packet Modulation Systems Using Differential Evolution Algorithm. *Wireless Personal
434 Communications*, 94(3), 1073–1086. <https://doi.org/10.1007/s11277-016-3670-5>
- 435 23. Park, D. C., & Kim, S. C. (2018). Partial Transmit Sequence Scheme for Envelope Fluctuation Reduction in
436 OFDMA Uplink Systems. *IEEE Communications Letters*, 22(8), 1652–1655.
437 <https://doi.org/10.1109/LCOMM.2018.2840542>

- 438 24. Al-Jawhar, Y. A., Ramli, K. N., Taher, M. A., Audah, L., Shah, N. S. M., Ahmed, M. S., & Hammoodi, A.
- 439 T. (2018). An enhanced partial transmit sequence based on combining hadamard matrix and partitioning
- 440 schemes in OFDM systems. *International Journal of Integrated Engineering*, 10(3), 1–7.
- 441 <https://doi.org/10.30880/ijie.2018.10.03.001>
- 442 25. Hung, H. L. (2011). Using evolutionary computation technique for trade-off between performance peak-to
- 443 average power ration reduction and computational complexity in OFDM systems. *Computers and Electrical*
- 444 *Engineering*, 37(1), 57–70. <https://doi.org/10.1016/j.compeleceng.2010.08.002>
- 445 26. Varahram, P., & Ali, B. M. (2011). A low complexity partial transmit sequence for peak to average power
- 446 ratio reduction in OFDM systems. *Radioengineering*, 20(3), 677–682.
- 447

Figures

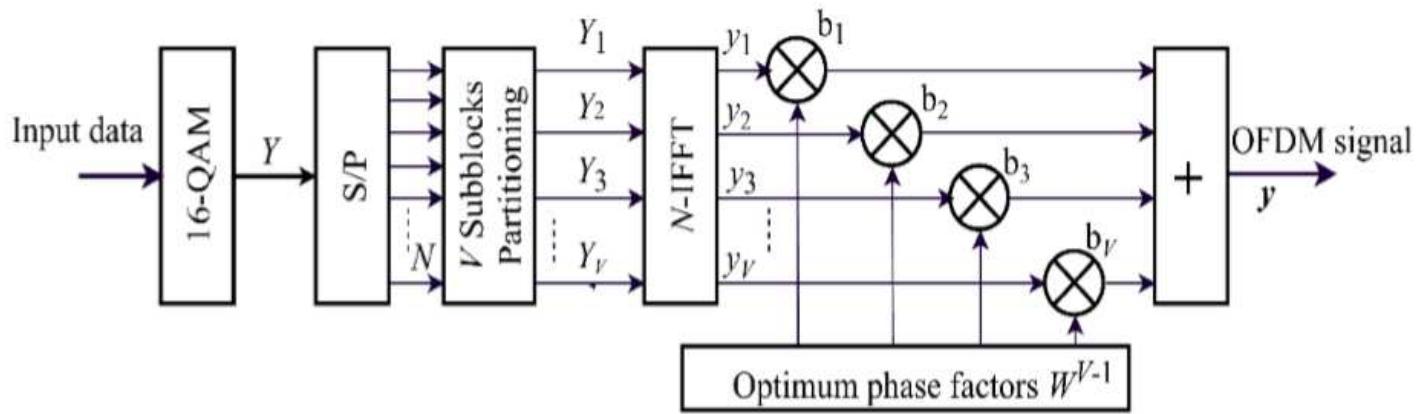


Figure 1

C-PTS block diagram [21]

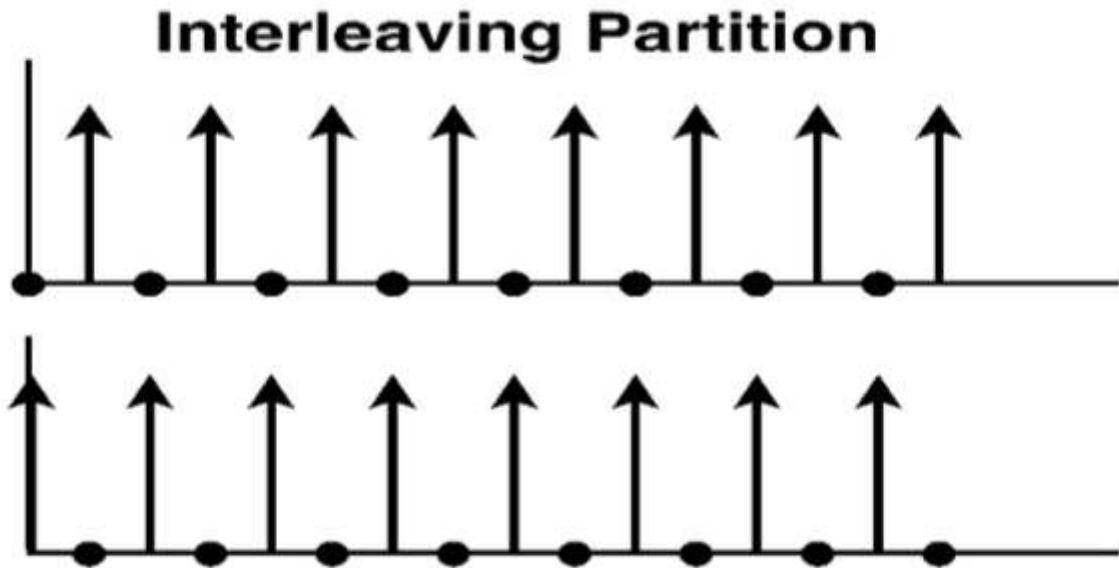


Figure 2

Interleaving segmentation scheme

Pseudo-random Partition

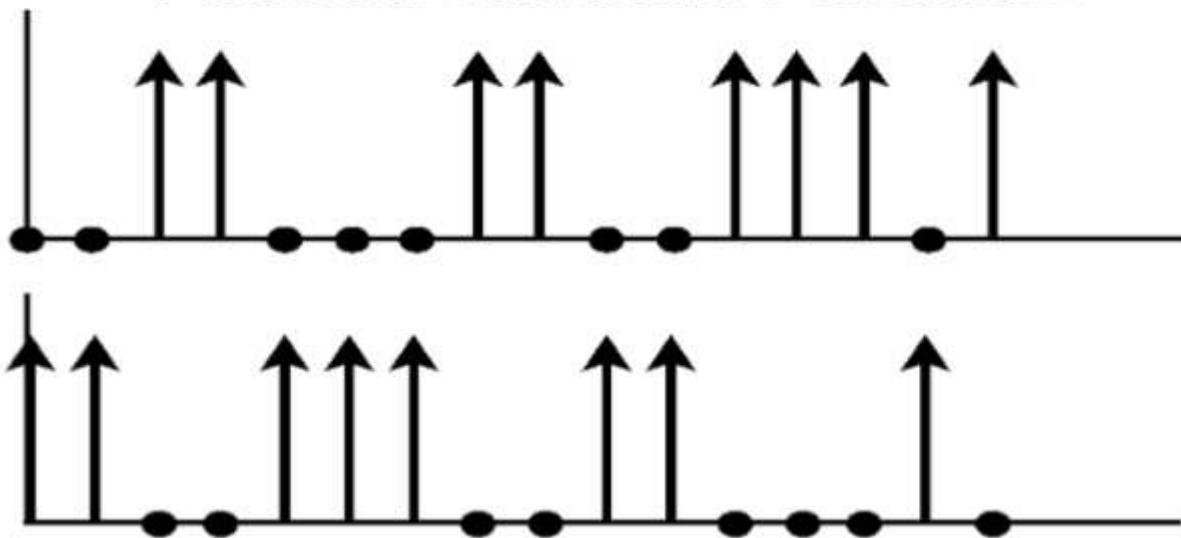


Figure 3

Pseudo-random segmentation scheme

Adjacent Partition

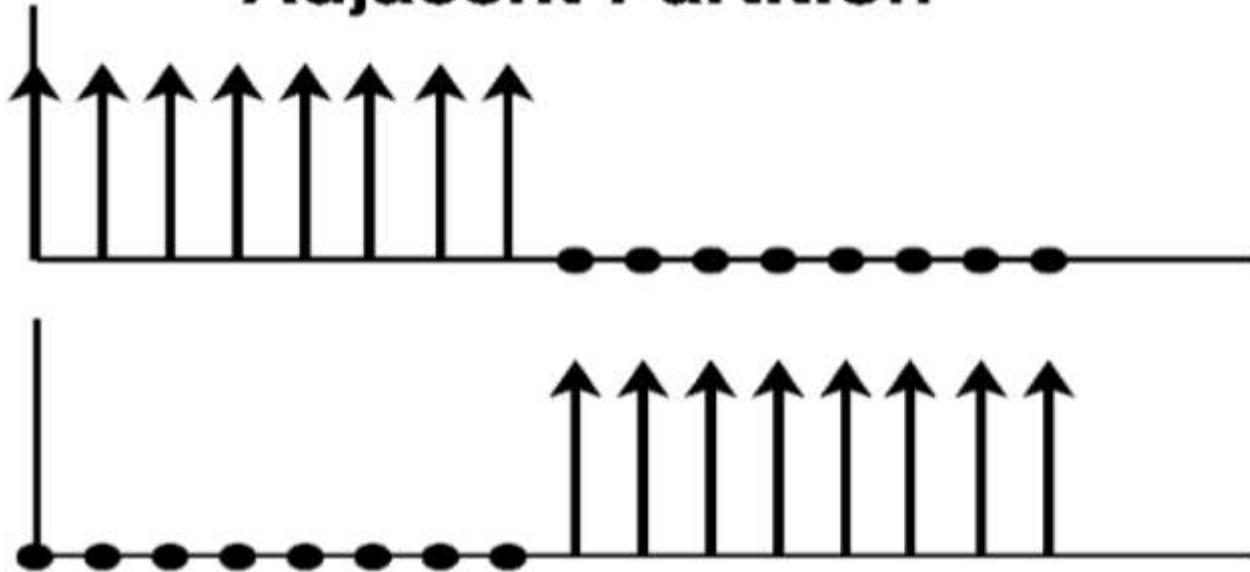


Figure 4

Adjacent segmentation scheme

Interleaving Matrix

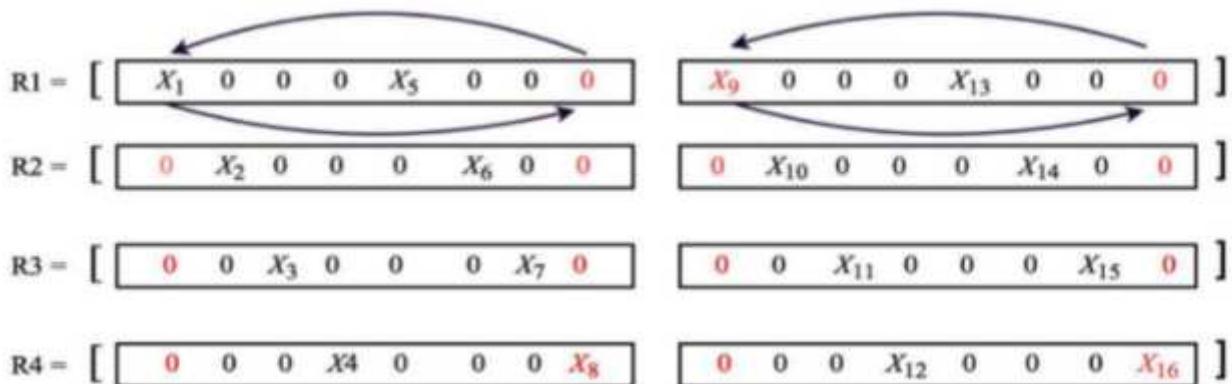
$$IL1 = [X_1 \ 0 \ 0 \ 0 \ X_5 \ 0 \ 0 \ 0 \ X_9 \ 0 \ 0 \ 0 \ X_{13} \ 0 \ 0 \ 0]$$

$$IL2 = [0 \ X_2 \ 0 \ 0 \ 0 \ X_6 \ 0 \ 0 \ 0 \ X_{10} \ 0 \ 0 \ 0 \ X_{14} \ 0 \ 0]$$

$$IL3 = [0 \ 0 \ X_3 \ 0 \ 0 \ 0 \ X_7 \ 0 \ 0 \ 0 \ X_{11} \ 0 \ 0 \ 0 \ X_{15} \ 0]$$

$$IL4 = [0 \ 0 \ 0 \ X_4 \ 0 \ 0 \ 0 \ X_8 \ 0 \ 0 \ 0 \ X_{12} \ 0 \ 0 \ 0 \ X_{16}]$$

Rotation Operation



Rotation Interleaving Segmentation Matrix

$$R-IL1 = [0 \ 0 \ 0 \ 0 \ X_5 \ 0 \ 0 \ X_1 \ 0 \ 0 \ 0 \ 0 \ X_{13} \ 0 \ 0 \ X_9]$$

$$R-IL2 = [0 \ X_2 \ 0 \ 0 \ 0 \ X_6 \ 0 \ 0 \ 0 \ X_{10} \ 0 \ 0 \ 0 \ X_{14} \ 0 \ 0]$$

$$R-IL3 = [0 \ 0 \ X_3 \ 0 \ 0 \ 0 \ X_7 \ 0 \ 0 \ 0 \ X_{11} \ 0 \ 0 \ 0 \ X_{15} \ 0]$$

$$R-IL4 = [X_8 \ 0 \ 0 \ X_4 \ 0 \ 0 \ 0 \ X_{16} \ 0 \ 0 \ X_{12} \ 0 \ 0 \ 0 \ 0 \ 0]$$

Figure 5

R-IL-PTS segmentation scheme when $N = 16$ and $V = 4$

Adjacent Matrix

$$\begin{aligned} \text{Ad1} &= [X_1 \ X_2 \ X_3 \ X_4 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ \text{Ad2} &= [0 \ 0 \ 0 \ 0 \ X_5 \ X_6 \ X_7 \ X_8 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ \text{Ad3} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ X_9 \ X_{10} \ X_{11} \ X_{12} \ 0 \ 0 \ 0 \ 0 \ 0] \\ \text{Ad4} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ X_{13} \ X_{14} \ X_{15} \ X_{16}] \end{aligned}$$

Rotation Operation

$$\begin{aligned} R1 &= [\boxed{X_1 \ X_2 \ X_3 \ X_4 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0}] \\ R2 &= [\boxed{0 \ 0 \ 0 \ 0 \ X_5 \ X_6 \ X_7 \ X_8 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0}] \\ R3 &= [\boxed{0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ X_{10} \ X_{11} \ X_{12} \ 0 \ 0 \ 0 \ 0 \ 0}] \\ R4 &= [\boxed{0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ X_{13} \ X_{14} \ X_{15} \ X_{16}}] \end{aligned}$$

Rotation Adjacent Segmentation Matrix

$$\begin{aligned} \text{R-Ad1} &= [0 \ X_2 \ X_3 \ X_4 \ 0 \ 0 \ 0 \ X_1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ \text{R-Ad2} &= [X_8 \ 0 \ 0 \ 0 \ X_5 \ X_6 \ X_7 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \\ \text{R-Ad3} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ X_{10} \ X_{11} \ X_{12} \ 0 \ 0 \ 0 \ 0 \ X_9] \\ \text{R-Ad4} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ X_{16} \ 0 \ 0 \ 0 \ X_{13} \ X_{14} \ X_{15} \ 0] \end{aligned}$$

Figure 6

R-Ad-PTS segmentation method, N = 16 and V=4

Pseudo-Random Matrix

$$PR1 = [0 \ X_6 \ 0 \ X_4 \ 0 \ 0 \ 0 \ X_{11} \ 0 \ 0 \ 0 \ 0 \ X_8 \ 0 \ 0 \ 0]$$

$$PR2 = [0 \ 0 \ 0 \ 0 \ 0 \ X_2 \ X_7 \ 0 \ 0 \ 0 \ X_1 \ 0 \ 0 \ 0 \ X_{10} \ 0]$$

$$PR3 = [X_{13} \ 0 \ 0 \ 0 \ X_5 \ 0 \ 0 \ 0 \ X_9 \ 0 \ 0 \ 0 \ X_{15} \ 0 \ 0 \ 0]$$

$$PR4 = [0 \ 0 \ X_{14} \ 0 \ 0 \ 0 \ 0 \ 0 \ X_{16} \ 0 \ 0 \ 0 \ X_3 \ 0 \ X_{12}]$$

Rotation Operation

$$R1 = [\boxed{0 \ X_6 \ 0 \ X_4 \ 0 \ 0 \ 0 \ X_{11}} \quad \boxed{0 \ 0 \ 0 \ 0 \ X_8 \ 0 \ 0 \ 0}]$$

$$R2 = [\boxed{0 \ 0 \ 0 \ 0 \ 0 \ X_2 \ X_7 \ 0} \quad \boxed{0 \ 0 \ X_1 \ 0 \ 0 \ 0 \ 0 \ X_{10} \ 0}]$$

$$R3 = [\boxed{X_{13} \ 0 \ 0 \ 0 \ X_5 \ 0 \ 0 \ 0} \quad \boxed{X_9 \ 0 \ 0 \ X_{15} \ 0 \ 0 \ 0 \ 0 \ 0}]$$

$$R4 = [\boxed{0 \ X_{14} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0} \quad \boxed{0 \ X_{16} \ 0 \ 0 \ 0 \ X_3 \ 0 \ X_{12}}]$$

Rotation Pseudo-Random segmentation Matrix

$$R-PR1 = [X_{11} \ X_6 \ 0 \ X_4 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ X_8 \ 0 \ 0 \ 0]$$

$$R-PR2 = [0 \ 0 \ 0 \ 0 \ 0 \ X_2 \ X_7 \ 0 \ 0 \ 0 \ X_1 \ 0 \ 0 \ 0 \ X_{10} \ 0]$$

$$R-PR3 = [0 \ 0 \ 0 \ 0 \ X_5 \ 0 \ 0 \ X_{13} \ 0 \ 0 \ 0 \ X_{15} \ 0 \ 0 \ 0 \ X_9]$$

$$R-PR4 = [0 \ 0 \ X_{14} \ 0 \ 0 \ 0 \ 0 \ X_{12} \ X_{16} \ 0 \ 0 \ 0 \ X_3 \ 0 \ 0]$$

Figure 7

PR-PTS segmentation method, N = 16 and V = 4

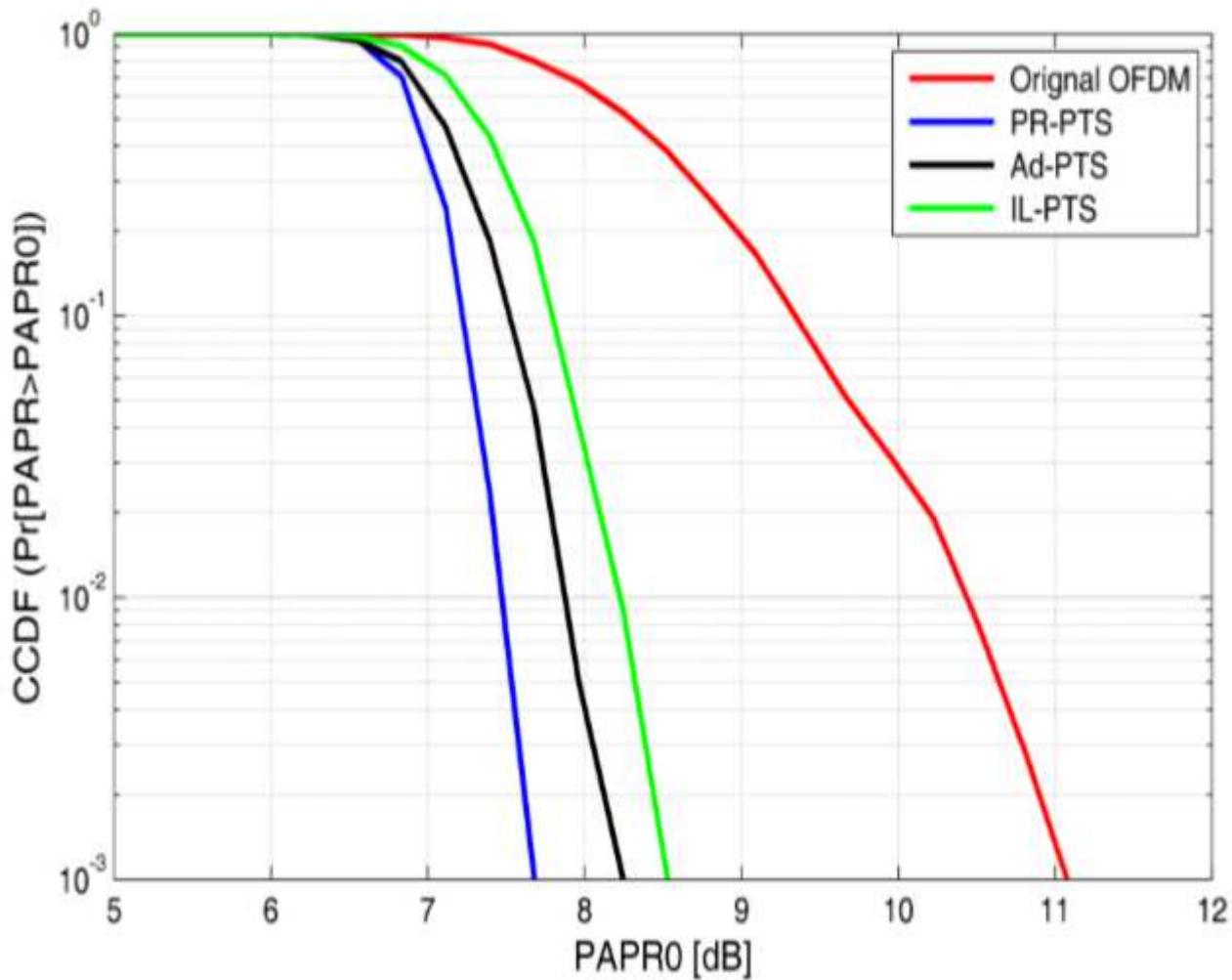


Figure 8

Comparison of the three ordinary partitioning methods, $N = 128$ and $V = 4$

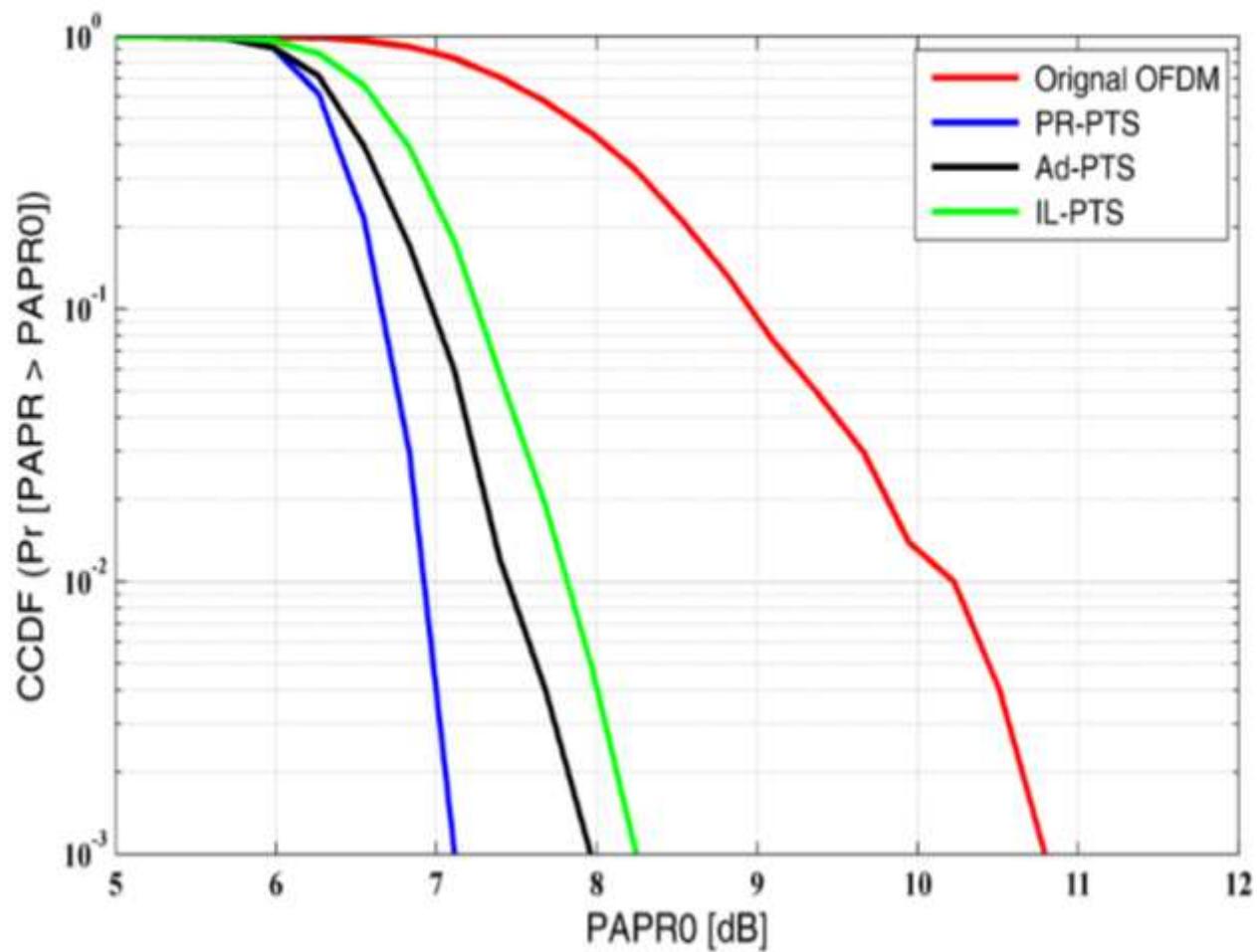


Figure 9

Comparison of the three ordinary partitioning methods, $N = 256$ and $V=4$

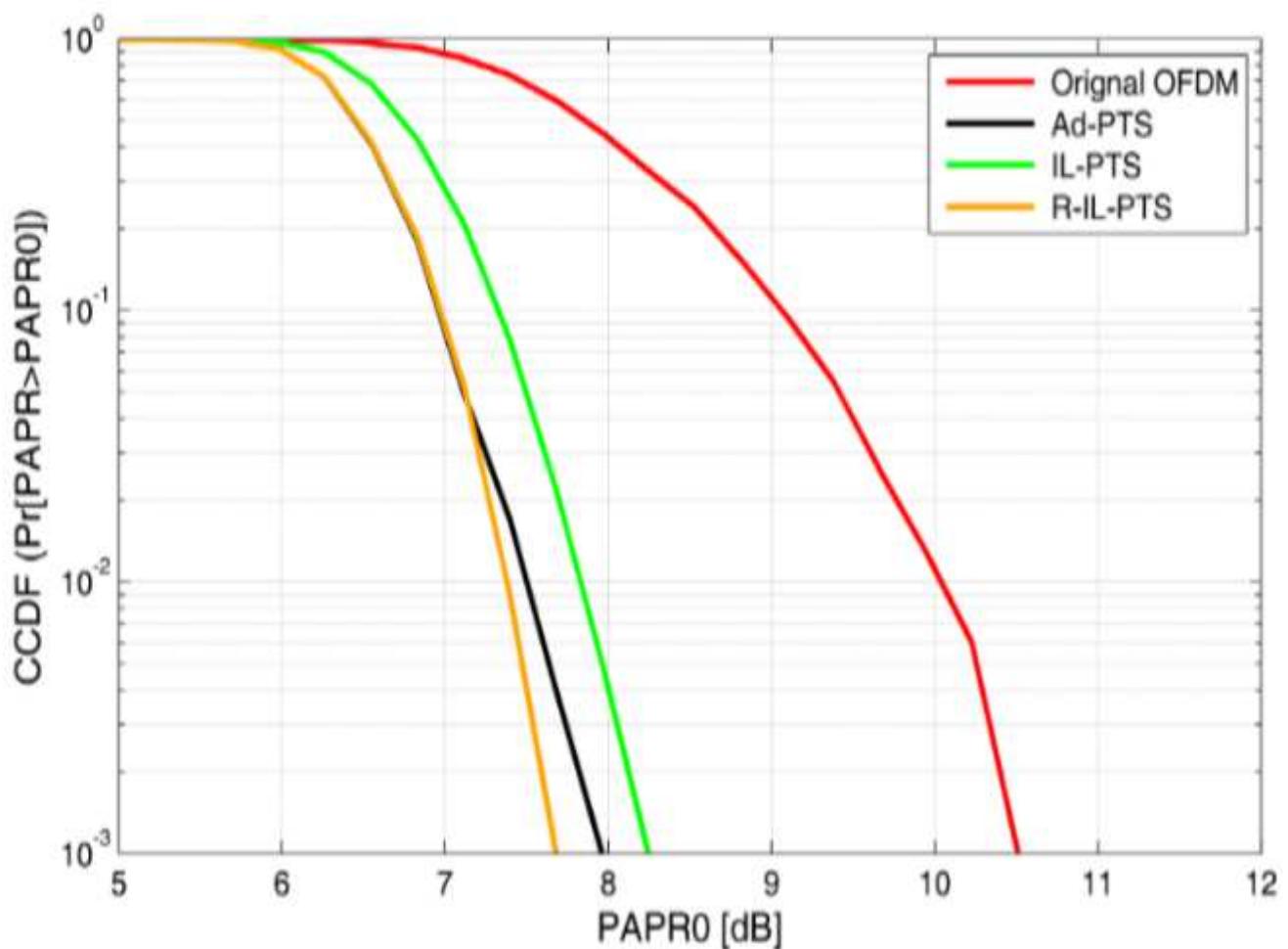


Figure 10

Comparison R-IL-PTS, IL-PTS and Ad-PTS, $N = 128$ and $V = 4$

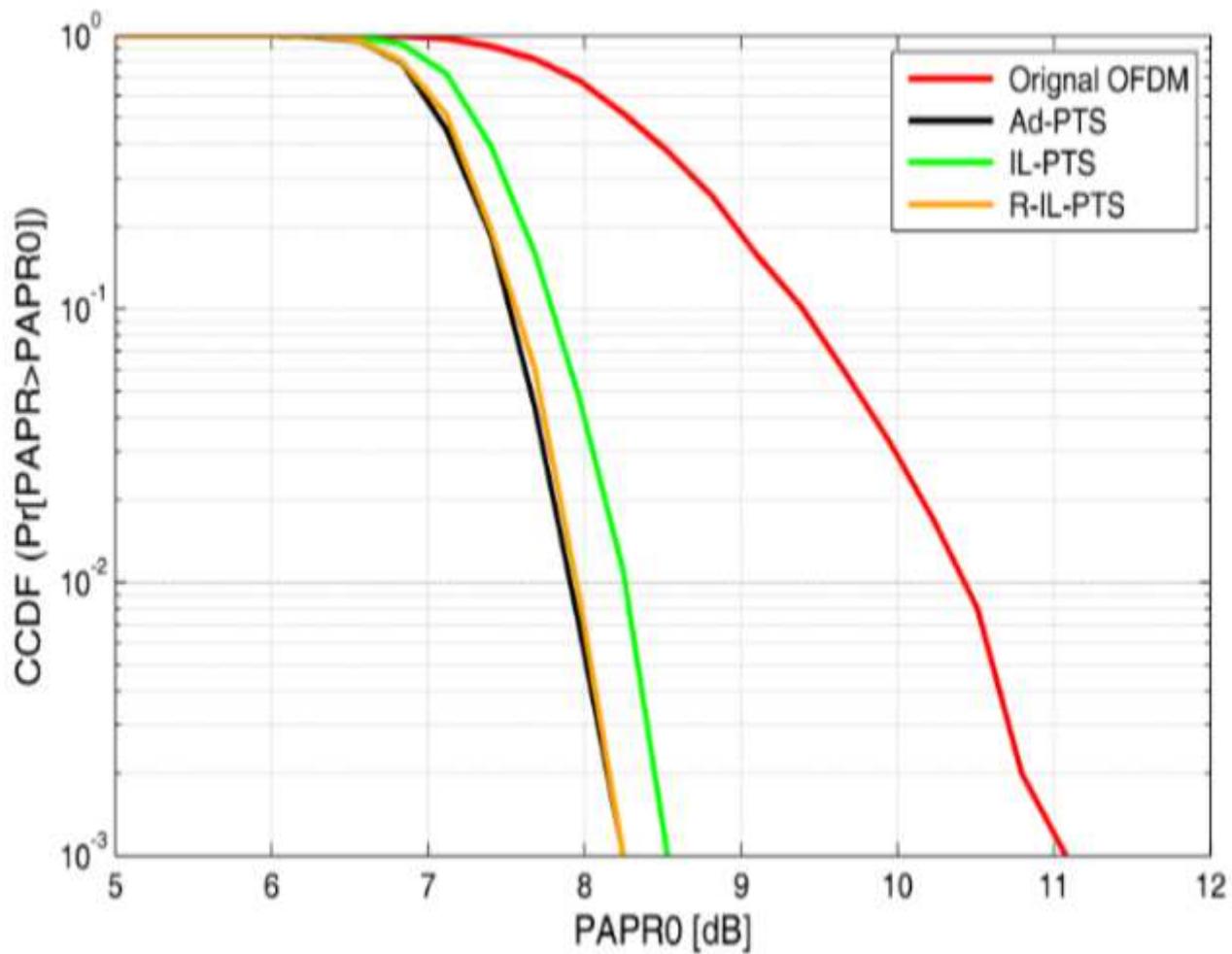


Figure 11

Comparison R-IL-PTS, IL-PTS and Ad-PTS, $N = 256$ and $V= 4$

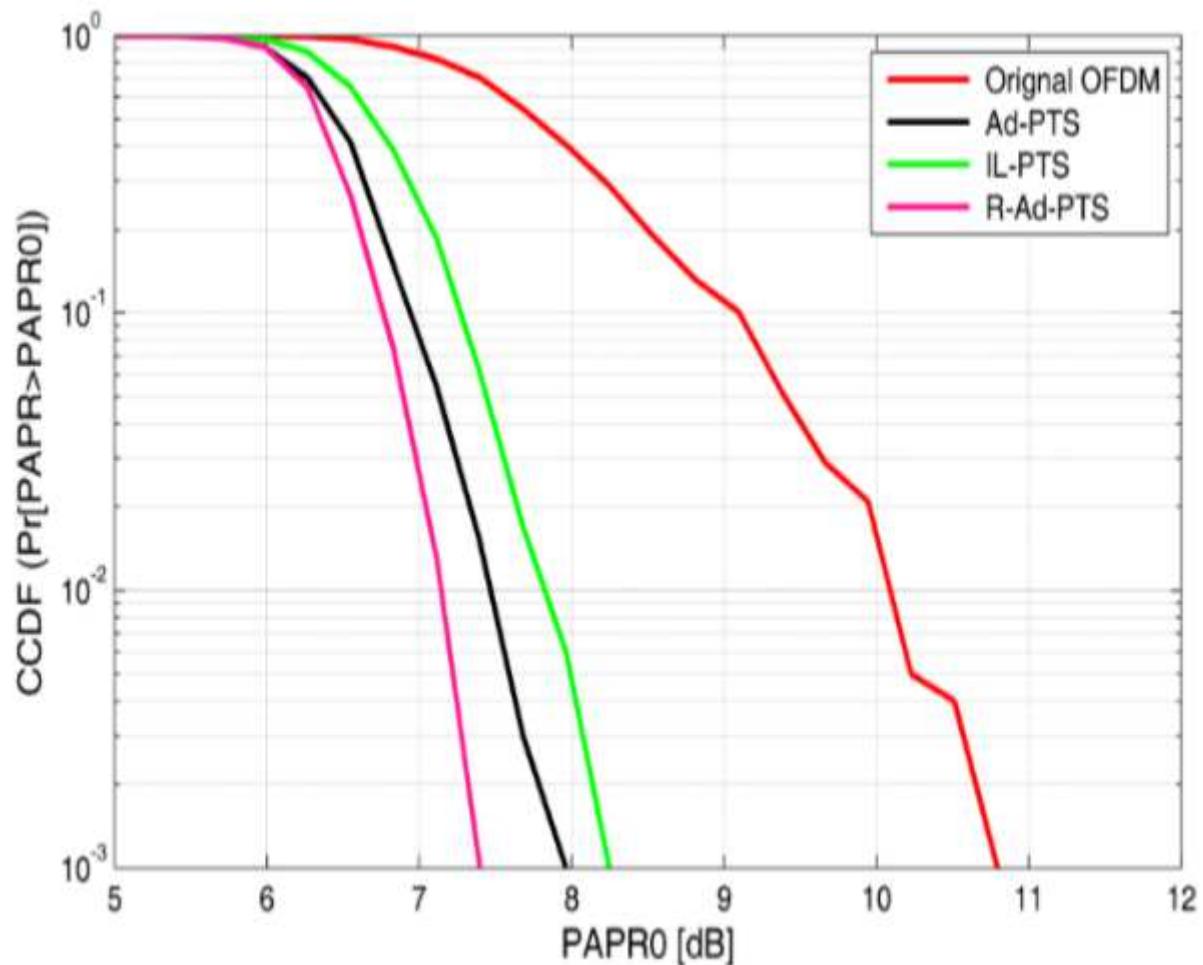


Figure 12

Comparison R-Ad-PTS, Ad-PTS and IL-PTS, $N = 128$ and $V = 4$

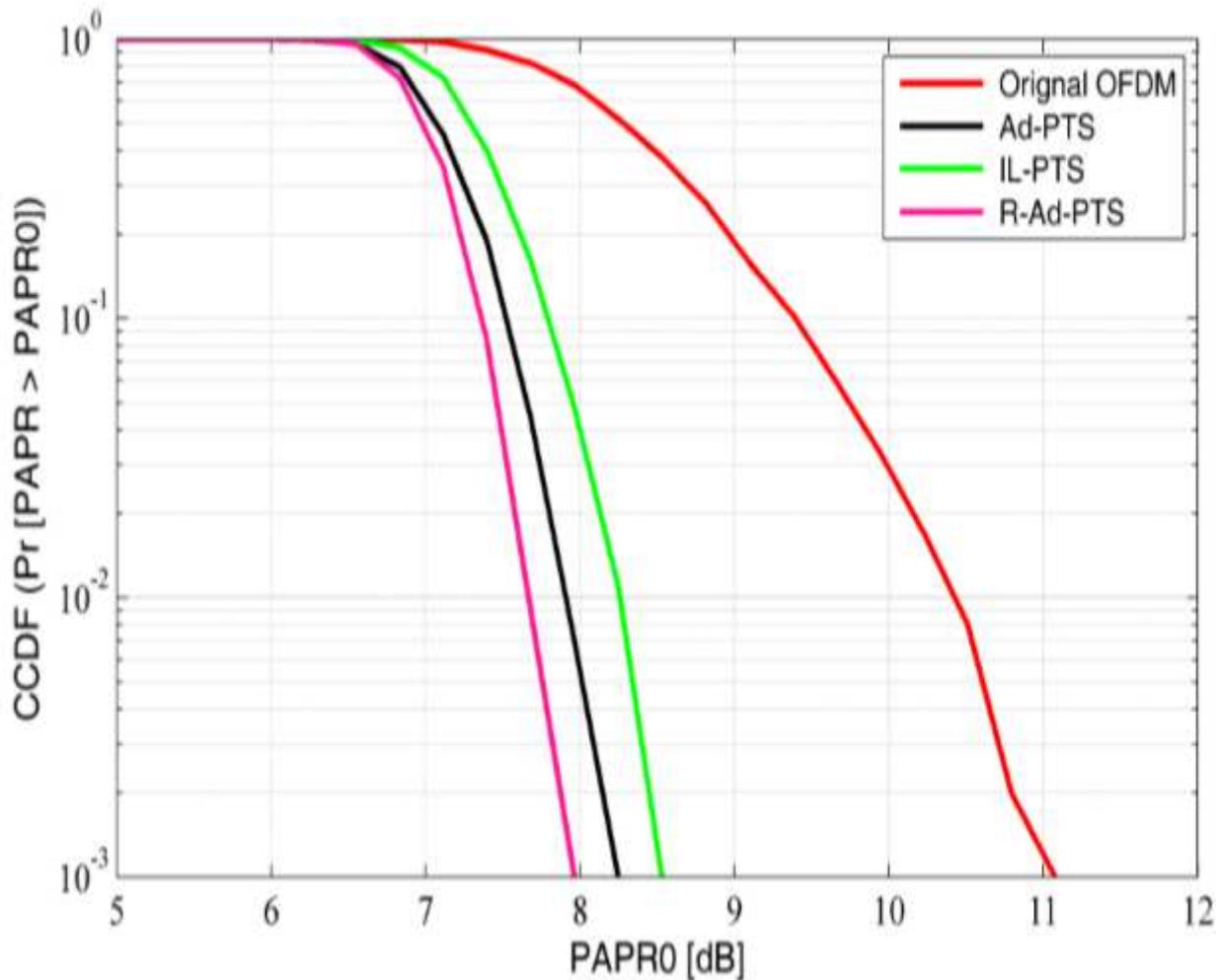


Figure 13

Comparison R-Ad-PTS, Ad-PTS and IL-PTS, $N = 256$ and $V= 4$

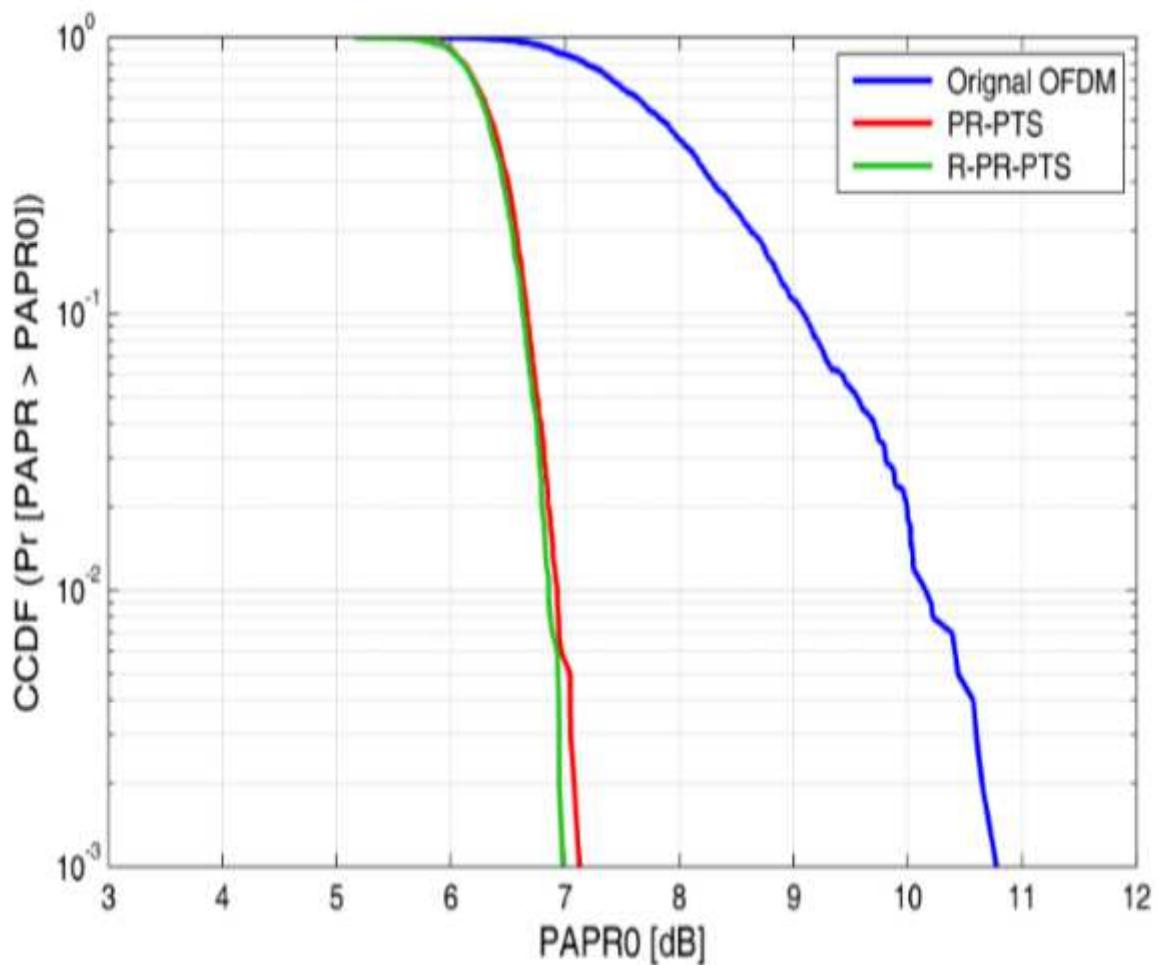


Figure 14

Comparison R-PR-PTS and PR-PTS, N = 128 and V= 4

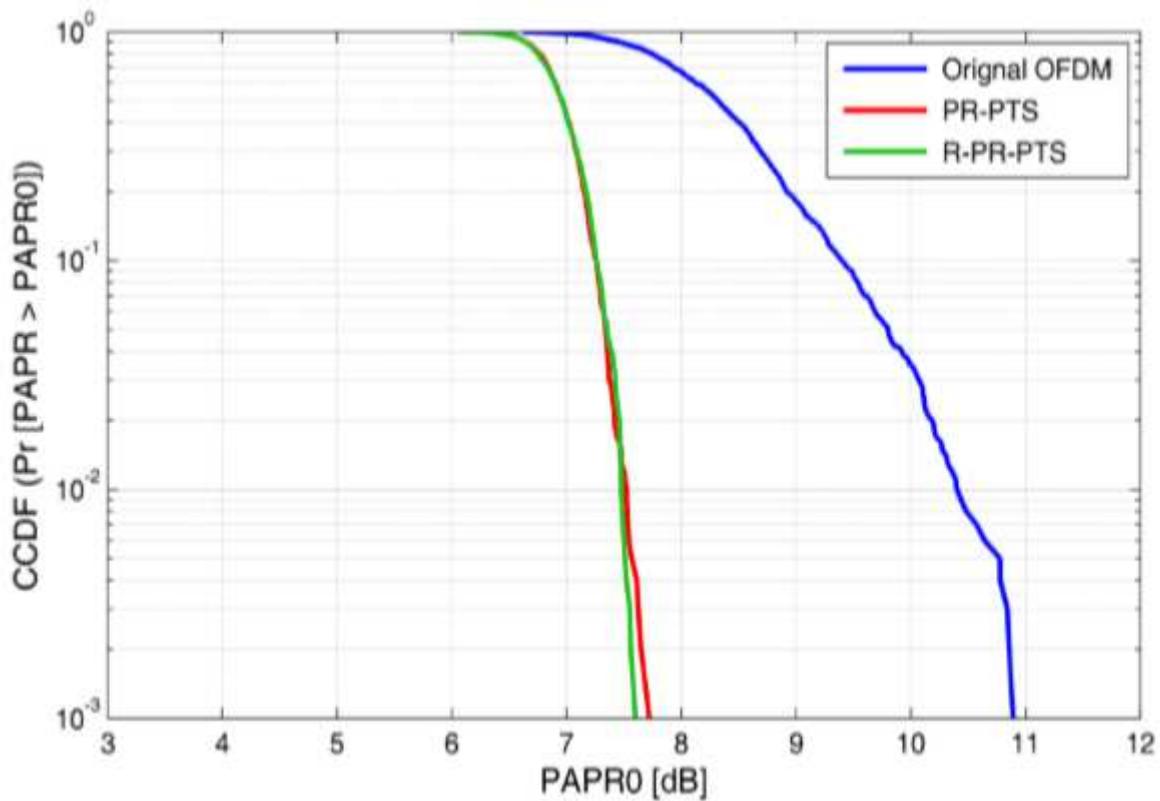


Figure 15

Comparison R-PR-PTS and PR-PTS, $N = 256$ and $V = 4$