Distributed Reed-Muller Coded-Cooperative Spatial Modulation

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Research Article

Keywords: Spatial modulation (SM), distributed Reed-Muller (DRM) codes, multiple input multiple output (MIMO), joint soft RM decoder, KP construction

Posted Date: March 16th, 2021

DOI: https://doi.org/10.21203/rs.3.rs-270805/v1

Noname manuscript No.
(will be inserted by the editor)

Distributed Reed-Muller coded-cooperative spatial modulation

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Received: date / Accepted: date

Abstract This paper proposes the distributed Reed-Muller coded spatial modulation (DRMC-SM) scheme based on Kronecker product (KP) construction. This special construction enabled an effective distribution of classical Reed-Muller (RM) code along source and relay nodes. The proposed DRMC-SM scheme not only offers robustness in bit error rate (BER) performance but also enhances the spectral efficiency due to additional antenna index transmission inculcated by spatial modulation (SM). The usefulness of KP construction over classical Plotkin (CP) construction in coded-cooperation is analysed with and without incorporating SM. An efficient criteria for selecting the optimum bits is adopted at relay node which eventually results in better weight distribution of mutually constructed (source and relay) RM code under proposed KP construction. The numerical results show that proposed KP construction outperforms CP construction by gain of 1 dB in signal to noise ratio (SNR) at bit error rate (BER) of 7×10^{-7}. Moreover, the proposed DRMC-SM scheme outperforms its non-cooperative Reed-Muller coded spatial modulation (RMC-SM) scheme as well as distributed turbo coded spatial modulation (DTC-SM) scheme in similar conditions. This prominent gain in SNR is evident due to path diversity, efficient selection of bits at relay node and the joint soft-in-soft-out (SISO) RM decoder deployed at the destination node.

Keywords Spatial modulation (SM) · distributed Reed-Muller (DRM) codes · multiple input multiple output (MIMO) · joint soft RM decoder · KP construction

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1 Introduction

Over a past decade, the MIMO techniques are widely deployed in wireless communication system to attain higher spectral efficiency [1,2] and ameliorate link reliability [3,4]. These techniques can be broadly categorized in two categories namely, spatial multiplexing and space time coding. Space time coding schemes [3,5] provide the low decoding complexity by using linear decoding and enhances the link reliability by transmitting same information through independent channels to the receiver. However, the symbol rate of such schemes is no more than 1 symbol per symbol duration regardless of number of transmit antennas. While the spatial multiplexing schemes use layered space-time approach like vertical bell lab layered space time (V-BLAST) scheme which provides higher data-rates [6]. The V-BLAST scheme allows all the transmit antennas to transmit their own stream of data simultaneously, which requires antenna synchronization and it produces high inter-antenna interference (IAI) at the receiver. As an effective remedy to this problem was suggested by Mesleh et al. who devised new type of MIMO scheme named as spatial modulation (SM) [7]. The SM exploited the unique idea of transmission by transmitting the information sequence not only via modulated symbols but also via antenna indices simultaneously. The modulated symbol is transmitted by using a single antenna known as a antenna index. Thus, the limitations of conventional MIMO schemes like antenna synchronization and IAI are efficiently averted by the employment SM that provokes an active transmit antenna during transmission [8]. The practical implementation of SM is discussed in [9]. In [10], the SM is employed into relay based systems. Moreover, different detection methods of SM along with their complexity analysis are discussed in [11].

Over the last three decades, channel coding has been proved as an effective means to improve the reliability of any communication channel. Therefore, the utilization of channel coding with cooperative schemes is an efficient method for enhancing the error performance of cooperative systems. These kind of cooperative schemes are named as coded-cooperative diversity which was pioneered by Hunter in 2002 [12]. In these schemes the channel codes at source and relay nodes mutually construct a powerful forward error-correction (FEC) code at the destination node. The source transmitted information is then decoded at the destination using joint decoding. During the last decade, various coded-cooperative schemes based on channel codes like polar codes [13,14], turbo codes [16], low density parity check (LDPC) codes [15], convolutional codes [17] have been efficiently developed. However, the performance gain provided by these schemes was based on lengthy information sequences. Furthermore, the complex encoding and decoding is required for such coded-cooperative schemes. Since there are many such application which require short length message sequences. Thus, a cooperation scheme based on RM code is proposed in [18], which offers better BER performance for small length information sequences with low encoding and decoding complexities. Moreover, the encoding of partial information sequence at the relay node reduces the latency of cooperative scheme. However, this scheme was only presented for binary phase shift keying (BPSK). Another RM coded-cooperative scheme based on rotated 4-QAM is reported in [19].

The construction of RM code in both the aforementioned schemes is based on CP construction where shorter block-length codes recursively construct a larger block-length code. As in the literature it is suggested that RM codes can be recursively constructed by various construction methods [20,21]. Therefore, we have employed KP construction [20] for proposed coded-cooperative design. This design is also based on Plotkin's construction but it is different from the CP construction. Similar kind of Plotkin's construction is used by an author in polar coded-cooperation [22]. In this construction, two short length codes can be extracted from a large length code which can be further employed in coded-cooperation like CP construction. Using this construction, the effective selection of the information bits at the relay yields a powerful code at the destination in comparison to CP construction. The intelligent selection of the information bits is performed using the efficient selection algorithm (ESA) proposed in [18]. Further, we utilize this RM code in conjunction with spectrally efficient SM technique. Hence, distributed Reed-Muller coded spatial modulation (DRMC-SM) scheme has been proposed in this manuscript. The effective utilization of SM makes DRMC-SM scheme spectrally efficient because information sequence is transmitted from both via 4-QAM symbols and antenna index simultaneously. Moreover, the deployment of soft SM demodulator along with soft RM decoder results in increased BER performance in the proposed cooperative communication system. An application of SM with space-time block code and its low complexity decoder is explained in [23]. Moreover, the bit-interleaved coded SM with joint iterative demodulation and soft decoding is detailed in findings of [24]. Furthermore, the performance analysis of trellis coded SM is carried out in [25]. Recently, a network coding scheme using SM for two way networks is reported in [26]. The novelty of this manuscript is briefed as follows

- The KP construction is effectively utilized for the construction of distributed Reed-Muller (DRM) code and its efficient employment in coded-cooperation.
- The DRMC-SM scheme for coded-cooperative and RMC-SM scheme for non-cooperative communication system have been devised.
- The powerful joint soft RM decoder (at the destination node) has been proposed for DRMC-SM scheme.
- The BER performance of the proposed scheme has also been presented for the imperfect channel case scenarios.

The renaming of the manuscript is structured as the following. Section 2 describes the basics of RM code and its distributed construction. Section 3 discusses the efficient sub-code formation by proposed construction. The preliminaries of spatial modulation and soft spatial modulation demodulator are explained in Section 4. The coded spatial modulation schemes named as DRMC-SM and RMC-SM are detailed in Section 5. The joint SISO RM decoder for DRMC-SM scheme is presented in Section 6. The simulation results of DRMC, DRMC-SM and RMC-SM schemes for different coded-cooperative scenarios are given in Section 7. Furthermore, the comparison of DRMC-SM scheme with DTC-SM scheme is also provided in this section. The manuscript is concluded in Section 8.

2 Construction of DRM codes

Classical Reed Muller codes belonged to a class of linear block codes whose rich structural properties and simple construction make it distinct over other block codes. Mathematically a binary Reed Muller code $\mathcal{R}(r,n)$ of order $r(0 \leq r \leq n)$ and code length $N = 2^n$ [21], where r and n correspond to positive integer values, is a set of all vectors $\mathbf{b}$, where $\mathbf{b}(\beta_1, \beta_2, \cdots, \beta_n)$ is a binary function which is an at most r degree polynomial [21]. The dimension and minimum hamming distance of RM code $\mathcal{R}(r,n)$ is defined by $u = \sum_{a=0}^{r} \binom{n}{a}$ and $d_{min} = 2^{n-r}$ [20], respectively.

The RM codes are rich in structural properties which allow them to be decomposed into component RM codes. Such codes are termed as Distributed Reed-Muller (DRM) codes and that can be deployed in coded-cooperation. The RM codes can be constructed by various construction techniques such as CP construction, KP construction or by algebraic construction [20,21]. In this paper, we have utilized KP construction for RM code construction. This construction has a built-in Plotkin's construction but there is subtle difference as compared to CP construction. In KP construction, a large length RM code $A_3(N_3, u_3, d_3)$ is decomposed into two short length RM codes $A_1(N_1, u_1, d_1)$ and $A_2(N_2, u_2, d_2)$, where N_k,u_k and $d_k (k = 1, 2, 3)$ that defines code length, information sequence length and code's minimum hamming distance. The obtained generator matrix through KP construction is given as

$$\mathbf{G}_{A_3} = \begin{bmatrix} \mathbf{G}_{A_1} & \mathbf{G}_{A_1} \\ \mathbf{0} & \mathbf{G}_{A_2} \end{bmatrix} \tag{1}$$

where $\mathbf{G}_{A_1}$, $\mathbf{G}_{A_2}$ and $\mathbf{G}_{A_3}$ are the generator matrices of Reed Muller codes i.e., A_1, A_2 and A_3, respectively. The generator matrix $\mathbf{G}_{A_3}$ has natural Plotkin's construction. Therefore, the large length RM code A_3 can be represented by short length RM codes A_1 and A_2 in Plotkin's form such as

$$A_3 = |A_1|A_2 + A_1| = \{|\mathbf{q}_1|\mathbf{q}_2 + \mathbf{q}_1| : \mathbf{q}_2 \in A_2, \mathbf{q}_1 \in A_1\}, \tag{2}$$

where addition operation is define over GF(2). The dimension and minimum hamming distance [20] of RM code A_3 are given by $u_3 = u_1 + u_2$ and $d_3 = min\{2d_1, d_2\}$, respectively.

The proposed KP construction of RM codes is opposite to CP construction of RM codes. The construction steps of RM code A_3 are detailed as follows

- For the required RM code $\mathcal{R}(r,n)$ of length $N = 2^n$, determine $N \times N$ matrix $\mathbf{B}^{\otimes n}$, where $\mathbf{B} \triangleq \left[\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right]$ and $\otimes$ defines the Kronecker product.
- Expurgate all those rows of matrix $\mathbf{B}^{\otimes n}$ with hamming weights less than 2^{n-r} to obtained the generator matrix $\mathbf{G}_{A_3}$ of RM code $\mathcal{R}(r,n)$.

- The obtained generated matrix $\mathbf{G}_{A_3}$ of Reed Muller code $\mathcal{R}(r,n)$ is already in Plotkin's form therefore the matrices $\mathbf{G}_{A_1}$ and $\mathbf{G}_{A_2}$ is extracted from matrix $\mathbf{G}_{A_3}$, where $\mathbf{G}_{A_1}$ is a $u_1 \times N/2$ generator matrix of Reed Muller code $\mathcal{R}(r,n-1)$ and $\mathbf{G}_{A_2}$ is a $u_2 \times N/2$ generator matrix of Reed Muller code $\mathcal{R}(r-1,n-1)$, besides $u_1 > u_2$.

It should be noted that the generator matrix $\mathbf{G}_{A_k}$,$(k = 1, 2, 3)$ obtained from KP construction is the permuted form of the generator matrix obtained from CP construction. Therefore, better codes in terms of BER can be constructed by exploiting its construction in coded-cooperation.

3 Efficient sub-code formation by proposed construction

The distributed Reed-Muller coded (DRMC) scheme for coded-cooperation is comprised of two RM codes $\mathcal{R}(r,n-1)$ and $\mathcal{R}(r-1,n-1)$. The RM code $\mathcal{R}(r,n-1)$ is placed at the source node which encodes u_1 information bits $\mathbf{p}_1 = [p_1^{(1)}, p_2^{(1)}, \cdots, p_{u_1}^{(1)}]$ into $N/2$ coded bits $\mathbf{q}_1 = [q_1^{(1)}, q_2^{(1)}, \cdots, q_{N/2}^{(1)}]$. The encoded sequence $\mathbf{q}_1 \in \mathcal{R}(r,n-1)$ is broadcasted to relay and destination nodes during first time-slot T_1. The relay node correctly decodes the information sequence $\hat{\mathbf{p}}_1$, i.e. $\hat{\mathbf{p}}_1 = \mathbf{p}_1$, whereas, the source to relay channel is considered as an ideal channel. The dimension of RM code $\mathcal{R}(r-1,n-1)$ which is placed at relay node is less than source RM code $\mathcal{R}(r,n-1)$. Therefore, only u_2 information bits $\mathbf{p}_2 = [p_1^{(2)}, p_2^{(2)}, \cdots, p_{u_2}^{(2)}]$ are encoded by relay into $N/2$ coded bits $\mathbf{q}_2 = [q_1^{(2)}, q_2^{(2)}, \cdots, q_{N/2}^{(2)}]$. In this coded-cooperative scheme, only source node is responsible for the generation of all the message bits while the relay node is not producing any extra information bits. Therefore, the u_2 information bits $\mathbf{p}_2$ for RM code $\mathcal{R}(r-1,n-1)$ must be selected wisely from u_1 information bits $\mathbf{p}_1$ i.e. $\mathbf{p}_2$ is a subset of $\mathbf{p}_1$. The coded sequences $\hat{\mathbf{q}}_1$ and $\mathbf{q}_2$ are XOR together by relay node and transmit to destination node in second time-slot T_2. In the destination, the $|\mathbf{q}_1|\hat{\mathbf{q}}_1 + \mathbf{q}_2|$ construction forms $\tilde{A}_3$ code with minimum hamming distance $d_{min}(\tilde{A}_3)$ and it can be represented as follows [18],

$$\tilde{A}_3 = |A_1|A_1 + A_2| : A_1 \in \mathcal{R}(r,n-1), A_2 \in \mathcal{R}(r-1,n-1) \quad (3)$$

where $\tilde{A}_3$ is a subcode of Reed Muller code $\mathcal{R}(r,n)$, i.e. $\tilde{A}_3 \subset \mathcal{R}(r,n)$ or $\tilde{A}_3 \subset A_3$. Furthermore, $d_{min}(\tilde{A}_3) \geq 2^{n-r}$ and the optimum sub-code is represented as A_{opt}. The optimum sub-code is determined by the algorithm proposed in [18]. The difference between the two constructions is demonstrated by the following example.

Example: Let $A_1 = \mathcal{R}(2,3)$ and $A_2 = \mathcal{R}(1,3)$ is considered as Reed Muller codes employed by DRMC scheme at source and relay nodes, respectively. The generator matrices of A_1 and A_2 RM codes are given as follows

$$\mathbf{G}^{'}_{A_1} = \begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,1\,1\,1\,1 \\ 0\,0\,1\,1\,0\,0\,1\,1 \\ 0\,1\,0\,1\,0\,1\,0\,1 \\ 0\,0\,0\,0\,0\,0\,1\,1 \\ 0\,0\,0\,0\,0\,1\,0\,1 \\ 0\,0\,0\,1\,0\,0\,0\,1 \end{bmatrix}, \quad \mathbf{G}^{'}_{A_2} = \begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,1\,1\,1\,1 \\ 0\,0\,1\,1\,0\,0\,1\,1 \\ 0\,1\,0\,1\,0\,1\,0\,1 \end{bmatrix}$$

Then the resultant RM code $A_3 = \mathcal{R}(2,4)$ is constructed using CP construction at the destination node. The generator matrix representation of RM code A_3 using (1) is defined as

$$\mathbf{G}^{'}_{A_3} = \begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,1\,1\,1\,1\,0\,0\,0\,0\,1\,1\,1\,1 \\ 0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,1 \\ 0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1 \\ 0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,0\,0\,1\,1 \\ 0\,0\,0\,0\,0\,1\,0\,1\,0\,0\,0\,0\,0\,1\,0\,1 \\ 0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,1\,0\,1\,0\,1 \end{bmatrix}$$

In case of proposed KP construction for coded-cooperation, the generator matrix of jointly constructed RM code $A_3 = \mathcal{R}(2,4)$ at the destination is determined by using the steps given in preceding section. The obtained generator matrix of code A_3 by the KP construction is given as follows

$$\mathbf{G}_{A_3} = \begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1 \\ 0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,1\,0\,0\,1\,1 \\ 0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,1\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1\,1\,1\,1\,0\,0\,0\,0\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,1\,0\,1\,0\,0\,0\,0\,0\,1\,0\,1 \\ 0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,0\,0\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,1\,0\,1\,0\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,1 \end{bmatrix}$$

The generator matrices of source and relay RM codes A_1 and A_2 are extracted from the generator matrix $\mathbf{G}_{A_3}$ such as

$$\mathbf{G}_{A_1} = \begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,1\,0\,1\,0\,1\,0\,1 \\ 0\,0\,1\,1\,0\,0\,1\,1 \\ 0\,0\,0\,1\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,1\,0\,1 \\ 0\,0\,0\,0\,0\,0\,1\,1 \end{bmatrix}, \quad \mathbf{G}_{A_2} = \begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,1\,0\,1\,0\,1\,0\,1 \\ 0\,0\,1\,1\,0\,0\,1\,1 \\ 0\,0\,0\,0\,1\,1\,1\,1 \end{bmatrix}$$

It is evident that the generator matrices $\mathbf{G}^{'}_{A_k}$ and $\mathbf{G}_{A_k}$ $(k = 1, 2, 3)$ are the row permutation of each other. For the CP construction, the optimum sub-code $A^{'}_{opt} \subset \mathcal{R}(2, 4)$ is selected with the help of efficient selection algorithm (ESA) [18]. On the basis of better weight distribution, this algorithm selects the best combination of $u_2 = 4$ information bits $\mathbf{p}_2 = [p_1^{(2)}, p_2^{(2)}, p_3^{(2)}, p_4^{(2)}]$ at relay node out of $u_1 = 7$ information bits $\mathbf{p}_1 = [p_1^{(1)}, p_2^{(1)}, \cdots, p_7^{(1)}]$ which is transmitted by source node. For CP construction the optimum selected bits at relay node are given as follows

$$p_1^{(2)} = p_2^{(1)}, p_2^{(2)} = p_3^{(1)}, p_3^{(2)} = p_6^{(1)}, p_4^{(2)} = p_7^{(1)}. \tag{4}$$

The optimum sub-code $A^{'}_{opt} \subset \mathcal{R}(2, 4)$ has the following weight distribution

$$W_{A^{'}_{opt}}(X) = 1 + 8X^4 + 32X^6 + 46X^8 + 32X^{10} + 8X^{12} + X^{16}. \tag{5}$$

Similarly, ESA is applied to the RM code constructed via KP construction yield the following optimum selected bits at relay node

$$p_1^{(2)} = p_4^{(1)}, p_2^{(2)} = p_5^{(1)}, p_3^{(2)} = p_6^{(1)}, p_4^{(2)} = p_7^{(1)}. \tag{6}$$

The optimum sub-code $A_{opt} \subset \mathcal{R}(2, 4)$ using KP construction has the following weight distribution

$$\bar{W}_{A_{opt}}(X) = 1 + 4X^4 + 32X^6 + 54X^8 + 32X^{10} + 4X^{12} + X^{16}. \tag{7}$$

It can be observed from (5) and (7) that error coefficient or the number of minimum hamming weight codewords is less for optimum sub-code $A_{opt} \subset \mathcal{R}(2, 4)$ which is caused by the KP construction.

In this paper, we have considered DRM code $\mathcal{R}(2, 5)$ for coded-cooperation. On the basis of minimum value of error coefficients K_1 and K_2 [18], the unique combinations of bit positions for relay node are obtained by ESA for each aforementioned RM construction. For sub-code $\mathcal{R}(2, 5)$ with proposed KP construction, the obtained unique combinations of bit positions by ESA are tabulated in Table 1. For the case of CP construction, the unique combinations of bit positions for sub-code $\mathcal{R}(2, 5)$ are given in [18].

For DRM code $\mathcal{R}(2, 5)$, the relay node selects $u_2 = 5$ information bits $\mathbf{p}_2 = [p_1^{(2)}, p_2^{(2)}, \cdots, p_5^{(2)}]$ out of $u_1 = 11$ information bits $\mathbf{p}_1 = [p_1^{(1)}, p_2^{(1)}, \cdots, p_{11}^{(1)}]$

transmitted by source node. On the basis of minimum value of error coefficients K_1 and K_2, the optimum combination of bit positions is selected for the CP construction which is given as [18]

$$\mathbf{p}_2 = [p_1^{(2)}, p_2^{(2)}, p_3^{(2)}, p_4^{(2)}, p_5^{(2)}] = [p_6^{(1)}, p_7^{(1)}, p_8^{(1)}, p_{10}^{(1)}, p_{11}^{(1)}], \tag{8}$$

and the obtained optimum sub-code $A'_{opt} \subset \mathcal{R}(2,5)$ using the sequence $\mathbf{p}_2$ mentioned in (8) has the following weight distribution

$$W_{A'_{opt}}(X) = 1 + 20X^8 + 416X^{12} + 1174X^{16} + 416X^{20} + 20X^{24} + X^{32}. \tag{9}$$

Similarly for proposed KP construction, the optimum combination of bit positions is selected from Table 1 based on the minimum value of error coefficients K_1 and K_2, which can be represented as

$$\begin{aligned} \mathbf{p}_2 &= [p_1^{(2)}, p_2^{(2)}, p_3^{(2)}, p_4^{(2)}, p_5^{(2)}] = [p_4^{(1)}, p_7^{(1)}, p_9^{(1)}, p_{10}^{(1)}, p_{11}^{(1)}], \\ \text{or } \mathbf{p}_2 &= [p_6^{(1)}, p_7^{(1)}, p_9^{(1)}, p_{10}^{(1)}, p_{11}^{(1)}], \end{aligned} \tag{10}$$

and the obtained optimum sub-code $A_{opt} \subset \mathcal{R}(2,5)$ using the selected combination of bits given in (10) consist of the following weight-distributions

$$W_{A_{opt}}(X) = 1 + 12X^8 + 448X^{12} + 1126X^{16} + 448X^{20} + 12X^{24} + X^{32}. \tag{11}$$

The weight distribution of the RM sub-codes A_{opt} and A'_{opt} are determined by exhaustive computer search. The weight distribution $W_{A_{opt}}(X)$ of the optimum subcode $A_{opt} \subset \mathcal{R}(2,5)$ constructed from the proposed KP construction is better than the weight distribution $W_{A'_{opt}}(X)$ of the optimum sub-code $A'_{opt} \subset \mathcal{R}(2,5)$ constructed from CP construction. This weight distribution of optimum sub-code $A_{opt} \subset \mathcal{R}(2,5)$ is the main cause of the enhanced BER performance. As, the KP construction has reduced the minimum hamming weight codewords from 20 to 12 that considerably enhances the bit error rate performance of coded-cooperative DRMC scheme.

Table 1 Combinations of bit indices which result in minimum value of K_1 and K_2

Serial No.	Sequence of bit indices	K_1	K_2
1	4 6 7 9 10	4	30
2	4 6 7 9 11	4	36
3	4 6 7 10 11	4	36
4	4 6 9 10 11	4	20
5	4 7 9 10 11	4	12
6	6 7 9 10 11	4	12

4 Spatial modulation

4.1 Preliminaries of SM

SM is a famous MIMO technique [7], which provides improve spectral efficiency due to the transmission of message bit sequence by both M-QAM symbols and antenna indices unlike tradition modulation schemes. The simple $N_t \times N_r$ SM technique with M-QAM modulation is illustrated in Fig. 1, where N_r and N_t represent no. of receive and transmit antennas, respectively. Initially, a train of $\zeta = \log_2(MN_t)$ bits is given to SM mapper that assigns $g = \log_2(M)$ bits ($\mathbf{Z}_{map}$ sequence) to M-QAM symbols z_m and $f = \log_2(N_t)$ bits ($\mathbf{Z}_{ant}$ sequence) to index $z_v \in [1 : N_t]$ of transmit antenna. It should be noted that ζ also defines the spectral efficiency of SM technique. The mapping of ζ bits to z_m and z_v is depend on a mapping table which is perfectly known to both source and destination nodes. An example of such a mapping table is given in [7]. The output of SM mapper can be represented as [11]

$$\mathbf{z}_{mv} = [0\ 0 \cdots z_m \cdots 0]^T ,$$

where $\mathbf{z}_{mv}$ is a complex sequence with N_t elements, z_m defines a complex symbol from M-QAM constellation with $E[|z_m|^2] = 1$, $m \in [1 : M]$, $v \in [1 : N_t]$ and T defines the transpose. Since a transmit antenna z_v is only take part in the transmission of z_m symbol in SM. Thus, the sequence $\mathbf{z}_{mv}$ has a non-zero element z_m at v-th position defining active transmit antenna while the rest of $N_t - 1$ zero elements defining the dormant transmit antennas.

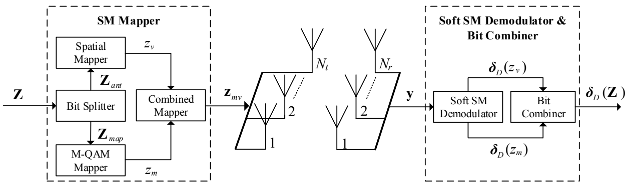

Fig. 1 Uncoded SM technique

4.2 Soft SM demodulator

The sequence $\mathbf{z}_{mv}$ after mapping is transmitted over channel $\mathbf{H}$. The received sequence is represented as

$$\begin{aligned}\mathbf{y} &= \mathbf{H}\,\mathbf{z}_{mv} + \mathbf{n}\,, \\ &= \mathbf{h}_v \mathbf{z}_m + \mathbf{n},\end{aligned} \tag{12}$$

where $\mathbf{y}$ shows $N_r \times 1$ received sequence, $\mathbf{H}$ demonstrates fading channel matrix of dimension $N_r \times N_t$, and $\mathbf{n}$ is $N_r \times 1$ AWGN sequence. Each element of $\mathbf{H}$

and $\mathbf{n}$ is identically independent distribution comprised of complex Gaussian distributed $\mathcal{CU}(0,1)$ and $\mathcal{CV}(0,\sigma^2)$, respectively. Moreover, $\mathbf{h}_v$ represents channel of active transmit antenna which is the v-th column of $\mathbf{H}$. After that the received sequence $\mathbf{y}$ undergoes soft SM demodulation. The process of soft SM demodulation is as follows. Let $\Lambda_{l,0}$ and $\Lambda_{l,1}$ represent the antenna index subsets whose every element is mapped by ζ bit sequences having 0 and 1 at l-th bit, respectively. Likewise, $\gamma_{l,0}$ and $\gamma_{l,1}$ represent the M-QAM symbol subsets whose every element is mapped by ζ bit sequences having 0 and 1 at l-th bit, respectively. Hence, the log-likelihood ratio's (LLR's) can be mathematically expressed as [24]

$$\begin{aligned}\delta_D(z_{v,l}) = \log\frac{P(z_{v,l}=0|\mathbf{y})}{P(z_{v,l}=1|\mathbf{y})} &= \log\frac{\sum_{\gamma}\sum_{\Lambda_{l,0}} P(\mathbf{y}|\mathbf{h}_v\, z_m)\prod_{\omega=1}^{f} P(z_{v,\omega})}{\sum_{\gamma}\sum_{\Lambda_{l,1}} P(\mathbf{y}|\mathbf{h}_v\, z_m)\prod_{\omega=1}^{f} P(z_{v,\omega})}, \\ &= \log\frac{\sum_{\gamma}\sum_{\Lambda_{l,0}} e^{-|\mathbf{y}-\mathbf{h}_v\, z_m|^2+\sum_{\omega=1}^{f}\log P(z_{v,\omega})}}{\sum_{\gamma}\sum_{\Lambda_{l,1}} e^{-|\mathbf{y}-\mathbf{h}_v\, z_m|^2+\sum_{\omega=1}^{f}\log P(z_{v,\omega})}},\end{aligned} \tag{13}$$

$$\delta_D(z_{m,l}) = \log\frac{P(z_{m,l}=0|\mathbf{y})}{P(z_{m,l}=1|\mathbf{y})} = \log\frac{\sum_{\gamma}\sum_{\Lambda_{l,0}} e^{-|\mathbf{y}-\mathbf{h}_v\, z_m|^2+\sum_{\omega=1}^{g}\log P(z_{m,\omega})}}{\sum_{\gamma}\sum_{\Lambda_{l,1}} e^{-|\mathbf{y}-\mathbf{h}_v\, z_m|^2+\sum_{\omega=1}^{g}\log P(z_{m,\omega})}}, \tag{14}$$

Finally, the bit-combiner concatenate the LLRs calculated from (13) and (14) to make the sequence $\boldsymbol{\delta}_D(\mathbf{Z})$. The slicer recovers the information sequence $\hat{\mathbf{Z}}$.For the coded system, the soft sequence $\boldsymbol{\delta}_D(\mathbf{Z})$ is useful for decoder which will be discussed in subsequent section.

5 Coded spatial modulation schemes

This section discusses the DRMC-SM scheme for coded-cooperative communication. Furthermore, it also explains the RMC-SM scheme for non cooperative communication.

5.1 Distributed Reed-Muller coded spatial modulation (DRMC-SM) scheme for coded cooperative communication

The rich recursive structure of RM code allows it to distribute over source and relay nodes. Therefore, an efficient DRMC-SM scheme is proposed for single relay coded cooperative communication as depicted in Fig. 2.

In DRMC-SM scheme, the message sequence $\mathbf{p}_1$ of source node (S) takes two consecutive time-slots for an end-to-end transmission. During time-slot T_1, the message sequence $\mathbf{p}_1$ of length u_1 is encoded by source node using RM code $\mathcal{R}(r, n-1)$. The encoded sequence $\mathbf{q}_1$ (a.k.a. $\mathbf{Z}^S$) of length $N_1 = 2^{n-1}$ is then undergo SM as described in Section 4. The output sequence $\mathbf{z}^S_{mv_1}(i_1) = [0,\cdots,z^S_m(i_1),\cdots,0]^T$ of SM mapper is send to relay (R) and destination (D)

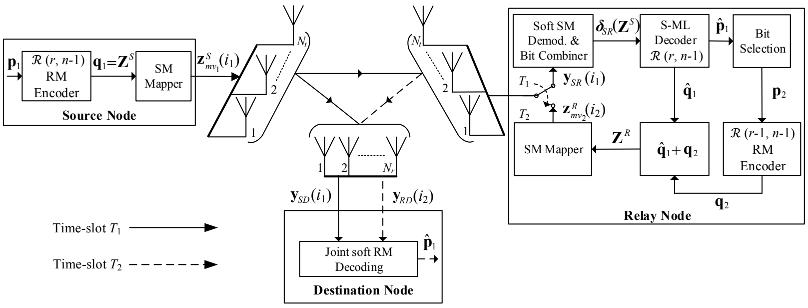

Fig. 2 Single relay DRMC-SM scheme for coded-cooperative communication systems

nodes, where $z_m^S(i_1)$ defines a complex symbol from M-QAM constellation having normalized $E[|z_m^S(i_1)|^2] = 1$, $m \in [1 : M]$, $v_1 \in [1 : N_t]$, $i_1 \in [1 : I_1]$ and $I_1 = N_1/log_2(MN_t)$ is the number of SM symbols in a codeword at source node.

At relay node, the received sequence $\mathbf{y}_{SR}(i_1)$ during time-slot T_1 can be modelled as

$$\begin{aligned} \mathbf{y}_{SR}(i_1) &= \mathbf{H}_{SR}\,\mathbf{z}_{mv_1}^S(i_1) + \mathbf{n}_{SR}(i_1), \\ &= \mathbf{h}_{SR}^{v_1}\, z_m^S(i_1) + \mathbf{n}_{SR}(i_1), \end{aligned} \tag{15}$$

where $\mathbf{H}_{SR} = [\mathbf{h}_{SR}^1, \mathbf{h}_{SR}^2, \cdots, \mathbf{h}_{SR}^{N_t}]$ is a source-to-relay (SR) slow fading $N_r \times N_t$ channel matrix. Each element of channel matrix $\mathbf{H}_{SR}$ defines a column vector like $\mathbf{h}_{SR}^{v_1} = [h_{1v_1}, h_{2v_1}, \cdots, h_{N_r v_1}]^T$. The channel matrix $\mathbf{H}_{SR}$ follows the Rayleigh fading channel model with each element distributed according to Gaussian distribution $\mathcal{CU}(0, 1)$. Moreover, $\mathbf{n}_{SR}(i_1) = [n_{SR}^1(i_1), n_{SR}^2(i_1), \cdots, n_{SR}^{N_r}(i_1)]$ is an AWGN vector with every element $n_{SR}^w(i_1)$,$w \in [1 : N_r]$ is distributed according to Gaussian distribution $\mathcal{CV}(0, \sigma^2)$. Similarly at destination node, the received sequence $\mathbf{y}_{SD}(i_1)$ during time-slot T_1 can be mathematically represented as

$$\begin{aligned} \mathbf{y}_{SD}(i_1) &= \mathbf{H}_{SD}\,\mathbf{z}_{mv_1}^S(i_1) + \mathbf{n}_{SD}(i_1), \\ &= \mathbf{h}_{SD}^{v_1}\, z_m^S(i_1) + \mathbf{n}_{SD}(i_1), \end{aligned} \tag{16}$$

where $\mathbf{H}_{SD}$, $\mathbf{h}_{SD}^{v_1}$ and $\mathbf{n}_{SD}(i_1)$ represent source-to-destination (SD) fading channel matrix of order $N_r \times N_t$, v_1-th fading channel vector and AWGN vector, respectively, defined in a similarly as $\mathbf{H}_{SR}$, $\mathbf{h}_{SR}^{v_1}$ and $\mathbf{n}_{SR}(i_1)$ in (15). All the I_1 RM coded SM symbols are send to relay and destination nodes in time-slot T_1.

During time-slot T_2, all I_1 received symbols at relay node are demodulated using soft SM demodulator and bit combiner. The soft maximum likelihood (S-ML) decoder of RM code $\mathcal{R}(r, n-1)$ is used to decode the soft coded sequence $\boldsymbol{\delta}_{SR}(\mathbf{Z}^S)$ which is provided by soft SM demodulator and bit combiner block. The decoded (detected) message sequence $\hat{\mathbf{p}}_1$ is partially encoded at relay node using Reed Muller code $\mathcal{R}(r-1,n-1)$ i.e. only u_2 message bits are chosen for encoding at relay node out of u_1 detected message bits. Since $u_2 < u_1$, therefore the selection of u_2 message bits depends upon the weight distribution criteria is described in Section 3. The effective selection of the information bits at relay node significantly enhances the bit error rate performance of the coded cooperative scheme [18]. The selected message sequence $\mathbf{p}_2$ of length u_2 is encoded using RM code $\mathcal{R}(r-1,n-1)$ to obtain the coded sequence $\mathbf{q}_2$ of length $N_2 = 2^{n-1}$. The coded sequence $\mathbf{q}_2$ and detected coded sequence $\hat{\mathbf{q}}_1$ are then GF(2) sum to get the coded sequence $\mathbf{Z}^R = \hat{\mathbf{q}}_1 + \mathbf{q}_2$. The SM is performed on the coded sequence $\mathbf{Z}^R$ and the output sequence $\mathbf{z}^R_{mv_2}(i_2) = [0,\cdots,z^R_m(i_2),\cdots,0]^T$ of SM mapper is transmitted to destination node, where $z^R_m(i_2)$ defines a complex symbol from M-QAM constellation having normalized $E[|z^R_m(i_2)|^2] = 1$, $m \in [1:M]$, $v_2 \in [1:N_t]$, $i_2 \in [1:I_2]$ and $I_2 = N_2/\log_2(MN_t)$. I_2 is the number of SM symbols in a codeword at relay node. During time-slot T_2, the received sequence $\mathbf{y}_{RD}(i_2)$ at destination node is modelled as follows

$$\begin{aligned} \mathbf{y}_{RD}(i_2) &= \mathbf{H}_{RD}\,\mathbf{z}^R_{mv_2}(i_2) + \mathbf{n}_{RD}(i_2), \\ &= \mathbf{h}^{v_2}_{RD}\, z^R_m(i_2) + \mathbf{n}_{RD}(i_2), \end{aligned} \tag{17}$$

where $\mathbf{H}_{RD}$, $\mathbf{h}^{v_2}_{RD}$ and $\mathbf{n}_{RD}(i_2)$ define relay-to-destination (RD) fading channel matrix of order $N_r \times N_t$, v_2-th fading channel vector and AWGN vector, respectively, which are represented in a same way as $\mathbf{H}_{SR}$, $\mathbf{h}^{v_1}_{SR}$ and $\mathbf{n}_{SR}(i_1)$ in (15). All I_1 and I_2 RM coded SM symbols are received by destination nodes during their respective time-slots which undergo joint soft RM decoding to obtain the detected message sequence $\hat{\mathbf{p}}_1$. The joint soft RM decoding is discussed in Section 6.

5.2 Reed-Muller coded spatial modulation (RMC-SM) scheme for non-cooperative communication system

A non-cooperative RMC-SM scheme also exploits the recursive structure of RM code. In RMC-SM scheme instead of placing the codes at relay and source nodes (as in DRMC-SM), both the codes are placed at source node as shown in Fig. 3. This scheme is also used a resonable benchmark for DRMC-SM scheme.

In this scheme, the message sequences $\mathbf{p}_1$ and $\mathbf{p}_2$ of length u_1 and u_2 are encoded by RM codes $\mathcal{R}(r,n-1)$ and $\mathcal{R}(r-1,n-1)$, respectively at source node. Since $\mathbf{p}_2$ is dependent on $\mathbf{p}_1$, so it is selected according to the same criteria used in DRMC-SM scheme which is explained in Section 3. The coded sequences $\mathbf{q}_1$ and $\mathbf{q}_2$ of each RM code are then GF(2) sum to get the coded sequence $\mathbf{q}_1 + \mathbf{q}_2$. This sequence is further concatenated with coded sequence $\mathbf{q}_1$ of RM code $\mathcal{R}(r,n-1)$ to construct the coded sequence $\mathbf{Z} = |\mathbf{q}_1|\mathbf{q}_1 + \mathbf{q}_2|$ of length $N = 2^n$. This sequence is undergone SM and transmitted to the destination node. The soft SM demodulation and S-ML decoding at the destination node finally estimates information bit sequence $\hat{\mathbf{p}}_1$.

6 Joint soft RM decoding

The joint soft Reed Muller decoding of DRMC-SM scheme is depicted in Fig. 4. The decoding is based on component like soft SM demodulator, bit combiner and S-ML decoder. In case of hard decoding, the soft metric components are replaced with the hard metric components.

At the destination node, soft SM demodulator provides LLRs $\boldsymbol{\delta}_{SD}(z_m(i_1))$ and $\boldsymbol{\delta}_{SD}(z_{v_1}(i_1))$ of M-QAM symbol and antenna index, respectively for each received sequence $\mathbf{y}_{SD}(i_1)$ during time-slot T_1. The LLRs of all I_1 RM coded SM symbols are combined by bit combiner block to construct the LLR sequence $\boldsymbol{\delta}_{SD}(\mathbf{Z}^S)$. In a similar way, the LLR sequence $\boldsymbol{\delta}_{RD}(\mathbf{Z}^R)$ is constructed by bit combiner block by combining all I_2 RM coded SM symbols during time-slot T_2.

The DRM code $\mathcal{R}(r,n)$ constructed from RM codes $\mathcal{R}(r,n-1)$ and $\mathcal{R}(r-1,n-1)$ can be decoded using an S-ML/MJL decoder in a single step [18, 19]. Therefore, a single joint coded sequence is formed by concatenating the output sequence of bit combiner blocks such as $\boldsymbol{\delta}_o = |\boldsymbol{\delta}_{SD}(\mathbf{Z}^S)|\boldsymbol{\delta}_{RD}(\mathbf{Z}^R)|$. The LLR sequence $\boldsymbol{\delta}_o$ is acted like a single codeword for RM decoder instead

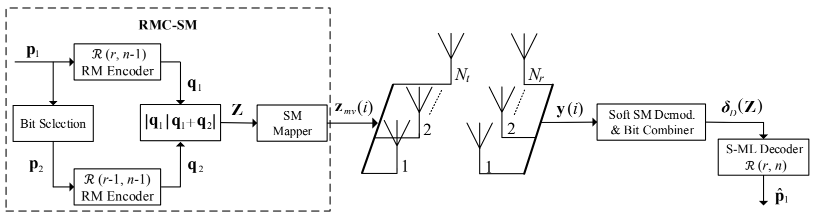

Fig. 3 Non-cooperative RMC-SM scheme

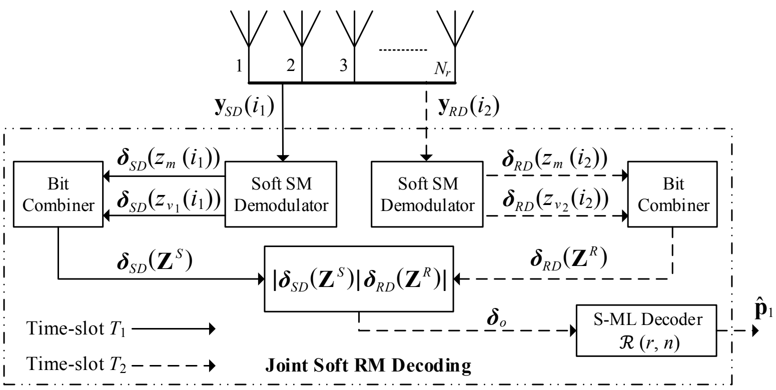

Fig. 4 Joint soft RM decoding for RMC-SM scheme

of two separate codewords $\boldsymbol{\delta}_{SD}(\mathbf{Z}^S)$ and $\boldsymbol{\delta}_{RD}(\mathbf{Z}^R)$. Different decoders of RM code are suggested in literature, MJL decoder [20] was the first RM decoder. A low complexity soft MJL decoder and a simple soft decoder for RM codes are proposed in [27,28]. In this paper, MJL and S-ML decoders are used for decoding. The decision metric of S-ML decoder can be defined as

$$\xi(\boldsymbol{\delta}_o, \mathbf{X}) = \big|\big||\boldsymbol{\delta}_{SD}(\mathbf{Z}^S)|\boldsymbol{\delta}_{RD}(\mathbf{Z}^R)| - |\mathbf{X}^S|\mathbf{X}^R|\big|\big|_F, \tag{18}$$

where $\mathbf{X}^S \in \{-1, +1\}$ and $\mathbf{X}^R \in \{-1, +1\}$ are the modulated/soft bit codewords of source and relay nodes, respectively. The well know Frobenius norm is denoted by $||.||_F$. The S-ML decoder generates the detected message sequence $\hat{\mathbf{p}} = |\hat{\mathbf{p}}_1|\hat{\mathbf{p}}_2|$ of length $u_3 = u_1 + u_2$ by using the joint LLR sequence $\boldsymbol{\delta}_o$. The source transmitted message sequence $\hat{\mathbf{p}}_1$ is recovered by selecting only the initial u_1 bits of detected message sequence $\hat{\mathbf{p}}$. In order to use the MJL decoder for the decoding of RM code, the whole process remains same except soft SM demodulator is replaced with hard SM demodulator [11]. For the details of MJL decoder are given in [20].

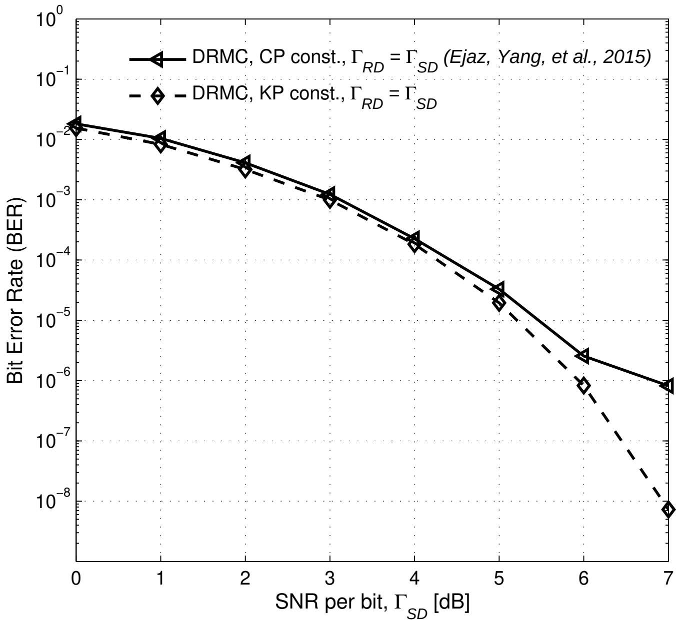

Fig. 5 BER performance of DRMC scheme for KP and CP construction over AWGN channel with optimum bit selection at relay and ideal SR channel

7 Numerical results

This section simulates and compares the bit error rate performance of DRMC, DRMC-SM and RMC-SM schemes. The optimally designed DRM code $\mathcal{R}(2,5)$ is employed in all simulations which is constructed using RM codes $\mathcal{R}(1,4)$ and $\mathcal{R}(2,4)$. The DRMC scheme is simulated over AWGN channel using B-PSK modulation while the DRMC-SM and RMC-SM schemes with 4-QAM modulation are simulated over slow Rayleigh fading channels. The term 'slow' defines that the channel remains static for an entire RM codeword and it known to the receiving end. Furthermore, the BER performance of DRMC-SM scheme is also evaluated for different transmit antennas. Hence, the spectral efficiency of DRMC-SM scheme for $N_t = 4$ and $N_t = 8$ with 4-QAM modulation is 4 b/s/Hz and 5 b/s/Hz, respectively. The SNR per bit (Γ_{SD}) of SD channel is used in all BER simulations and the code rate R_D is defined according to the code constructed at destination node i.e. $R_D = 11/32$ [18].

The coded-cooperative DRMC scheme is simulated using CP and KP constructions over AWGN channel with optimum bit selection at relay node, ideal SR channel ($\Gamma_{SR} = \infty$) and the relay node does not enjoy any additional gain in SNR over source node ($\Gamma_{RD} = \Gamma_{SD}$). The optimally selected bits at the relay node for both constructions are given as (8) and (10). The simu-

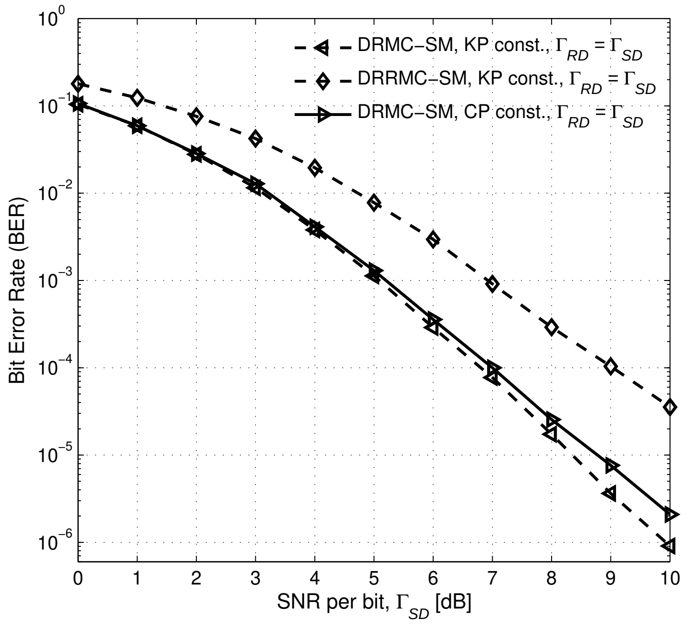

Fig. 6 Comparison between the BER performance of DRMC-SM scheme under KP and CP construction over slow Rayleigh fading channel with optimum and random bit selection at relay, ideal SR channel and $N_t = N_r = 4$

lated results in Fig. 5 clearly indicate the better BER performance of DRMC scheme with KP construction over the simulated SNR region. At low SNR region, the bit error rate performance of DRMC scheme with KP construction is slightly better than the DRMC scheme with CP construction. However, at high SNR region the difference between two schemes becomes more prominent. The DRMC scheme with KP construction produces a gain of about 1 dB in SNR over CP constructed DRMC scheme at BER$= 7 \times 10^{-7}$. This enhanced BER performance of DRMC scheme is only made possible due to better weight distribution offered by KP construction over existing CP construction.

The spectrally efficient DRMC-SM scheme is analysed over slow Rayleigh fading channel with ideal SR channel ($\Gamma_{SR} = \infty$), $N_t = N_r = 4$, $\Gamma_{RD} = \Gamma_{SD}$ and joint soft RM decoder at the destination. The comparison between the DRMC-SM schemes with KP and CP construction under optimum bit selection at relay is shown in Fig. 6. The KP construction performs better than CP construction even for DRMC-SM scheme as well. At SNR=10 dB, the DRMC-SM scheme with KP construction and DRMC-SM scheme with CP construction attain BER$= 9 \times 10^{-7}$ and BER$= 2 \times 10^{-6}$, respectively. Furthermore, we have compared the proposed DRMC-SM schemes under the optimum and random bit selection scenarios at relay node. The scheme with random bit selection at relay is named as Distributed Random Reed-Muller Coded

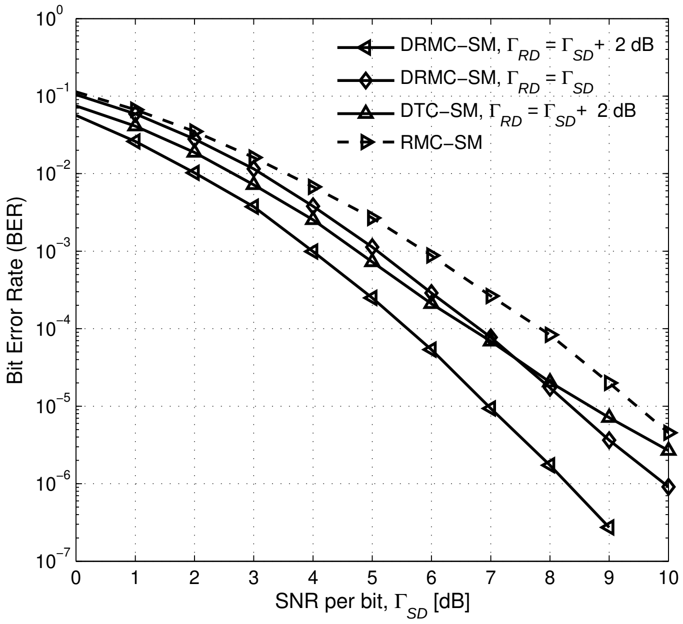

Fig. 7 BER performance of DRMC-SM, RMC-SM and DTC-SM schemes over slow Rayleigh fading channel with optimum bit selection at relay under ideal SR channel and $N_t = N_r = 4$

Spatial Modulation (DRRMC-SM) scheme. The DRMC-SM scheme obtains a bit error rate performance gain of approx. 2.2 dB over DRRMC-SM scheme at bit error rate of 3.5×10^{-5}. It shows the efficacy of DRMC-SM scheme over DRRMC-SM scheme. Thus, the proposed DRMC-SM scheme with KP construction is opted for further analysis.

For four transmit antennas ($N_t = 4$) BER performance of DRMC-SM and RMC-SM schemes under slow Rayleigh fading channel is depicted in Fig. 7. In this simulation soft SM demodulator along with S-ML decoder is used and ideal SR channel ($\Gamma_{SR} = \infty$) is considered. It can be observed from Fig. 7 that the coded cooperative DRMC-SM scheme performs better than the non cooperative RMC-SM scheme. In case relay enjoys no gain in SNR over source node, i.e. $\Gamma_{RD} = \Gamma_{SD}$, then the coded-cooperative DRMC-SM scheme provides a 1.2 dB better BER performance over non cooperative RMC-SM scheme at BER$= 4.5 \times 10^{-6}$. If relay is given an extra gain of 2 dB in SNR over source node, i.e. $\Gamma_{RD} = \Gamma_{SD} + 2$ dB, then the bit error rate performance of coded cooperative DRMC-SM scheme is further enhanced as it yields 2.5 dB performance gain over non cooperative RMC-SM scheme at BER$= 4.5 \times 10^{-6}$. Moreover, the coded cooperative DRMC-SM scheme is also compared with the equivalent coded cooperative DTC-SM scheme under similar conditions such as $R_D = 1/3$, $N = 33$ and $N_t = N_r = 4$ etc. In DTC-SM scheme, source node uses a recursive systematic convolutional (RSC) encoder having the code rate

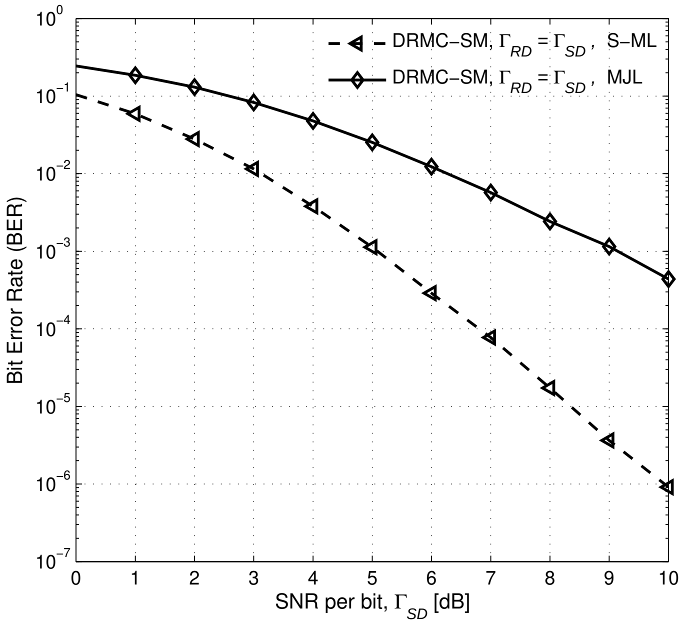

Fig. 8 Joint soft RM decoding vs joint hard RM decoding of DRMC-SM scheme under ideal SR channel and $N_t = N_r = 4$

1/2 along with SM mapper while relay node employs the same RSC encoder with an interleaver and SM mapper but with punctured systematic bit. The symmetric RSC encoders having generator matrix $\mathbf{G} = [1, 5/7]_8$ are employed at source and relay nodes. The turbo decoder (MAP) utilizes eight decoding iterations to detect the message sequence. Under similar conditions DRMC-SM scheme performs better than the DTC-SM scheme and at BER$= 2.5 \times 10^{-6}$ it attains a BER performance gain of 2.2 dB. The evident gain in performance of DRMC-SM scheme is due to the optimally design of relay node (effective bit selection) and the joint soft RM decoding performed at destination node.

In order to show the effectiveness of the joint soft RM decoding, we compare the bit error rate performance of DRMC-SM scheme under joint soft RM decoding and joint hard RM decoding. The simulated result shown in Fig. 8 that DRMC-SM scheme with joint soft RM decoding provides a gain of 4.3 dB at BER$= 4.5 \times 10^{-4}$ as compared to DRMC-SM scheme with joint hard RM decoding. The gain in bit error rate performance of DRMC-SM scheme with joint soft RM decoding is obvious due to deployment of soft SM demodulator and S-ML decoder. Since in joint hard RM decoding, the hard SM demodulator and MJL decoder is used which degrades the overall bit error rate performance of DRMC-SM scheme.

The DRMC-SM scheme is further analysed for the practical scenario. For the case of practical scenario the SR channel is assumed to be the non-ideal

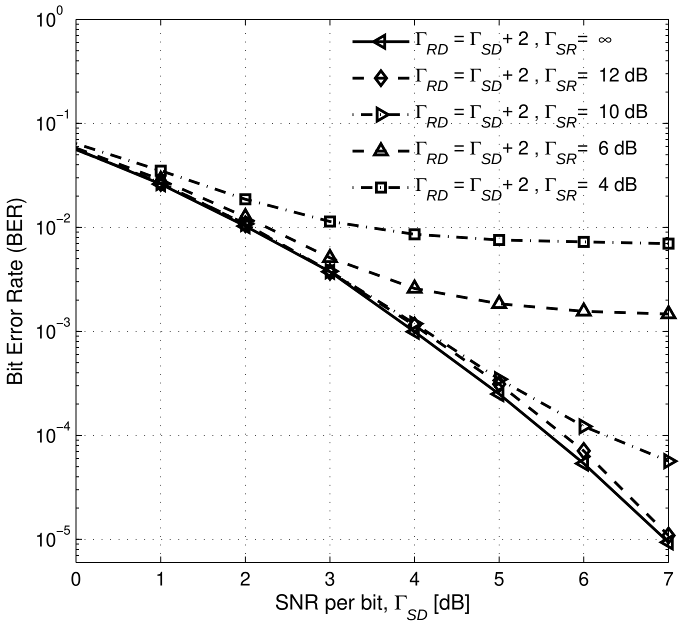

Fig. 9 BER performance of DRMC-SM scheme under non-ideal SR channel and $N_t = N_r = 4$

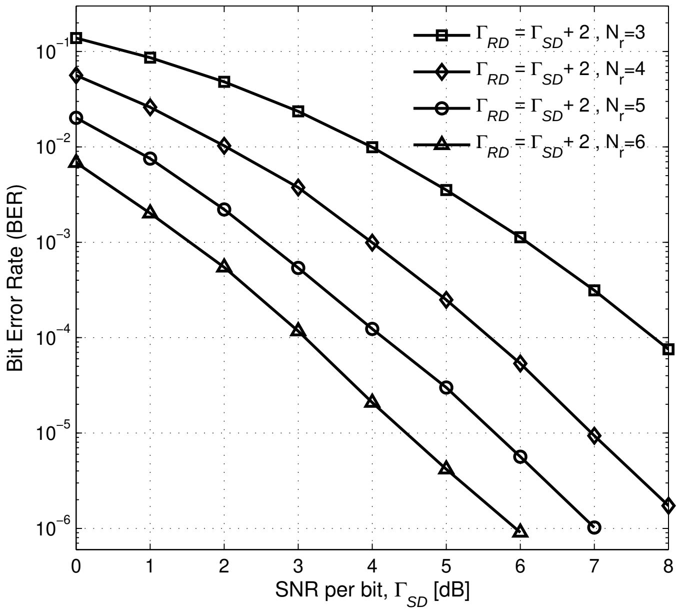

Fig. 10 The effect of receive antennas on the BER performance of DRMC-SM scheme with four transmit antennas

$\Gamma_{SR} \neq \infty$ and $\Gamma_{RD} = \Gamma_{SD} + 2$ dB. It is a known fact that the error performance of coded-cooperative scheme degrades due to lack of cooperation between source and relay node [12]. This degradation may be due to bad or noisy SR channel. Therefore, we analyse the DRMC-SM scheme for $\Gamma_{SR} = 4$ dB and $\Gamma_{SR} = 6$ dB. It can be observed from Fig. 9 that the relay is in outage for both non-ideal SR channels $\Gamma_{SR} = 4$ dB and $\Gamma_{SR} = 6$ dB, which significantly degrades the BER performance of DRMC-SM scheme. In such cases relay propagates the error to the destination node due to the incorrect detection of source information. This problem can be control by the employing any error propagation control mechanism at relay such as cyclic redundancy check (CRC) proposed in [12]. The interested reader may refer to [29] for the details of error propagation control mechanism. Further, we analyse the DRMC-SM scheme for $\Gamma_{SR} = 10$ dB and $\Gamma_{SR} = 12$ dB. It can be seen that for the case of $\Gamma_{SR} = 10$ dB, the BER performance of DRMC-SM scheme is improved. It achieves the BER$= 6 \times 10^{-6}$ at SNR=7 dB which is 1 dB away from the bit error rate performance of ideal DRMC-SM scheme ($\Gamma_{SR} = \infty$). For the case of $\Gamma_{SR} = 12$ dB, the bit error rate performance of DRMC-SM scheme is enhanced significantly and approximately reached the bit error rate performance of ideal DRMC-SM scheme ($\Gamma_{SR} = \infty$).

In Fig. 10, the effect of no. of receive antennas in DRMC-SM scheme is examined. The bit error rate performance of DRMC-SM scheme is further

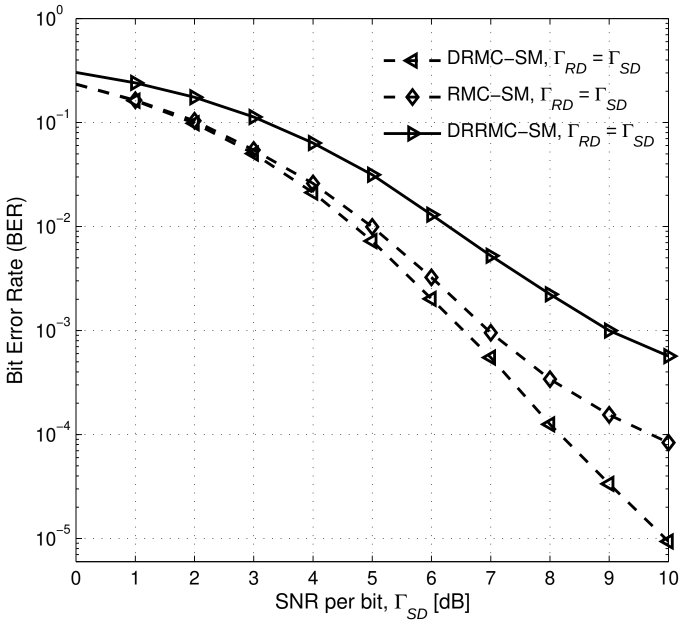

Fig. 11 BER performance of DRMC-SM, DRRMC-SM and RMC-SM schemes over slow Rayleigh fading channel, ideal SR channel, $N_t = 8$ and $N_r = 4$

improved by the addition of some receive antennas. In case receive antennas are five or six, a gain ranges from 1.5 dB to 2.5 dB is attained over four receive antennas by DRMC-SM scheme at BER$= 10^{-5}$. However, if the no. of receive antennas decreases from four to three, the BER performance degradation of about 2.3 dB is observed at BER$= 8\times10^{-5}$. Thus, there is a trade off between the bit error rate performance of DRMC-SM scheme and number of receive antennas.

Finally, the DRMC-SM scheme is analysed for eight transmit antennas ($N_t = 8$) which has the spectral efficiency of 5 b/s/Hz. It is observed from Fig. 11 that the optimum bit selection at relay node is also effective for DRMC-SM scheme with eight transmit antennas. Under similar conditions, the DRMC-SM scheme beats the DRRMC-SM scheme with the performance gain of 3 dB at BER$= 6 \times 10^{-4}$. Further, the DRMC-SM scheme is compared to RMC-SM scheme under the assumption of $\Gamma_{RD} = \Gamma_{SD}$ and optimum bit selection. DRMC-SM scheme performs better than RMC-SM scheme. At SNR=10 dB, the DRMC-SM and RMC-SM schemes attain BER$= 9 \times 10^{-6}$ and BER$= 8 \times 10^{-5}$, respectively. This shows the supremacy of DRMC-SM scheme over non-cooperative RMC-SM scheme irrespective of number of transmit antennas.

8 Conclusion

This manuscript presented KP construction of RM codes for coded cooperative communication system. This construction has offered better weight distribution for jointly constructed RM code at the destination node in comparison to CP construction. Therefore, the DRMC scheme with KP construction achieves 1 dB BER performance gain over traditional DRMC scheme having CP construction over AWGN channel. However, this scheme is spectrally inefficient. Thus, we extended the proposed scheme to spectrally efficient DRMC-SM scheme that not only transmit the coded information via modulated symbols but also via transmit antenna indices. The numerical results confirm that the DRMC-SM scheme performs better than the non-cooperative RMC-SM scheme due to the joint RM code construction, efficient bit selection (at relay), joint soft RM decoding and path diversity. Moreover, we also compared DRMC-SM scheme with DTC-SM scheme under similar conditions. The proposed DRMC-SM scheme has also shown the superior BER performance in comparison to DTC-SM scheme. The dominance of joint soft RM decoder is evaluated by comparing it with joint hard RM decoder. The DRMC-SM scheme is further investigated for more practical coded-cooperative scenarios by considering non-ideal SR channel. Finally, the effect of transmit and receive antennas on the bit error rate performance of DRMC-SM scheme is also analysed.

Acknowledgements This work is sponsored by National Natural Science Foundation of China under the contract No. 61771241.

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Figures

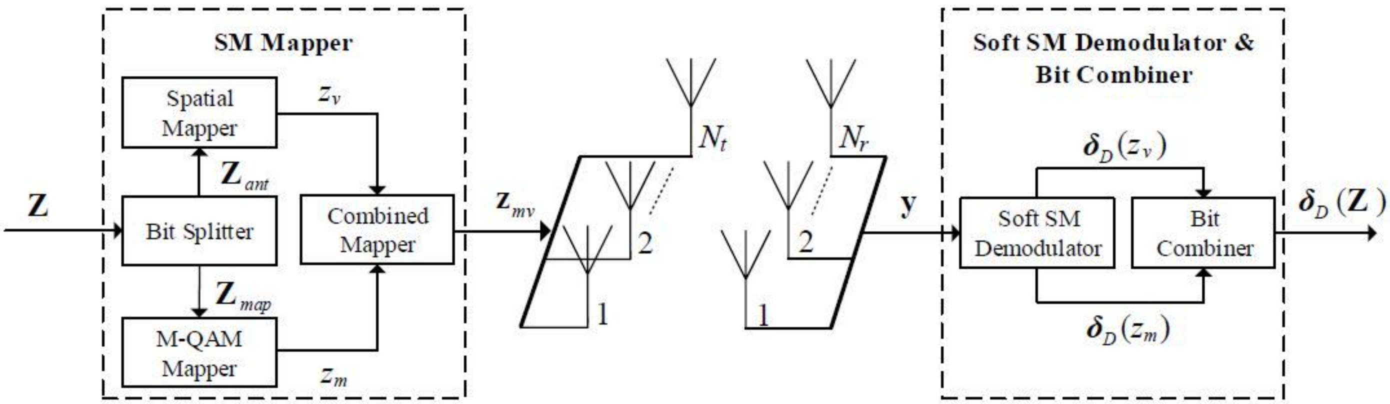

Figure 1

Uncoded SM technique

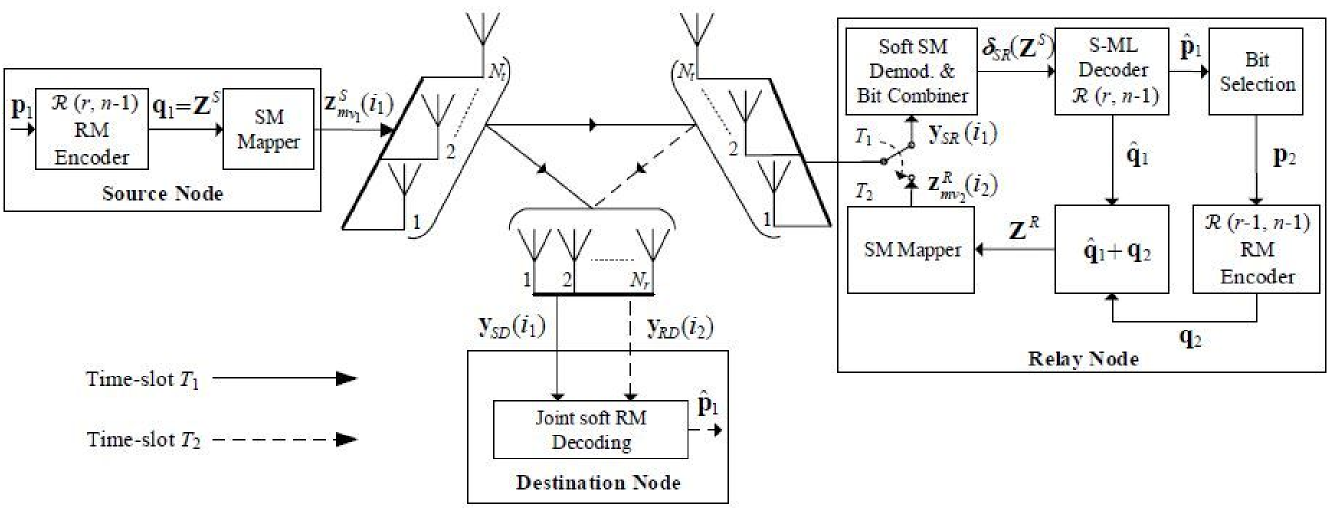

Figure 2

Single relay DRMC-SM scheme for coded-cooperative communication systems

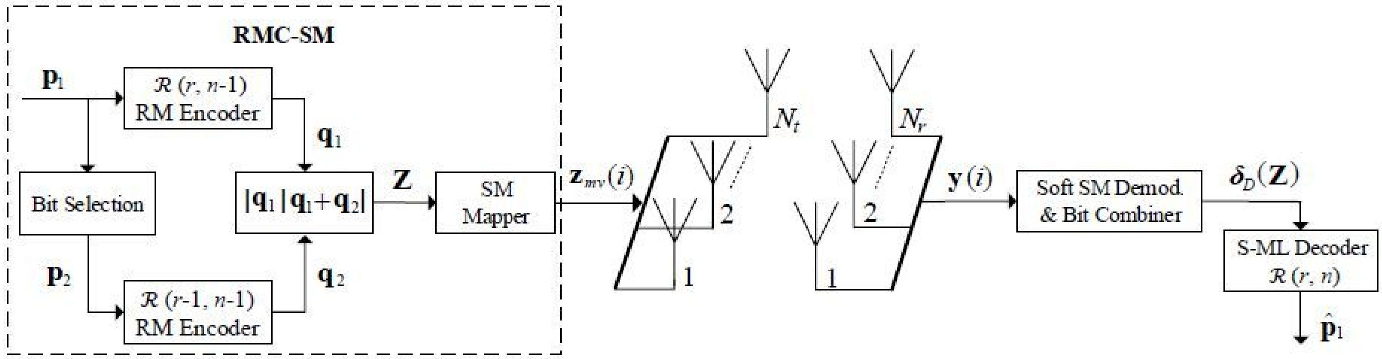

Figure 3

Non-cooperative RMC-SM scheme

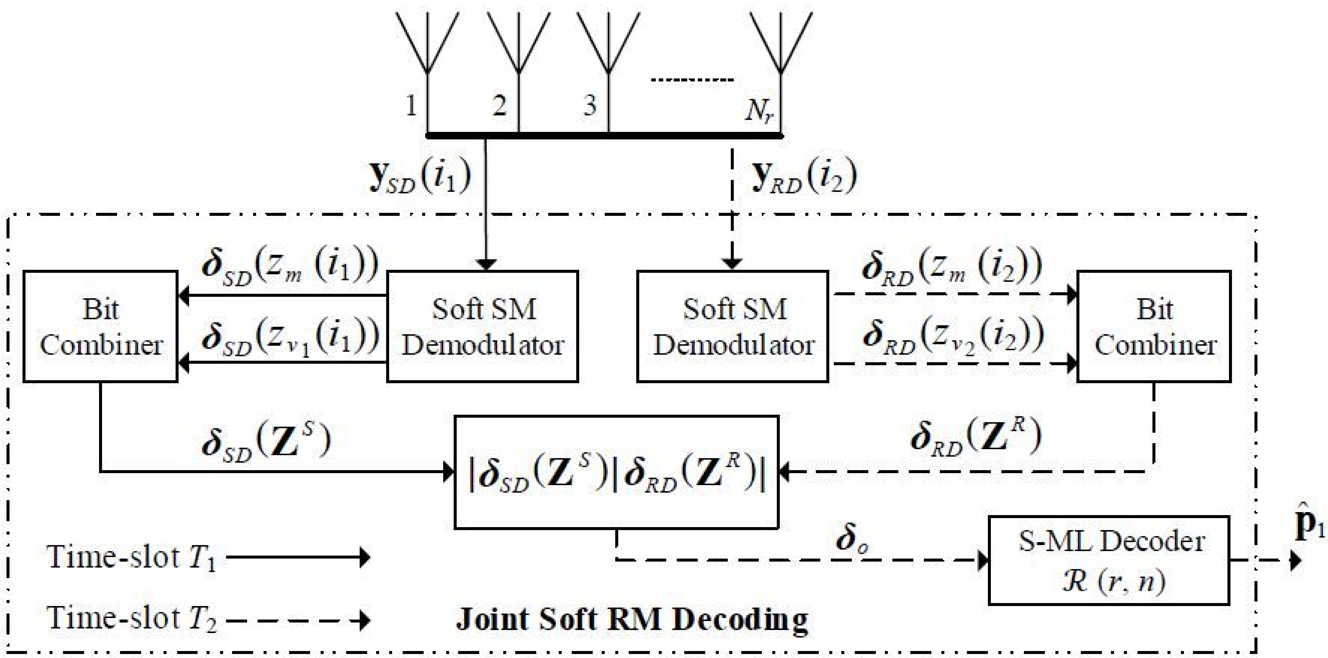

Figure 4

Joint soft RM decoding for RMC-SM scheme

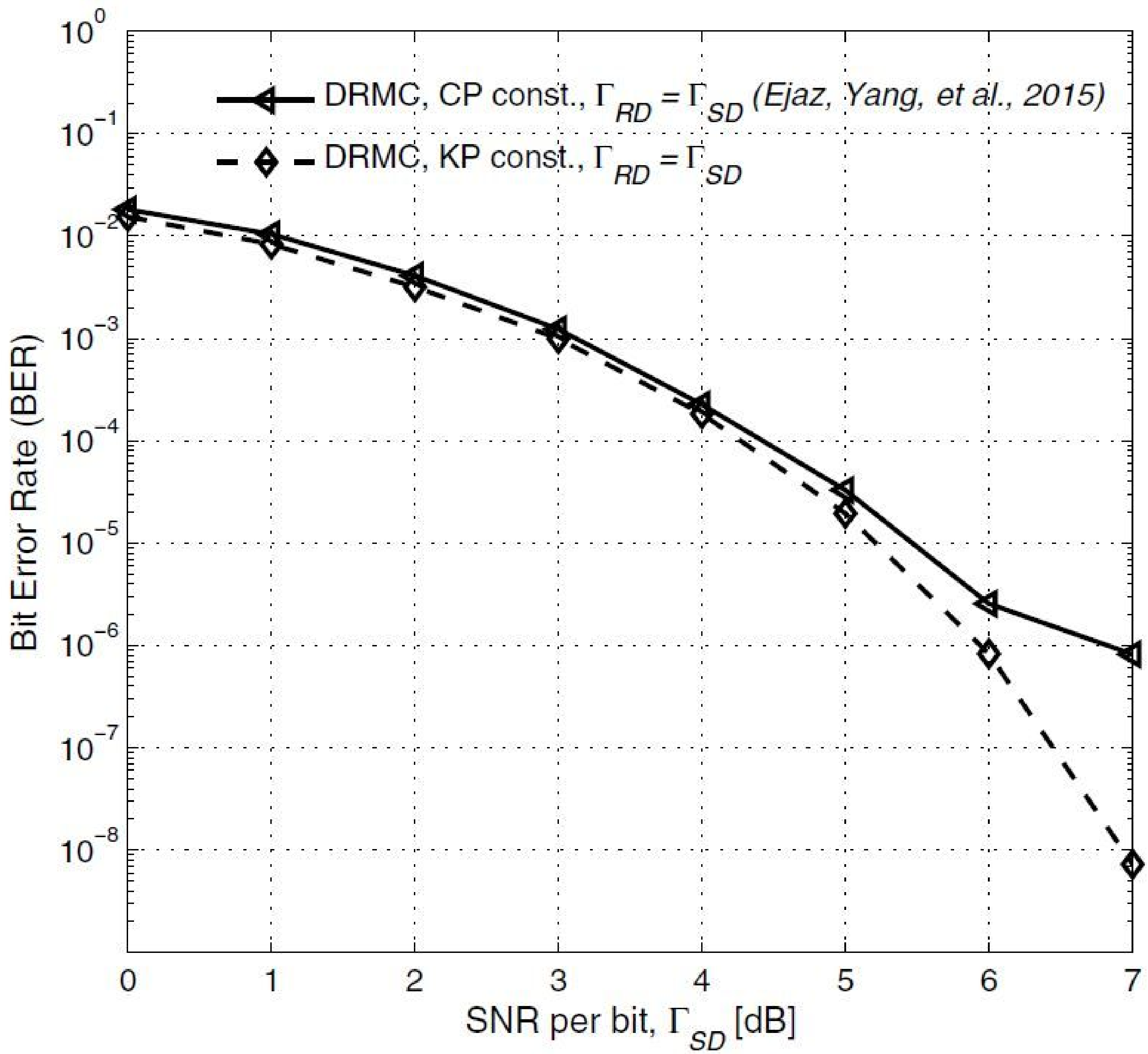

Figure 5

BER performance of DRMC scheme for KP and CP construction over AWGN chan- nel with optimum bit selection at relay and ideal SR channel

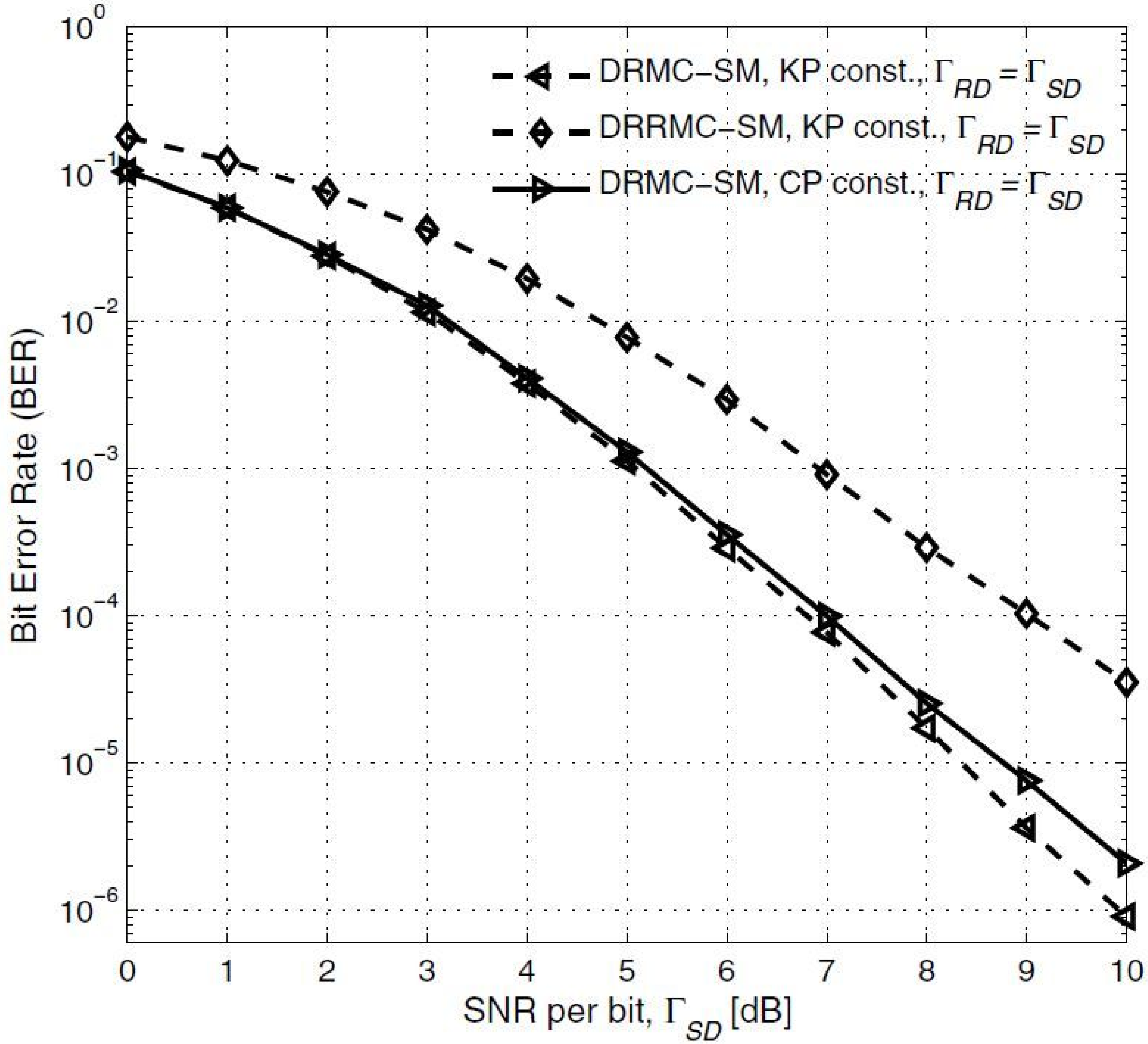

Figure 6

Comparison between the BER performance of DRMC-SM scheme under KP and CP construction over slow Rayleigh fading channel with optimum and random bit selection at relay, ideal SR channel and Nt = Nr = 4

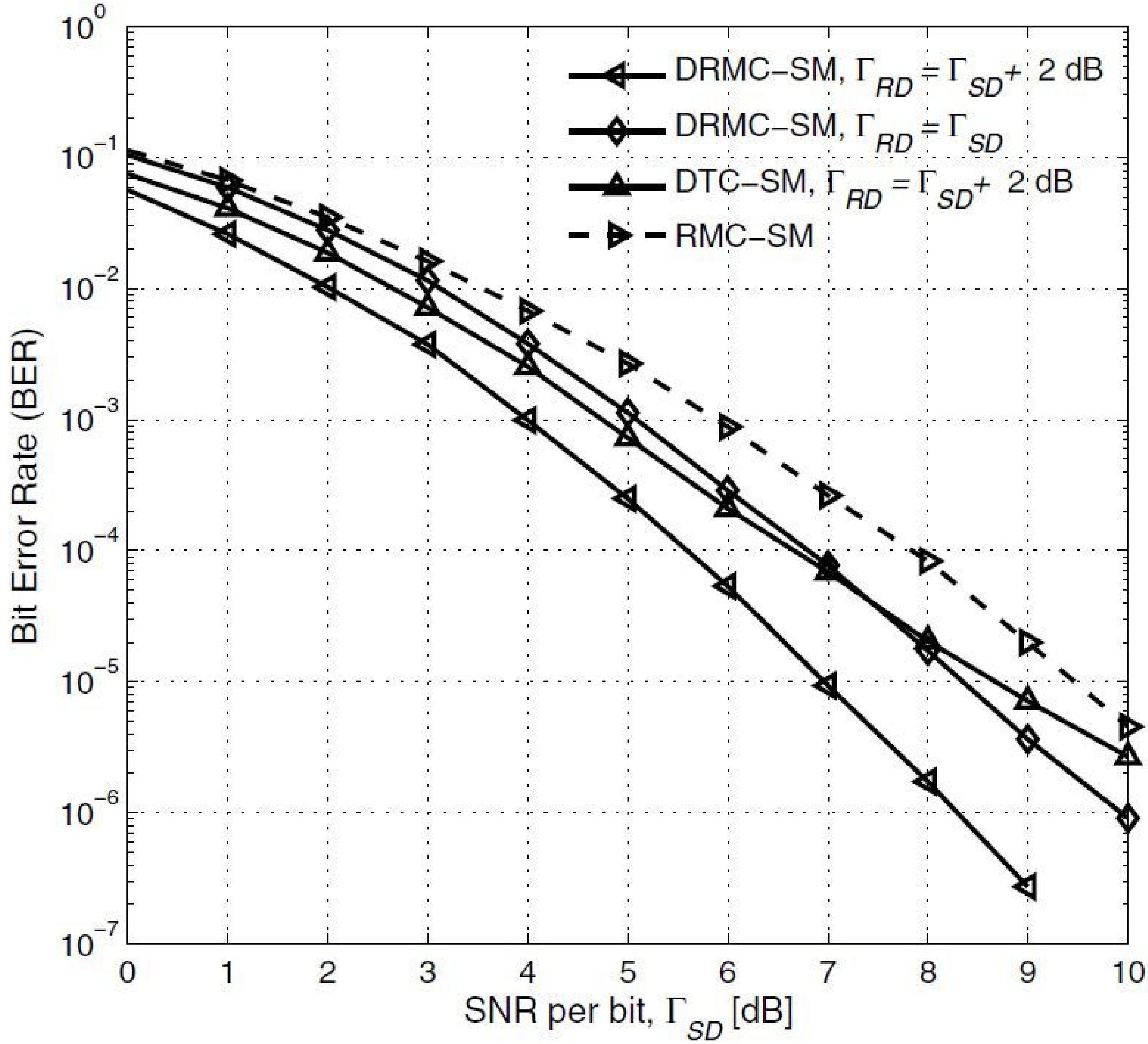

Figure 7

BER performance of DRMC-SM, RMC-SM and DTC-SM schemes over slow Rayleigh fading channel with optimum bit selection at relay under ideal SR channel and Nt = Nr = 4

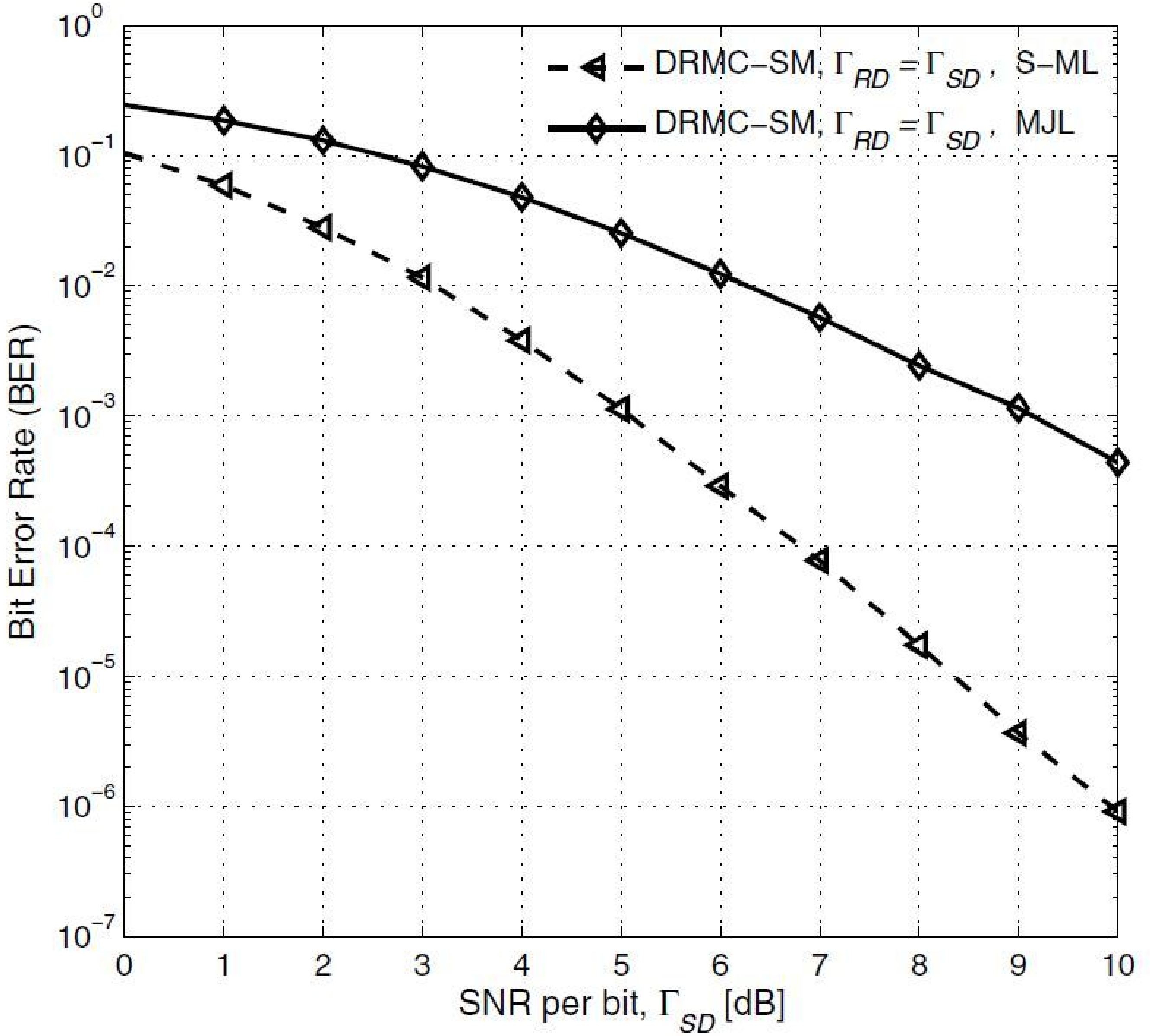

Figure 8

Joint soft RM decoding vs joint hard RM decoding of DRMC-SM scheme under ideal SR channel and Nt = Nr = 4

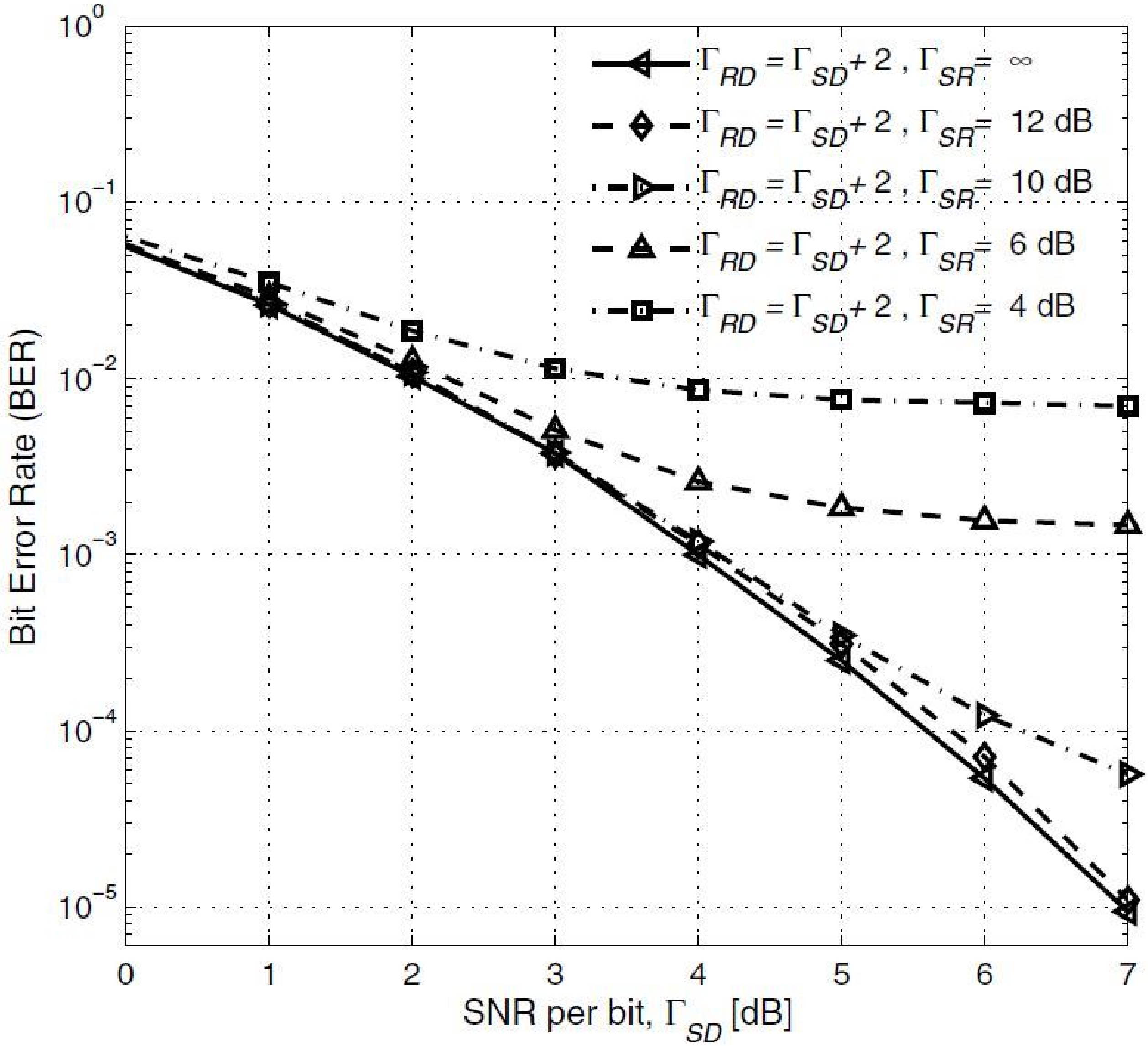

Figure 9

BER performance of DRMC-SM scheme under non-ideal SR channel and Nt = Nr = 4

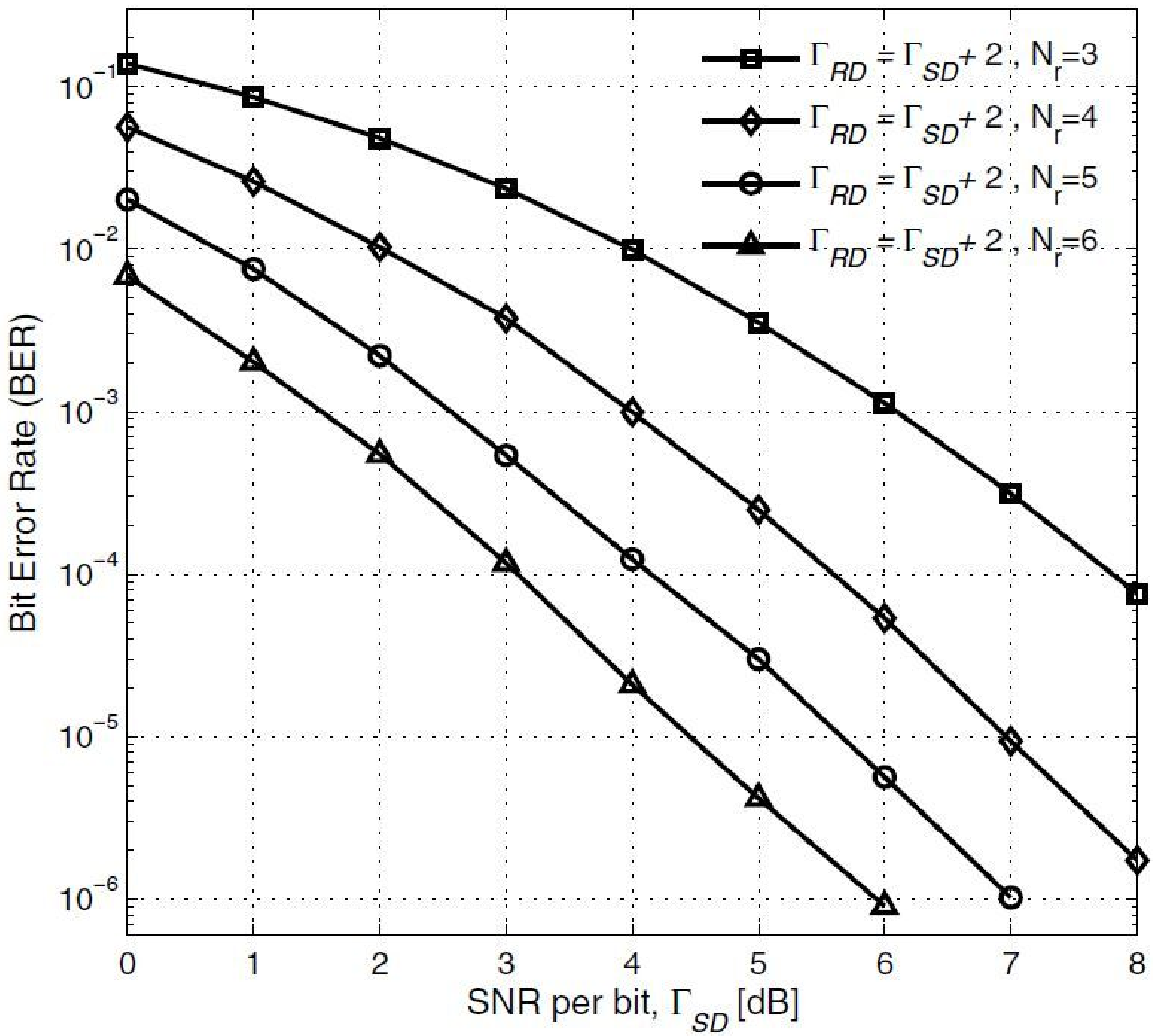

Figure 10

The effect of receive antennas on the BER performance of DRMC-SM scheme with four transmit antennas

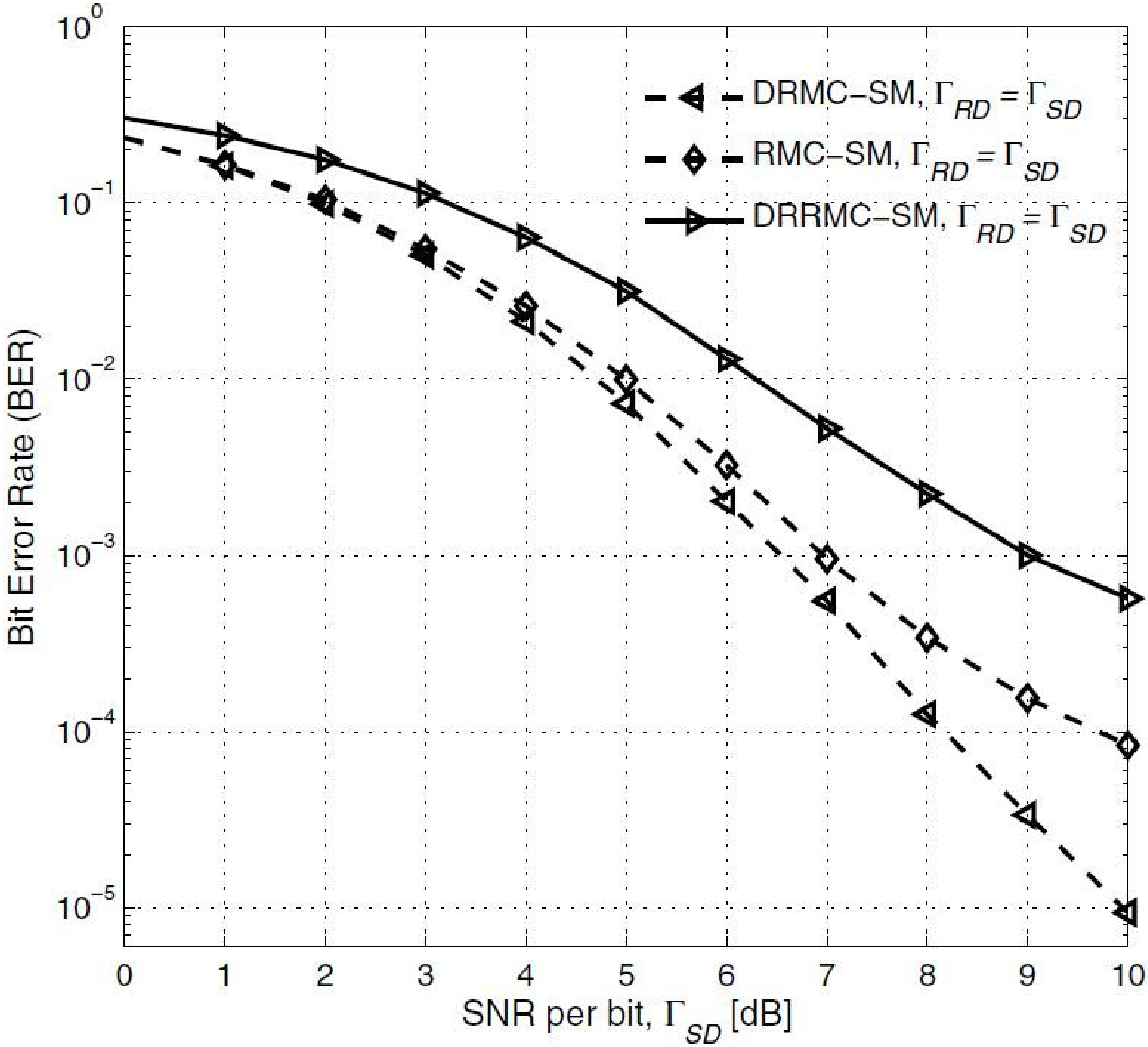

Figure 11

BER performance of DRMC-SM, DRRMC-SM and RMC-SM schemes over slow Rayleigh fading channel, ideal SR channel, Nt = 8 and Nr = 4