Distributed Robust Tracking Control for Multiple Euler–Lagrange Systems With Full-state Constraints and Input Saturation via Event-triggered Control

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Distributed robust tracking control for multiple Euler–Lagrange systems with full-state constraints and input saturation via event-triggered control

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Abstract

In this paper, the issue of distributed tracking control is studied for multiple Euler–Lagrange systems in presence of external disturbances and input saturation. Specifically, the full-state constraints, input saturation, communication delay, and unmeasured velocity are also considered simultaneously. Firstly, an adaptive distributed state observer is introduced to obtain the leader's time-varying position information, at the same time, a delay function is employed to compensate the communication delay. Moreover, the event-triggered control scheme is developed to reduce communication source and computation load, and the anti-saturation compensation algorithm is exploited to compensate for the influence of system saturation. Thirdly, an adaptive law is designed to offset external disturbances. What's more, the high-gain observer is used to estimate the unmeasured velocities. Theorem analysis shows that the system errors can converge to zero. Finally, numerical simulations are present to verify the effectiveness of the proposed control strategy.

Keywords: Multiple Euler-Lagrange system; event-triggered control; distributed control; input saturation

1. Introduction

Compared with the traditional single agent system, multi-agent system composed of many agents can perform more complex tasks with obvious advantages and high efficiency. Compared with a single complex and expensive agent, multiple simple agents can effectively reduce the work cost. When a single agent cannot complete the specified task, using multiple agents to work together to complete the given task can effectively improve work efficiency. When one agent goes wrong in the multi-agent system, other agents can still complete the task effectively, which can improve the flexibility and robustness of the system. When the overall performance of the system needs to be improved, just increase the number of agents on the basis of the original system which can avoid large-scale adjustment of the whole system. Multi-agent system plays a more and more important role in more and more scenes, such as industrial production, aerospace, multi satellite communication, missile cluster target tracking, fighter formation and so on [1-13].

For the research of multi-agent system, many researchers in related fields have carried out the corresponding research work, mainly reflected in their published papers or books. In many literatures, the research of multi-agent system is mainly about linear system, that is, the system described by linear equation. However, in practical application scenarios, most of them are nonlinear systems, such as multiple Euler-Lagrange equations. There are two kinds of communication topologies in multi-agent system, one is undirected graph, the other is directed graph. The communication between agents in undirected graph is

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bidirectional, and the weight can be set according to the importance of communication. The communication between agents in digraph is unidirectional, and the weight can also be set appropriately. But if there are packet loss and network failure problems, undirected graph communication topology cannot deal with it well. And because the communication of digraph is orderly, it is relatively easy to implement [14-26]. Many papers focus on the communication topology of digraph when they study multi-agent system. Three adaptive group consensus control schemes were proposed for EI systems with uncertain parameters [3]. One is based on directed acyclic topology, and the other two are based on directed balanced bipartite graph. Although the consistency can be guaranteed by mathematic operation, this leaderless control method means that the whole system has no leader and the state of each agent is limited because of that the common input they want to reach is related to the initial state of the system. If a leader is added, it means that the whole system will follow the trajectory of the leader. Obviously, the state of the follower cannot be limited by the initial state, which can not only increase the diversity of system motion forms, but also simplify the steps and procedures in the design of control strategy [8, 27]. The problem of coordinated tracking control based on directed communication topology is studied by using backstepping method [28, 29]. For the nonlinear parameters, the method of parameter linearization is adopted. At the same time, the neural network is used to fit the nonlinear uncertainties. Through mathematical reasoning and numerical simulation, it is concluded that the whole system can achieve consistency, that is, the position tracking error is bounded. However, this algorithm does not consider the problem of system delay and saturation compensation, so it may not achieve the ideal position tracking effect in practical application. The system delay is considered in reference [15, 30], and the delay function is introduced into the adaptive distributed observer to achieve the effect of delay compensation [31]. At the same time, the system instability caused by saturation is also considered. Similarly, the saturation compensation algorithm is introduced to counter compensate the part of the actual control input that exceeds the maximum control rate, so that the control can make the action of each agent in a reasonable range. However, because it is not easy to measure the speed of each agent in the actual system, it is necessary to observe the speed of the agent through the observer and transfer the estimation errors to the some uncertain parameters related to velocities of EI equation, which is more in line with the needs of the actual system [26, 30, 32]. In the large-scale real multi-agent control system, for example, in the workshop where a large number of manipulators are responsible for product processing, due to the limited communication of the whole system, it will lead to the problem of insufficient communication bandwidth, that is, the problem of limited bandwidth in multi manipulator communication. The event-triggered control scheme greatly reduces the amount of data transmission between agents [32], so that the agents do not need to keep the communication state all the time, but transmit the data related to the control information at the trigger time to update the current control rate of each agent, which greatly avoids the problem of communication channel congestion in the large-scale agent cooperative control, and can promote the whole intelligent system [33]. The system keeps the same state after a limited number of trigger times and achieves good control effect [34,35]. In the multi-agent system based on event-triggered control, the update control rate of each agent is only executed when the trigger conditions are met, which cleverly avoids the problems caused by bandwidth limitation. Thus, a coordinated control method for multiple Euler-Lagrange systems based on event-triggered scheme is proposed to solve the bandwidth problem of the nonlinear systems.

In this paper, based on the adapted event- triggered mechanism, we study the consensus tracking control of multiple Euler–Lagrange systems in directed communication topology. The problem of communication delay and input saturation are both considered. At the same time, a high-gain observer is introduced to observe the actual speed of the agent, which skillfully avoids the problem of difficult speed measurement. Through stability analysis and numerical simulation, the proposed algorithm in this paper can achieve good position tracking performance and ensure the consensus of the system.

The contributions of this paper are as follows: (1) The state constraints, input saturation and communication delay are all considered in multiple Euler–Lagrange system. (2) The event trigger mechanism is designed to reduce data transmission and avoid the impact of bandwidth. (3) A high-gain observer is used to observe the speed of each agent for the feedback loop. (4) An adaptive disturbances suppression algorithm is employed to counteract the influence of relatively large external environment interference on the system.

The rest of this study is as follows. In the second section, the dynamic model of Euler-Lagrange system and some preparatory knowledge used in this paper are given. The description of the event triggering scheme and the design of the control

law are shown in the third section. In the fourth section, the effectiveness of cooperative control of multi manipulator system is verified by simulation experiments. The conclusion is in Section 5.

2. Preliminaries

2.1. Euler-Lagrange dynamics

The multi-agent systems are composed of n followers described by $(1, 2, \dots, n)$ and one dynamic leader (denoted by 0). The EL equation of motion for the i th follows is described as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i + w_i \quad (1)$$

where $q_i, \dot{q}_i, \ddot{q}_i \in R^p$ are the generalized coordinate, velocity and acceleration, respectively, $M_i(q_i) \in R^{p \times p}$ is the symmetric positive definite inertia matrix, $C_i(q_i, \dot{q}_i) \in R^{p \times p}$ is the Coriolis and centrifugal terms, $g_i(q_i) \in R^p$ denotes the gravitational force, $\tau_i \in R^p$ is the control input, and $w_i \in R^p$ represents the external disturbances (including unmodeled dynamics, environment disturbance, and noise). In this research, assume that $M_i(q_i), C_i(q_i, \dot{q}_i)$ and $g_i(q_i)$ are all unknown.

Assumption 1. The disturbance w_i is bounded, i.e., the limit is $w_{\max}$ which is predesigned, such that $\|w_i\| \leq w_{\max}$.

Some characteristic of this equation are as follows:

Property 1. The matrix $(\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i))$ is skew-symmetric, i.e., for all $a_i \in R^p$, it can satisfy that $a_i^T [(\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i))] a_i = 0$. where a_i is shown as a bounded arbitrary constant vector

Property 2. Two positive constants $k_{\max}$ and $k_{\min}$, such that $k_{\max} I_p \leq M_i(q_i) \leq k_{\min} I_p$, where I_p denotes the $p \times p$ identity matrix.

According to [30], the dynamic leader's generalized coordinate based on our assumption, expressed as q_0 is generated as:

$$\dot{v} = Sv \quad q_0 = Ev \quad (2)$$

where $v \in R^m$ is the auxiliary state variable, $S \in R^{m \times m}$ and $E \in R^{p \times m}$ are constant real matrices. When the leader's position coordinates are known, v can be determined. At the same time, S and E are globally known by all the follows so that the follower can get the leader's position information easily by solving the distributed observer.

The following assumption is given.

Assumption 2. All the eigenvalues of matrix S are semi-simple with zero real parts.

2.2. Graph theory

When we study the consensus cooperative control problem of multi-agent systems, we usually describe the communication network between agents with a graph. It can be seen that algebraic graph theory is an indispensable theoretical tool to study the above problems. The following will summarize the graph theory knowledge and related concepts needed in this paper.

Generally speaking, multi-agent itself can be represented by nodes, and the interaction between agents can be represented by edges. According to the direction of each edge in the graph, it can be divided into digraph and undirected graph. Suppose the system contains n agents, A directed graph G can be expressed as $(V, \mathcal{E})$, where $V = (1, 2, \dots, n)$ denotes a finite set of nonempty nodes and n denotes the number of nodes also known as the order of a graph. $\mathcal{E} \subseteq V \times V$ is a directed edge set composed of nodes. In a digraph G , an ordered array $(i, j) \in \mathcal{E}$ denotes a directed edge, which describes that node j obtains information from node i , where i is called the father of j and j is called the child of i . Node i and node j are called neighbor nodes. Compared with digraph, in undirected graph, the ordered array $(i, j) \in \mathcal{E}$ indicates that node i and node j can communicate information each other, that is, for any $(i, j) \in \mathcal{E}$, all have $(j, i) \in \mathcal{E}$. Undirected graph is a special kind of digraph.

Adjacency matrix of digraph is described as $A = [a_{ij}] \in R^{n \times n}$, The matrix represents the weight of each node in communication, suppose $(i, j) \in \mathcal{E}, a_{ij} = 1$, otherwise, $a_{ij} = 0 (i \neq j)$, in addition, $a_{ii} = 0$ [15].

The communication topology used in this paper is a digraph with five nodes, one leader node and four followers nodes. Based on the specific situation of this paper, we make the following assumption.

Assumption 3. The directed paths exist, which are from the leader to all the followers.

3. Control design via event-triggered scheme

3.1. Distributed observer design for followers

According to the dynamic leader's generalized coordinate, for S and E are globally known by all the follows. The following adaptive distributed observers are designed, by observing v , the tracking trajectory of each follower is obtained.

$$\dot{c}_i = Sc_i - k \sum_{j=0}^n [a_{ij} (c_i(t-T) - c_j(t-T))] \quad (3)$$

where $i=1, \dots, n$, $c_i \in R^m$ denotes the estimate of v for the i th follower, and T denotes the constant communication delay among agents. here, assume that $c_0 = v$.

Lemma 1. For system (2), suppose assumptions 1 and 2 hold. If the communication delay T satisfy $0 \leq \dot{T}(t) \leq b < 1$, and positive definite matrices G, Y make the following inequalities hold [15],

$$J = \begin{pmatrix} G & -2\mu_2(H \otimes I_m) \\ 0 & -(1-\dot{T})G \end{pmatrix} < 0, L = \begin{pmatrix} Y & -2\mu_2(H \otimes I_m) \\ 0 & -(1-\dot{T})Y \end{pmatrix} < 0 \quad (4)$$

Therefrom, $\lim_{t \rightarrow \infty} \tilde{v} = \lim_{t \rightarrow \infty} c_i - v_i = \lim_{t \rightarrow \infty} c_i - v = 0$.

3.2. Control design and the event-triggering mechanism

This study aims to ensure the state variables $q_i, \dot{q}_i$, i.e.,

$$\lim_{t \rightarrow \infty} (q_i - q_0) = 0, \lim_{t \rightarrow \infty} (\dot{q}_i - \dot{q}_0) = 0 \quad (5)$$

Meanwhile, this system also needs to make certainty that the state variables is limited and will not exceed the constraint boundary, i.e., for $\forall t \geq 0$, the inequalities $|q_i| \leq b_{di}$ always hold, where $i=1, \dots, n$. Furtherly, the system uncertainties and control saturation question are taken into consideration. The Anti-windup compensator is employed to directly compensate for the saturation difference. The saturation function is as follows

$$\text{windup}(u) = [\text{windup}(u_1), \text{windup}(u_2), \dots, \text{windup}(u_n)]^T \quad (6)$$

where $\text{windup}(u_i) = \text{sgn}(u_i) \min\{u_{i\max}, |u_i|\}$, $i=1, 2, \dots, n$, $u_{i\max}$ is the maximum value of the control input. So it is obviously that there will be a difference Δu between the desired control input and the actual control input, which is described as

$$\Delta u = \text{windup}(u) - u \quad (7)$$

Assumption 4. When the input saturation takes place, if the control input is unbounded, the difference will be unbounded and the system will be out of control. Therefore, the following conditions need to be satisfied

$$\|\Delta u\|_2 \leq \lambda \quad (8)$$

where λ denotes the difference limit, which is a positive constant.

In practical systems, input saturation may exist. In order to counteract the effect of input saturation, a saturation compensation auxiliary system is used, denoted as follow

$$\begin{cases} \dot{\zeta}_1 = -F_1 \zeta_1 + \zeta_2 \\ \dot{\zeta}_2 = -F_2 \zeta_2 + M^{-1} \Delta u \end{cases} \quad (9)$$

where F_1, F_2 are all diagonal matrices whose diagonal elements are all normal number. ζ_1, ζ_2 are the output of the auxiliary system.

According to (2) and (3), we define the auxiliary variable as follows

$$q_{ai} = E c_i \quad (10)$$

Define the error variable as

$$\begin{aligned} \varepsilon_{1i} &= q_i - q_{ai} - \zeta_{1i} \\ \varepsilon_{2i} &= \dot{q}_i - \dot{q}_{ai} - \alpha_i - \zeta_{2i}, i=1, 2, \dots, n \end{aligned} \quad (11)$$

where virtual control function α can be shown as $\alpha = -K_i \varepsilon_{1i} - F_1 \zeta_1$, $K_i = \text{diag}(k_{11}, k_{12}, \dots, k_{1n})$. what's should be mentioned is that all the diagonal elements of K_i are positive constants.

At the moment, let $x_1 = q_i, x_2 = \dot{q}_i$, so the state equation is obtained according to (1) as follows

$$\dot{x}_i(t) = A x_i + B u_i + c, i=1, 2, \dots, N \quad (12)$$

where $A = \begin{pmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ 0_{2 \times 2} & M_i^{-1} C_i \end{pmatrix}$, $B = \begin{bmatrix} 0_{2 \times 2} \\ M_i^{-1} \end{bmatrix}$, $c = M_i^{-1} w_i - M_i^{-1} g_i$. A, B and c are all nonlinear matrix which are composed of nonlinear elements related to state variables.

In this study, compared with [14] and [15], we use triggering time instants $\{t_k^i\}, k = 0, 1, \dots$, to denote the instant when the trigger condition of agent i is satisfied. So assume that each agent updates the controller at its own triggering time instants.

Firstly, define the Lyapunov function candidate as follows

$$V_1 = \sum_{i=1}^n \frac{1}{2} \ln \frac{b_{li}^2}{b_{li}^2 - \varepsilon_{li}^2} + \frac{1}{2} \zeta_1^T \zeta_1 \quad (13)$$

where $b_l = b_a - A_0 = [b_{l1}, b_{l2}, \dots, b_{ln}]$ is the constraint boundary of ε_l .

When we take the derivative of the above function, we have

$$\dot{V}_1 = \sum_{i=1}^n \frac{\varepsilon_{li}(\varepsilon_{2i} + \zeta_{2i} + \alpha_i - \dot{\zeta}_{li})}{b_{li}^2 - \varepsilon_{li}^2} + \zeta_1^T \dot{\zeta}_1 \quad (14)$$

Then take Eq. (9) into Eq. (14), we have

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^n \frac{\varepsilon_{li}(\varepsilon_{2i} - k_{li}\varepsilon_{li})}{b_{li}^2 - \varepsilon_{li}^2} - \zeta_1^T F_1 \zeta_1 + \zeta_1^T \zeta_2 \leq -\sum_{i=1}^n \frac{k_{li}\varepsilon_{li}^2}{b_{li}^2 - \varepsilon_{li}^2} + \sum_{i=1}^n \frac{\varepsilon_{li}\varepsilon_{2i}}{b_{li}^2 - \varepsilon_{li}^2} - \zeta_1^T F_1 \zeta_1 + \frac{1}{2} \zeta_1^T \zeta_1 + \frac{1}{2} \zeta_2^T \zeta_2 \\ &\leq -\sum_{i=1}^n \frac{k_{li}\varepsilon_{li}^2}{b_{li}^2 - \varepsilon_{li}^2} + \sum_{i=1}^n \frac{\varepsilon_{li}\varepsilon_{2i}}{b_{li}^2 - \varepsilon_{li}^2} \end{aligned} \quad (15)$$

Secondly, choose the following Lyapunov function candidate

$$V_2 = V_1 + \sum_{i=1}^n \frac{1}{2} \ln \frac{b_{2i}^2}{b_{2i}^2 - \varepsilon_{2i}^2} + \frac{1}{2} \zeta_2^T \zeta_2 + \frac{1}{2\nu_0} \tilde{\gamma}^2 \quad (16)$$

Take the derivative of function (16),

$$\dot{V}_2 = \dot{V}_1 + \sum_{i=1}^n \frac{1}{2} \ln \frac{\varepsilon_{2i}\dot{\varepsilon}_{2i}}{b_{2i}^2 - \varepsilon_{2i}^2} + \frac{1}{2} \zeta_2^T \dot{\zeta}_2 + \frac{1}{2\nu_0} \tilde{\gamma}^2 \quad (17)$$

The event-based controller can be designed as

$$u_i(t) = -K \sum_{j=0}^N a_{ij} (x_{li}(t_k^i) - x_{lj}(t_k^i)) + M(-F_2 \zeta_{2i} + \dot{\alpha}_i + \ddot{q}_{ai} - \delta_1 \varepsilon_{li} - \delta_2 \varepsilon_{2i} - \frac{\hat{\gamma} \varepsilon_{2i}}{2\zeta^2}) + C_i x_{2i} + g_i \quad t \in [t_k^i, t_{k+1}^i) \quad (18)$$

where $x_{li}(t_k^i) = q_i(t_k^i)$, $x_{lj}(t_k^i) = q_j(t_k^i)$.

By defining the measurement error of each agent as $e_i(t) = q_i(t_k^i) - q_i(t)$, $t \in [t_k^i, t_{k+1}^i)$, the controller (18) can be rewritten

as

$$u_i(t) = -K \sum_{j=0}^N a_{ij} (x_{li}(t) - x_{lj}(t) + e_i(t) - e_{ij}(t)) + M(-F_2 \zeta_{2i} + \dot{\alpha}_i + \ddot{q}_{ai} - \delta_1 \varepsilon_{li} - \delta_2 \varepsilon_{2i} - \frac{\hat{\gamma} \varepsilon_{2i}}{2\zeta^2}) + C_i x_{2i} + g_i \quad (20)$$

where $x_{li}(t) = q_i(t)$, $x_{lj}(t) = q_j(t)$, $e_{ij}(t) = q_j(t_k^i) - q_j(t)$, $t \in [t_k^i, t_{k+1}^i)$, $\delta_1 = \sum_{i=1}^{n_l} \frac{b_{2i}^2 - \varepsilon_{2i}^2}{b_{li}^2 - \varepsilon_{li}^2}$, n_l denotes the total number of

joints per manipulator.

The adaptive law is designed as follows

$$\dot{\gamma} = \nu_0 \sum_{i=1}^n \frac{\varepsilon_{2i}^2}{b_{2i}^2 - \varepsilon_{2i}^2} - \nu_0 \nu_1 \tilde{\gamma} \quad (21)$$

where ν, ν_0, ν_1 are all positive constants.

Theorem 1. For communication delay T , let the derivative of T satisfy that $0 \leq \dot{T} \leq d \leq 1$, If two positive definite matrices G, Y can satisfy (4), and the initial conditions satisfy that $\varepsilon_{1i}(0) \in \Omega_{01} \triangleq \{|\varepsilon_{1i}| < b_{1i}\}$, $\varepsilon_{2i}(0) \in \Omega_{02} \triangleq \{|\varepsilon_{2i}| < b_{2i}\}, i=1,2,\dots,n$. Then the controller (20), adaptive law (21) and the distributed observer (3) can make certain that x_{1i}, x_{2i} will converge to the set $\Omega_{x_1} \triangleq \{x_{1i} < b_{a1}, i=1,2,\dots,n\}$, $\Omega_{x_2} \triangleq \{x_{2i} < b_{a2}, i=1,2,\dots,n\}$, the related error $\varepsilon_1, \varepsilon_2, \tilde{\gamma}, \zeta_1, \zeta_2$ will be uniformly bounded, and ε_1 can finally converge to the sets $\varepsilon_1 \in \Omega_{\varepsilon_1} \triangleq \{|\varepsilon_{1i}| \leq b_{1i} \sqrt{1 - e^{-2(V_2(0)+C/\beta)}}\}$, ε_2 can finally converge to the sets $\varepsilon_2 \in \Omega_{\varepsilon_2} \triangleq \{|\varepsilon_{2i}| \leq b_{2i} \sqrt{1 - e^{-2(V_2(0)+C/\beta)}}\}, i=1,\dots,n$. At the same time, the control parameters can be chosen to make the followers track the leader asymptotically.

Now substitute controller (30) and adaptive law (21) into (17), we have

$$\begin{aligned} \dot{V}_2 &= -\sum_{i=1}^n \frac{k_{1i} \varepsilon_{1i}^2}{b_{1i}^2 - \varepsilon_{1i}^2} - \sum_{i=1}^n \frac{k_{2i} \varepsilon_{2i}^2}{b_{2i}^2 - \varepsilon_{2i}^2} + \sum_{i=1}^n \frac{\varepsilon_{2i} I_i (D - \frac{\tilde{\gamma} \varepsilon_{2i}}{2\zeta_2^2})}{b_{2i}^2 - \varepsilon_{2i}^2} - \bar{F}_1 \|\zeta_1\|^2 + \frac{1}{2} \zeta_2^T \zeta_2 - \zeta_2^T F_2 \zeta_2 + \zeta_2^T M^{-1} \Delta u - \frac{1}{\nu_0} \tilde{\gamma} \times (\nu_0 \sum_{i=1}^n \frac{\varepsilon_{2i}^2 / 2\zeta_2^2}{b_{2i}^2 - \varepsilon_{2i}^2} - \nu_0 \nu_1 \tilde{\gamma}) \\ &\leq -\sum_{i=1}^n \frac{k_{1i} \varepsilon_{1i}^2}{b_{1i}^2 - \varepsilon_{1i}^2} - \sum_{i=1}^n \frac{k_{2i} \varepsilon_{2i}^2}{b_{2i}^2 - \varepsilon_{2i}^2} + \sum_{i=1}^n \frac{\|\varepsilon_{2i}\| d_m - \frac{\tilde{\gamma} \varepsilon_{2i}^2}{2\zeta_2^2} + \frac{\tilde{\gamma} \varepsilon_{2i}^2}{2\zeta_2^2}}{b_{2i}^2 - \varepsilon_{2i}^2} - \bar{F}_1 \|\zeta_1\|^2 + \frac{1}{2} \zeta_2^T \zeta_2 - \zeta_2^T V_2 \zeta_2 + \zeta_2^T M^{-1} \Delta u + \nu_1 \tilde{\gamma} \tilde{\gamma} \end{aligned} \quad (22)$$

where $\bar{F}_1 = \lambda_{\min}(F_1) - \frac{1}{2}$ is positive.

According to **Property 2** which describes that $0 < M_{m,i} \leq \|M_i(q_i)\| \leq M_{M,i}$, there exists a positive constant m , making

$0 \leq \|M^{-1}\|_2 \leq m$ hold, then according to Young's inequality, we can have

$$\zeta_2^T M^{-1} \Delta u \leq \frac{1}{2} \zeta_2^T \zeta_2 + \frac{1}{2} \bar{m}^2 \theta^2 \quad (23)$$

$$\|\varepsilon_{2i}\| d_m \leq \frac{\gamma \varepsilon_{2i}^2}{2\zeta_2^2} + \frac{\zeta_2^2}{2} \quad (24)$$

Then, substitute (23) and (24) into (21), we have

$$\begin{aligned} \dot{V}_2 &\leq -\sum_{i=1}^n \frac{k_{1i} \varepsilon_{1i}^2}{b_{1i}^2 - \varepsilon_{1i}^2} - \sum_{i=1}^n \frac{k_{2i} \varepsilon_{2i}^2}{b_{2i}^2 - \varepsilon_{2i}^2} + \sum_{i=1}^n \frac{\zeta_2^2 / 2}{b_{2i}^2 - \varepsilon_{2i}^2} - \bar{F}_1 \|\zeta_1\|^2 + \frac{1}{2} \zeta_2^T \zeta_2 - \zeta_2^T F_2 \zeta_2 + \frac{1}{2} \zeta_2^T \zeta_2 + \frac{1}{2} \bar{m}^2 \theta^2 + \nu_1 \tilde{\gamma} \tilde{\gamma} \\ &\leq -\sum_{i=1}^n \frac{k_{1i} \varepsilon_{1i}^2}{b_{1i}^2 - \varepsilon_{1i}^2} - \sum_{i=1}^n \frac{k_{2i} \varepsilon_{2i}^2}{b_{2i}^2 - \varepsilon_{2i}^2} + \sum_{i=1}^n \frac{\zeta_2^2 / 2}{b_{2i}^2 - \varepsilon_{2i}^2} - \bar{F}_1 \|\zeta_1\|^2 - \bar{F}_2 \|\zeta_2\|^2 + \frac{1}{2} \bar{m}^2 \theta^2 - \nu_1 \tilde{\gamma}^2 + \nu_1 (\frac{\kappa}{2} \tilde{\gamma}^2 + \frac{1}{2\kappa} \gamma^2) \end{aligned} \quad (25)$$

where $\bar{F}_2 = \lambda_{\min}(F_2) - 1$ is positive.

Lemma 2. For any vector $k_b \in R^n$, which contain normal number, if $|e| < k_b$, then we can make the inequality

$$\ln \frac{k_b^T k_b}{k_b^T k_b - e^T e} \leq \frac{e^T e}{k_b^T k_b - e^T e} \text{ holds.}$$

According to **lemma 2.**, we can simplify (25) as

$$\begin{aligned} \dot{V}_2 &\leq -\sum_{i=1}^n \frac{1}{2} \ln \frac{k_{1i} b_{1i}^2}{b_{1i}^2 - \varepsilon_{1i}^2} - \sum_{i=1}^n \frac{1}{2} \ln \frac{k_{2i} b_{2i}^2}{b_{2i}^2 - \varepsilon_{2i}^2} + \sum_{i=1}^n \frac{\zeta^2/2}{b_{2i}^2 - \varepsilon_{2i}^2} - \bar{F}_1 \|\zeta_1\|^2 - \bar{F}_2 \|\zeta_2\|^2 + \frac{1}{2} \bar{m}^2 \theta^2 - \nu_1 \tilde{\gamma}^2 + \nu_1 \left(\frac{\kappa}{2} \tilde{\gamma}^2 + \frac{1}{2\kappa} \gamma^2 \right) \\ &\leq -\beta V_2 + F \end{aligned} \quad (26)$$

$$\text{where } \beta = \min \left\{ 2K_{1i}, 2K_{2i}, 2\bar{F}_j, 2\nu_0\nu_1 \left(1 - \frac{\kappa^2}{2} \right) \right\}, j=1, 2, i=1, 2, \dots, n, F = \frac{1}{2} \bar{m}^2 \theta^2 - \nu_1 \tilde{\gamma}^2 + \nu_1 \left(\frac{\kappa}{2} \tilde{\gamma}^2 + \frac{1}{2\kappa} \gamma^2 \right).$$

If the initial condition satisfies $\varepsilon_{1i}(0) \in \Omega_{\varepsilon_1}, \varepsilon_{2i}(0) \in \Omega_{\varepsilon_2}$, for arbitrary $t > 0$, the inequalities $|\varepsilon_{1i}| < b_{1i}, |\varepsilon_{2i}| < b_{2i}$ can be obtained. Meanwhile, $\varepsilon_{1i}, \varepsilon_{2i}, \tilde{\lambda}, \zeta_{1i}, \zeta_{2i}$ ($i=1, 2, \dots, n$) are all bounded. Let $\|\zeta_{1i}\| \leq \delta_{1i}, \|\zeta_{2i}\| \leq \delta_{2i}$. According to (11), one can obtain that

$$\begin{aligned} |q_i| &= |\varepsilon_{1i} + \zeta_{1i} + q_{ai}| < |b_{1i} + \delta_{1i} + A_{0i}| = b_{1i} + \delta_{1i} + A_{0i} \\ |\dot{q}_i| &= |\varepsilon_{2i} + \zeta_{2i} + \dot{q}_{ai} + \alpha_i| < |b_{2i} + \delta_{2i} + k_{1i} b_{1i}| = b_{2i} + \delta_{2i} + k_{1i} b_{1i} \triangleq b_{ci} + A_{1i} \end{aligned} \quad (27)$$

At the moment, take the integral of (26), have

$$0 \leq V_2 \leq (V_2(0) - \frac{F}{\beta}) e^{-\beta t} + \frac{F}{\beta} \leq V_2(0) + \frac{F}{\beta} \quad (28)$$

From (16), get

$$\frac{1}{2} \ln \frac{b_{1i}^2}{b_{1i}^2 - \varepsilon_{1i}^2} \leq V_2(0) + \frac{F}{\beta} \quad (29)$$

Now, take the exponent arithmetic on both sides, then make the following inequality hold.

$$|\varepsilon_{1i}| \leq b_{1i} \sqrt{1 - e^{-2V_2(0) + F/\beta}} \leq b_{1i}, \forall t > 0 \quad (30)$$

Similarly, we can have $|\varepsilon_{2i}| \leq b_{2i} \sqrt{1 - e^{-2V_2(0) + F/\beta}} \leq b_{2i}, \forall t > 0$. In result, we can get the conclusion that $\varepsilon_1, \varepsilon_2$ will converge to zero by choosing appropriate parameters. At the same time, the tracking error will converge to any small number which is around zero.

3.4. Design of high gain speed observer

Lemma 3. Suppose the system output $y(t)$ and $y^{(n)}(t)$ are bounded, which can satisfy $|y^{(k)}| < Y_K$, where Y_K is a positive constant. The following linear systems can be considered

$$\begin{aligned} \mathcal{G}\dot{\psi}_i &= \psi_{i+1}, i=1, \dots, n-1 \\ \mathcal{G}\dot{\psi}_n &= -\bar{\eta}_1 \psi_n - \bar{\eta}_2 \psi_{n-1} - \bar{\eta}_3 \psi_{n-2} - \dots - \bar{\eta}_{n-1} \psi_2 - \psi_1 + x_1(t) \end{aligned} \quad (31)$$

In the above formula (40), $\mathcal{G}$ is a random small positive constant number, parameter η_1 to η_{n-1} can be obtained by the Herwitz polynomials $s^n + \bar{\eta}_1 s^{n-1} + \dots + \bar{\eta}_{n-1} s + 1$. Then we can get the following equation

$$\xi_k = \frac{\psi_k}{\mathcal{G}^{k-1}} - x_1^{(k-1)} = -\mathcal{G}\Psi^k, k=1, \dots, n-1 \quad (32)$$

where $\Psi = \psi_n + \bar{\eta}_1 \psi_{n-1} + \dots + \bar{\eta}_{n-1} \psi_1$, $\Psi^{(k)}$ denotes k -order derivative of Ψ . Meanwhile, there exist positive number t^* and h_k , which can satisfy that $\forall t > t^*$, $\|\xi_k\| \leq \mathcal{H}_k$, $k=1,2,3,\dots,n$.

In the design of multi-agent cooperative control law, the velocity vector of each agent is required to be known, however, the velocity is not easy to measure with high precision, so a velocity observer is introduced to estimate the velocity vector of each agent. According to **Lemma 3**, $\psi_{k+1}/\mathcal{G}^k$ can converge to $x_1^{(k)}$, where $x_1^{(k)}$ denotes the k -order derivative of x_1 , and due to that ξ_k converge 0, its k -order derivative is bounded. So it is effective to exploit $\psi_{k+1}/\mathcal{G}^k$ as observer to estimate the output signal which can be derived n times. The observer is designed as follows

$$\begin{aligned} \mathcal{G}\dot{\psi}_1 &= \psi_2 \\ \mathcal{G}\dot{\psi}_2 &= -\bar{\eta}_1 \psi_2 - \psi_1 + x_1 \end{aligned} \quad (33)$$

Consequently, $\psi_2/\mathcal{G}$ can be used to evaluate $\dot{x}_1(t)$.

4. Simulations

A multiple manipulator system is composed of five manipulators. Each manipulator has two joints, one of which is the leader and the other four are followers, the communication topology is given in **Fig.1**. In order to achieve the goal of consistent information transmission with the leader, event-triggered control scheme is used to solve the problem of limited bandwidth of mass data transmission. Due to the anti-saturation compensation, the system tends to be uniform and stable. The high gain observer makes the system estimate the real-time speed information of each manipulator, and the observation error converges quickly.

Fig. 1. Communication topology

The equation of state originated from (1) of the system is described as follows

$$\dot{x}_i(t) = Ax_i + Bu_i + c, i=1,2,\dots,N \quad (34)$$

where $A = \begin{pmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ 0_{2 \times 2} & M_i^{-1}C_i \end{pmatrix}$, $B = \begin{bmatrix} 0_{2 \times 2} \\ M_i^{-1} \end{bmatrix}$, $c = M_i^{-1}w_i - M_i^{-1}g_i$.

Table 1 Dynamic Parameters

Parameter	Manipulator1	Manipulator2	Manipulator3	Manipulator4
l_{i1} (m)	1	0.95	1	1.02
l_{i2} (m)	0.98	1	0.96	1
m_{i1} (kg)	1.02	0.96	1.01	1.04
m_{i2} (kg)	1.12	1.15	1.07	1.09
J_{i1} (kg m ²)	0.23	0.21	0.19	0.21
J_{i2} (kg m ²)	0.41	0.40	0.42	0.41

where $x_i = [x_{i1}^T, x_{i2}^T]^T$, $x_{i1} = [q_{i1}, q_{i2}]^T$, $x_{i2} = [\dot{q}_{i1}, \dot{q}_{i2}]^T$, $w = 0.2 \cdot u$.

$$M_i(q_i) = \begin{pmatrix} Y_{i1} + Y_{i2} + 2Y_{i3} \cos q_{i2} & Y_{i2} + Y_{i3} \cos q_{i2} \\ Y_{i2} + Y_{i3} \cos q_{i2} & Y_{i2} \end{pmatrix}$$

$$C_i(q_i, \dot{q}_i) = \begin{pmatrix} -Y_{i3}(\sin q_{i2})\dot{q}_{i2} & -Y_{i3}(\sin q_{i2})(\dot{q}_{i1} + \dot{q}_{i2}) \\ Y_{i3}(\sin q_{i2})\dot{q}_{i1} & 0 \end{pmatrix}$$

$$G_i(q_i) = \begin{pmatrix} Y_{i4}g \cos q_{i1} + Y_{i5}g \cos(q_{i1} + q_{i2}) \\ Y_{i5}g \cos(q_{i1} + q_{i2}) \end{pmatrix},$$

$$Y_{i1} = J_{i1} + m_{i2}l_{i1}^2, Y_{i2} = 0.25m_{i2}l_{i2}^2 + J_{i2}, Y_{i3} = 0.5m_{i2}l_{i1}l_{i2}, Y_{i4} = (0.5m_{i1} + m_{i2})l_{i1},$$

$$Y_{i5} = 0.5m_{i2}l_{i2},$$

$g = 9.8m/s^2$ is the acceleration of gravity, m_{i1} and m_{i2} are masses of two links of the manipulators. l_{i1} and l_{i2} are the length of the link and the distance from center of mass to the joints, respectively. Some initial parameters are given as follows:

$$q_{11}(0) = 2\pi/5, q_{12}(0) = -\pi/3, q_{21}(0) = \pi/5, q_{22}(0) = -\pi/6$$

$$q_{31}(0) = 4\pi/5, q_{32}(0) = \pi/6, q_{41}(0) = 3\pi/5, q_{42}(0) = \pi/3$$

Let $\dot{q}_{i1}(0) = \dot{q}_{i2}(0) = 0$. Set the control input maximum as 30 Nm. Set the parameters of the event triggering scheme as follows:

$$D = [d_1 \ d_2 \ d_3 \ d_4]^T = [0.1 \ 0.1 \ 0.1 \ 0.1]^T,$$

$$\Delta = [b_1 \ b_2 \ b_3 \ b_4]^T = [0.2 \ 0.2 \ 0.2 \ 0.2]^T,$$

$$\chi = 0.16, Z = [\rho_1 \ \rho_2 \ \rho_3 \ \rho_4]^T = [0.15 \ 0.15 \ 0.15 \ 0.15]^T$$

The trajectories of leaders are given as follows:

$$q_{01}(t) = q_{d1} = \pi/6 \sin(wt + \pi/2) + \pi/2, q_{02}(t) = q_{d2} = 2\pi/3 \sin(wt + \pi/2)$$

$$\text{where } w = 0.1\pi, q_0(0) = [0, 0]^T, \dot{q}_0(0) = [0, 0]^T.$$

Communication time-varying delay defined as $T = 0.1 + 0.06 \sin(0.2t)$.

The defined generalized coordinate of the leader q_0 is generated by Eq. (2), where $S = \begin{bmatrix} 0 & 0.1\pi & 0 \\ -0.1\pi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$$v = \begin{bmatrix} \sin(\omega t) \\ \sin(\omega t + \pi/2) \\ 1 \end{bmatrix}, E = \begin{bmatrix} 0 & \pi/6 & \pi/2 \\ 0 & 2\pi/3 & 0 \end{bmatrix}.$$

The parameters of distributed observers and controllers are chosen as

$$k = 1, F_1 = \text{diag}(20, 20), F_2 = \text{diag}(20, 20), K = 0.1, \delta_1 = \text{diag}(6, 6), \delta_2 = \text{diag}(5, 3), b_1 = [3, 3]^T, b_2 = [18, 10]^T$$

$$\mathcal{G} = 0.01, \bar{\eta}_1 = [1, 1]^T.$$

The parameters of adaptive law proposed in this paper are $\nu = 5, \nu_0 = 20, \nu_1 = 10$.

The simulation results are shown in **Figs. 2-13**. **Fig. 2** shows the position tracking error of joint 1 of the four manipulators. **Fig. 3** shows the position tracking error of joint 2 of the four manipulators. **Fig. 4** shows the velocity error of joint 1 of the four manipulators. Likely, **Fig. 5** shows the velocity error of joint 2 of the four manipulators. **Figs. 6-9** clearly show that the amount of data transmission is greatly reduced, and the average trigger interval time is 0.9957s. The use of event-triggered control scheme greatly avoids the problem of communication channel congestion in the large-scale agent cooperative control. The system keeps the same state after a limited number of trigger times and achieves good control performance. **Fig. 10** and **Fig. 11** clearly show that the control torques acting on the manipulators. The control torques in the initial stage reach the upper bound of the control input. This is because the control instructions calculated by the control law exceed the saturation limit. However, the anti-windup technique has effectively eliminated the effect of control saturation problem to ensure a good control performance. The control torques can reduce within 15 Nm when the coordinated tracking process reach stable, meantime, the control input of joint 1 and joint 2 are smooth without chattering. Thus, the use of event-triggered control scheme can greatly reduces the amount of data transmission between agents, and the satisfied control performance can be obtained.

The literature [15] have also studied similar problems. However, it does not consider the influence of the system communication bandwidth and the problem that the speed state quantity is not easy to obtain directly. Compared with [15], the problem of communication under the condition of limited communication bandwidth is considered in this paper, and event trigger mechanism is exploited to make data transmission only occur at the trigger time, which greatly reduces the number of data transmission. **Figs. 6-9** clearly shows the Triggering instants and interval times of the four manipulators. Moreover, a high gain observer is introduced to estimate the speed state, and the estimated value is brought into the control rate formula. According to the analysis of the simulation results in **Fig. 2** and **Fig. 3**, the consistency error of the four manipulators converges quickly, so this paper can stably realize coordinated tracking as desired.

Fig. 2. Position tracking errors q_{ei1} , $i=1-4$

Fig. 3. Position tracking errors q_{ei2} , $i=1-4$

Fig. 4. Velocity tracking errors v_{ei1} , $i=1-4$

Fig. 5. Velocity tracking errors v_{ei2} , $i=1-4$

Fig. 6. Triggering instants and interval times of manipulator 1

Fig. 7. Triggering instants and interval times of manipulator 2

Fig. 8. Triggering instants and interval times of manipulator 3

Fig. 9. Triggering instants and interval times of manipulator 4

Fig. 10. Control input of link 1 u_{i1} , $i=1-4$

Fig. 11. Control input of link 2 u_{i2} , $i=1-4$

Fig. 12 and **Fig. 13** show that the auxiliary errors do not exceed b_1 , and finally converges to 0 stably, which illustrate that state constraints are also obtained.

Fig. 12. The auxiliary error variable $\epsilon_{1i1}, i=1-4$

Fig. 13. The auxiliary error variable $\epsilon_{1i2}, i=1-4$

5. Conclusions

This study discussed a novel event-triggered control strategy which is developed for multiple Euler–Lagrange systems with external disturbances, communication delay, and control input saturation. The event-triggered control scheme greatly reduces the amount of data transmission between agents, so that the agents do not need to keep the communication state all the time, but transmit the data related to the control information at the trigger time to update the current control law of each agent, which greatly avoids the problem of communication channel congestion in the large-scale agent cooperative control. The system keeps the same state after a limited number of trigger times and achieves good control performance. The use of high gain observer also solves the problem of speed difficult to measure. The adaptive law is introduced to compensate the interferences when the external disturbances are relatively large. For heterogeneous multi-agent system, event-triggered cooperative control will be studied in the future.

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Declaration of Interest Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Figures

Figure 1

Communication topology

Figure 2

Position tracking errors ei1 q ,i=1-4

Figure 3

Position tracking errors 2 ei q , i=1-4

Figure 4

Velocity tracking errors v_{ei1} , $i=1-4$

Figure 5

Velocity tracking errors 2 eiv , i=1-4

Figure 6

Triggering instants and interval times of manipulator 1

Figure 7

Triggering instants and interval times of manipulator 2

Figure 8

Triggering instants and interval times of manipulator 3

Figure 9

Triggering instants and interval times of manipulator 4

Figure 10

Control input of link 1 $1u$, $i=1-4$

Figure 11

Control input of link 2 $i2 u, i=1-4$

Figure 12

The auxiliary error variable $\epsilon_{1i}, i = 1-4$

Figure 13

The auxiliary error variable ϵ_{1i2} , $i = 1-4$