

# The Weibull Distribution on Reliability Evaluation of Mechanical Property for Impaction Bone Grafting

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## Research article

**Keywords:** Weibull modulus; reliability; elastic modulus; impaction bone grafting

**Posted Date:** May 15th, 2020

**DOI:** <https://doi.org/10.21203/rs.3.rs-27901/v1>

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# Abstract

**Background:** The mechanical properties of bone grafts used to reconstruct defects during revision surgery, and their long-term reliability, are important in terms of treatment outcomes. However, few relevant studies have appeared. Herein, we use a new method, the Weibull distribution, to evaluate reliability.

**Methods:** We created impacted bone grafts of two distinct sizes (7–10 mm and small slurry) using an impacting apparatus. The elastic moduli were evaluated in vitro and the Weibull moduli ( $m$  values) were calculated.

**Results:** The elastic moduli fitted the Weibull distribution well. Large bone grafts (7–10 mm) exhibited high elastic moduli but low Weibull moduli, suggestive of poor reliability.

**Conclusions:** The Weibull distribution was useful to evaluate the reliability of bone graft mechanical properties. A high Weibull modulus indicated high reliability and thus low variability. Both the values and reliability of mechanical properties must be considered when contemplating bone grafting.

## Introduction

Impaction bone grafting is widely used to reconstruct bone defects during revision hip arthroplasty. However, there have been few studies on reliability, and in those that have appeared the elastic moduli (stiffness values) differed widely, suggesting that the mechanical properties varied [1–3, 5, 6, 14]. The clinical outcomes have been generally favorable, but somewhat variable, and the optimal choice of reconstructive bone graft remains unclear [4, 9, 11, 13]. The values and reliabilities of mechanical properties are important in terms of surgical outcomes; evaluation of graft reliability is essential. The Weibull distribution statistically explores the stability of mechanical properties [8, 17, 18]. A high Weibull modulus,  $m$ , indicates low variability (high reliability), and *vice versa*. Previously, we noted that the elastic moduli of large bone grafts were higher than those of small slurry bone grafts in various tests [16]. Here, we used the Weibull modulus to evaluate the reliability of mechanical properties.

## Methods

### Materials

After 20 patients who had undergone primary hip arthroplasty gave written informed consent, we retrieved bone grafts from femoral heads, cut the heads into two halves, and randomly divided them into a large bone group (7–10 mm) and small slurry bone group (about 2 mm). The half-heads of the former group were cut into cubes of dimensions  $7 \times 7 \times 7$ ,  $8 \times 8 \times 8$ ,  $9 \times 9 \times 9$  and  $10 \times 10 \times 10$  mm; those of the latter group were reamed using an acetabular reamer (38 mm) of diameter ca. 2 mm. Both grafts were of pure cancellous bone. The bone grafts were mixed to minimize within-group differences, divided into 5-g samples (mean weight of 7–10 mm grafts 5.02 g,  $n = 10$ ; mean weight of small slurry bone grafts 5.00 g,

n = 10) and stored at  $-80^{\circ}\text{C}$ . Before testing, grafts were thawed at room temperature for 2 h and then impacted in an apparatus resembling that of Bavadekar et al. [1], with minor modifications (Fig.1). Each sample was placed in a tube 14.7 mm in inner diameter and 24.3 mm in outer diameter; the tube wall was sufficiently thick to resist transverse expansion of the samples. Each sample was then intermittently impacted with a solid mass (1,220 g) dropped from a height of 40 cm to mimic the hammer/impactor system. The tube walls contained vents through which marrow and liquid could be extruded. We measured the elastic moduli (Z 2.5 apparatus; Zwick GmbH & Co., Ulm, Germany) at impactions 3, 5, 10, 20, 30, 40, and 50 [16]. The compression velocity was set to 0.5 mm/min. The compression force was limited to 80 N and the graft displacement to 0.3 mm. Testing was immediately terminated when a plateau in either the compression force or displacement was achieved. Each elastic modulus (in MPa) was calculated by reference to the curve between 60 and 98% of the maximal load. As a bone graft is a visco-elastoplastic material exhibiting time-dependent creep and recoil, we delivered impacts at 1-min intervals [7, 15].

## Statistical Analysis

Changes in the elastic modulus were analyzed by two-way ANOVA, performed using GraphPad Prism software (version 5.0; GraphPad Software, Inc., San Diego, CA, USA). A P-value  $<0.05$  was taken to reflect significance.

To obtain elastic moduli after impactions 3, 20, and 50, we first specified discrete thresholds ( $x_i$  values; e.g., from 0 to 10 in steps of 2) and then recorded  $n_i$  values (the numbers of samples) with test values  $X < x_i$ . Division of  $n_i$  by  $N$  (the sample size) yielded an estimate of the probability distribution function,  $F(x_i) = P(X < x_i) = n_i/N$ . The probability density function,  $f(x)$ , was the derivative of  $F(x)$ . The data were fitted using the Weibull function,  $F(x) = 1 - \exp[-(x/\beta)^m]$ . We plotted  $\ln[-\ln(1-F(x))]$  (the usual double-logarithmic form of the Weibull expression) versus  $\ln(x)$ , and determined the Weibull modulus  $m$  from the slope of the linear fit, and  $\beta$  from the intercept [ $= -m\ln(\beta)$ ]. The fitting parameters, the mathematical expectation  $u = \beta\Gamma(1+1/m)$ , and the standard deviation (see Equation 1 in the Supplementary Files)

## Results

One sample of each group failed during the test. Parameters for elastic modulus are shown in Table 1.

Table 1  
Parameters for elastic modulus

Property $\varepsilon$	$m$	$\beta$	$\sigma$	$u$	$\sigma/u$
Small slurry bone ( $n_I = 3, n_S = 9$ )	1.853	4.459	2.22	3.96	0.56
Small slurry bone ( $n_I = 20, n_S = 9$ )	3.296	14.16	4.24	12.7	0.334
Small slurry bone ( $n_I = 50, n_S = 9$ )	1.785	27.43	14.14	24.4	0.579
Large bones ( $n_I = 3, n_S = 9$ )	1.448	68.5	43.58	62.12	0.702
Large bones ( $n_I = 20, n_S = 9$ )	1.818	59.45	30.11	52.84	0.57
Large bones ( $n_I = 50, n_S = 9$ )	1.559	52.48	30.91	47.17	0.66

$\varepsilon$ , elastic modulus;  $m$ , Weibull modulus;  $\beta$ , fitting parameter;  $\sigma$ , standard deviation;  $u$ , mathematical expectation;  $n_I$ , denotes the number of impaction;  $n_S$ , denotes the number of samples.

## Elastic modulus

The elastic modulus of the 7–10-mm bone grafts was higher than that of the small slurry bone grafts at each test level ( $P < 0.001$ , two-way ANOVA). The mean elastic moduli of the 7–10 mm bone grafts ranged from 49.64 to 68.06 MPa. The highest mean elastic modulus of small slurry bone was 29.12 MPa, and was achieved only after 50 impactions (Fig. 2) [16].

## Weibull distribution

The probability functions of the elastic moduli followed the Weibull distribution (Fig. 3). The  $m$  values of small slurry bone grafts at all test levels were higher than those of large bone grafts. Thus, small slurry bone was more stable (more reliable) than larger bone (Fig. 4).

## Discussion

We used the Weibull distribution to evaluate the reliability of bone graft mechanical properties. A high reliability, indicated by a high  $m$ , indicates that the mechanical value distribution is narrow and the outcomes predictable. We tested mechanical properties to 50 impactions; no further change in elastic modulus was apparent. Also, it is unusual for an orthopedist to impart 50 impactions considering the time required and the risk of fracture [1].

Large bone grafts (7–10 mm) are widely used, as recommended by Schreurs et al. [10, 12]. Unexpectedly, we found that the elastic modulus of the 7–10 mm bone grafts did not increase during the first 20 impactions. We used experimental equipment similar to that of Bavadekar et al. [3], but with larger bone grafts. The inner diameter of the tube was twice that of the (smallest) 7-mm bone grafts. The cubes could thus readily form different structures within the tube. When impacted, the inner structures of the particles,

and the inter-particle voids, can change. The effects of such changes on the elastic modulus could not be observed but may be associated with particle shape; further research is necessary to confirm this. However, the steady-state curve attained after 30 impactions was similar to that of other reports, suggesting that further impaction was not required [1, 5, 6]. The irregular particles of small slurry bone became increasingly stiffer during impaction. However, even after 50 impactions, the elastic modulus remained lower than that of the 7–10-mm bone grafts, indicating poorer resistance to elastic deformation and complex stresses.

The elastic moduli of the large bone grafts ranged more widely than those of the small slurry bone grafts at each test level, suggesting that the *in vitro* data might be unreliable. Using the Weibull distribution, we confirmed that the elastic modulus of 7–10-mm bone grafts was less reliable than that of small slurry bone grafts, reflecting variation in the free volume (inter-particle voids). The samples were of near-identical weight, but the large bone grafts were of much higher volume because the inter-particle voids were larger and the free volume fraction thus greater, associated with low-level homogeneity and poor uniformity (i.e., a low  $m$  value). However, although the elastic modulus of small slurry bone grafts was more reliable, we cannot conclude that the mechanical properties of this material were better. The elastic modulus remained low during testing, suggesting that the initial stability was poor.

## Conclusion

We suggest that acceptable grafts should exhibit high mechanical values and reliability. When the elastic moduli of different graft materials do not differ significantly, a graft with a high Weibull modulus  $m$ , reflecting high-level reliability, should be preferred.

## Declarations

### Ethical Approval and Consent to participate

This study was approved by the Ethics Committee of The First Affiliated Hospital of Zhejiang Chinese Medical University. Written informed consents were obtained from all patients.

### Consent for publication

Written informed consents were obtained from all patients.

### Availability of supporting data

The datasets used and analyzed during the current study are available from the corresponding author on reasonable request.

### Competing interests

The authors declare that they have no competing interests.

## Funding

This work was supported by grants from the National Natural Science Foundation of China (No. 81572124).

## Authors' contributions

Conceived and designed the experiment: RH; performed the experiments: MY, LT, and CY; analyzed the data: MY and LT; contributed materials: MY, LT, and CY; and wrote the manuscript: MY. All authors read and approved the final manuscript.

## Acknowledgments

Not applicable.

## Authors' information

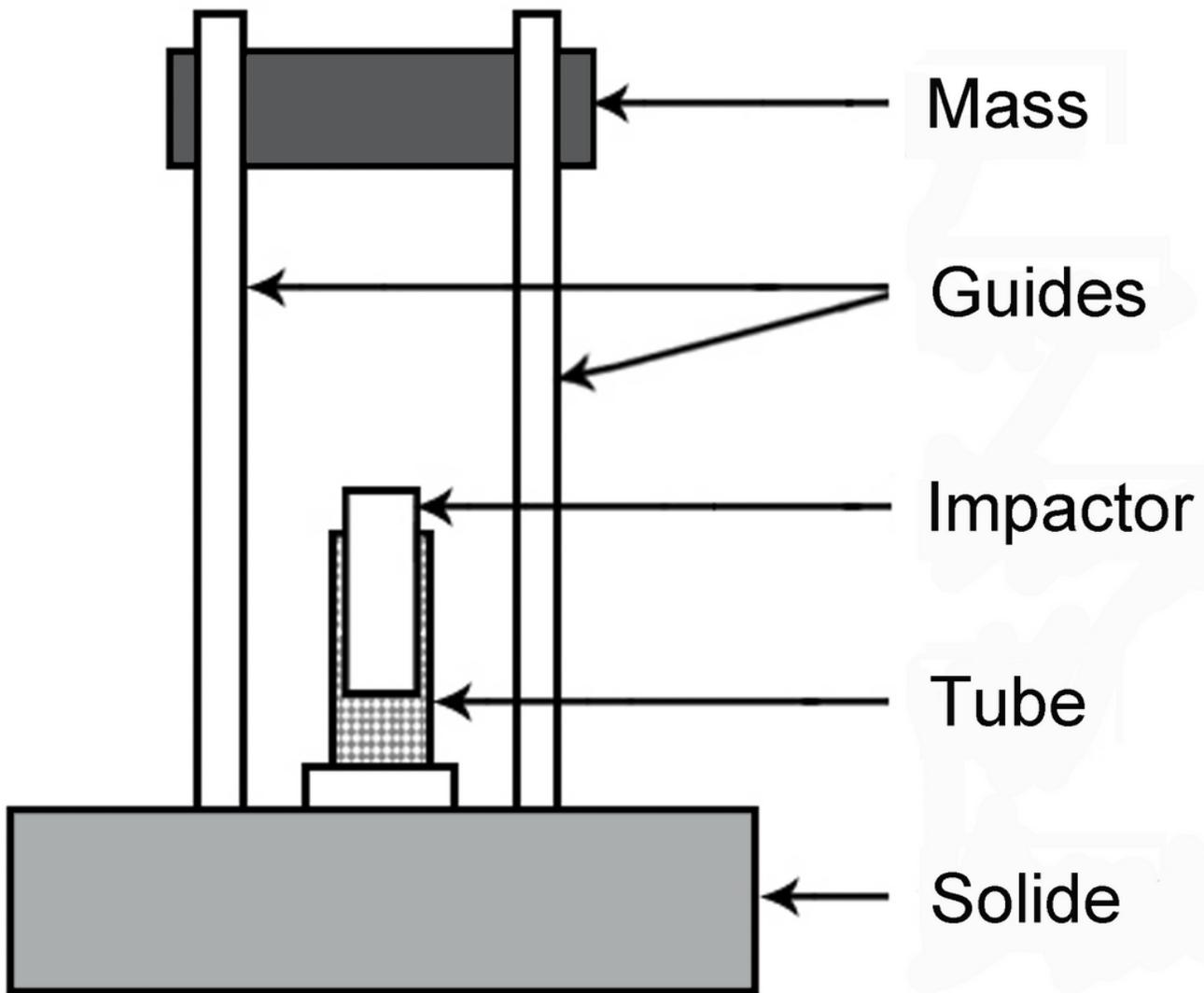
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## Figures



**Figure 1**

A graft in a tube was impacted by an impactor driven by a mass. Marrow and liquid emerged through vents 1.5-mm in diameter.

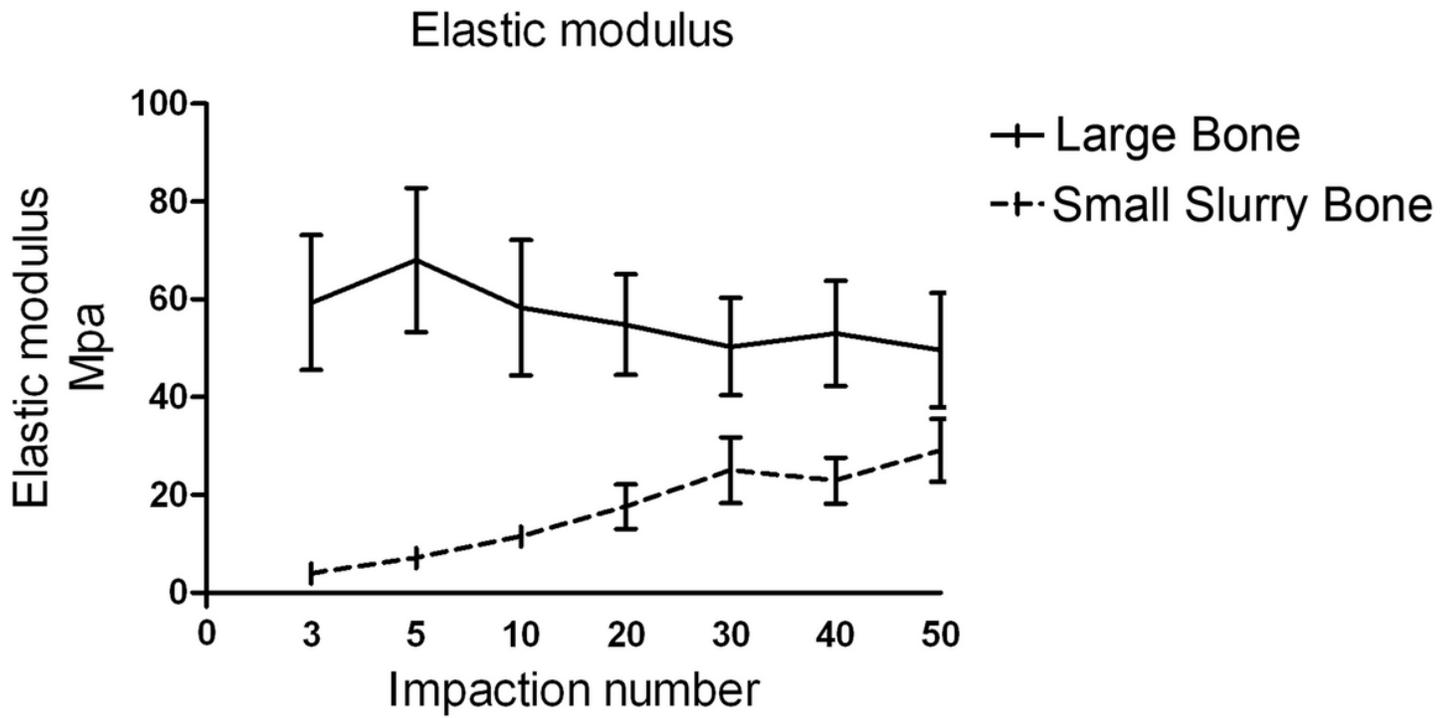
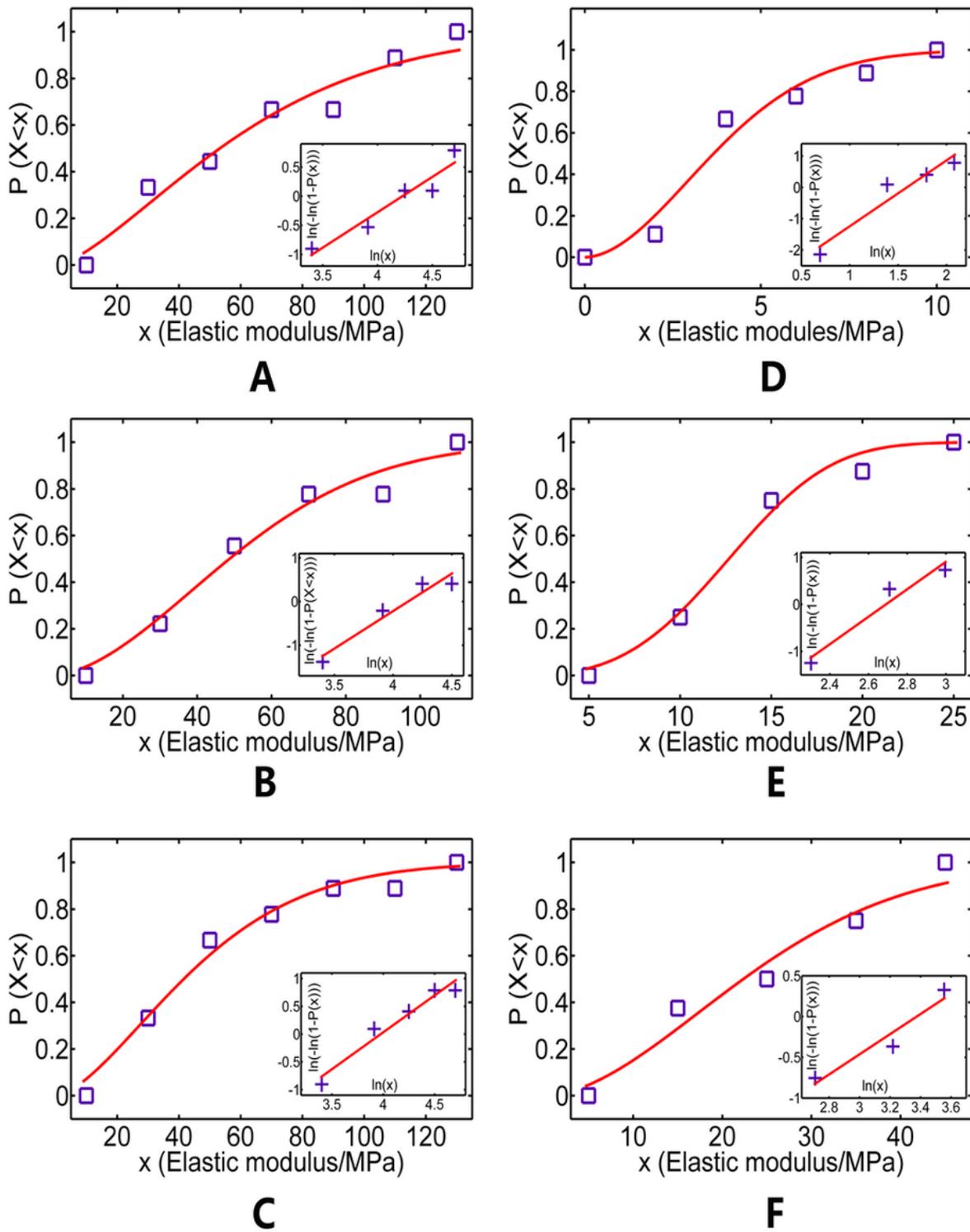


Figure 2

Changes in the elastic moduli of the two types of grafts during impactation.  $P < 0.001$  at each test level.



**Figure 3**

The elastic modulus probability distribution functions for large bone grafts at impactions 3 (A), 20 (B), and 50 (C) fitted by the Weibull function; and that for small slurry bone grafts at impactions 3 (D), 20 (E), and 50 (F), also fitted by the Weibull function. Inset:  $\ln[-\ln(1-F(x))]$  versus  $\ln(x)$  plot, where  $x$  is the elastic modulus and  $F(x)=P(X < x)$ .

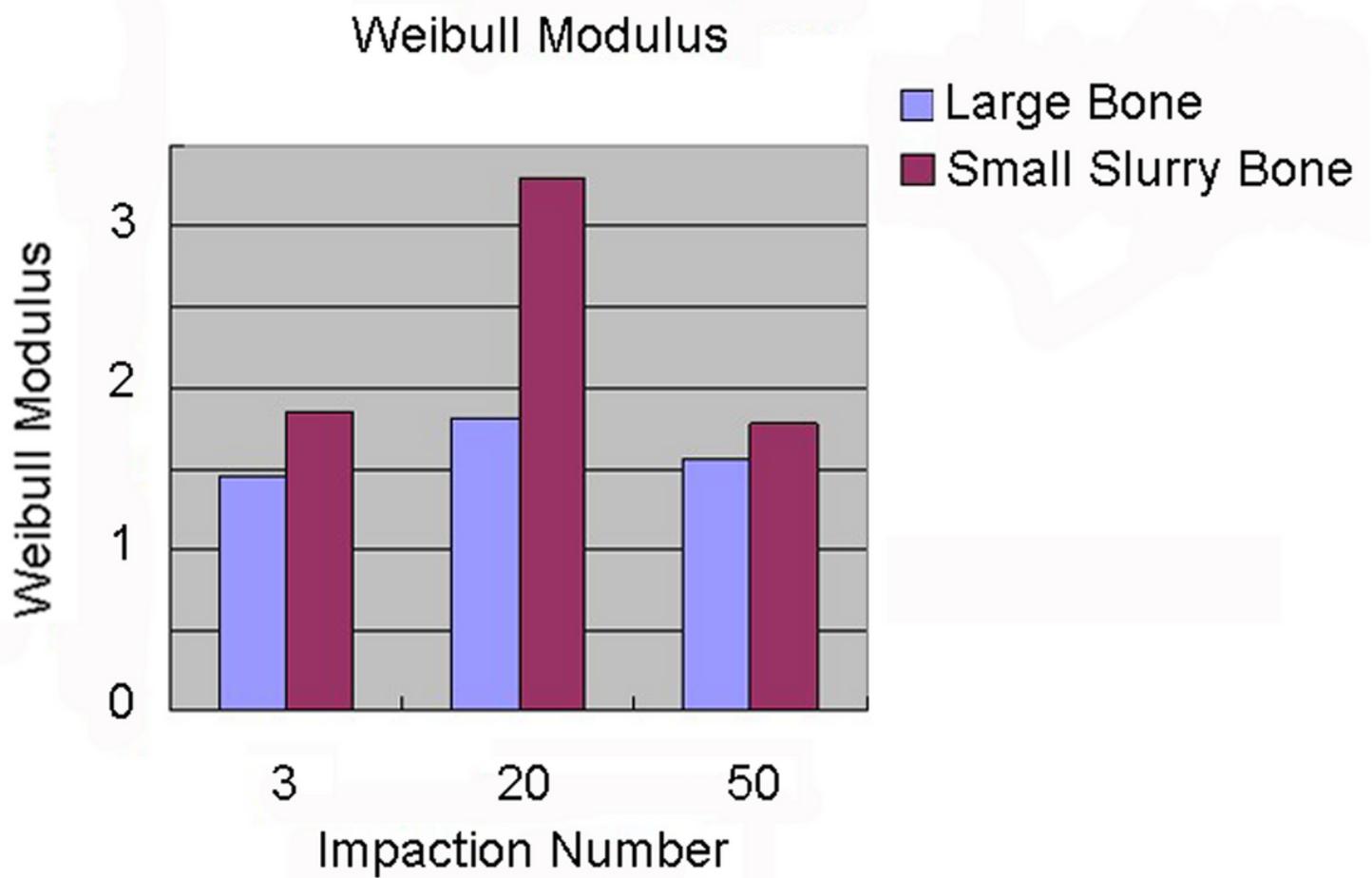


Figure 4

The Weibull moduli (m values) of the two types of bone grafts.

## Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [Equation1.pdf](#)