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Abstract Applying geometry to the analysis and interpretation of basic probability assignment(BPA) is a unique research direction in evidence theory. Though the geometric representation of BPA has been proposed, the visualization method of BPA is still lack of sufficient research. In this paper, we propose a new BPA visualization method based on the vector representation of the BPA to illustrate the image of BPA directly. The basic point and the uncertain vectors can be obtained by the given BPA firstly, and then we connect these components to construct the image of BPA. Through the image of BPA, we can effectively analyze the interaction effect of focal elements in BPA, and observe the potential characteristics of BPA directly. Meanwhile, the geometric meanings of parameters in the vector representation of the BPA can be explained. Finally, the advantages and applications have been studied and discussed.

Keywords Dempster-Shafer evidence theory · basic probability assignment visualization · basic probability assignment vector · uncertain vector

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1 Introduction

In the actual decision-making process Fei and Deng (2020); Song et al. (2015a); Jiang and Wei (2018), the strength of uncertainty dramatically affects the reliability of decision-making. Therefore, accurate measurement and reduction of uncertainty have become an essential topic in the field of computational science and artificial intelligence Song et al. (2016); Wang and Song (2018). In the past decades, many theories have been proposed to explain and analyze uncertainty, such as evidence theory Luo and Deng (2019); Li et al. (2019), D number Fan et al. (2016); liu and Deng (2019), Z number Liu et al. (2019b); Li et al. (2020), fuzzy set Zavadskas et al. (2015); Nguyen et al. (2015), and else Erkmén and Stephanou (1990); Han and Deng (2018). These studies have been widely used in some practical scenarios, such as industrial alarm system Xu et al. (2018, 2016), data clustering Liu et al. (2016); Xu et al. (2017), pattern recognition Ding et al. (2016); Lu et al. (2018).

Dempster-Shafer evidence theory, called as D-S evidence theory Dempster (2008); Shafer (2016), as a classical and efficient tool to model the uncertainty from multiple information sources, has been receiving increasing attention. Through assigning the probability value (called basic probability assignment, BPA) to a set of events instead of a single event, the D-S evidence theory can capture the uncertainty in the case of incomplete and conflicting information. However, in the complex and high conflict situation, the original D-S evidence theory often leads to unreasonable results. Hence, in the recent years, some improvements and relevant theories have emerged, for example, generalized D-S theory Xiao (2020); Gao and Deng (2020), linguistic D numbers Seiti et al. (2020); Mo and Deng (2019); Zhao and Deng (2019), which also has been applied in complex networks analysis Wen and Deng (2020); Wen et al. (2020), classifier design Denoeux (1995, 2000), environmental assessment Kang et al. (2019); Gao and Deng (2019).

As an extension of probability, the BPA in D-S evidence theory plays a vital role in representing the possibility of hypothesis and capturing uncertainty. However, compared with BPAs or probabilities, people are more sensitive to the shape or images of objects. Therefore, in recent years, some scholars had made some efforts to explain D-S evidence theory with geometry Ha et al. (1998); Black (1997); Daniel (2006). For example, Cuzzolin (2008) introduced the belief space and built a geometric approach to the evidence theory. Luo and Deng (2020) proposed the BPA vector (BPAV) as a new vector and geometry interpretation of BPA, which is much short than Cuzzolin (2008), and this representation is more comfortable to analyze.

However, Luo and Deng (2020) focused on the vector representation and the D-S fusion rules under this representation and lacked the analysis of BPA image. Hence, in this paper, we proposed a new BPA visualization method based on Luo's vector representation. In this method, the BPA image was drawn by constructing a base point and uncertain vectors, and gradually adding the uncertain vector from the base point. Finally, we could obtain the image of BPA in the BPAV space and observe the features of the BPA

directly. Meanwhile, the parameters, like $\kappa(H_1|H_1, H_2)$ Luo and Deng (2020), could have a reasonable geometric interpretation in the proposed method. In the end, we used some examples to show the visualization result in 2-elements and 3-elements situation.

The rest of this paper is organized as follows: Section 2 starts with a brief introduction of the basic concepts, such as: evidence theory, basic probability assignment vector and else. The proposed visualization method is presented in Section 3. Section 4 gives the numerical examples to testify our proposed method and some discussion about the results. The final conclusion is in Section 5.

2 Preliminaries

2.1 Dempster-Shafer Evidence Theory

Dempster-Shafer Evidence Theory as a standard mathematical tool to model multiple-source information fusion problem Dempster (2008); Shafer (2016) has been widely researched many years. In recent years, many researchers continuously enrich the content of D-S theory, such as fusion rules Song et al. (2015b); Denoeux (2008); Dutta (2018), conflict management methods Cai et al. (2020); Liu et al. (2019a); Murphy (2000), and entropy Yan and Deng (2020); Li et al. (2019). Moreover, the proposed method is also based on the theoretical framework, so the D-S theory is briefly introduced as follows.

The framework of discernment(FOD), represented as Θ , is an exhaustive set of all hypotheses of a random variable and these hypotheses are mutually exclusive. In general, the FOD with N elements is expressed as

$$\Theta = \{H_1, H_2, H_3, \dots, H_N\} \quad (1)$$

where H_i represents the i-th hypothesis. The power set of Θ , called as 2^Θ , contains the all possible subsets, like:

$$2^\Theta = \{\emptyset, \{H_1\}, \{H_2\}, \{H_1, H_2\}, \dots, \Theta\} \quad (2)$$

A basic probability assignment function(BPA) is a mapping from 2^Θ to $[0, 1]$, defined as

$$m : 2^\Theta \rightarrow [0, 1] \quad (3)$$

and the BPA needs to satisfy the next condition as:

$$\begin{aligned} \sum_{A \in 2^\Theta} m(A) &= 1 \\ m(\emptyset) &= 1 \end{aligned} \quad (4)$$

2.2 Vector Representation of BPA

Given a FOD $\Theta = \{H_1, H_2, H_3, \dots, H_N\}$, a BPA can be represented as a vector $\mathbf{M}$, called as the basic probability assignment vector (BPAV) Luo and Deng (2020):

$$\mathbf{M} = (M_1, M_2, M_3, \dots, M_N) \quad (5)$$

where

$$M_j = \sum_{A_i \subseteq 2^\Theta} m(A_i) \kappa(H_j | A_i) \quad (j = 1, 2, 3, \dots, N) \quad (6)$$

M_j is called probability assignment quantity (PAQ) Luo and Deng (2020), and these PAQs satisfy the normalization condition:

$$\sum_{j=1}^N M_j = 1 \quad (7)$$

According to (5) and (6), for a specific BPA, the corresponding BPAV is relevant to the variable parameters $\kappa(H_j | A_i)$. Following the definition of BPAV Luo and Deng (2020), these variable parameters should satisfy the conditions as:

$$\begin{aligned} \kappa(H_j | A_i) &= 0 & \text{if } H_j \notin A_i \\ \kappa(H_j | A_i) &\in [0, 1] & \text{if } H_j \in A_i \\ \sum_{H_j \in A_i} \kappa(H_j | A_i) &= 1 & \text{for a fixed } A_i \in \Theta \end{aligned} \quad (8)$$

It is worth noting that for a certain BPA, not only one determined BPAV corresponds to it, but all BPAVs that satisfy the above conditions. For example, if there is a BPA in the binary FOD $\Theta = \{H_1, H_2\}$, like $m(H_1) = 0.5, m(H_2) = 0.3, m(\Theta) = 0.2$, the BPAV of this BPA can be expressed as:

$$\begin{aligned} M &= (m(H_1) + \kappa(H_1 | \Theta)m(H_1 | \Theta), \quad m(H_2) + \kappa(H_2 | \Theta)m(H_2 | \Theta)) \\ &= (0.5 + 0.2\kappa(H_1 | \Theta), \quad 0.3 + 0.2\kappa(H_2 | \Theta)) \end{aligned} \quad (9)$$

where $\kappa(H_1 | \Theta) + \kappa(H_2 | \Theta) = 1$ and $\kappa(H_1 | \Theta), \kappa(H_2 | \Theta) \in [0, 1]$. Hence, with the different value of the variable parameters, the BPAV $\mathbf{M}$ can obtain different results in the $M_1 - M_2$ Cartesian coordinate system. The boundary points of the BPAV are easy to obtain by setting the variable parameter as the extreme values, such as: $\kappa(H_1 | \Theta) = 0, \kappa(H_2 | \Theta) = 1$.

The meanings of these variable parameters, like $\kappa(H_j | A_i)$, represent the proportion of focal elements with multiple subsets A_i to each independent single subset H_j , such as $m(H_1, H_2, H_3)$ to H_1, H_2, H_3 . The meaning of the PAQs represents the probability.

3 The Proposed Method

In this section, we give some necessary concepts firstly, and then the proposed visualization method will be introduced. In order to describe the method accurately, we give the specific process with an example.

3.1 The Basic Geometry Concepts

To represent the geometry of the BPA in the N-dimension space, we need to define some basic concepts: BPAV space, constraint plane, basic point, uncertain vector and uncertain geometry. The BPAV space and constraint plane are used to describe the external environment of BPAV, and the other concepts are used to depict the image of BPAV.

Definition 3.1 BPAV space P_b is an N-dimension space when the FOD has N elements. Each axis of the BPAV space represents a corresponding probability assignment quantity (PAQ).

For example, for a FOD $\Theta = \{H_1, H_2, H_3\}$, the three axis separately represent the M_1, M_2, M_3 .

Definition 3.2 According to (7), all BPAVs must be in the one plane in the P_b , called constraint plane.

For example, for a FOD $\Theta = \{H_1, H_2, H_3\}$, the constraint plane is $M_1 + M_2 + M_3 = 1$. At the same times, the constraint planes in the 2-elements and 3-elements FOD are shown in the Figure 1.

Fig. 1 The constraint plane and BPAV space

In Figure 1, the black line and black plane represent the constraint plane in the corresponding BPAV space.

Definition 3.3 Basic point is the start point of constructing uncertain geometry, and it is defined as $A = (m(H_1), m(H_2), \dots, m(H_N))$ where $m(H_j)$ is the BPA of the H_j .

Definition 3.4 Uncertain vector is a particular type of vector with the same dimension as the FOD. It can be expressed as $[x_1, x_2, \dots, x_N]$, and raise from the focal elements containing over one element.

For example, there is a focal element like $m(H_1, H_2) = 0.3$ and $FOD = \{H_1, H_2, H_3\}$, the corresponding uncertain vector is $V_{H_1-H_2} = [a_1, a_2, 0]$ and $\|V_{H_1-H_2}\|_1 = 0.3$.

Definition 3.5 Uncertain geometry is the image of BPA in BPAV space, which is built by the basic point and the uncertain vectors. We can start from

the basic point A , and then we go through all the uncertain vectors to get the endpoint B . All endpoints B will make up a polygon in the BPAV space, and that is uncertain geometry.

Then, we will take an example to show these definitions by a certain BPA intuitively. Given a FOD $\Theta = \{H_1, H_2, H_3\}$, the BPA is $m(H_1) = 0.3, m(H_2) = 0.1, m(H_1, H_2) = 0.3, m(H_1, H_3) = 0.2, m(\Theta) = 0.1$. The basic point A and the uncertain vectors are:

$$\begin{aligned} A &= (m(H_1), m(H_2), m(H_3)) = (0.3, 0.1, 0.0) \\ V_{H_1-H_2} &= [a_1, a_2, 0] \quad \|V_{H_1-H_2}\|_1 = 0.3 \\ V_{H_1-H_3} &= [b_1, 0, b_2] \quad \|V_{H_1-H_3}\|_1 = 0.2 \\ V_{H_1-H_2-H_3} &= [c_1, c_2, c_3] \quad \|V_{H_1-H_2-H_3}\|_1 = 0.1 \end{aligned} \quad (10)$$

where $V_{H_1-H_2}, V_{H_1-H_3}, V_{H_1-H_2-H_3}$ are the uncertain vectors and correspond with the focal elements $m(H_1, H_2), m(H_1, H_3), m(H_1, H_2, H_3)$. The symbol $\|\bullet\|$ represents the L1-norm. The parameters a_i, b_i, c_i is over 0.

3.2 BPA Visualization Method

When we have the above definition, the BPA visualization method can be proposed. Due to the constraint in Eq.(7), all BPAVs converted from this BPA must be in the constraint plane. The constraint plane represents the maximal valid range of BPAV, and the uncertain geometry represents the image of BPA in the BPAV space.

Then, the proposed BPA visualization method can be concluded as two steps:

1. Calculating the basic point and uncertain vectors by the given BPA;
2. Making the discretization for each uncertain vector;
3. Calculating the endpoint by starting from the basic point and passing each type of uncertain vectors;
4. The image of BPA can be constructed by all endpoints.

Next, the details of the above steps will be shown by a specific example. A BPA is $m(H_1) = 0.3, m(H_2) = 0.1, m(H_1, H_2) = 0.3, m(H_1, H_3) = 0.2, m(\Theta) = 0.1$. Hence, the basic point A and the corresponding uncertain vectors $V_{H_1-H_2}, V_{H_1-H_3}, V_{\Theta}$ are easy to obtain by the definition, and the results are shown in Eq.(10).

Because the uncertain vector is not definite, but a set of vectors satisfying the conditions. Therefore, we need to discretize the uncertain vectors to get definite vectors. For example, an uncertain vector V_{H_1, H_2} can be discretized as $[0.3, 0, 0]$ or $[0.1, 0.2, 0]$ or $[0, 0.3, 0]$ or else. The image of these uncertain vectors are in Figure 2.

Fig. 2 The shape of uncertain vectors

The blue line and red line are separately the images of V_{H_1, H_3} and V_{H_1, H_2} , and the green triangle is the image of V_Θ .

After acquiring the point A and uncertain vectors, the next step is to construct the uncertain geometry. We start from the base point, through all the uncertain vector $V_{H_1-H_3}$, $V_{H_1-H_2}$, V_Θ jumps, and finally reach all the endpoints. These endpoints make up the uncertain geometry. Taking a point $(0.5, 0.4, 0.1)$ in the uncertain geometry as an example demonstrates the details. The process is as follows: (the order of summing is unimportant)

Table 1 The process of obtaining a point in the uncertain geometry

step	start point	uncertain vector	endpoint
1	$(0.3, 0.1, 0.0)$	$V_{H_1-H_3} = (0.2, 0.0, 0.0)$	$(0.5, 0.1, 0.0)$
2	$(0.5, 0.1, 0.0)$	$V_{H_1-H_2} = (0.0, 0.3, 0.0)$	$(0.5, 0.4, 0.0)$
3	$(0.5, 0.4, 0.0)$	$V_\Theta = (0.0, 0.0, 0.1)$	$(0.5, 0.4, 0.1)$

Through the process in Table 1, we can obtain a point in the uncertain geometry. It is easy to prove the point is on the constraint plane $M_1 + M_2 + M_3 = 1$ and these uncertain vectors $V_{H_1-H_3}$, $V_{H_1-H_2}$, V_Θ satisfy the definition. We draw the uncertain geometry and constraint plane together, shown in Figure 3.

Fig. 3 The BPA visualization result

In Figure 3, the red image represents the uncertain geometry, and the black plane is the constraint plane. The image of BPA in bpav space is the superposition of several uncertain vectors starting from the base point.

The complexity of the uncertain geometry is relevant to the level of complexity of the corresponding BPA. The more complex the BPA, the more complex the geometry. This method can intuitively display the geometric characteristics of BPA in BPAV space, and give geometric meaning to the parameters of BPAV.

4 Experiments

In this section, we will compare our proposed visualization method with the existing methods Cuzzolin (2008); Luo and Deng (2020) in two situations. Because it is difficult to draw the geometry more than 3-D directly, this paper mainly presents the visualization results in the case of 2-elements(2D) and 3-elements(3D).

4.1 A case of 2-elements

In 2-elements case, the FOD can be denoted as $FOD = \{H_1, H_2\}$. A general BPA can be represent as:

$$m(H_1) = a, \quad m(H_2) = b, \quad m(H_1, H_2) = 1 - a - b; \quad (11)$$

According to the our proposed method, this BPA can be transformed into BPAV and the image is shown in Fig.4

Fig. 4 the 2-elements case

In Fig.4, the basic point is $A = (a, b)$ and the uncertain vector only has one. The red line represents the image of this BPA, and $\overrightarrow{AD}$ denotes a case of the uncertain vector $V_{H_1-H_2}$. The points B and C are corresponding to the extreme values of the uncertain vector.

In 2-elements situation, our visualization method will obtain the same results Luo and Deng (2020); Cuzzolin (2008). Though the shapes are the same, the geometric interpretations are different. Following Luo and Deng (2020), we need to obtain the boundary points B, C firstly, and then confirm the $\overline{BC}$ is the image of BPA. However, our proposed method confirms the $\overline{BC}$ by the uncertain vector, basic point and constraint plane firstly, and then the boundary points will be obtained.

Furthermore, Combining the form of the uncertain vector and the calculation of BPAV, we can find that the uncertain vector can be constructed by parameters $\kappa(H_1|H_1, H_2)$ and $\kappa(H_2|H_1, H_2)$. In Figure 4, the uncertain vector

$\overrightarrow{AD}$ can be denoted as:

$$V_{H_1-H_2} = [a_1, a_2] \quad \|V_{H_1-H_2}\|_1 = 1 - a - b \quad (12)$$

where $a_1, a_2 \in [0, 1 - a - b]$. If we use the proposed visualization method to calculate the end point $D = (\hat{a}, \hat{b})$, we can gain the next formula:

$$\begin{aligned} \hat{a} &= m(a) + a_1 = m(a) + \kappa(H_1|H_1, H_2)(1 - a - b) \\ \hat{b} &= m(b) + a_2 = m(b) + \kappa(H_2|H_1, H_2)(1 - a - b) \end{aligned} \quad (13)$$

where $\kappa(H_1|H_1, H_2), \kappa(H_2|H_1, H_2) \in [0, 1]$.

According to Eq.(13), we can design a vector $[\kappa(H_1|H_1, H_2), \kappa(H_2|H_1, H_2)]$, and that vector must be the same as the uncertain vector $V_{H_1-H_2}$. Hence, the calculation of BPAV can be decomposed by the uncertain vectors and the base point, meanwhile, the process of superposition is the BPA visualization method. The result also can be obtained by the same method in 3-elements case.

4.2 A case of 3-elements

In 3-elements case, the FOD can be denoted as $FOD = \{H_1, H_2, H_3\}$. There is already a detailed example to illustrate the process of BPA visualization method in Section 3.2. Therefore, we will directly discuss the advantages of our proposed method.

The advantages of the proposed visualization method can be concluded as two main points. The first one is the proposed method can provide a tool to directly exhibit the combination effect of multiple focal elements, such as the $m(H_1, H_2)$ and $m(H_1, H_3)$ in the $FOD = \{H_1, H_2, H_3\}$. The second point is that the image of BPA can provide more latent characteristics than the "probability" form. Then, we will separately provide an experiment to illustrate these advantages.

4.2.1 The combination of the focal elements

For convenience, we use the same BPA in Section 3.2, and the BPA is $m(H_1) = 0.3, m(H_2) = 0.1, m(H_1, H_2) = 0.3, m(H_1, H_3) = 0.2, m(\Theta) = 0.1$. According to the proposed method, the basic point and the uncertain vectors are obtained as Eq.(10) and the images of the uncertain vectors are shown in Figure 2.

When we wonder the effect of the combination of multiple focal elements, the traditional methods often cannot directly represent. However, In our proposed method, We can analyze the influence of multiple focal elements on BPA itself by superimposing the uncertainty vector corresponding to multiple focal elements and show the influence in the form of an image.

For example, we wonder the combination effect of the $m(H_1, H_2) = 0.3$ and $m(H_1, H_3) = 0.2$. We just need to add two corresponding uncertain vectors $V_{H_1-H_2}, V_{H_1-H_3}$ from the base point $(0.3, 0.1, 0)$, and then we will obtain Figure 5.

Fig. 5 The combination effect

The red region is the image of this BPA, and the green region is the superposition of $V_{H_1-H_2}, V_{H_1-H_3}$. From Figure 5, we can find the combination of $V_{H_1-H_2}, V_{H_1-H_3}$ is a parallelogram, and the uncertain vector V_Θ can control the position of this parallelogram in the red region.

If we change the combination of uncertain vectors, we can get different images to show the joint effect of different focal elements. This method can validly exhibit the function of each focal element for the BPA, and provide a new tool to analyze the inner structure of BPA.

4.2.2 The characteristic of the visualization method

In Dempster-Shafer evidence theory, more types of uncertainty can be distinguished as two classes Yager (1983). The first one is called conflict, which denotes the sets with an empty intersection, and the other one is called as non-specificity, which corresponds to the sets with cardinality greater than one.

The proposed visualization method focuses on highlighting the non-specificity of BPA, rather than describing all the characteristics of BPA equally. Because the shape of BPA image is controlled by uncertain vectors, which are generated by the focal elements like $m(H_1, H_2), m(H_1, H_2)$, and the BPAs of a single set only control the position of the image. Hence, the image of BPA mainly represents the non-specificity, and the conflict can be represented by the intersection and the other region between two BPAs.

Then we use the three BPAs to show the above characteristic, and there are as follows: $(FOD : \Theta = \{H_1, H_2, H_3\})$

Table 2 The three BPAs

order	$m(H_1)$	$m(H_2)$	$m(H_3)$	$m(H_1, H_2)$	$m(H_1, H_3)$	$m(H_2, H_3)$	$m(\Theta)$
1	$\frac{1}{19}$	$\frac{1}{19}$	$\frac{1}{19}$	$\frac{3}{19}$	$\frac{3}{19}$	$\frac{3}{19}$	$\frac{7}{19}$
2	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
3	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0

The first BPA denotes the maximum Deng entropy distribution Kang and Deng (2015), and the second BPA represents the uniform distribution in evidence theory, and the third BPA represents the uniform distribution in probability theory. The images of these BPAs are shown in Figure 6.

Fig. 6 The image of the BPA in Table.2

Because the image of the third BPA in Table.2 is a single point $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, it is no necessary to give the image directly. But we can still use it to compare with other BPAs. According to Figure 6, we can find that the shape of the image is decided by the focal elements with cardinality greater than one, and the images of the first BPA and the second BPA are similar. Hence, the proportion of the $m(H_1, H_2) : m(H_1, H_3) : m(H_2, H_3)$ also affect the shape of the image.

5 Conclusion

In this paper, we proposed a new BPA visualization method to transform a BPA into an image in the BPAV space, and this method can work in the 2D or 3D situation which has potential to be extended to N-dimension. Through

the proposed method, we can directly research the characteristics of the BPA and have a new tool to exhibit the BPA by geometry.

Our contributions can be concluded as two points. The first one is the proposed visualization method itself which handles the challenge of exhibiting a BPA directly, and the necessary geometric concepts are defined. In order to describe the algorithm clearly and accurately, we give a specific example to show the algorithm process. The second point is that the proposed method has the ability to analyze the combined effect of multiple focal elements for a BPA, which is hard for the traditional method.

Although the proposed method can implement the BPA visualization function and the corresponding analysis method is introduced, with the increasing of the number of uncertain vectors, the complexity of drawing the image is increasing rapidly. For example, if we have four uncertain vectors and take 100 points for each vector, we will obtain the 100^4 points in the end. The problem will be solved in the future.

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Author contributions

HFL performed all experiments and wrote the manuscript. ZMP and YD contributed to the conception of the study and provided critical revisions.

Compliance with ethical standards

Conflict of interest All the authors certify that there is no conflict of interest with any individual or organization for the present work. This article does not contain any studies with human participants or animals performed by any of the authors.

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Figures

(a) 2-elements

(b) 3-elements

Figure 1

The constraint plane and BPAV space

Figure 2

The shape of uncertain vectors

Figure 3

The BPA visualization result

Figure 4

the 2-elements case

Figure 5

The combination effect

(a) Maximum deng entropy distribution

(b) Uniform distribution

Figure 6

The image of the BPA in Table.2