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Horizontal gravity in ocean Ekman transport

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Abstract

Three-dimensional gravity vector field $\mathbf{g}$ ($= \mathbf{i}g_\lambda + \mathbf{j}g_\phi + \mathbf{k}g_z$) in geodesy has been greatly simplified to a uniform vertical vector ($-g_0\mathbf{k}$) in oceanography with (λ, ϕ, z) the (longitude, latitude, height), $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ the corresponding unit vectors, and $g_0 = 9.81 \text{ m/s}^2$. Recent studies by the author show such simplification incorrect. The horizontal gravity is important in ocean dynamics. Along the same path, the horizontal gravity is included into the classical Ekman layer dynamics with constant eddy viscosity and depth-dependent-only density $\rho(z)$ represented by an e-folding near-inertial buoyancy frequency. A new Ekman spiral and in turn a new formula for the Ekman transport are obtained. With the horizontal gravity data from the global static gravity model EIGEN-6C4 and the surface wind stress data from the Comprehensive Ocean-Atmosphere Data Set (COADS), the Ekman transport due to the horizontal gravity is crucial and cannot be neglected.

Introduction

The seminal paper by Ekman [1] laid the foundation for the modern oceanography through modeling the turbulent mixing in upper ocean as a diffusion process similar to molecular diffusion, with an eddy viscosity K , which was taken as a constant with many orders of magnitude larger than the molecular viscosity. The turbulent mixing generates ageostrophic component of the upper ocean currents (called the Ekman spiral), decaying by an e-folding over a depth as the current vector rotate to the right (left) in the northern (southern) hemisphere through one radian. Along with the Ekman spiral, the Ekman transport was identified.

The Ekman theory was established using the uniform gravity ($-g_0\mathbf{k}$, $g_0 = 9.81 \text{ m s}^{-2}$) with $\mathbf{k}$ the unit vector in vertical (positive upward). However, the real gravity is well-known in geodesy as a three-dimensional vector field [$\mathbf{g} = \mathbf{g}_h + g_z\mathbf{k}$] with $\mathbf{g}_h$ the horizontal gravity vector. Recent research [2-6] show that the horizontal gravity $\mathbf{g}_h$ is comparable to the horizontal pressure gradient force, Coriolis force, and surface wind stress curl. Thus, the feasibility of using the uniform gravity ($-g_0\mathbf{k}$) in the ocean Ekman layer dynamics needs to be investigated. Oceanographers are referred to the appendices for information about the horizontal gravity.

Dynamic equation with real gravity

Steady-state large-scale ocean dynamic equation with the Boussinesq approximation (replacing density ρ by a constant ρ_0 except ρ being multiplied by the gravity) and the real gravity is given by [3-6]

$$\rho_0 [2\mathbf{\Omega} \times \mathbf{U}] = -\nabla p + \rho \mathbf{g} + \rho_0 \mathbf{F} \quad (1)$$

if the pressure gradient force, gravity, and friction are the only real forces. Here, ∇ is the three-dimensional vector differential operator in the polar spherical coordinates $\mathbf{\Omega} = \Omega(\mathbf{j} \cos \varphi + \mathbf{k} \sin \varphi)$, is the Earth rotation vector with $\Omega = 2\pi/(86164 \text{ s})$ the Earth rotation rate; ρ is the density; $\rho_0 = 1,028 \text{ kg/m}^3$, is the characteristic density; $\mathbf{U} = (u_\lambda, u_\varphi)$, is the horizontal velocity vector; p is the pressure; $\mathbf{F}$ is turbulent diffusive force due to the vertical shear represented by

$$\mathbf{F} = \frac{\partial}{\partial z} \left(K \frac{\partial \mathbf{U}}{\partial z} \right); \quad (2)$$

and $\mathbf{g}$ is the real gravity represented by [see Eq.(E7)]

$$\mathbf{g}(\lambda, \varphi, z) \approx \mathbf{g}_h - g_0 \mathbf{k}, \quad \mathbf{g}_h \approx g_0 \nabla_h N(\lambda, \varphi) \quad (3)$$

where $R = 6.3781364 \times 10^6 \text{ m}$, is the Earth radius; N is the geoid height relative to the normal Earth (i.e., reference spheroid); and $\nabla_h \equiv \mathbf{i} \frac{\partial}{R \cos \varphi \partial \lambda} + \mathbf{j} \frac{\partial}{R \partial \varphi}$ is the horizontal vector differential operator.

Let the horizontal velocity $\mathbf{U}$ be decomposed into geostrophic and ageostrophic (Ekman) velocity,

$$\mathbf{U} = \mathbf{U}_g + \mathbf{U}_E, \quad \rho_0 [2\mathbf{\Omega} \times \mathbf{U}_g] = -\nabla_h p \quad (4)$$

where $\mathbf{U}_E = (u_E, v_E)$ is the Ekman velocity. After substitution of (2)-(4) into (1), the horizontal component of (1) is represented by

$$\rho_0 [2\mathbf{\Omega} \times \mathbf{U}_E] = \rho \mathbf{g}_h + \rho_0 \frac{\partial}{\partial z} \left(K \frac{\partial \mathbf{U}_E}{\partial z} \right) + \rho_0 \frac{\partial}{\partial z} \left(K \frac{\partial \mathbf{U}_g}{\partial z} \right) \quad (5)$$

Baroclinicity (i.e., non-zero horizontal density gradient) and spatially varying eddy viscosity K affect the Ekman layer dynamics [7]. To limit the study on the effect of the

horizontal gravity, the eddy viscosity K is assumed constant and the density varies in vertical only, i.e., there is no vertical shear of the geostrophic current

$$\partial \mathbf{U}_g / \partial z = 0. \quad (6)$$

Furthermore, a special density $\rho(z)$ is selected for this study as the e-folding near-inertial buoyancy frequency [8],

$$\Theta(z) = \Theta_0 \exp\left(\frac{z}{d}\right), \quad \Theta^2(z) \equiv -\frac{g_0}{\rho_0} \frac{d\rho}{dz}, \quad \Theta_0 = 5.24 \times 10^{-3} \text{ s}^{-1}, \quad d = 1.3 \text{ km} \quad (7a)$$

The second formula in (7a) becomes

$$ds(z)/dz = -\frac{\Theta_0^2}{g_0} e^{2z/d}, \quad s(z) \equiv \rho / \rho_0, \quad (7b)$$

Substitution of (6) and (7b) into (5) leads to the Ekman dynamic equation with including the horizontal gravity

$$2\mathbf{\Omega} \times \mathbf{U}_E = s(z)\mathbf{g}_h + K \frac{\partial^2 \mathbf{U}_E}{\partial z^2} \quad (8)$$

The horizontal gravity, Ekman velocity, vertical shear of the Ekman velocity, and the Ekman volume transport are defined in complex variables

$$G_h = g_\lambda + i g_\varphi, \quad U = u_E + i v_E, \quad W = \frac{\partial U}{\partial z}, \quad M = \rho_0 \int_{-\infty}^0 U dz, \quad i \equiv \sqrt{-1} \quad (9)$$

where (g_λ, g_φ) are the longitudinal and latitudinal components of the horizontal gravity. The Ekman equation (8) is converted into the complex form

$$\frac{\partial^2 U}{\partial z^2} - i \frac{f}{K} U = -\frac{G_h}{K} s(z). \quad (10)$$

Differentiation of (10) with respect to z and use of (7b) and (9) lead to the equation for the vertical shear of the Ekman velocity

$$\frac{\partial^2 W}{\partial z^2} - i \frac{f}{K} W = \frac{G_h \Theta_0^2}{K} \exp\left(\frac{2z}{d}\right) \quad (11)$$

Boundary conditions

The turbulent momentum flux should be continuous at the ocean surface ($z = 0$),

$$\rho_0 K W \Big|_{z=0} = \tau_\lambda + i \tau_\varphi \quad (12)$$

where $(\tau_\lambda, \tau_\varphi)$ are the surface wind stress components. The Ekman velocity U and vertical shear W need to be satisfied by the lower boundary condition,

$$|W| \text{ finite as } z \rightarrow -\infty \quad (13a)$$

$$|U| \rightarrow 0 \text{ as } z \rightarrow -\infty \quad (13b)$$

Ekman solutions in complex form

Eq.(11) with the boundary conditions (12) and (13a) has an exact solution

$$W(z) = \frac{1}{\rho_0 K} \left[(\tau_\lambda + i \tau_\varphi) - \frac{B G_h}{g_0} (2\chi^2 + i) \right] e^{(1+i)\pi z/D_E} + \frac{B}{\rho_0 K} \frac{G_h}{g_0} (2\chi^2 + i) e^{2z/d} \quad (14)$$

where

$$D_E \equiv \pi \sqrt{\frac{2K}{|f|}}, \quad \chi \equiv \frac{D_E}{\pi d} = \frac{\sqrt{2K/|f|}}{d}, \quad B \equiv \frac{\rho_0 \Theta_o^2 D_E^2}{2\pi^2 g_0 (4\chi^4 + 1)} \quad (15)$$

Here, D_E is the Ekman layer depth; and χ is proportion to the ratio between the Ekman layer depth (D_E) and the e-folding depth (d) of the buoyancy frequency Θ . Integration of (14) with respect to z from $z = -\infty$ to z and use of the lower boundary condition (13b) lead to the Ekman spiral,

$$U(z) = \frac{D_E}{2\pi \rho_0 K} \left\{ (\tau_\lambda + \tau_\varphi) + i(\tau_\varphi - \tau_\lambda) - B G_h \left[(2\chi^2 + 1) - i(2\chi^2 - 1) \right] \right\} e^{(1+i)\pi z/D_E} + \frac{B d G_h}{2\rho_0 K} (2\chi^2 + i) e^{2z/d} \quad (16)$$

Integration of (16) with respect to z from $-\infty$ to 0 leads to the complex form of the Ekman transport

$$M = \rho_0 \int_{-\infty}^0 U dz = \frac{1}{f} (\tau_\phi - i\tau_\lambda) - \frac{BG_h}{f} (1 - i2\chi^2) + \frac{d^2 BG_h}{4K} (2\chi^2 + i) \quad (17)$$

Simplification due to small parameter χ

The eddy viscosity K inferred from Ekman theory and the time-averaged stress was directly estimated from $0.054 \text{ m}^2 \text{ s}^{-1}$ [9] obtained from the field measurements acquired from a surface mooring set in the western Sargasso Sea (34°N , 70°W) as part of the Long Term Upper Ocean Study Phase 3 (LOTUS-3) during the summer of 1982, to $0.006 \text{ m}^2 \text{ s}^{-1}$ [10] obtained from the low-frequency current measurements in the Strait of Georgia, British Columbia. The smaller value ($0.006 \text{ m}^2 \text{ s}^{-1}$) may be treated as a lower bound of the eddy viscosity [9]. With the larger value of $K = 0.054 \text{ m}^2 \text{ s}^{-1}$, the parameter χ is estimated by

$$\chi = \frac{\sqrt{2K/|f|}}{d} = \frac{0.02095}{\sqrt{|\sin \phi|}}, \text{ for } K = 0.054 \text{ m}^2 \text{ s}^{-1}, d = 1.3 \text{ km} \quad (18)$$

where the parameter χ varies from 0.071 at $\phi=5^\circ$ (N or S) to 0.021 at $\phi=90^\circ$ (N or S). The maximum values of χ^2 and χ^4 (at $\phi=5^\circ$ S or N) are estimated by

$$\chi^2 \leq 0.5036 \times 10^{-2}, \quad \chi^4 \leq 0.2536 \times 10^{-4} \quad (19)$$

It is reasonable to delete χ^4 in (15) and the parameter B becomes

$$B \equiv \frac{\rho_0 \Theta_o^2 D_E^2}{2\pi^2 g_0} = \frac{\rho_0 K \Theta_o^2}{|f| g_0} \quad (20)$$

After deleting terms with χ^2 , the Ekman profile (16) and the Ekman transport (17) become

$$U(z) = \frac{D_E}{2\pi\rho_0 K} \left\{ (\tau_\lambda + \tau_\varphi) + i(\tau_\varphi - \tau_\lambda) - BG_h(1+i) \right\} e^{(1+i)\pi z/D_E} + \frac{BdG_h}{2\rho_0 K} i e^{2z/d} \quad (21)$$

$$M = \rho_0 \int_{-\infty}^0 U dz = \frac{1}{f} (\tau_\varphi - i\tau_\lambda) - \frac{BG_h}{f} + \frac{d^2 BG_h}{4K} i \quad (22)$$

Ekman transport

The Ekman transport (22) is transformed back into the vector form

$$\mathbf{M} = -\mathbf{k} \times \frac{\boldsymbol{\tau}}{f} + \left(\frac{\rho_0 \Theta_0^2 d^2}{4fg_0} \right) \mathbf{k} \times \mathbf{g}_h - \frac{\rho_0 \Theta_o^2 K}{f|f|g_0} \mathbf{g}_h \quad (23)$$

which shows the contribution of the horizontal gravity $\mathbf{g}_h$ to the Ekman transport. Let the Ekman transport ($\mathbf{M}$) be separated into two parts

$$\mathbf{M} = \mathbf{M}_w + \mathbf{M}_G \quad (24)$$

with

$$\mathbf{M}_w = -\mathbf{k} \times \frac{\boldsymbol{\tau}}{f} \quad (25)$$

due to the surface wind stress (i.e., classical Ekman transport), and

$$\mathbf{M}_G = \mathbf{k} \times \frac{1}{f} \left(\frac{\rho_0 \Theta_0^2 d^2}{4g_0} \mathbf{g}_h \right) - \frac{\rho_0 \Theta_o^2 K}{f|f|g_0} \mathbf{g}_h, \quad (26)$$

due to the horizontal gravity. The Ekman transport due to the horizontal gravity $\mathbf{M}_G$ has along $\mathbf{g}_h$ component, $-\rho_0 \Theta_o^2 K / (f |f| g_0) \mathbf{g}_h$, and across $\mathbf{g}_h$ component, $[(\rho_0 \Theta_o^2 d^2) / (4 f g_0)] \mathbf{g}_h$. The ratio between the two component magnitudes is given by

$$\frac{[\rho_0 \Theta_o^2 K / (f |f| g_0)] \mathbf{g}_h}{[(\rho_0 \Theta_o^2 d^2) / (4 f g_0)] \mathbf{g}_h} = \frac{K}{4 d^2 |f|} = \frac{\chi^2}{8} \ll 1 \quad (27)$$

where (19) is used. Thus, the along $\mathbf{g}_h$ component is negligible. The Ekman transport due to the horizontal gravity is given by

$$\mathbf{M}_G \simeq \mathbf{k} \times \frac{1}{f} \left(\frac{\rho_0 \Theta_o^2 d^2}{4 g_0} \mathbf{g}_h \right) \quad (28)$$

The Ekman transport due to both surface wind stress and horizontal gravity (23) becomes

$$\mathbf{M} = -\mathbf{k} \times \frac{1}{f} \left(\boldsymbol{\tau} - \frac{\rho_0 \Theta_o^2 d^2}{4 g_0} \mathbf{g}_h \right) \quad (29)$$

Substitution of (3) into (29) leads to

$$\mathbf{M} = -\mathbf{k} \times \frac{1}{f} \left(\boldsymbol{\tau} - \frac{\rho_0 \Theta_o^2 d^2}{4} \nabla_h N \right) \quad (30)$$

A nondimensional E number is defined by

$$E \equiv \frac{\rho_0 \Theta_o^2 d^2 |\mathbf{g}_h|}{4 g_0 |\boldsymbol{\tau}|} = \frac{\rho_0 \Theta_o^2 d^2 |\nabla_h N|}{4 |\boldsymbol{\tau}|} \quad (31)$$

to represent relative importance of horizontal gravity versus wind stress on the Ekman transport. Note that the non-dimensional E number is independent of the eddy viscosity K , but dependent on $\Theta_0^2 d^2$, $|\nabla_h N|$, and $|\boldsymbol{\tau}|$.

Data sources

Two datasets were used to identify importance of the horizontal gravity on the Ekman layer dynamics in addition to the surface wind stress: (a) global static geoid undulation (N) dataset from the EIGEN-6C4 model [11] (<http://icgem.gfz-potsdam.de/home>) for computing the horizontal gravity $\mathbf{g}_h$, (b) climatological annual mean surface wind stress (τ_λ, τ_ϕ) data from the Comprehensive Ocean-Atmosphere Data Set (COADS) [12] (<http://iridl.ldeo.columbia.edu/SOURCES/.DASILVA/.SMD94/.climatology/>) for computing the surface wind stress curl. The two datasets have the same horizontal resolution: $1^\circ \times 1^\circ$. With these two datasets, the climatological annual mean Ekman transport is calculated with (25) due to the surface wind stress ($\mathbf{M}_W$), and with (28) due to the horizontal gravity ($\mathbf{M}_G$) for the global oceans except for the equatorial region ($5^\circ\text{S} - 5^\circ\text{N}$) where the geostrophic balance does not exist. In addition, the global E -number is also computed with (31).

Global Ekman transport

The calculated global Ekman transport has different patterns due to wind stress $\mathbf{M}_W$ (Fig. 1a) and due to horizontal gravity $\mathbf{M}_G$ (Fig. 2a). The intensities of the Ekman transport components $|\mathbf{M}_W|$ (Fig. 1b) and $|\mathbf{M}_G|$ (Fig. 2b) have different horizontal distributions and strengths. The histogram of $|\mathbf{M}_W|$ (Fig. 1c) shows near Gamma distribution with the shape parameter of 1 and scale parameter of 2, and with the mean and standard deviation (946.8,

993.9) $\text{kg m}^{-1}\text{s}^{-1}$. However, the histogram of $|\mathbf{M}_G|$ (Fig. 2c) shows near Weibul distribution with the shape parameter of 1.5 and scale parameter of 10, and with the mean and standard deviation (4,098, 5,836) $\text{kg m}^{-1}\text{s}^{-1}$. With the density $\rho(z)$ given by (6) (i.e., Garrett-type e-folding near-inertial buoyancy frequency), the global mean Ekman transport is around four times larger due to the horizontal gravity $|\mathbf{M}_G|$ than due to the surface wind stress $|\mathbf{M}_W|$.

Global non-dimensional E number

Larger Ekman transport due to the horizontal gravity $|\mathbf{M}_G|$ than due to the surface wind stress $|\mathbf{M}_W|$ is also shown in the world ocean distribution of E values (Figure 3a). The histogram of E (Figure 3b) indicates a positively skewed distribution with a long tail extending to values larger than 10. The statistical characteristics of E are 3.222 as the mean, 2.295 as the standard deviation, 0.9958 as the skewness, and 3.256 as the kurtosis. The statistics show that the horizontal gravity cannot be neglected in comparison to the surface wind stress curl in the Ekman layer dynamics.

Conclusion

Importance of the horizontal gravity component in the ocean Ekman transport is demonstrated using the dynamic equations including the horizontal gravity. New formulas are obtained for the Ekman spiral, and Ekman transport with forcing due to the horizontal gravity in addition to the surface wind stress. With the constant eddy viscosity K and the e-folding type depth-dependent buoyancy frequency (no horizontal density gradient), new equations for the Ekman spiral and Ekman transport have been derived including both surface wind stress and horizontal gravity. The Ekman transports due to wind stress ($\mathbf{M}_W$) and horizontal gravity ($\mathbf{M}_G$) are identified using the two independent datasets: COADS for wind stress ($\boldsymbol{\tau}$), and EIGEN-6C4 geoid height (N) for the horizontal gravity. Note that the

larger Ekman transport due to the horizontal gravity than due to the surface wind stress is only for the specially selected density field represented by the e-folding near-inertial buoyancy frequency, not for the density in the real ocean. However, it shows that the horizontal gravity is an important forcing term in the ocean Ekman layer dynamics.

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Author contributions

PCC designed the project, obtained the datasets, conducted the computation, and wrote the manuscript. The source codes for plotting Figures 1a, 1b, 1c, 2a, 2b, 2c, 3a, 3b, F1a, F1b, F1c, F1d, and F1e were written by the author's research group using the Matlab Version R2019b (<https://www.mathworks.com/products/matlab.html>.)

Competing interests

The author declares no competing interests.

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Figure 1. Climatological annual mean Ekman transport due to surface wind stress $\mathbf{M}_w$ (unit: $\text{kg m}^{-1}\text{s}^{-1}$) calculated using the COADS data: (a) vector plot of $\mathbf{M}_w$, (b) contour plot of $|\mathbf{M}_w|$, and (c) histogram of $|\mathbf{M}_w|$. The source codes for the plots were written by the author's research group using the Matlab Version R2019b (<https://www.mathworks.com/products/matlab.html>.)

Figure 2. Climatological annual mean Ekman transport due to horizontal gravity $\mathbf{M}_G$ (unit: $\text{kg m}^{-1}\text{s}^{-1}$) calculated using the EIGEN-6C4 geoid height (N) data: (a) vector plot of $\mathbf{M}_G$, (b) contour plot of $|\mathbf{M}_G|$, and (c) histogram of $|\mathbf{M}_G|$. The source codes for the plots were written by the author's research group using the Matlab Version R2019b (<https://www.mathworks.com/products/matlab.html>.)

Figure 3. Climatological annual mean non-dimensional E -number calculated using the COADS annual mean surface wind stress and the EIGEN-6C4 geoid undulation (N) data: (a) contour plot of E , (b) histogram of E . The source codes for the plots were written by the author's research group using the Matlab Version R2019b (<https://www.mathworks.com/products/matlab.html>.)

Figures

Figure 1

Climatological annual mean Ekman transport due to surface wind stress M_w (unit: $\text{kg m}^{-1}\text{s}^{-1}$) calculated using the COADS data: (a) vector plot of M_w , (b) contour plot of $|M_w|$, and (c) histogram of $|M_w|$. The source codes for the plots were written by the author's research group using the Matlab Version R2019b (<https://www.mathworks.com/products/matlab.html>.) Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

Figure 2

Climatological annual mean Ekman transport due to horizontal gravity MG (unit: $\text{kg m}^{-1}\text{s}^{-1}$) calculated using the EIGEN-6C4 geoid height (N) data: (a) vector plot of MG , (b) contour plot of $|MG|$, and (c) histogram of $|MG|$. The source codes for the plots were written by the author's research group using the Matlab Version R2019b (<https://www.mathworks.com/products/matlab.html>.) Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

Figure 3

Climatological annual mean non-dimensional E-number calculated using the COADS annual mean surface wind stress and the EIGEN-6C4 geoid undulation (N) data: (a) contour plot of E, (b) histogram of E. The source codes for the plots were written by the author's research group using the Matlab Version R2019b (<https://www.mathworks.com/products/matlab.html>.) Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the

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