A Recursive Frequency Smoothing Method of Cyclic Spectrum Density Estimation

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Research

Keywords: Cyclic spectrum density estimation, Frequency smoothing, Computational cost, Recursive method

Posted Date: May 18th, 2020

DOI: <https://doi.org/10.21203/rs.3.rs-28727/v1>

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1 A Recursive Frequency Smoothing Method of Cyclic 2 Spectrum Density Estimation

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9 **Abstract:** Cyclic spectrum density estimation plays a significant role in cyclostationary signals
10 analysis. Generally, the methods of cyclic spectrum density estimation are mainly focus in time-domain,
11 such as Fast fourier transform Accumulation Method (FAM) and Strip Spectral Correlation Algorithm
12 (SSCA). In this paper, based on the principle of frequency smoothing methods, an improved method in
13 frequency-domain was proposed, which could reduce the computational cost greatly by using recursion.
14 The recursive method was based on the fact that the range difference of smoothing window was small
15 when window slid. In other words, we could get a new point of cyclic spectrum density just by
16 computing the complex conjugate products of newly coming into window. In addition, the simulation
17 results were given at last, which showed the good performance of our proposed method.

18 **Keywords:** Cyclic spectrum density estimation; Frequency smoothing; Computational cost; Recursive
19 method
20

21 1. Introduction

22 Many cyclostationary signals are widely used in engineering fields, such as communications,
23 telemetry, radar and sonar. Traditionally, these signals are treated as non-stationary. However, it is more
24 suitable to be modeled as cyclostationary because their statistics are periodic due to modulating, sampling
25 and other signal processing operations. As a consequence, the analysis of cyclostationary get a lot of
26 attention in signal processing and it is widely used in many fields, including signal reconnaissance,
27 modulation recognition and sonar detection [1-4]. In the analysis of cyclostationary signal, the cyclic

spectrum density estimation plays an important role for its calculation cost and implementation diversity. Generally, there are two types of algorithms to estimate cyclic spectrum density. One is time smoothing algorithms, such as Fast fourier transform Accumulation Method (FAM) and Strip Spectral Correlation Algorithm (SSCA). The other one is frequency smoothing algorithms, such as Frequency Smoothing Method (FSM) [5-7].

The basic estimation methods in time smoothing algorithms and frequency smoothing algorithms are both given in Ref. [5]. The performance and computational efficiency of the commonly used estimation methods are analyzed and compared explicitly in Ref. [6]. In addition, a conclusion is drawn that the computational cost of frequency smoothing algorithms is greater than that of time smoothing algorithms^[6]. As a consequence, almost all the estimation methods in succedent studies are focus in time smoothing algorithms, and the frequency smoothing algorithms are ignored actually.

In recent years, the study of cyclic spectrum density mainly focuses on these fields: characteristic of cyclic spectrum density, optimization of implementation and application of cyclic spectrum density. Cyclostationary Generalized Frequency Division Multiplexing (GFDM) signal properties in flexible cognitive radio is analyzed in Ref. [8]. The algorithms to calculate cyclic spectrum density is optimized in Refs. [9-12], while all these algorithms are based on time smoothing method. Cyclic spectrum density can be used to detect signal feature^[13] and signal classification^[14]. With regards to implementation, time smoothing algorithms generally employ a coarse-grain channelizer, where the bandwidth of each channel is a large fraction of the analysis bandwidth. On the other hand, frequency smoothing algorithms generally employ a fine-grain channelizer, where the bandwidth of each channel is a small fraction of the analysis bandwidth [15]. That is to say, if a higher frequency resolution is required under the same analysis bandwidth, the performance of frequency smoothing algorithms is better. Therefore, it is necessary to study an improved frequency smoothing algorithm by reducing its computational cost.

The primary objective of this paper is to introduce a recursive approach method for calculating cyclic spectrum density based on frequency smoothing method, and compare the computational cost and performance with other algorithms. The proposed recursive frequency smoothing algorithms has reduced computational cost successfully, what's more, it has good performances in detecting cyclic frequency at different Signal to Noise Ratio (SNR) levels.

The rest of the paper is organized as follows. In Section 2, the definition of cyclic spectrum density is described. Section 3 presents implementation and improvement of frequency smoothing method.

Section 4 analyses and compares different algorithms in computational cost and performance. Section 5 shows some simulation results by using the proposed recursive frequency smoothing method and SSCA. At last, the paper ends with the conclusions provided by Section 6.

2. Definition of Cyclic Spectrum Density

Suppose $x(t)$ is a non-stationary complex signal with zero mean, the time-vary autocorrelation of $x(t)$ can be defined as [16]

$$R_x(t, \tau) = E\{x(t)x^*(t - \tau)\}, \quad (1)$$

where τ is the time delay between $x(t)$ and $x^*(t)$, $x^*(t)$ is the conjugation of $x(t)$.

If statistics of $x(t)x^*(t - \tau)$ is periodic with period T_0 , sample mean can be used to estimate universal mean. Suppose sample time is $t + nT_0$, where n is a integer and it is between $-N$ and N , the autocorrelation function can also be written as

$$R_x(t, \tau) = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N x(t + nT_0)x^*(t + nT_0 - \tau), \quad (2)$$

where $R_x(t, \tau)$ is a periodic function with a period of T_0 . When N tends to infinity, Eq. (2) is equivalent to Eq. (1). It can also be expanded into a Fourier series

$$R_x(t, \tau) = \sum_{m=-\infty}^{\infty} R_x^\alpha(\tau) e^{+j\frac{2\pi}{T_0}mt} = \sum_{m=-\infty}^{\infty} R_x^\alpha(\tau) e^{+j2\pi\alpha t}, \quad (3)$$

where $\alpha = \frac{m}{T_0}$ and $j = \sqrt{-1}$. The Fourier coefficients are given by

$$R_x^\alpha(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} R_x(t, \tau) e^{-j2\pi\alpha t} dt, \quad (4)$$

Substitute Eq. (2) into Eq. (4), and let $T = (2N+1)T_0$, we can obtain,

$$R_x^\alpha(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t - \tau) e^{-j2\pi\alpha t} dt = \langle x(t)x^*(t - \tau) e^{-j2\pi\alpha t} \rangle_t, \quad (5)$$

where the coefficient $R_x^\alpha(\tau)$ is called cyclic autocorrelation with a cyclic frequency of α , and it is also a function of τ .

74 Whereas in practice, a symmetrical form is more common used

$$y_\tau(t) = x(t + \tau/2)x^*(t - \tau/2), \quad (6)$$

75 So, the cyclic autocorrelation with symmetrical form can be written as

$$R_x^\alpha(\tau) = \left\langle x(t + \tau/2)x^*(t - \tau/2)e^{-j2\pi\alpha t} \right\rangle_t, \quad (7)$$

76 Cyclic spectrum density can be acquired from the definition of cyclic autocorrelation. That is to
77 say, it can be got from the Fourier transform of cyclic autocorrelation $R_x^\alpha(\tau)$

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau) e^{-j2\pi f\tau} d\tau, \quad (8)$$

78 where $S_x^\alpha(f)$ is called cyclic spectrum density or spectral correlation. $S_x^\alpha(f)$ is a function of α and
79 f , where α is cyclic frequency that represent the time-varied characteristics of signal, and f is
80 frequency. When $\alpha = 0$, $S_x^0(f)$ becomes a spectrum of a stationary signal.

81 If we re-express Eq. (7) as

$$R_x^\alpha(\tau) = \left\langle [x(t + \tau/2)e^{-j\pi\alpha(t+\tau/2)}][x(t - \tau/2)e^{j\pi\alpha(t-\tau/2)}]^* \right\rangle_t, \quad (9)$$

82 and set

$$\begin{cases} u(t) = x(t)e^{-j\pi\alpha t} \\ v(t) = x(t)e^{j\pi\alpha t} \end{cases}, \quad (10)$$

83 the cyclic autocorrelation in Eq. (9) can be expressed as the cross-correlation of $u(t)$ and $v(t)$.

$$\begin{aligned} R_x^\alpha(\tau) &= R_{uv}(\tau) = \left\langle u(t + \tau/2)v^*(t - \tau/2) \right\rangle_t \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t + \tau/2)v^*(t - \tau/2) dt \end{aligned} \quad (11)$$

84 Then $S_x^\alpha(f)$ is the cross-spectral density of $U(f)$ and $V^*(f)$ from Eq. (11),

$$S_x^\alpha(f) = U(f)V^*(f), \quad (12)$$

85 where $U(f)$ and $V(f)$ are the Fourier transform of $u(t)$ and $v(t)$, respectively.

86 $U(f) = X(f + \alpha/2)$ and $V(f) = X(f - \alpha/2)$, and $X(f)$ is the Fourier transform of $x(t)$.

87 3. Implementation and Improvement of Frequency Smoothing Method

88 In engineering calculation, the average T in Eq. (11) is impossible of being infinite. Therefore,
 89 the issue of cyclic spectrum density estimation must be studied at the condition of the limited T .

90 If the center time is t , the Fourier transform of the signal $x(t)$ is

$$X_T(t, f) = \int_{t-T/2}^{t+T/2} x(u) e^{-j2\pi fu} du, \quad (13)$$

91 The time-varied spectrum of $U(f)$ and $V(f)$ can be acquired by shifting frequency $\pm\alpha/2$ of
 92 $X_T(t, f)$,

$$\begin{cases} U_T(t, f) = X_T(t, f + \alpha/2) \\ V_T(t, f) = X_T(t, f - \alpha/2) \end{cases}, \quad (14)$$

93 Because the time of $x(t)$ is infinite, the time of $U_T(t, f)$ and $V_T(t, f)$ is also infinite, their cross
 94 correlation spectrum can be acquired by doing average processing with limited window width Δt , so

$$S_{uv,T}(t, f)_{\Delta t} = \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} \frac{1}{T} U_T(t, f) V_T^*(t, f) dt, \quad (15)$$

95 where T denotes the length of Fourier transform, and Δt denotes the window width of time-average.

$$\begin{aligned} S_x^\alpha(f) &= S_{uv}(f) = \lim_{\Delta t \rightarrow \infty} \lim_{T \rightarrow \infty} S_{uv,T}(t, f)_{\Delta t} \\ &= \lim_{\Delta t \rightarrow \infty} \lim_{T \rightarrow \infty} \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} \frac{1}{T} U_T(t, f) V_T^*(t, f) dt \end{aligned}, \quad (16)$$

96 Let $\Delta f = \frac{1}{T}$, then

$$S_x^\alpha(f) = \lim_{\Delta t \rightarrow \infty} \lim_{\Delta f \rightarrow 0} \frac{1}{\Delta t} \int_{t-\Delta t/2}^{t+\Delta t/2} \Delta f U_{1/\Delta f}(u, f) V_{1/\Delta f}^*(u, f) du, \quad (17)$$

97 Therefore, Eq. (17) is a time smoothing implementation of cyclic spectrum density. In Eq. (17),

$$\begin{cases} U_{1/\Delta f}(t, f) = X_{1/\Delta f}(t, f + \alpha/2) \\ V_{1/\Delta f}(t, f) = X_{1/\Delta f}(t, f - \alpha/2) \end{cases}, \quad (18)$$

$$X_{1/\Delta f}(t, f) = \int_{t-1/2\Delta f}^{t+1/2\Delta f} x(u) e^{-j2\pi fu} du, \quad (19)$$

98 a frequency smoothing implementation is

$$S_x^\alpha(f) = \lim_{\Delta f \rightarrow 0} \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta f} \int_{f-\Delta f/2}^{f+\Delta f/2} \frac{1}{\Delta t} U_{\Delta t}(t, v) V_{\Delta t}^*(t, v) dv, \quad (20)$$

$$\begin{cases} U_{\Delta t}(t, f) = X_{\Delta t}(t, f + \alpha/2) \\ V_{\Delta t}(t, f) = X_{\Delta t}(t, f - \alpha/2) \end{cases}, \quad (21)$$

$$X_{\Delta t}(t, f) = \int_{t-\Delta t/2}^{t+\Delta t/2} x(u) e^{-j2\pi fu} du, \quad (22)$$

99 In Ref. [1], it has proved that Eq. (17) and Eq. (22) is equivalent when $\Delta t \Delta f \square 1$.

100 The discrete expression of cyclic spectrum density estimation using frequency smoothing method
101 is given as follows [1]

$$S_{x_M}^\alpha(f)_{\Delta f} = \frac{1}{M} \sum_{\gamma=-(M-1)/2}^{(M-1)/2} \frac{1}{N} X_{\Delta t}(t, f + \frac{\alpha}{2} + \gamma F_s) X_{\Delta t}^*(t, f - \frac{\alpha}{2} + \gamma F_s), \quad (23)$$

$$X_{\Delta t}(t, f) = \sum_{k=0}^{N-1} a_r(kT_s) x(t - kT_s) e^{-j2\pi f(t - kT_s)}, \quad (24)$$

102 where $a_r(n)$ is data window function, M is the number of frequency smoothing range (number of
103 frequency samples in rectangle window), N is the number of time samples, $F_s = 1/NT_s$ is the
104 frequency sample increment, T_s is the time sampling increment, $\Delta f = MF_s$ is the width of rectangle
105 smoothing window in frequency-domain, and Δt is length of time series and $\alpha = 2mF_s$ is the cyclic
106 frequency. The algorithm architecture is depicted in Figure 1. In this algorithm, $f = (p+l)/(2N)$ and
107 $\alpha = (p-l)/N$, where p is a certain frequency of $X_{\Delta t}(t, f)$ and l is a certain frequency of
108 $X_{\Delta t}^*(t, f)$. For simplicity, the treatment in this paper is restricted to cyclic spectrum density analysis of
109 a single real-valued signal.

Figure 1. Frequency smoothing method architecture

According to Eq. (23), the cyclic spectrum density is equal to the accumulation of M conjugated products of Fourier transform of $x(t)$ for fixed spectrum frequency f_0 and cycle frequency α_0 .

Since $S_x^\alpha(f)$ is the function of both cyclic frequency α and spectrum frequency f , the two variables can change respectively for acquiring estimation of $S_x^\alpha(f)$. As depicted in Figure 2, the fixed cycle frequency α_0 is the distance between two smoothing windows, f_0 and $f_0 + F_s$ are the centers of window before and after window movement, respectively, where F_s is the increment of spectrum frequency sample.

Figure 2. Sketch of smoothing window move-fixed α

When f is fixed, α_0 and $\alpha_0 + 2F_s$ are the distances of the center of window before and after window movement, respectively, where $2F_s$ is the increment of cyclic frequency samples. The sketch of smoothing window is depicted in Figure 3.

Figure 3. Sketch of smoothing window move-fixed f

Although the range is changed after the window movement, the conjugated multiply relationship between each pair of spectrums is not changed if they still located in the window. Moreover, the content in window is changed very small before and after the movement. That is to say, the difference between the successive cyclic correlations is small. Based on this principle, the computational cost can be reduced effectively. The detailed deduce procedure is shown below.

According to Eq. (23), $S_{x_M}^\alpha(f)_{\Delta f}$ is a function of two variables with α and f , where these two variables are both in frequency domain. The increment of spectrum frequency and cyclic frequency are

133 F_s and $2F_s$, respectively. The range of f and α are $0 \leq NF_s$ and $0 \leq 2NF_s$, respectively. If α is
 134 fixed and f move with F_s , we can obtain

$$S_{x_M}^\alpha(f_0 + F_s)_{\Delta f} = \frac{1}{M} \sum_{\gamma=-(M-1)/2}^{(M-1)/2} \frac{1}{N} X_{\Delta}(f_0 + F_s + \frac{\alpha}{2} + \gamma F_s) X_{\Delta}^*(f_0 + F_s - \frac{\alpha}{2} + \gamma F_s), \quad (25)$$

135 Assume $p = \gamma + 1$, then

$$\begin{aligned} S_{x_M}^\alpha(f_0 + F_s)_{\Delta f} &= \frac{1}{M} \sum_{p=-(M-1)/2+1}^{(M-1)/2+1} \frac{1}{N} X_{\Delta}(f_0 + \frac{\alpha}{2} + pF_s) X_{\Delta}^*(f_0 - \frac{\alpha}{2} + pF_s) \\ &= \frac{1}{M} \sum_{p=-(M-1)/2}^{(M-1)/2} \frac{1}{N} X_{\Delta}(f_0 + \frac{\alpha}{2} + pF_s) X_{\Delta}^*(f_0 - \frac{\alpha}{2} + pF_s) \\ &\quad + \frac{1}{MN} [X_{\Delta}(f_0 + \frac{\alpha}{2} + \frac{M+1}{2}F_s) X_{\Delta}^*(f_0 - \frac{\alpha}{2} + \frac{1+M}{2}F_s) - X_{\Delta}(f_0 + \frac{\alpha}{2} + (-\frac{M-1}{2})F_s) X_{\Delta}^*(f_0 - \frac{\alpha}{2} + (-\frac{M-1}{2})F_s)] \\ &= S_{x_M}^\alpha(f_0)_{\Delta f} + \frac{1}{MN} [X_{\Delta}(f_0 + \frac{\alpha}{2} + \frac{M+1}{2}F_s) X_{\Delta}^*(f_0 - \frac{\alpha}{2} + \frac{M+1}{2}F_s) - X_{\Delta}(f_0 + \frac{\alpha}{2} + \frac{1-M}{2}F_s) X_{\Delta}^*(f_0 - \frac{\alpha}{2} + \frac{1-M}{2}F_s)] \end{aligned} \quad (26)$$

136 In fact, this is an iterative process

$$\begin{aligned} S_{x_M}^\alpha(f_0 + F_s)_{\Delta f} - S_{x_M}^\alpha(f_0)_{\Delta f} \\ = \frac{1}{MN} [X_{\Delta}(f_0 + \frac{\alpha}{2} + \frac{M+1}{2}F_s) X_{\Delta}^*(f_0 - \frac{\alpha}{2} + \frac{M+1}{2}F_s) - X_{\Delta}(f_0 + \frac{\alpha}{2} + \frac{1-M}{2}F_s) X_{\Delta}^*(f_0 - \frac{\alpha}{2} + \frac{1-M}{2}F_s)] \end{aligned} \quad (27)$$

137 When α is fixed, due to the window movement, the difference between two cyclic spectrum
 138 density points is equal to the difference between the conjugated products of the points that just come into
 139 and go out window. As a result of iterative, we just need to compute the difference of two conjugated
 140 products instead of M complex conjugated products for acquiring the cyclic spectrum density in a new
 141 point. Furthermore, if the memory is sufficient, only one conjugated complex product which comes into
 142 the smoothing window justly is needed to be computed.

143 At the same time, due to the symmetry of the cyclic spectrum density, it is that
 144 $S_{x_M}^\alpha(-f)_{\Delta f} = S_{x_M}^\alpha(f)_{\Delta f}$ and $S_{x_M}^{-\alpha}(f)_{\Delta f} = S_{x_M}^{\alpha*}(f)_{\Delta f}$. Then only one quadrant values need to be
 145 computed, and the other quadrant values can be acquired by shifting. What's more, according to the
 146 above iterative, the initial value is different when α is different. For a fixed α , the initial value is the
 147 accumulation of M complex conjugated products when the distance of window center is α_0 and
 148 $f = 0$, as shown in Eq. (28)

$$S_{x_{\Delta t}}^{\alpha_0}(0)_{\Delta t} = \frac{1}{M} \sum_{\gamma=-(M-1)/2}^{(M-1)/2} \frac{1}{N} X_{\Delta t}(\frac{\alpha_0}{2} + \gamma F_s) X_{\Delta t}^*(\frac{\alpha_0}{2} + \gamma F_s), \quad (28)$$

Through the above improvement works, the computational cost of FSM can be reduced greatly.

4. Analysis and Comparison of Different Algorithms

Some algorithms of cyclic spectrum density estimation are compared in this section.

Each N-point Fast Fourier Transform (FFT) requires $\frac{N}{2} \log_2 N$ complex floating point multiplies

or $2N \log_2 N$ real floating point multiplies. For convenience, we only compare the multiplication

operations between different algorithms, and the multiplication operations are converted into real floating

point multiplies. The computational cost of FAM and SSCA come from Ref. [15]. Here, $P = N/4$ and

$N' = P/4$.

1 FAM

Applying the window..... $2PN'$

First FFT..... $2PN' \log_2 N'$

Down-conversion..... $4PN'$

Multiplication..... $4PN'^2$

Second FFT..... $2PN'^2 \log_2 P$

2 SSCA

Applying the window..... $2PN'$

First FFT..... $2PN' \log_2 N'$

Down-conversion..... $4PN'$

Multiplication..... $4N'^2$

Second FFT..... $2NN' \log_2 N$

3 FSM

M is the width of frequency smoothing window, and is N/16 in usual.

FFT..... $2N \log_2 N$

Applying the window..... $8M(\frac{N}{2} - \frac{M}{2})(\frac{N}{2} - \frac{M}{2})$

4 Recursive FSM

M is the width of frequency smoothing window, and is N/16 in usual.

FFT..... $2N \log_2 N$

176 Recursion..... $4(\frac{N}{2} - \frac{M}{2})(\frac{N}{2} - \frac{M}{2})$

177 The comparison of computational cost for the commonly used algorithms is shown in Table 1 and
 178 Figure 4. It can be observed that computational cost of FSM is the most, FAM is the second, and SSCA
 179 is the least. This conclusion agrees with that of Ref. [6]. However, the computational cost of recursive
 180 FSM is reduced greatly, and is better than that of SSCA.

181 **Table 1.** Computational cost of different algorithms.

Algorithms	Computational cost
FAM	$6PN' + 2PN' \log_2 N' + 4PN'^2 + 2PN'^2 \log_2 P$
SSCA	$6PN' + 2PN' \log_2 N' + 4N'^2 + 2NN' \log_2 N$
FSM	$2N \log_2 N + 8M(\frac{N}{2} - \frac{M}{2})(\frac{N}{2} - \frac{M}{2})$
Recursive FSM	$2N \log_2 N + 4(\frac{N}{2} - \frac{M}{2})(\frac{N}{2} - \frac{M}{2})$

182
 183 **Figure 4.** Comparison of different algorithms

184 5. Simulation Results and Discussion

185 Cyclic spectrum density is an effective tool in modulation recognition field. Take Binary Phase
 186 Shift Keying (BPSK) signal as an example,

$$x(t) = \cos(2\pi f_c t + \phi(t) + \phi_0) = b(t) \cos(2\pi f_c t + \phi_0), \quad (29)$$

187 where $b(t) = \sum_{n=-\infty}^{\infty} b_n q(t - nT_0 - t_0)$, and b_n is a random sequence whose value is ± 1 (corresponding to
 188 the phase 0 and π), and $q(t)$ is rectangular pulse.

189 Its theoretical cyclic spectrum density is

$$S_{BPSK}^{\alpha}(f) = \begin{cases} \frac{1}{4T_0} [Q(f + f_c + \frac{\alpha}{2})Q(f + f_c - \frac{\alpha}{2}) + Q(f - f_c + \frac{\alpha}{2})Q(f - f_c - \frac{\alpha}{2})] \exp(-j2\pi\alpha t_0), \alpha = \frac{p}{T_0} \\ \frac{1}{4T_0} [Q(f + \frac{p}{2T_0})Q(f_c - \frac{p}{2T_0})] \exp(-j2\pi t_0 \frac{p}{T_0} \pm 2\phi_0), \alpha = \frac{p}{T_0} \pm 2f_c \end{cases}, \quad (30)$$

190 That is to say, the cyclic spectrum density has a peak at $\alpha = \frac{p}{T_0}$ or $\alpha = \frac{p}{T_0} \pm 2f_c$, p is an integer,
 191 and $Q(f)$ is the Fourier transform of $q(t)$. α contains information about carrier frequency and code
 192 rate, and can be used in BPSK modulation recognition.

193 In order to evaluate the performance of the proposed recursive FSM, simulations are carried out in
 194 certain conditions. Assume parameters of the signal as follows: BPSK modulation, sample frequency
 195 $f_s = 1200\text{kHz}$, carrier frequency $f_c = 150\text{kHz}$, code rate $f_0 = 7500\text{Hz}$, white Gaussian noise and
 196 calculation points $N = 1024$. The cyclic spectrum densities calculated by SSCA and recursive FSM at
 197 different SNR levels are shown in Figure 5-7. According to Eq. (29), cyclic frequency α can take a
 198 section of $S_{BPSK}^{\alpha}(f)$ for $f = 0$, which can be used in signal detection and modulation recognition. The
 199 cyclic frequency α calculated by SSCA and recursive FSM respectively at different SNR levels are
 200 shown in Figure 8-10.

201 **Figure 5.** SNR=5dB. Cyclic spectrum density calculated by (a) SSCA and (b) recursive FSM.

202

Figure 6. SNR=0dB. Cyclic spectrum density calculated by (a) SSCA and (b) recursive FSM.

203

Figure 7. SNR=-5dB. Cyclic spectrum density calculated by (a) SSCA and (b) recursive FSM.

204

Figure 8. SNR=5dB. The cyclic frequency α calculated by (a) SSCA and (b) recursive FSM.

Figure 9. SNR=0dB. The cyclic frequency α calculated by (a) SSCA and (b) recursive FSM.

Figure 10. SNR=-5dB. The cyclic frequency α calculated by (a) SSCA and (b) recursive FSM.

Under the same SNR level, the peaks of recursive FSM in Figure 8-10 are more obvious. Especially at low SNR, the cyclic frequency of SSCA is blurred, while that of recursive FSM is still clear. That means it is easier to estimate symbol rate by using recursive FSM. This is because FSM is implemented directly in frequency domain, and there are no calculating errors in frequency-domain due to windowing.

We believe that recursive FSM can reduce the computational cost and has a good performance compared to the traditional algorithms, and this algorithm can be used in some occasions with strict computing resources, such as satellite load, embedded system, portable instrument, and et al. However, we concentrate on only the computation of the cyclic frequency, and have simulations at limited SNR levels.

6. Conclusions

The definition of cyclic spectrum density is introduced briefly and some different estimation methods are given in this paper. On the base of cyclic spectrum density estimation method analysis, an improved frequency smoothing method using iterative process to reduce the computational cost is proposed. In addition, by comparing the simulation results of SSCA and recursive FSM at the different SNR levels, recursive FSM has a good performance to acquire cyclic frequency which can be used in signal recognition.

Abbreviations

FAM: Fast fourier transform Accumulation Method; SSCA: Strip Spectral Correlation Algorithm; FSM: Frequency Smoothing Method; GFDM: Generalized Frequency Division Multiplexing; SNR: Signal to Noise Ratio; FFT: Fast Fourier Transform; BPSK: Binary Phase Shift Keying.

Ethics approval and consent to participate

Not applicable.

Acknowledgements

Not applicable.

Authors' contributions

Conceptualization: X.Z.F. and J.Z.D.; Methodology: X.Z.F., J.Z.D., J.Y.C., S.H.Y. and A.A.J.; Writing – original draft: X.Z.F.; Writing – review & editing: X.Z.F., J.Z.D., J.Y.C., S.H.Y. and A.A.J.

Funding

This research received no external funding.

Availability of data and materials

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Consent for publication

Not applicable.

Competing Interests

The authors declare that they have no competing interests.

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Figures

Figure 1
Frequency smoothing method architecture

Figure 2
Sketch of smoothing window move-fixed a

Figure 3

ketch of smoothing window move-fixed f

Figure 4

Comparison of different algorithms

Figure 5

SNR=5dB. Cyclic spectrum density calculated by (a) SSICA and (b) recursive FSM.

Figure 6

SNR=0dB. Cyclic spectrum density calculated by (a) SSICA and (b) recursive FSM.

Figure 7

SNR=-5dB. Cyclic spectrum density calculated by (a) SSCA and (b) recursive FSM.

Figure 8

SNR=5dB. The cyclic frequency a calculated by (a) SSCA and (b) recursive FSM.

Figure 9

SNR=0dB. The cyclic frequency α calculated by (a) SSCA and (b) recursive FSM.

Figure 10

SNR=-5dB. The cyclic frequency α calculated by (a) SSCA and (b) recursive FSM.