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Double-Hoist Dynamics and Oscillation Control of a Quadcopter Carrying a Swinging and Twisting Load

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Abstract

Quadcopters can serve as aerial cranes that are able to suspend large-size loads below the fuselage for material-handling services. Double-hoist mechanisms help quadcopter to transport bulky loads effectively. However, double-hoist mechanisms exhibit strong coupling effect among the quadcopter's attitude, load swing, and load twisting. Different from the single-hoist mechanisms, no effects have been directed at quadcopter slung loads with double-hoist mechanisms. A novel nonlinear dynamics model of a quadcopter carrying a distributed-mass load by double-hoist mechanisms is given in this article. This explicit model captures the coupling between the quadcopter and load motion. Then a novel control method is proposed to reduce the swing and twisting of the load simultaneously. The simulation results illustrate that the control method succeeds in limiting the unwanted oscillations of quadcopter attitudes, load swing and twisting.

Keywords

Quadcopter slung load, double hoist, load swing and twisting, modeling, vibration control.

1. Introduction

Quadcopters suspending heavy and bulky loads might be used in construction, transportation, and rescue missions. Quadcopters working as aerial cranes provide essential material-handling services throughout the world [1-4]. Unfortunately, a cable-suspended slung load constitutes a flexible system that degrades effectiveness and safety. Undoubtedly, the operator's commanded

motions and external disturbances, such as wind gusts and aerodynamic interaction cause undesirable oscillations of the quadcopter and load. Therefore, the study of dynamics and control of aerial cranes is essential for safe and efficient material-handling operations.

Modeling of quadcopters with external suspended loads has received broad attention [5-7]. Numerous references modeled the suspended load as the single-hoist dynamics, which is more suitable for small-size loads. Much of the literature has discussed the solutions to the challenging problems posed by the suspended-load swing of quadcopters. Active mechanisms were studied to realize oscillation reduction. However, the drawback of such mechanisms is that they are complicated to implement in realistic conditions. In order to reduce the swing, effective feedback control schemes use measurement and estimation of the suspended-load oscillations in closed-loop systems [8-18]. Unfortunately, accurately sensing the suspended-load is difficult, and the feedback controller may conflict with actions of the operator. Optimal trajectory planning [19-28], flatness-based control [29], and command shaping techniques [30] have been used to reduce the oscillations by modifying the pilot commands to create a low-swing motion. The approaches can suppress the residual oscillations, but it does not address external disturbances.

An increasing number of examples show that quadcopters might transport large-size objects under double-hoist mechanism. The bulky loads will be suspended by two cables below the quadcopters. The double-hoist mechanisms exhibit stronger coupling effect between the quadcopter and load than the single-hoist mechanism. However, little attention has been focused on quadcopter slung load with double-hoist configuration. Therefore, it is essential to study the double-hoist dynamics and corresponding control in the quadcopter suspended load. This article provides a nonlinear dynamics model of quadcopters carrying distributed-mass loads by double-hoist mechanisms. This model contains the double-hoist impact on the quadcopter and distributed-mass load. The dynamic equations can easily be used by readers to design controllers or to perform simulations.

The coupling effect among attitudes of aerial vehicles, load swing in the flight direction, and load twisting about the suspension cables is significant. An obvious challenge is that the load twisting must be limited. Some articles [31-32] reported helicopter slung loads, in which considered aerodynamic devices including vertical aerodynamic surfaces or fins to reduce the load twisting. However, this kind of control method is not easy to carry out in every transport. Disturbance rejection is another challenge. For example, wind gusts or aerodynamic effects from rotors may

cause load swing and twisting, which result in oscillations of vehicle attitudes because of coupling impacts. However, the difficulty of accurately sensing the load states is the barrier for rejecting external disturbances toward the application of previous control methods. A new control system is designed in this article. This proposed controller is utilized to reduce the swing and mitigate the twisting of the load simultaneously. Meanwhile, the controller can also reject the external disturbances without measuring the load oscillations on-the-fly.

A primary contribution of this article is a dynamic model of aerial vehicles transporting distributed-mass loads by double-hoist mechanisms. To the best of our knowledge, it is a novel model using double-hoist dynamics of the slung-load below a quadcopter's fuselage. This model is provided in complete dynamic equations so that it can be used by other researchers. In addition, it includes the coupling among the vehicle attitude, load swing and twisting. The new control method is another contribution of this article. The control method suppresses the load swing and twisting, stabilizes the quadcopter's attitudes, rejects the external disturbances, and is easy to implement. Simulation results demonstrate that the new controller performs well under different flight distances and different load length and mass. The modeling approach and control method are also applicable to the dynamics and control of other types of aerial cranes, such as tiltrotor or helicopter transporting large loads.

The rest of this article is organized as following. The nonlinear dynamic model of a quadcopter with a distributed-mass load is derived in Section 2. A method for controlling the quadcopter attitudes, and suppressing the suspended-load swing and twisting is proposed in Section 3. Numerous simulations are performed to validate the disturbance rejection, effectiveness, and robustness of the proposed control method in Section 4. Finally, conclusions are drawn in Section 5.

2. Modeling

Fig. 1 shows a schematic representation of a quadcopter transporting a load. The quadcopter uses a gyrostad $\mathbf{D}$, meaning a rigid body with three fixed-axis $\mathbf{D}_x$, $\mathbf{D}_y$, $\mathbf{D}_z$ in the model. The mass of the quadcopter is m_D , and the centroidal principal axis moments of inertia about the $\mathbf{D}_x$, $\mathbf{D}_y$, $\mathbf{D}_z$ axis are I_{xx} , I_{yy} and I_{zz} , respectively. The basic transformations are Newtonian $\mathbf{N}_x$, $\mathbf{N}_y$ and $\mathbf{N}_z$. Motion of the quadcopter is divided into the displacement, x , along the $\mathbf{N}_x$ direction, the displacement, y , along the $\mathbf{N}_y$ direction, the displacement, z , along the $\mathbf{N}_z$ direction, yaw attitude,

Fig. 1. Model of a quadcopter transporting a load by double-hoist mechanisms.

ψ , pitch attitude, θ , and the roll attitude, ϕ . The inertial coordinates $\mathbf{N}_x \mathbf{N}_y \mathbf{N}_z$ can be converted to the moving Cartesian coordinates $\mathbf{D}_x \mathbf{D}_y \mathbf{D}_z$ of the quadcopter by rotating the yaw attitude, ψ , pitch attitude, θ , and the roll attitude, ϕ , respectively. The quadrotors produce thrust force, F_z , along $\mathbf{D}_z$ direction and thrust moments, M_ϕ, M_θ, M_ψ , about $\mathbf{D}_x, \mathbf{D}_y, \mathbf{D}_z$ directions.

A rigid link attaches a connection point, S , to the quadcopter center of mass, D^* , along the $\mathbf{D}_z$ direction. The distance between the quadcopter center of mass, D^* , and the point, S , is $l_c \mathbf{D}_z$. Another rigid link, which is parallel to the $\mathbf{D}_x \mathbf{D}_y$ plane, connects two suspension points, P, Q , to the point, S . The distance between quadcopter center, D^* , and suspension point, Q , is $l_A \mathbf{D}_x + l_B \mathbf{D}_y + l_C \mathbf{D}_z$, while that between the quadcopter center, D^* , and suspension point, P , is $-l_A \mathbf{D}_x -$

$l_B \mathbf{D}_y + l_C \mathbf{D}_z$. The coupling effects between quadcopter attitudes and load oscillations may vary by changing link lengths, l_A, l_B, l_C .

Two massless and inelastic cables, $\mathbf{C}_1$ and $\mathbf{C}_2$, of same length, l_S , suspend below the quadcopter with swing angles, $\beta_1, \gamma_1, \beta_2, \gamma_2$, to orient the pendulum. A uniformly distributed-mass load ($\mathbf{B}$) of mass, m_L , length, l_L , and centroidal moment of inertia, I_L , is attached to the cables. The load is slender such that the cross-sectional area is assumed to be small. The centroid of the load is located at its center, B^* . The slope angle, δ , of the load is the angle between the load direction and $\mathbf{D}_x \mathbf{D}_y$ plane, while the twist angle, ε , is the angle between the load direction and $\mathbf{D}_x \mathbf{D}_z$ plane. Therefore, the coordinates $\mathbf{D}_x \mathbf{D}_y \mathbf{D}_z$ can also be converted to the moving Cartesian coordinates $\mathbf{B}_x \mathbf{B}_y \mathbf{B}_z$ of the load by rotating the twist angle, ε , and the slope angle, δ , respectively.

Inputs to the near-hover model are the thrust force, F_z , and thrust moments, M_ψ, M_θ, M_ϕ . Outputs are the quadcopter displacements, x, y, z , the attitudes, ψ, θ, ϕ , the swing angles, $\beta_1, \gamma_1, \beta_2, \gamma_2$, the twist angle, ε , and the slope angle, δ . The generalized speeds, u_r , ($r=1,2,\dots,12$) in [33] are chosen as:

$$u_1 = \dot{x}, u_2 = \dot{y}, u_3 = \dot{z}, u_4 = \dot{\phi}, u_5 = \dot{\theta}, u_6 = \dot{\psi}. \quad (1)$$

$$u_7 = \dot{\beta}_1, u_8 = \dot{\gamma}_1, u_9 = \dot{\beta}_2, u_{10} = \dot{\gamma}_2, u_{11} = \dot{\varepsilon}, u_{12} = \dot{\delta}. \quad (2)$$

where Eq. (1) is the generalized speed of quadcopter, and Eq. (2) is the generalized speed of the load. The basis transformation matrix among the bodies $\mathbf{D}$ (Quadcopter), $\mathbf{C}_1$ (Cable 1), $\mathbf{C}_2$ (Cable 2), and $\mathbf{B}$ (Load) are:

$$\begin{Bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \\ \mathbf{D}_z \end{Bmatrix} = \begin{bmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \sin \phi \cos \theta \\ \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi & \cos \phi \cos \theta \end{bmatrix} \begin{Bmatrix} \mathbf{N}_x \\ \mathbf{N}_y \\ \mathbf{N}_z \end{Bmatrix} \quad (3)$$

$$\begin{Bmatrix} \mathbf{C}_{1x} \\ \mathbf{C}_{1y} \\ \mathbf{C}_{1z} \end{Bmatrix} = \begin{bmatrix} \cos \beta_1 & 0 & -\sin \beta_1 \\ \sin \beta_1 \sin \gamma_1 & \cos \gamma_1 & \cos \beta_1 \sin \gamma_1 \\ \sin \beta_1 \cos \gamma_1 & -\sin \gamma_1 & \cos \beta_1 \cos \gamma_1 \end{bmatrix} \begin{Bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \\ \mathbf{D}_z \end{Bmatrix} \quad (4)$$

$$\begin{Bmatrix} \mathbf{C}_{2x} \\ \mathbf{C}_{2y} \\ \mathbf{C}_{2z} \end{Bmatrix} = \begin{bmatrix} \cos \beta_2 & 0 & -\sin \beta_2 \\ \sin \beta_2 \sin \gamma_2 & \cos \gamma_2 & \cos \beta_2 \sin \gamma_2 \\ \sin \beta_2 \cos \gamma_2 & -\sin \gamma_2 & \cos \beta_2 \cos \gamma_2 \end{bmatrix} \begin{Bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \\ \mathbf{D}_z \end{Bmatrix} \quad (5)$$

$$\begin{Bmatrix} \mathbf{B}_x \\ \mathbf{B}_y \\ \mathbf{B}_z \end{Bmatrix} = \begin{bmatrix} \cos \varepsilon \cos \delta & \sin \varepsilon \cos \delta & -\sin \delta \\ -\sin \varepsilon & \cos \varepsilon & 0 \\ \cos \varepsilon \sin \delta & \sin \varepsilon \sin \delta & \cos \delta \end{bmatrix} \begin{Bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \\ \mathbf{D}_z \end{Bmatrix} \quad (6)$$

The angular velocity of quadcopter in the Newtonian frame $\mathbf{N}$ is:

$${}^N\boldsymbol{\omega}^D = (u_4 - u_6 \sin \theta)\mathbf{D}_x + (u_5 \cos \varphi + u_6 \sin \varphi \cos \theta)\mathbf{D}_y + (u_6 \cos \varphi \cos \theta - u_5 \sin \varphi)\mathbf{D}_z \quad (7)$$

The velocity of quadcopter center of mass D^* in the Newtonian frame $\mathbf{N}$ is:

$${}^N\mathbf{v}^{D^*} = u_1\mathbf{N}_x + u_2\mathbf{N}_y + u_3\mathbf{N}_z \quad (8)$$

The angular velocity of the load is:

$${}^N\boldsymbol{\omega}^B = {}^N\boldsymbol{\omega}^D + {}^D\boldsymbol{\omega}^B = {}^N\boldsymbol{\omega}^D - u_{11} \sin \delta \mathbf{B}_x + u_{12} \mathbf{B}_y + u_{11} \cos \delta \mathbf{B}_z \quad (9)$$

The angular velocity of cables $\mathbf{C}_1, \mathbf{C}_2$ are:

$${}^N\boldsymbol{\omega}^{C_1} = {}^N\boldsymbol{\omega}^D + {}^D\boldsymbol{\omega}^{C_1} = {}^N\boldsymbol{\omega}^D + u_8 \mathbf{C}_{1x} + u_7 \cos \gamma_1 \mathbf{C}_{1y} - u_7 \sin \gamma_1 \mathbf{C}_{1z} \quad (10)$$

$${}^N\boldsymbol{\omega}^{C_2} = {}^N\boldsymbol{\omega}^D + {}^D\boldsymbol{\omega}^{C_2} = {}^N\boldsymbol{\omega}^D + u_{10} \mathbf{C}_{2x} + u_9 \cos \gamma_2 \mathbf{C}_{2y} - u_9 \sin \gamma_2 \mathbf{C}_{2z} \quad (11)$$

The velocity of the load center of mass B^* is:

$${}^N\mathbf{v}^{B^*} = {}^N\mathbf{v}^{D^*} + {}^N\boldsymbol{\omega}^D \times (l_A \mathbf{D}_x + l_B \mathbf{D}_y + l_C \mathbf{D}_z) + {}^N\boldsymbol{\omega}^{C_2} \times l_S \mathbf{C}_{2z} - {}^N\boldsymbol{\omega}^B \times 0.5l_L \mathbf{B}_x \quad (12)$$

The derivative of Eqs. (7)(9) with respect to time derives the angular acceleration, ${}^N\boldsymbol{\alpha}^D$, of quadcopter, and angular acceleration, ${}^N\boldsymbol{\alpha}^B$, of load, respectively. The derivative of Eqs. (8)(12) with respect to time yields the acceleration, ${}^N\mathbf{a}^{D^*}$, of quadcopter center of mass, and acceleration, ${}^N\mathbf{a}^{B^*}$, of load center of mass, respectively. Meanwhile, we extract the r^{th} partial velocities $\mathbf{v}_r^{D^*}$ in [33], the r^{th} partial angular velocities $\boldsymbol{\omega}_r^D$ of the quadcopter as coefficients of u_r from Eqs. (7)(8), the r^{th} partial angular velocities of load $\boldsymbol{\omega}_r^B$ from Eq. (9), and r^{th} partial velocities $\mathbf{v}_r^{B^*}$ of the load center of mass B^* from Eq. (12).

The generalized inertia forces in [33] are:

$$F_r^* = -m_D \cdot {}^N\mathbf{a}^{D^*} \cdot \mathbf{v}_r^{D^*} - (\mathbf{I}_D \cdot {}^N\boldsymbol{\alpha}^D + {}^N\boldsymbol{\omega}^D \times \mathbf{I}_D \cdot {}^N\boldsymbol{\omega}^D) \cdot \boldsymbol{\omega}_r^D \quad (r=1,2,\dots,12) \quad (13)$$

$$- m_L \cdot {}^N\mathbf{a}^{B^*} \cdot \mathbf{v}_r^{B^*} - (\mathbf{I}_B \cdot {}^N\boldsymbol{\alpha}^B + {}^N\boldsymbol{\omega}^B \times \mathbf{I}_B \cdot {}^N\boldsymbol{\omega}^B) \cdot \boldsymbol{\omega}_r^B$$

where $\mathbf{I}_D, \mathbf{I}_B$ are the inertia matrix of Quadcopter ($\mathbf{D}$) and load ($\mathbf{B}$), respectively. They satisfy:

$$\mathbf{I}_D = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}, \quad \mathbf{I}_B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I_L & 0 \\ 0 & 0 & I_L \end{bmatrix}. \quad (14)$$

A part of generalized active forces in [33] is due to the force of gravity on the quadcopter and load. Another part of the generalized active forces is due to the thrust force and moments produced by the rotors. Thus, the generalized active force is:

$$F_r = (m_D g \mathbf{N}_z - F_z \mathbf{D}_z) \cdot \mathbf{v}_r^{D^*} + (M_\varphi \mathbf{D}_x + M_\theta \mathbf{D}_y + M_\psi \mathbf{D}_z) \cdot \boldsymbol{\omega}_r^D + m_L g \mathbf{N}_z \cdot \mathbf{v}_r^{B^*} \quad (r=1,2,\dots,12) \quad (15)$$

where $F_z \mathbf{D}_z$ is the thrust force along the $\mathbf{D}_z$ direction. In order to maintain the quadcopter flight altitude, the magnitude of the thrust force along the $\mathbf{N}_z$ direction is $F_z \mathbf{D}_z \cdot \mathbf{N}_z$ for balancing gravity of the quadcopter and load.

The quadcopter, cables, and load generate a three-dimensional four-bar linkage, in which contains three velocity constraints:

$$u_7 l_s \cos \beta_1 \cos \gamma_1 - u_8 l_s \sin \beta_1 \sin \gamma_1 - u_9 l_s \cos \beta_2 \cos \gamma_2 + u_{10} l_s \sin \beta_2 \sin \gamma_2 - u_{11} l_L \sin \varepsilon \cos \delta - u_{12} l_L \cos \varepsilon \sin \delta = 0 \quad (16)$$

$$u_8 l_s \cos \gamma_1 - u_{10} l_s \cos \gamma_2 - u_{11} l_L \cos \varepsilon \cos \delta + u_{12} l_L \sin \varepsilon \sin \delta = 0 \quad (17)$$

$$u_7 l_s \sin \beta_1 \cos \gamma_1 + u_8 l_s \cos \beta_1 \sin \gamma_1 - u_9 l_s \sin \beta_2 \cos \gamma_2 - u_{10} l_s \cos \beta_2 \sin \gamma_2 + u_{12} l_L \cos \delta = 0 \quad (18)$$

The Kane's equation in [33] describes that the sum of generalized inertia forces in Eq. (13) and generalized active forces in Eq. (15) should be limited to zero. Then the nonlinear dynamic equations of the motion in Fig. 1 yield:

$$[\mathbf{M}] \{ \dot{\mathbf{U}} \} + \{ \mathbf{Z} \} = 0 \quad (19)$$

where $\mathbf{M}$ is the mass matrix, $\dot{\mathbf{U}}$ is the column matrix of derivatives of generalized speed u_r with respect to time, and $\mathbf{Z}$ is the column matrix of gravity terms, centrifugal and Coriolis terms, and input terms.

The dynamical model in Eq. (19) includes twelve system states, $x, y, z, \varphi, \theta, \psi, \beta_1, \gamma_1, \beta_2, \gamma_2, \varepsilon, \delta$, and four inputs, $F_z, M_\varphi, M_\theta, M_\psi$. Therefore, it is an under-actuated system that requires a sophisticated control system. The control objectives consist of three parts, i) attitude control of the quadcopter, ii) swing and twisting suppression of the distributed-mass load, and iii) external-disturbance rejection. To achieve the objectives, a combination of feedback control and hybrid filter will be presented in this article. A feedback controller regulates the quadcopter attitude and rejects external disturbances, while a hybrid filter attenuates the load swing and twisting caused by pilot commands.

Fig. 2. Combined control architecture.

3. Oscillation Suppression

This section presents a combined control scheme including a feedback controller and three prefilters. A model-following controller (MFC) regulates the quadcopter's attitude by following the states of a prescribed model and attenuating the tracking errors, while three prefilters reduces both the load swing and twisting by modifying the pilot commands.

The configuration of the combined feedback and prefilter control is shown in Fig. 2. The operator generates pilot commands, ψ_r , θ_r , ϕ_r , via the remote-control transmitter. Three prefilters (hybrid filter) modify the pilot commands to produce modified commands, ψ_s , θ_s , ϕ_s . The measurement of the quadcopter attitude, ψ , θ , ϕ , is used to adjust the thrust moments M_ψ , M_θ , M_ϕ , in a feedback control loop. Then, the attitudes are forced to track the modified commands by the MFC controller. The frequencies and corresponding damping ratios are estimated by system parameters and are applied to design the three filters. The combined feedback controller and prefilter drive the aerial cranes towards the desired position with minimal oscillations of the quadcopter attitude, load swing and twisting.

3.1 MFC Controller for Attitude Regulation

The dynamics of a quadcopter slung load in Eq. (19) is too complicated to analytically design an MFC controller. Oscillations of the quadcopter and load are assumed to be small, and the model is also assumed to undergo planar motions. Then three linearized models can be derived in the $\mathbf{D}_x\mathbf{D}_y$, $\mathbf{D}_x\mathbf{D}_z$, $\mathbf{D}_y\mathbf{D}_z$ planes. Three explicit MFC controllers are designed to control the corresponding attitude of the quadcopter from the three linearized models. The prescribed model of the MFC controller is described as:

$$\begin{cases} \ddot{\psi}_m + 2\zeta_m \omega_m \dot{\psi}_m + \omega_m^2 \psi_m = \omega_m^2 \psi_s \\ \ddot{\theta}_m + 2\zeta_m \omega_m \dot{\theta}_m + \omega_m^2 \theta_m = \omega_m^2 \theta_s \\ \ddot{\phi}_m + 2\zeta_m \omega_m \dot{\phi}_m + \omega_m^2 \phi_m = \omega_m^2 \phi_s \end{cases} \quad (20)$$

where ζ_m is the damping ratio of the prescribed model, ω_m is the frequency of the prescribed model, ψ_s , θ_s , ϕ_s , are modified commands, ψ_m , θ_m , ϕ_m , are model outputs. A prescribed model with reasonable damping ratio (0.707) and reasonable settling time ($\leq 4s$) is utilized. The desired closed-loop poles were designed to be approximately $-1 \pm j1$. Thus, the frequency, ω_m , was 1.41 rad/s by using the pole placement method. In addition, the asymptotic tracking control law of the MFC controller is expressed as:

$$\begin{cases} M_\psi = B_\psi \cdot \ddot{\psi}_m + k_{\psi d} \cdot (\dot{\psi}_m - \dot{\psi}) + k_{\psi p} \cdot (\psi_m - \psi) \\ M_\theta = B_\theta \cdot \ddot{\theta}_m + k_{\theta d} \cdot (\dot{\theta}_m - \dot{\theta}) + k_{\theta p} \cdot (\theta_m - \theta) \\ M_\phi = B_\phi \cdot \ddot{\phi}_m + k_{\phi d} \cdot (\dot{\phi}_m - \dot{\phi}) + k_{\phi p} \cdot (\phi_m - \phi) \end{cases} \quad (21)$$

where $k_{\psi p}$, $k_{\psi d}$, $k_{\theta p}$, $k_{\theta d}$, $k_{\phi p}$, $k_{\phi d}$, are control gains, and B_ψ , B_θ , B_ϕ , are coefficients. Suitable tracking errors can be achieved by designing control gains. The coupling effect between the aerial vehicle and suspended load decreases the real part of the eigenvalues in Eq. (21) and increases the system damping. Given that the quadcopter mass, m_D , and moment of inertia, I_{xx} , I_{yy} , I_{zz} are 85 kg, 4.5 kg·m², 4.5 kg·m², 6 kg·m², respectively. Thus, the prescribed eigenvalues of attitudes ψ , θ , ϕ , in Eq. (21) are selected to be $-12 \pm j4$, $-6 \pm j2$, and $-6 \pm j2$, respectively. Therefore, the control gains, $k_{\psi p}$, $k_{\psi d}$, $k_{\theta p}$, $k_{\theta d}$, $k_{\phi p}$, $k_{\phi d}$, were calculated by the pole placement technique to be 960, 144, 180, 54, 431.6, 129.5, respectively. Note that the stability and transient performance of explicit MFC controllers in Eqs. (20-21) have been described in [34-35].

Resulting from the three linearized models and the MFC controller, each plane includes two linearized frequencies and corresponding damping ratios. The first-mode damping ratio is near zero, while the second-mode damping ratio is near one. The second-mode oscillations can be neglected because corresponding amplitudes may damp quickly. Actually, the three-dimensional four-bar linkage shown in Fig. 1 includes three natural frequencies with near-zero damping, and the MFC controller adds another three frequencies with high damping. Therefore, only three first-mode frequencies and damping ratios in three planar motions should be considered because they exhibit low frequency and near zero damping.

The linearized frequencies are dependent on the system parameters, such as quadcopter mass, m_D , cable length, l_s , load length, l_L , load mass, m_L , and suspension distances, l_A , l_B , l_C . The cable

Fig. 3. Linearized frequencies and damping for various load length and mass.

length, l_s , and suspension distance, l_c , have fundamental influence on linearized frequencies. Increasing cable length, l_s , and suspension distances, l_c , decrease three linearized frequencies sharply. The load length and mass might also have some impacts on the linearized frequencies and damping. The three frequencies and damping in the $D_x D_y$, $D_x D_z$, $D_y D_z$ planes induced by various load length and mass are shown in Fig. 3. The cable length, l_s , and suspension distances, l_A , l_B , l_C , were set to 5 m, 0 m, 1 m, 0.1 m, respectively. The load length ranged from 2.5 m to 5.5 m. While the linear mass density of the load was fixed at 5 kg/m in this article, increasing load length increases load mass. Both the frequency and damping ratio in the $D_y D_z$ plane increase as the load length and mass increase. The frequency in the $D_x D_z$ plane changes slightly, while the corresponding damping ratio increases sharply with increasing load length. In the $D_x D_y$ plane, increasing load length decreases the frequency significantly and increases the damping ratio.

The frequency and damping in the $D_y D_z$ plane are more related with the load swing, β_1 , β_2 , and pitch attitude, θ , that in the $D_x D_z$ plane are more related with the load swing, γ_1 , γ_2 , load slope, δ , and roll attitude, ϕ , and that in the $D_x D_y$ plane are more related with the load twisting, ε , and yaw attitude, ψ . The frequency in the $D_y D_z$ plane approaches the frequency in the $D_x D_z$ plane near the load length of 3.57 m, where the internal resonance might arise. The same phenomenon will occur near load length of 4.71 m and 4.96 m.

3.2 Hybrid Filter for Suppressing Load Oscillations

The load swing and twisting caused by external disturbance might also induce oscillations of quadcopter attitude because of coupling effect between quadcopter and load. The damping effect

in the MFC controller can attenuate the oscillations of the quadcopter attitude. Then the load oscillations can also be reduced by MFC controller because of the coupling effect. However, the MFC controller should not be used to suppress the load swing and twisting caused by pilot commands. This is because the settling time for attenuating load oscillations by MFC controller would be very long.

In addition to regulating the quadcopter's attitude with the MFC controller, it is useful to design the prefilter to suppress load oscillations induced by pilot commands. The prefilter is a combination of discrete- and continuous-time function, which inherent in the limited motion of the quadcopter results in a limited response to oscillations of the suspended-load.

The nonlinear dynamics of the quadcopters slung loads in Eq. (19) with the MFC controller in Eqs. (20)(21) can be approximated near the equilibria as three second-order systems with linearized frequencies and damping ratios shown in Fig. 3. The response of a second-order harmonic oscillator resulting from an impulse, $A_1\delta(n-0)$, at time zero and one continuous function, $c(\tau)$, is:

$$f(t) = A_1 \frac{\omega_k}{\sqrt{1-\zeta_k^2}} e^{-\zeta_k \omega_k t} \sin\left(\omega_k \sqrt{1-\zeta_k^2} t\right) + \int_{\tau=0}^{+\infty} c(\tau) \frac{\omega_k}{\sqrt{1-\zeta_k^2}} e^{-\zeta_k \omega_k (t-\tau)} \sin\left(\omega_k \sqrt{1-\zeta_k^2} (t-\tau)\right) d\tau \quad (22)$$

where ω_k, ζ_k are linearized frequency and damping ratio shown in Fig. 3. The vibrational amplitude of the response (22) is given by:

$$D(t) = \frac{\omega_k}{\sqrt{1-\zeta_k^2}} e^{-\zeta_k \omega_k t} \sqrt{[S(\omega_k, \zeta_k)]^2 + [C(\omega_k, \zeta_k)]^2} \quad (23)$$

where

$$S(\omega_k, \zeta_k) = \int_{\tau=0}^{+\infty} c(\tau) e^{\zeta_k \omega_k \tau} \sin\left(\omega_k \sqrt{1-\zeta_k^2} \tau\right) d\tau \quad (24)$$

$$C(\omega_k, \zeta_k) = A_1 + \int_{\tau=0}^{+\infty} c(\tau) e^{\zeta_k \omega_k \tau} \cos\left(\omega_k \sqrt{1-\zeta_k^2} \tau\right) d\tau \quad (25)$$

The vibration amplitudes can be reduced to zero when Eqs. (24) and (25) are limited to zero. In order to ensure that the modified pilot-commands reach the same set-point as the original commands, the unit-gain constraint should be satisfied:

$$A_1 + \int_{\tau=0}^{+\infty} c(\tau) d\tau = 1 \quad (26)$$

The impulse and continuous function should be positive because the negative magnitude would annoy the operator. Setting Eq. (24) and Eq. (25) to zero and solving unit-gain constraint in Eq. (26) in a time-optimal solution yields a hybrid filter, $h(\tau)$, of impulse and continuous function:

$$h(\tau) = \begin{cases} A_1, & \tau = 0 \\ c_1 \omega_k \cdot e^{-\zeta_k \omega_k \tau}, & \frac{(1-r)\pi}{\omega_k \sqrt{1-\zeta_k^2}} \leq \tau \leq \frac{(1+r)\pi}{\omega_k \sqrt{1-\zeta_k^2}} \\ 0, & \text{others} \end{cases} \quad (27)$$

where r is the modified factor of the continuous function. The magnitudes, c_1 , A_1 , are given by:

$$c_1 = \frac{1}{\left(\frac{2}{\sqrt{1-\zeta_k^2}} \sin(r\pi) + \frac{e^{-\pi\zeta_k/\sqrt{1-\zeta_k^2}}}{\zeta_k} \left(e^{r\pi\zeta_k/\sqrt{1-\zeta_k^2}} - e^{-r\pi\zeta_k/\sqrt{1-\zeta_k^2}} \right) \right)} \quad (28)$$

$$A_1 = \frac{2c_1}{\sqrt{1-\zeta_k^2}} \sin(r\pi) \quad (29)$$

The hybrid filter (27) includes one impulse and one exponential function. The duration of the hybrid filter (27) is:

$$T_h = \frac{(1+r)\pi}{\omega_k \sqrt{1-\zeta_k^2}} \quad (30)$$

The hybrid filter (27) is different from the input shaper (a series of impulses) and command smoother (continuous function). The duration of hybrid filter (27) can be designed by changing the modified factor, r . The modified factor, r , has a large effect on the frequency insensitivity and duration of the hybrid filter. Increasing modified factor, r , increases frequency insensitivity and duration of the hybrid function. The 5% frequency insensitivity defined in [36] for the hybrid filter (27) ranges from 0.9680 to 1.0321 when modified factor, r , is set to 0.1.

Dynamic analyses indicate that oscillations with three linearized frequencies and damping ratios in the Fig. 3 should be suppressed. Therefore, three hybrid filters with three linearized frequencies and damping ratios are placed in series to attenuate swing and twisting of the load. The MFC controller shown in Eqs. (20) and (21) regulates the quadcopter's attitudes and rejects external disturbances, and three hybrid filters shown in Eq. (27) attenuates the load oscillations caused by pilot commands.

By double-hoist mechanisms and two suspension links shown in Fig. 1, the swing angles, β_1 , γ_1 , β_2 , γ_2 , the slope angle, δ , and the twist angle, ε , are fully coupled with yaw attitude, ψ , pitch

attitude, θ , and the roll attitude, φ . The coupling effects benefit disturbance rejection. This is because the MFC controller stabilizes the attitudes, and will also reduces the load oscillations by the coupling impact. Therefore, the load oscillations caused by external disturbances, which do not need to measure on-the-fly, can be suppressed by using the double-hoist mechanisms and two suspension links. Single-hoist configuration and cable connection directly to aerial vehicles cannot cause fully coupling effect between the load oscillations and quadcopter attitudes.

4. Numerical Verification

In the mid 2000's, A prefilter on pilot commands has been experimentally verified on large-scale aerial vehicles, and then is used by Sikorsky. Meanwhile, it was demonstrated that the prefilter is difficult to achieve good experimental results on small-scale aerial vehicles. The work of experimental verification on large-scale quadcopter is massive. Therefore, this article only presented the numerical verification of disturbance rejection, effectiveness, and robustness of the control method. The combined MFC controller and prefilter will be applied to the dynamic model in Eq. (19) to test the dynamic behavior. The quadcopter mass, m_D , and moment of inertia, I_{xx} , I_{yy} , I_{zz} , suspension distances, l_A , l_B , l_C , cable length, l_S , and linear mass density of the load in this section are fixed at 85 kg, $4.5 \text{ kg} \cdot \text{m}^2$, $4.5 \text{ kg} \cdot \text{m}^2$, $6 \text{ kg} \cdot \text{m}^2$, 0 m, 1 m, 0.1 m, 5 m, 5 kg/m, respectively.

4.1 Disturbance Rejection

External disturbances might cause load oscillations in many cases. The first group of simulations is conducted to investigate the control performance of rejecting external disturbances. Fig. 4 shows a simulated response to a non-zero initial condition caused by external disturbances when the load length is 4 m. The initial values of load swing, β_1 , γ_1 , β_2 , γ_2 , load slope δ , load twisting ε , were set to -6.8182° , 15.6532° , 2.9851° , -6.4229° , -2.6012° , 78.0025° , respectively. Meanwhile, the initial values of three quadcopter attitudes were set to be zero. Then load oscillations would also induce oscillations of quadcopter attitudes by coupling effect between the quadcopter and load.

The frequencies of attitude, ψ , and load twisting, ε , are similar because they are related with the linearized frequency in the $\mathbf{D}_x\mathbf{D}_y$ plane shown in Fig. 3. Other frequencies in Fig. 4 are similar because frequencies in the $\mathbf{D}_y\mathbf{D}_z$ and $\mathbf{D}_x\mathbf{D}_z$ planes are similar in this case. The damping effect inherent in the proposed control scheme attenuated oscillations of quadcopter attitude, load swing and twisting, which approach their corresponding equilibria. Double-hoist mechanisms proposed

Fig. 4. Simulated response to a non-zero initial condition. a) quadcopter attitude, φ , θ , ψ ; b) load swing and slope, γ_1 , γ_2 , δ ; c) load swing and twisting, β_1 , β_2 , ε .

in Fig. 1 create fully coupling effect between the quadcopter and load. The presented controller reduced oscillations of quadcopter and load caused by external disturbances using the coupling impact.

4.2 Effectiveness of Oscillation Suppression

A non-linear simulation was performed to move the quadcopter flying about the $\mathbf{D}_z$ direction, and then flying forward along the $\mathbf{D}_y$ direction. At the beginning of flight, the acceleration drives the quadcopter to reach the desirable flight direction in the $\mathbf{D}_z$ direction, and then move at a constant flight speed along the $\mathbf{D}_y$ direction. At the end of travel, the quadcopter was decelerated

to stop at a desired distance. This process simulates normal straight-line flight along a required direction.

There are two stages in the flight system response. The transient stage of the response is defined as the period when the quadcopter flies forward. The residual stage is defined as the period when the quadcopter attempts to stop and hovers in the sky. The maximum peak-to-peak deflections during the transient and residual stages are referred to as the transient deflection and residual amplitude, respectively.

The simulated response for a yaw angle of 6° about the $\mathbf{D}_z$ direction and a flight distance of 60 m along the $\mathbf{D}_y$ direction is shown in Fig. 5 when load length, l_L , was set to 4 m. The transient deflection, residual amplitude, and settling time of quadcopter's attitude, φ , with the MFC controller were 12.53° , 1.05° , 46.42 s, respectively, whereas, those with the combined MFC controller and three prefilters were reduced to 6.76° , 0.26° , 0.37 s, respectively. The settling time of quadcopter attitudes is defined as the time required for the residual response of the quadcopter's attitude to settle within 0.1° . Thus, the combined MFC controller and three prefilters obviously reduced the oscillations of the quadcopter's attitude. The oscillation frequency of quadcopter's attitude in Fig. 5a is predicted accurately by the corresponding result in the Fig. 3.

With the MFC controller, the transient deflection, residual amplitude, settling time of the load swing, γ_1 , were 10.93° , 8.09° , 58.82 s, and those of the load swing, β_1 , were 2.20° , 1.28° , 11.84 s. The settling time of load oscillations is defined as time required for the residual oscillations of the load to settle within 0.5 degree. With the combined MFC controller and three prefilters, the transient deflection and residual amplitude of the load swing, γ_1 , were only 0.97° and 0.20° , while those of the load swing, β_1 , were 0.19° and 0.0008° . The combined MFC controller and three prefilters totally suppressed the settling time of the load swing, γ_1 , β_1 . The oscillation frequencies of the load are similar to the frequencies of quadcopter's attitude in Fig. 5. This is explained by the coupling effect between the quadcopter and the load.

The transient deflection, residual amplitude, settling time of the load twisting, ε , with the MFC controller were 5.39° , 2.94° , 72.12 s, and those with the combined MFC controller and three prefilters were 0.46° , 0.002° , 0.0 s. Meanwhile, the transient deflection, residual amplitude, settling time of the load slope, δ , with the MFC controller were 5.46° , 4.05° , 39.82 s, and those with the combined MFC controller and three prefilters were 0.49° , 0.10° , 0.0 s. Thus, the

Fig. 5. Simulated response to a flight distance. a) quadcopter attitude, φ, ψ ; b) load swing, γ_1, β_1 ; c) load slope and twisting, δ, ε .

combined control method limited the load twisting and slope to low values, thereby allowing for safer operation.

A battery of simulations was performed to investigate the effectiveness of the combined controller for varying flight distances. The load mass, m_L , and load length, l_L , were set to 20 kg, and 4 m, respectively. The residual amplitude and settling time of the quadcopter's attitude, φ , are presented in Fig. 6a as a function of flight distance. Due to in phase or out of phase interactions between the oscillations caused by the acceleration and deceleration, the residual amplitude and settling time of the quadcopter's attitude, φ , with the MFC controller has a periodic amplitude. Additionally, frequency of the quadcopter's attitude, φ , is restricted by the frequency in Fig. 3

a) quadcopter attitude, φ, ψ .

b) load swing, γ_1, β_1 .

c) load slope and twisting, δ, ε .

Fig. 6. Residual amplitude and settling time against flight distance. a) quadcopter attitude, φ, ψ ; b) load swing, γ_1, β_1 ; c) load slope and twisting, δ, ε .

because of coupling effect among attitude, φ , load swing, γ_1, γ_2 , and load slope, δ . Both the magnitude of peaks and troughs in the residual amplitude and settling time decrease with the increasing flying distance. This is mainly because the MFC controller provides a damping effect. Finally, the residual amplitude and settling time of the quadcopter's attitude, φ , stabilized at approximately 0.93° and 42.5 s. Both the residual amplitude and settling time of the quadcopter's attitude, ψ , decrease slowly as the flight distances increase. When the combined MFC controller and three prefilters is utilized, the residual amplitude and the settling time of attitude, φ , fell below 0.3° and 0.7 s. Meanwhile, the combined control method eliminated all of residual amplitude and

settling time of attitude, ψ . Thus, the combined MFC controller and three prefilters reduces the residual amplitude and settling time of the quadcopter attitudes to very low levels for a wide range of flight motions.

Fig. 6b shows the residual amplitude and settling time of the load swing, γ_1, β_1 . The simulated results of the load swing, γ_1 , are similar to that of the quadcopter attitude, φ , observed in Fig. 6a. Moreover, the frequency, γ_1 , is also restricted by the frequency linked to the value in Fig. 3. As the flight distance increases, the residual amplitude and settling time of the load swing, γ_1 , with the MFC controller settled to approximately 7.3 ° and 54.9 s. Both the residual amplitude and settling time of the load swing, β_1 , decrease slightly as the flight distances increase. However, with the use of the combined MFC controller and three prefilters, the residual amplitude of the load swing, γ_1, β_1 , were limited to less than 0.2 ° and 0.001 °, respectively. The combined control method also suppresses all of the settling time of the load swing, γ_1, β_1 , to very low levels over a wide range of flight motions.

The results for load twisting, ε , and slope, δ , are shown in Fig. 6c. The dynamics of the load slope are also similar to the that of the quadcopter attitude, φ , and the load swing, γ_1 . The residual amplitude and settling time of the load twisting changed slightly for load length shown. The combined MFC controller and three prefilters limited the residual amplitude of the load twisting and slope to below 0.01 ° and 0.1 ° for all the cases shown in Fig. 6c. The combined control method attenuated an average of 99.9 % and 97.7 % more residual amplitude of the load twisting and slope than the MFC controller and suppresses total settling time.

4.3 Robustness to Modeling Errors in System Parameter

An additional set of simulations was performed to assess the robustness of the combined control scheme for various modeling errors in the system parameters. The flight distance was set to 60 m along the $\mathbf{D}_y$ direction after a yaw angle of 6 ° about the $\mathbf{D}_z$ direction. The load length was varied from 2.5 m to 5.5 m in the simulations. The load mass increased as the load length increased because of constant load density. However, the modeled load length and mass for the combined controller were fixed at 4 m, and 20 kg, respectively. The corresponding design frequencies and damping ratios for the combined controller were 1.4880 rad/s, 0.025, 1.5001 rad/s, 0.021, 1.6667 rad/s, 0.009.

The residual amplitude and settling time of the quadcopter attitude for the fixed design frequency are shown in Fig. 7a. By using the MFC controller, increasing load length increases the residual amplitude of quadcopter attitude, φ , and decreases corresponding settling time. Meanwhile, the

a) quadcopter attitude, ϕ , ψ .

b) load swing, γ_1 , β_1 .

c) load slope and twisting, δ , ϵ .

Fig. 7. Residual amplitude and settling time against load length and mass. a) quadcopter attitude, ϕ , ψ ; b) load swing, γ_1 , β_1 ; c) load slope and twisting, δ , ϵ .

residual amplitude of quadcopter attitude, ψ , increases slowly as the load length increases. The settling time of quadcopter attitude, ψ , shows no significant change before 3.7 m. The settling time does rise from 3.7 m, and reaches the local maximum at 4.7 m. After this peak point, the settling time falls. Under the same conditions, the combined control method suppressed the residual amplitude and settling time of quadcopter attitude, ϕ , better than the MFC controller by an average of 75.7% and 99.3%, and those of quadcopter attitude, ψ , by an average of 99.0% and 100%.

Fig. 7b shows the residual amplitude and settling time of the load swing, γ_1 , β_1 . With the MFC controller, as the load length increases, the residual amplitude and settling time of the load swing,

γ_1 , decreases. Increasing load length increases the residual amplitude of the load swing, β_1 , slightly. Meanwhile, the settling time of the load swing, β_1 , keeps zero until 3.5 m, and then increases slowly. The combined MFC controller and three prefilters limited the residual deflection and settling time of the load swing, γ_1 , β_1 , to under 0.2 °, 0.0 s, 0.1 °, 0.0 s, respectively, for the parameter ranges shown.

The simulated load twisting and slope are shown in Fig. 7c. When using the MFC controller, the residual amplitude of the load twisting changes slightly and the settling time of the load twisting decreases with increasing load length. The residual amplitude of load slope contains a peak as the load length is varied. The settling time of load slope decreases with increasing load length. Therefore, the combined control scheme reduced residual amplitude and settling time of the load twisting over the MFC controller by an average of 98.0% and 100%, and those of the load slope by an average of 97.6% and 100%.

Figures. 4-7 demonstrates that the combined MFC controller and three prefilters is beneficial for reducing the oscillations of the quadcopter's attitude, load swing, and load twisting. Therefore, the combined control method provides a safer environment to operate the quadcopter slung load, and a higher efficiency for transferring loads to the desired position precisely.

5. Conclusions

The dynamic effects and oscillation suppression of quadcopters transporting large-size loads were presented in this article. A nonlinear dynamic model of a quadcopter carrying a distributed-mass slender load with double-hoist dynamics was derived using Kane's method. The model includes the motions of the quadcopter's attitude, load swing, and load twisting. A combined MFC controller and three prefilters was presented for controlling the quadcopter's attitude, load swing, and load twisting. The MFC controller adjusts the quadcopter's attitudes and rejects external disturbances, while the hybrid filter reduces the load swing and twisting caused by pilot commands. Numerous simulations verify the effectiveness and robustness of the combined control scheme for various flight distances and system parameters.

This article only provides numerical verification. Because the prefilter is difficult to achieve good experimental results on small-scale aerial vehicles in the laboratory, the future study would be experimental verification of theoretical findings on the large-scale quadcopter (dozens of kilograms).

Availability of Data and Materials

The datasets supporting the conclusions of this article are included within the article.

Competing interests

The authors declare no competing financial interests.

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Authors' contributions

Kuo Zhu: Methodology, Investigation, Writing - Original Draft.

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Figures

Figure 1

Model of a quadcopter transporting a load by double-hoist mechanisms.

Figure 2

Combined control architecture.

Figure 3

Linearized frequencies and damping for various load length and mass.

Figure 4

Simulated response to a non-zero initial condition. a) quadcopter attitude, φ , θ , ψ ; b) load swing and slope, γ_1 , γ_2 , δ ; c) load swing and twisting, β_1 , β_2 , ϵ .

Figure 5

Simulated response to a flight distance. a) quadcopter attitude, φ , ψ ; b) load swing, γ_1 , β_1 ; c) load slope and twisting, δ , ε .

a) quadcopter attitude, φ, ψ .

b) load swing, γ_1, β_1 .

c) load slope and twisting, δ, ε .

Figure 6

Residual amplitude and settling time against flight distance. a) quadcopter attitude, φ, ψ ; b) load swing, γ_1, β_1 ; c) load slope and twisting, δ, ε .

a) quadcopter attitude, ϕ , ψ .

b) load swing, γ_1 , β_1 .

c) load slope and twisting, δ , ϵ .

Figure 7

Residual amplitude and settling time against load length and mass. a) quadcopter attitude, ϕ , ψ ; b) load swing, γ_1 , β_1 ; c) load slope and twisting, δ , ϵ .