

On the Bound of Cumulative Return in Trading Series

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On the Bound of Cumulative Return in Trading Series

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Abstract

There is no doubt that cumulative return is one of fundamental concerns in financial markets. In this paper, we first reveal the upper bound of cumulative return, and then propose a method to evaluate the performance of trading strategies by using proposed upper bound. Furthermore, with the help of bootstrap methodology, we conduct numerous experiments on distinct international stock markets, including developed markets and emerging markets, to verify the validity of the proposed upper bound. And both the theoretical and empirical results show that the effectiveness of the proposed upper bound and reveal its significant potentials on evaluating performance of trading rules.

Keywords: Bound, Bootstrap , Return, Technical Indicator, Time Series

I. Introduction

As a significant part of financial markets, stock market undoubtedly becomes the focus of many analysts and investors. However, considering the stock price is always affected by political, macro or micro economy events, the stock investment is often very risky [1]. While driven by high profits, lots of investors still involve in stock investment, including professional and unprofessional ones. For professional investors, fundamental analysis and technical analysis are two major approaches in making decisions in stock markets [2]. Fundamental analysis is based on expectations about future asset's prices upon market fundamentals and economic factors, including macroeconomic, industrial and business variables. But technical analysis extrapolates trend or statistically relevant characteristics from past stock prices, which is based on the assumption that historical behavior has an effect on the future stock price. Empirical evidence has shown that both fundamental and technical analysis can help to achieve great profits in stock market investment [3]. However, for unprofessional individual investors, lack of generous financial advantage and information superiority always make them become victims when institutional investors manipulate stock price [7]. Unwilling to accept such a fact, some of them try to seek help from technical trading rules in investment (since a qualitative analysis on macroeconomics fundamentals is usually subjective and hard to assess [8]). Nevertheless, the effectiveness of technical trading rules still remains controversy [9-15], and the setting of parameters of a certain trading rules always have a great impact on return. Therefore, how to choose proper technical trading rules and abandon invalid ones is of great importance for investors. That is exactly what we have focused on in this paper. We reveal the upper bound of return, which offer a new way to assess the performance of a technical trading rule.

As one of fundamental concerns in stock markets, the related literature to cumulative return is extensive, especially on cross sectional return [16], but that to the bound of cumulative return in trading time series is quite rare. As for the bound of cumulative return, prior works are often based on pricing of assets, but seldom focuses on trading behaviors. For example, [17] provided the theoretical upper bounds, and found the empirical R^2 s are more than theoretical upper bounds on predictive regression. Similarly, an important bound on cumulative return on Mean-Variance model was earlier studied by [18], which is focusing on is portfolio selection. Different from these prior works, the demonstrated bound in our work aims to control the trading risk by selecting proper trading rules in investment.

In this paper, we first deduce the upper bound of cumulative return in theory and prove the correctness of several corollaries from it. Then, based on bootstrap methodology, we investigate the effectiveness of the bound in evaluating the performance of technical trading rules on both emerging market and developed market. In short, our study makes several contributions as follows:

1. Deduce the upper bound of cumulative return in theory, revealing the limitation of maximal possible profit of a stock trading time series.
2. Find that when the mean of return rate series (\bar{r}) is less than the transaction cost rate (k), the more trades, the more losses. When the number of trades increases to infinity, the cumulative returns will be likely to converge close to zero.
3. Present a new way to assess the performance of trading strategies, which can help unprofessional investors to select proper technical trading rules.

The rest of this paper is organized as follows. Section 2 first introduces the data set, then shows the theoretical background of the bound of cumulative return, several common technical trading rules, and bootstrap methodology. In Section 3, we explore the correctness and effectiveness of the proposed upper bound by conducting numerous experiments on distinct stock indices. Finally, Section 4 makes our main conclusion and a prospect of future work.

II. Theoretical Model on Bound of Cumulative Return

Assume S is a time series, the return of the i -th trade without considering transaction cost is defined as,

$$r_i = \frac{S(s_i) - S(b_i)}{S(b_i)} \quad (1)$$

Here, $S(*)$ is the price at time $*$. s_i and b_i denotes the time of conducting i -th buy and sell trades, respectively. When taking transaction costs into consideration, the cumulative return $R(n)$ can be calculated by:

$$R(n) = \prod_{i=1}^n \left(1 + \frac{S(s_i) - S(b_i) - T_i}{S(b_i)}\right) \quad (2)$$

Where n is the number of trades, and T_i is the transaction costs in the i -th trade which can be approximately considered as $T_i \approx S(s_i) \cdot k$, and here k is regarded as a constant, denoting transaction cost rate. Further, we can obtain:

$$R(n) = \prod_{i=1}^n (1 - k) \frac{S(s_i)}{S(b_i)} = \prod_{i=1}^n (1 - k)(1 + r_i) \quad (3)$$

Before deriving the bounds of cumulative return $R(n)$, we first introduce D-I inequality which is proposed by Dragomir and Ionescu[19]. As a preliminary knowledge, we show the D-I inequality

as follows:

Theorem 1(D-I inequality) Let $f: I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable convex function on I , $x_i \in I$ and $p_i \geq 0 (i = 1, \dots, n)$ with $P_n := \sum_{i=1}^n p_i > 0$. Then we have the inequality as follow,

$$0 \leq \frac{1}{P_n} \sum_{i=1}^n p_i f(x_i) - f\left(\frac{1}{P_n} \sum_{i=1}^n p_i x_i\right) \quad (4)$$

Where $f'(x_i)$ is the first derivation of $f(x)$ at x_i , and $f'(x_i) = \frac{\partial f(x_i)}{\partial x}$.

Then, with the help of the D-I inequality, we exhibit the upper bound of cumulative return $R(n)$ as Theorem 2:

Theorem 2(Upper bound) $\forall n > 0, \exists R(n)$ satisfies the following inequality:

$$R(n) \leq [(1 - k)(1 + \bar{r}_i)]^n \quad (5)$$

Here $\bar{r} = \frac{1}{n} \sum_{i=1}^n r_i$, k is transaction cost rate.

Proof Let $G(n) = -\ln R(n)$, according to Eq 3, then

$$G(n) = -\ln \prod_{i=1}^n (1 - k)(1 + r_i) = n \cdot [-\ln(1 - k)] + \sum_{i=1}^n [-\ln(1 + r_i)] \quad (6)$$

In the D-I inequality, we let $p_i = 1$, so $P_n := \sum_{i=1}^n p_i = n$, and $f(x) = -\ln x$. It is easily to certify that $f(x)$ is a differentiable convex function on $(0, +\infty)$, because the second derivative of $f(x)$ is greater than 0 when $x \in (0, +\infty)$ (see Eq 7).

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial(\partial(-\ln x)/\partial x)}{\partial x} = \frac{\partial(-1/x)}{\partial x} = \frac{1}{x^2} > 0 \quad (7)$$

Let $x_i = 1 + r_i (-1 \leq r_i)$, the Eq 4 can be transformed into:

$$-n \cdot \ln \left[\frac{1}{n} \sum_{i=1}^n (1 + r_i) \right] \leq \sum_{i=1}^n [-\ln(1 + r_i)] \quad (8)$$

With the assistance of Eq 8 and the Eq 6, $G(n)$ can be further derived:

$$G(n) \geq n \cdot [-\ln(1 - k)] - n \cdot \ln(1 + \bar{r}) = -\ln[(1 - k)(1 + \bar{r}_i)]^n \quad (9)$$

Where $\bar{r} = \frac{1}{n} \sum_{i=1}^n r_i$, and considering that $G(n) = -\ln R(n)$, we can further obtain

$$R(n) \leq [(1 - k)(1 + \bar{r}_i)]^n \quad (10)$$

Then, the upper bound of $R(n)$, has been proved.

End of proof.

According to Eq 5, we can derive some corollaries about cumulative returns as follows:

Corollary 1 Limitation of Zero

$$R(n) \leq 1 \text{ and } \lim_{n \rightarrow +\infty} R^n = 0, \quad \text{if } \bar{r} \leq k$$

Proof Because $\bar{r} \leq k$, therefore

$$R(n) \leq [(1 - k)(1 + \bar{r}_i)]^n \leq (1 - k^2)^n \leq 1 \quad (11)$$

because, $k \in (0, 1)$, $(1 - k^2) \in (0, 1)$, so $R(n) \leq 1$. Further,

$$0 \leq \lim_{n \rightarrow +\infty} R^n \leq \lim_{n \rightarrow +\infty} (1 - k^2)^n = 0 \quad (12)$$

End of proof

Obviously, the Corollary 1 reveals the that when the number of trades n increases to infinity, the cumulative returns will converge to 0, which means investors will lose all the money in

investment. In addition, we show the trend of upper bound with the increase of k and n in Fig 2. Easy to find when $\bar{r} \leq k$, the declining situation of the upper bound with n is small, while when n is large enough, the upper bound would become close to zero. This is consistent with Corollary 1.

Fig 1. When $\bar{r} \leq k$, the upper bound will decrease with the increase of k and n . Here the left subgraph shows the situation when n is small (range from 0 to 100), while the right subgraph shows the situation that when n is larger enough (range from 10,000 to 100,000). As visible, the upper bound exhibits a clear decreasing trend with the n and k increase.

Corollary 2 Critical unprofitable condition

$$\text{Let } \gamma = \frac{k}{1-k}, \quad \bar{r} \leq \gamma \text{ is a sufficient condition to make } R(n) \leq 1.$$

Proof If $\bar{r} \leq \gamma = \frac{k}{1-k}$, we can derive the Eq 5 as follows,

$$R(n) \leq [(1-k)(1+r_i)]^n \leq \left[(1-k) \left(1 + \frac{k}{1-k} \right) \right]^n = 1^n = 1 \quad (13)$$

End of proof

As for corollary 2, we illustrate a critical profitable line (CPL: $\bar{r} = \gamma = k/(1-k)$) as the watershed of trading risk, as shown in Fig 3, when the part over CPL is the possible profitable area, but that below CPL is an absolute loss area without chance of achieving return. Note that the ordinate denotes the logarithm of the mean of return series, i.e., $\log(\bar{r})$, in Fig 3. The proposed CPL is new to related literature and able to be acted as a decision measurement of profitability in financial trading activity.

Fig 2. The critical profitable line. Note that we here use a base 10 logarithmic scale for the y-axis and a linear scale for the x-axis. The details see text.

III. Experiments and Discussion

A. Data Materials

In this paper, the profitability of proposed technical trading rules is assessed on four important financial market indices, which are Dow Jones Industrial Average (DJIA), FTSE 100 Index (FTSE), Nikkei 225 (N225) and SSE Composite Index (SCI), respectively. And we download these daily indices from Yahoo Finance, a popular publicly available source of data. Further, we show the temporal evolution of these stock indices in Fig 1.

- DJIA, from September, 12th 1988 to September, 09th 2018, for a total of 7562 days;
- FTSE, from September, 12th 1988 to November, 26th 2018, for a total of 7663 days;
- N225, from September, 13th 1988 to November, 26th 2018, for a total of 7408 days;
- SCI, from January, 04th 2000 to November, 26th 2018, for a total of 5464 days.

Fig 3. Temporal evolution of four stock indices. From top to bottom, we show the DJIA, FTSE, N225 and SCI indices. See text for further details.

B. Technical trading rules

As we all know, technical trading rules are widely used in stock market investment. In this part, we will introduce several the most popular technical trading rules, which are moving average convergence divergence(MACD), commodity channel index(CCI), rate of change(ROC), and random trading strategy(RND), respectively. As the simplest trading strategy [8], the random trading strategy makes trade decision (buy or sell) at time t completely at random (t follows a uniform distribution with a mean of 15). The other technical trading rules and their formula are presented in Table 1. Notably, the parameter setting of these trading rules is according to [20].

Table 1. Details of technical trading rules.

Oscillator	Formula	Parameter	Technical rules	
			Buy	Sell
MACD	$\text{MACD}(m, n) = \text{EMA}(n) - \text{EMA}(m)$ $\text{EMA}(n) = \left[\frac{(C_t - \text{EMA}_{t-1})}{n+1} \right] + \text{EMA}_{t-1}$	n=12, m=26	MACD ↗ 0	MACD ↘ 0
CCI	$\text{CCI}(n) = \frac{M - M(n)}{d(n) \times 0.015}$ $M = \frac{H + L + C}{3}$ $d(n) = \frac{1}{n} \sum_{i=0}^{n-1} M_{t-i} - \bar{M}_t(n) $	n=9	CCI ↗ -100	CCI ↘ 100
ROC	$\text{ROC}(n) = \left(\frac{C_t}{C_{t-n}} - 1 \right) \times 100$	n=13	ROC ↗ 0	ROC ↘ 0

^aC, L, H are close price, low price, high price, respectively; ↗, ↘ mean upwards/downwards cross.

C. Bootstrap methodology

In order to reduce the influence of "luck" and make results more convincing, we adopt bootstrap methodology [23] in experiments. The main steps of the bootstrap methodology in this paper are described as follows.

1. Resample: For each experiment, randomly choose entering and exiting points, forming a test period $[\text{enterpoint}_i, \text{exitpoint}_i]$;
2. Generate return series and compute cumulative return: In each test period, use a certain trading rule to make trades and obtain the corresponding return series, then compute cumulative return R_i by Eq.3;
3. Repeat step 1 and step 2: Repeat the above two steps for M times, then get the estimation of \bar{R} , $\bar{R} = \frac{1}{M} \sum_{i=0}^M R_i$. In this paper, we take M as 1000.

In this section, with the help of bootstrap methodology, we first investigate the validity of the proposed upper bound in different stock indices and then provide a method to evaluate the performance of trading strategies. Besides, we conduct numerous experiments on distinct stock indices to testify the validity of the proposed method.

D. Validation of the Upper Bound

In order to testify the validity of proposed upper bound in real stock indices, we choose DJIA as an example to illustrate the influence of the number of trade on cumulative return and its upper bound. Fig 4 shows the results. Easily find that regardless of transaction cost rate and technical trading rules, the corresponding upper bound hold, meaning the validity of upper bound is verified.

Fig 4. Validation of the upper bound of cumulative return in DJIA. Here left subgraph shows the cumulative return and its upper bound with the different trading strategies ($k = 0.005$). While right subgraph shows the cumulative return and its upper bound with transaction cost rate. k_1, k_2 and k_3 take 0.003, 0.005 and 0.007, respectively. Notably, UB represents the upper bound. As visible, the upper bound always holds, regardless of trading strategies and transaction cost rate.

E. Performance Evaluation of the Typical Indicators

Former part has proved the validity of the upper bound. Here we will propose a method to assess the performance of trading strategies. First of all, we should compute \bar{r} of the trading strategy in the selected stock index. To some extent, the \bar{r} can reflect the potential profitability of the technical trading rule. Then evaluating the performance of trading strategies according to the relationship between \bar{r} and k . If $k > \bar{r}$, meaning the trading strategy will perform poorly in this stock and we should abandon it.

Then, we will test the effectiveness of this method by experiments. First, we divide each stock index into two parts equally, which are training set and testing set. By using the bootstrap methodology, we calculate the \bar{r} in training set and regard it as the \bar{r} of this stock index. Table 2 presents the \bar{r} of several technical trading rules in different stock indices. Furthermore, we will consider transaction costs. Transaction cost is an important factor that affects the cumulative return and always consists of two major components: explicit costs and implicit costs. The former is the direct costs of trading, such as broker commissions and taxes, while the latter involves in indirect costs such as the influence of the trade price and the opportunity cost of failing to execute the order, which is difficult to measure [14]. By the way, it should be noted that in many markets, especially in the emerging markets, the implicit costs are even higher than the explicit costs [24]. In simplicity, we only take explicit costs into consideration and approximately measure transaction costs by transaction cost rate k in this paper. Here we show the influence of k on $R(n)$ in different stock indices, and set the value of k range from 0.001 to 0.01 in Fig 5. Undoubtedly, the cumulative return shows a clear downward tendency when the k increase gradually, this is in accordance with common sense. We also notice that when $k > \bar{r}$, the cumulative return will less than 1. For example, the \bar{r} of ROC in N225 is 0.0005(see Table 2), which is apparently less than k , thus the

cumulative return will less than 1. Fig 5 supports our analysis.

Table 2. The \bar{r} of used technical trading rules in stock indices

Trading rules	Stock indices			
	DJIA	FTSE	N225	SCI
CCI	0.0046	0.0052	0.0002	-0.0021
MACD	0.0065	0.0023	0.0059	0.0251
RND	0.0048	0.0028	0.0004	0.0048
ROC	0.0038	0.0016	0.0005	0.0149

Fig 5. The influence of k on cumulative return in stock indices. From top to bottom, from left to right, we display the results of ROC, RND, CCI and MACD strategies, respectively. Further analysis can be seen in text.

In addition, we also investigate the influence of the number of trades n on $R(n)$. For simplicity, here we take DJIA as an example and exhibit results in Fig 6. From Table 2, we have known the \bar{r} of the ROC, CCI, RND and MACD in DJIA are 0.0038, 0.0046, 0.0048, 0.0065, respectively. When k takes 0.007, exist $k > \bar{r}$, therefore according to Corollary 1, the $R(n)$ ought to show an approximately downward tendency when n increases gradually. Apparently, Fig 6 ($k = 0.007$) supports our Corollary 1. Besides, according to Corollary 2, when $k = 0.007$, $\bar{r} < \gamma = k/(1 - k)$ stands, meaning the trading rules cannot achieve profits in DJIA. As for when n is small, there exists $R(n) > 1$, we believe it mainly caused by the "luck". With n increases, the influence of the "luck" will diminish gradually. Other subgraph in Fig 6 deliver that if $k < \bar{r}$, it has a large probability that $R(n)$ will increase with the n increases. Therefore, the relationship between k and \bar{r} is a crucial factor in evaluating profitability of trading strategy in investment. If satisfying the condition $k < \bar{r}$, the technical trading rule should not be adopted in the stock index. Worthy to mention that the value of k tends to be correlated with the stock market. For example, in the Chinese stock market, the transaction costs mainly consist of stamp duty (0.1% of the turnover), commission (no more than 0.3% of the turnover) and transfer fees (only charge in the Shanghai market, which is 0.006% of the denomination)[15].

Fig 6. The influence of the number of trades n on cumulative return in DJIA. From top to bottom, from left to right, k takes 0.001, 0.005, 0.003 and 0.007 respectively. See text for further analysis.

IV. Conclusion

In this paper, we first reveal the upper bound of cumulative return in theory. Then we derive and prove several corollaries from the upper bound, concluding that when the mean of return rate series \bar{r} less than transaction cost rate k , the more trades, the more losses. Further, we propose a method to assess the performance of trading strategies by using the proposed upper bound. In order to test the validity of our method, with the help of bootstrap methodology, we conduct numerous experiments on SCI, DJIA, N225, and FTSE to evaluate the performance of four common technical trading rules. At last, the results verify our method. In short, we conclude that the

relationship between k and \bar{r} is a crucial factor in evaluating profitability of trading strategy in investment. It is worth mentioning that we do not deny that technical trading rules with certain parameters can make great profits in some indexes, thus we expect our method can help to alleviate the risk for unprofessional investors in choosing trading strategies. What's more, we advise unprofessional individual investors who have less investment experience to stay away from stock markets or to seek guidance from professional institutional investors, in case of unnecessary loss.

One of frontier application fields of our work is high-frequency trading, for an instance, the upper bound can be directly used to construct theoretical bounds in the work of [25], in which the authors developed an automatic high-frequency trading system assisted by a particle swarm optimization algorithm. On the other hand, our finding is also able to help to avoid some unnecessary trades of trading strategies by designing a specific stop-loss mechanism. In future work, we should pay more attention to combine our finding with some trading strategies to provide a better guidance for unsophisticated investors in stock investments.

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Declarations

Competing interests: The authors declare no competing interests.

Figures

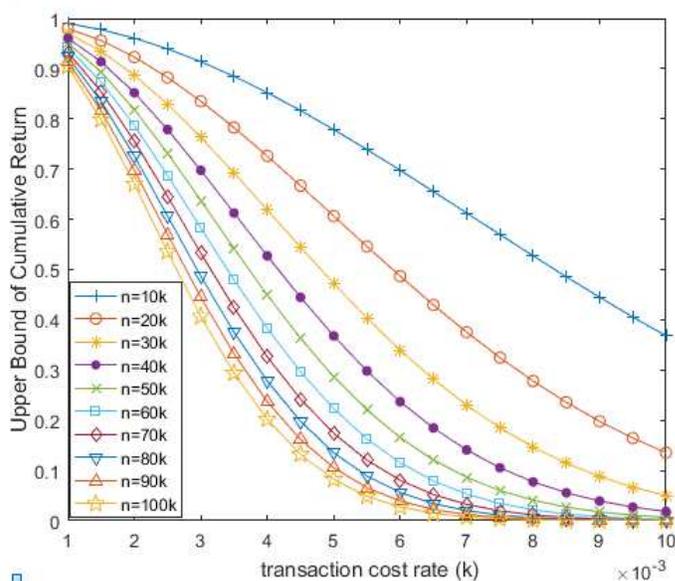
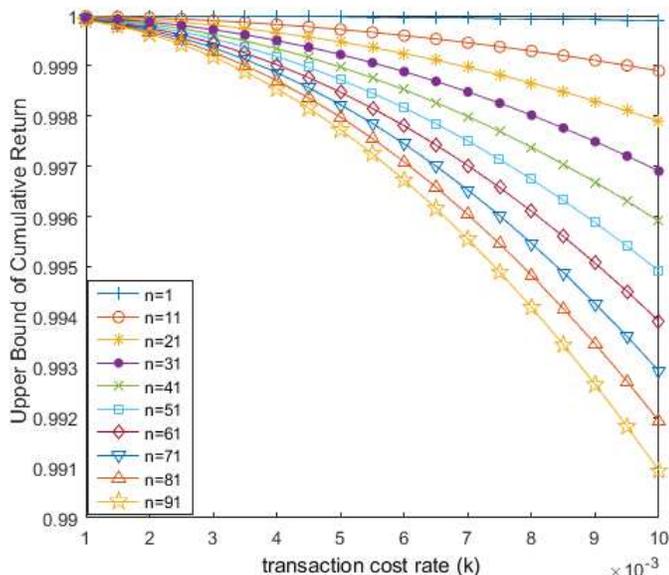


Figure 1

When mean of r is less than and equal to k , the upper bound will decrease with the increase of k and n . Here the left subgraph shows the situation when n is small (range from 0 to 100), while the right subgraph shows the situation that when n is larger enough (range from 10,000 to 100,000). As visible, the upper bound exhibits a clear decreasing trend with the n and k increase.

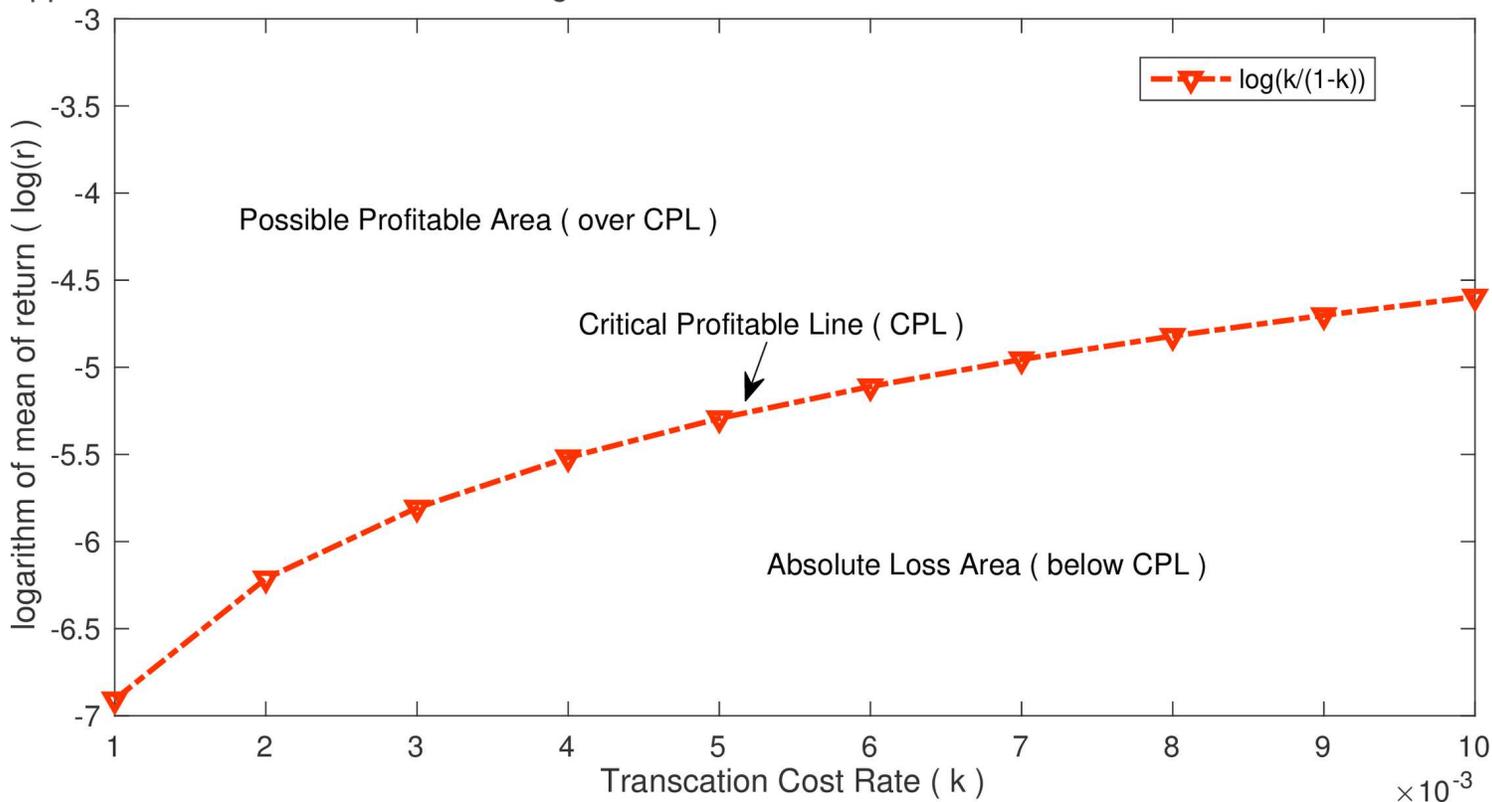


Figure 2

The critical profitable line. Note that we here use a base 10 logarithmic scale for the y-axis and a linear scale for the x-axis. The details see text.

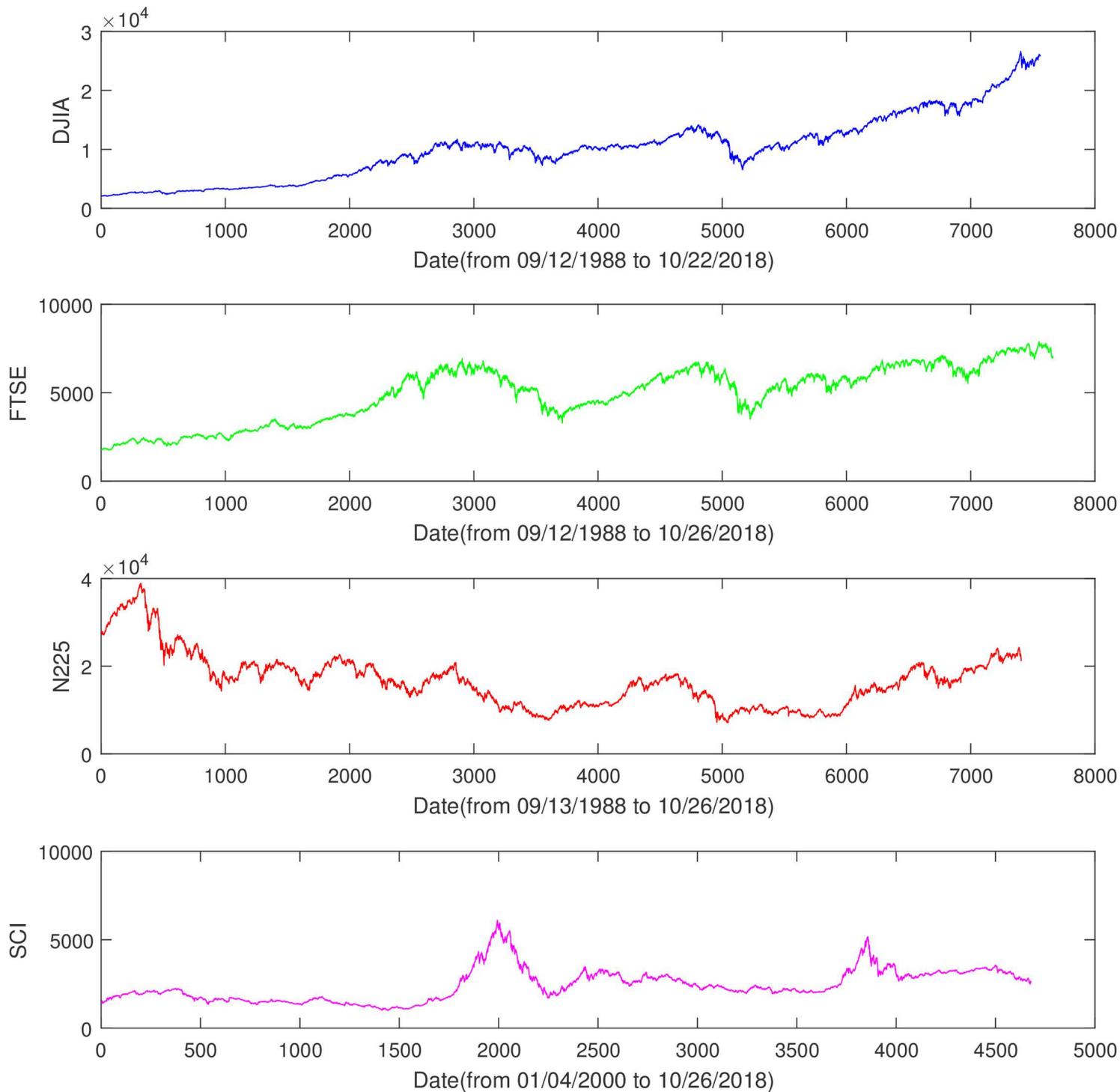


Figure 3

Temporal evolution of four stock indices. From top to bottom, we show the DJIA, FTSE, N225 and SCI indices. See text for further details.

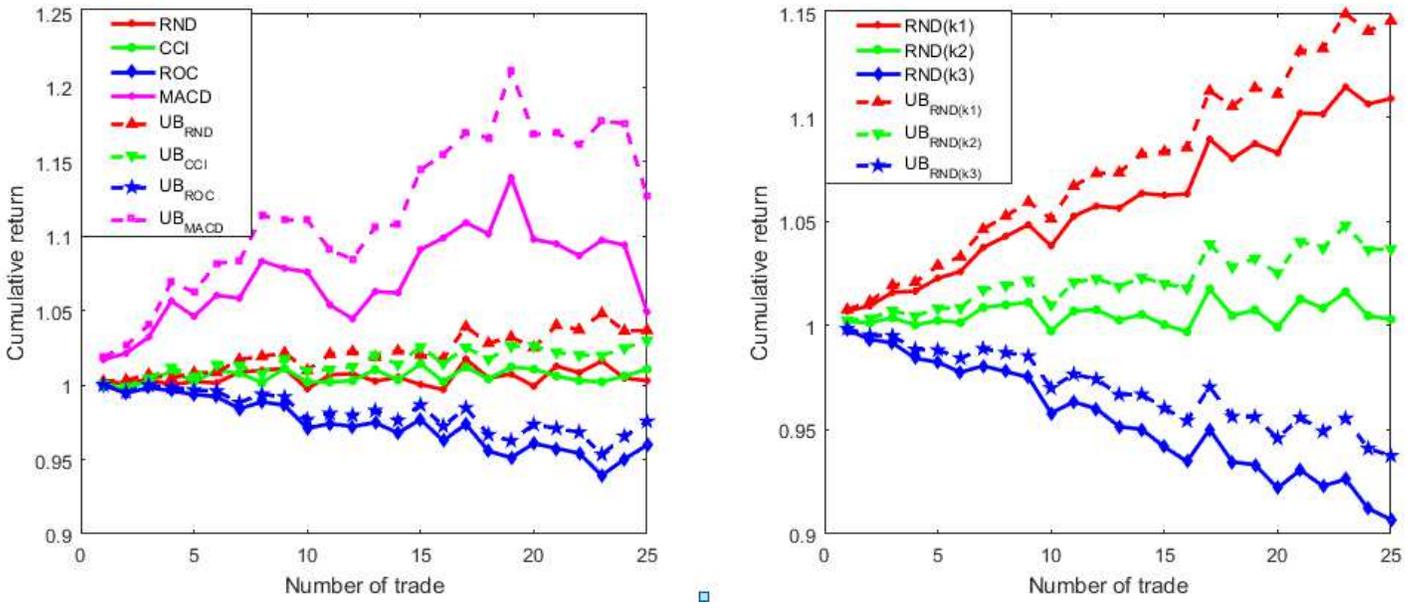


Figure 4

Validation of the upper bound of cumulative return in DJIA. Here left subgraph shows the cumulative return and its upper bound with the different trading strategies ($k=0.005$). While right subgraph shows the cumulative return and its upper bound with transaction cost rate. k_1, k_2 and k_3 take 0.003, 0.005 and 0.007, respectively. Notably, UB represents the upper bound. As visible, the upper bound always holds, regardless of trading strategies and transaction cost rate.

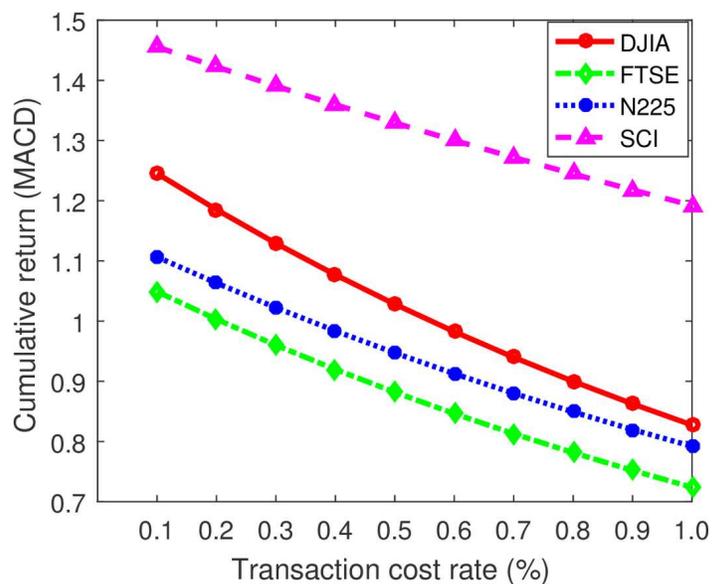
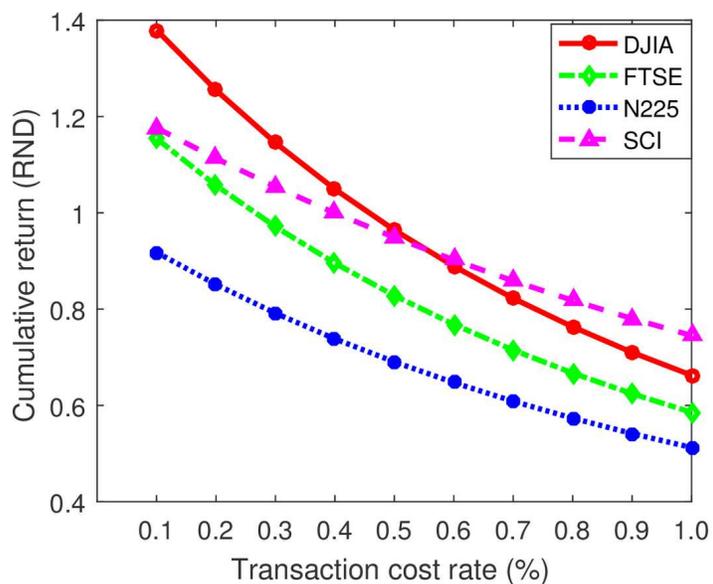
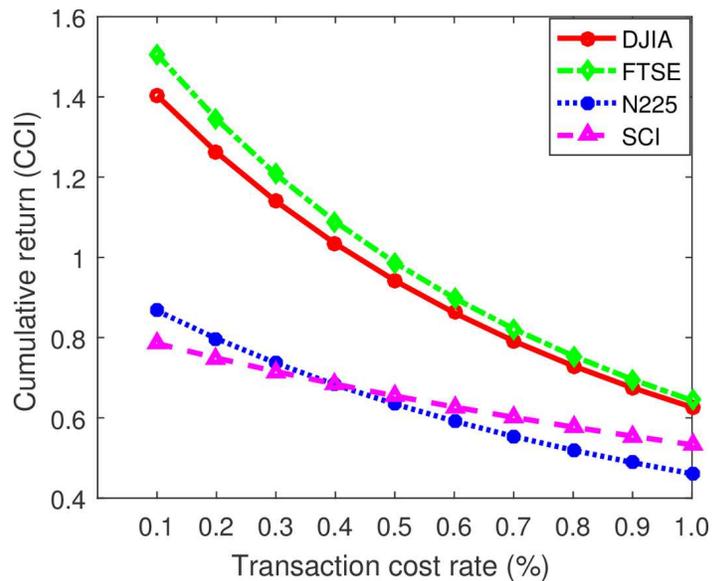
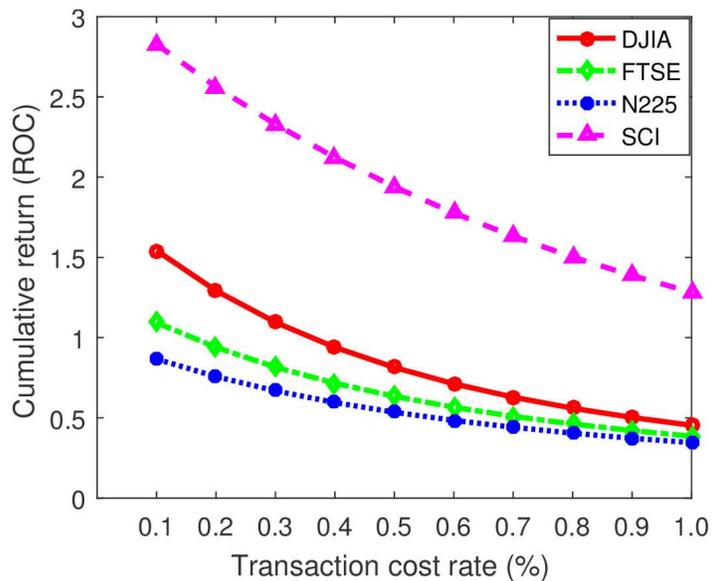


Figure 5

The influence of k on cumulative return in stock indices. From top to bottom, from left to right, we display the results of ROC, RND, CCI and MACD strategies, respectively. Further analysis can be seen in text.

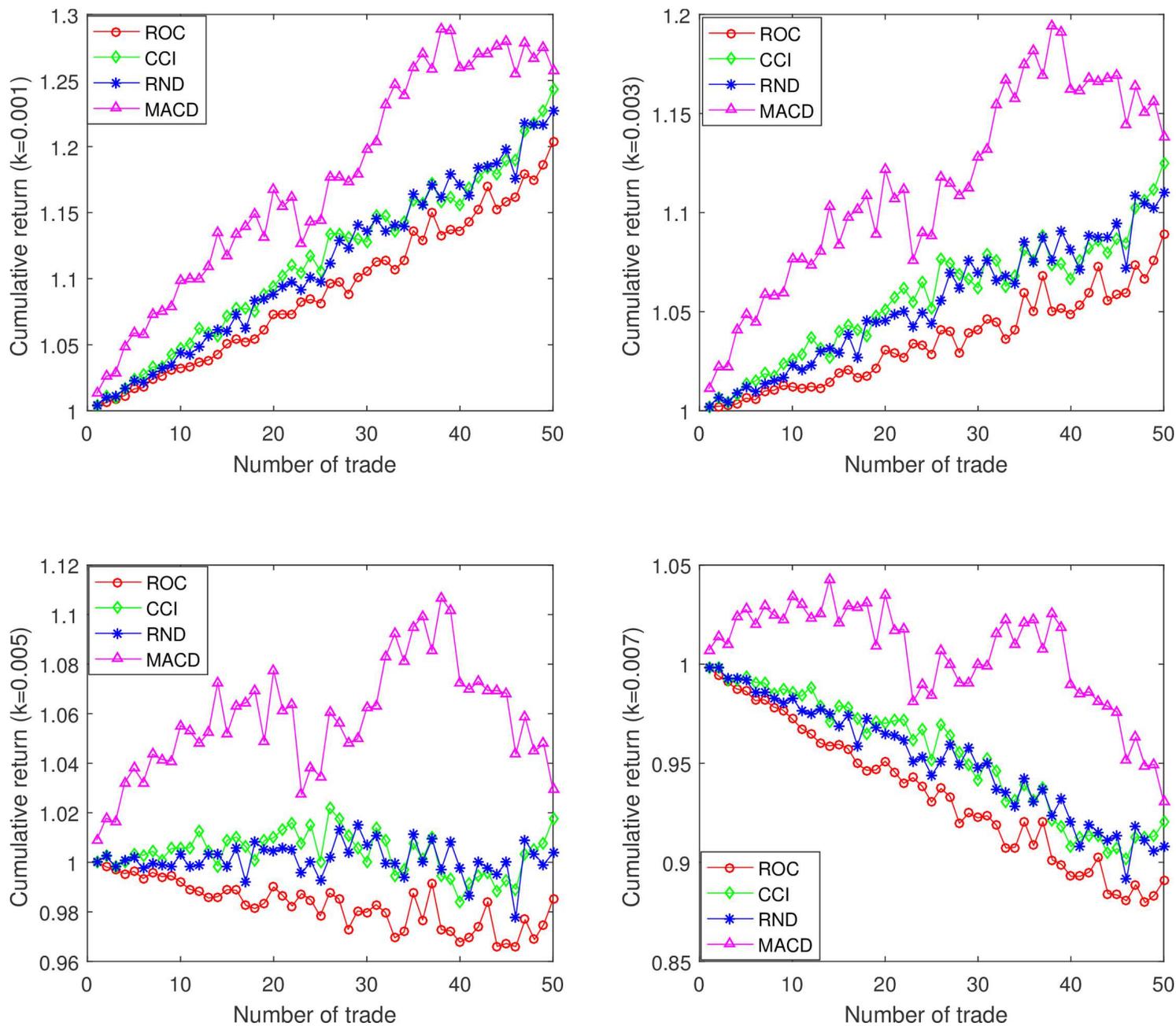


Figure 6

The influence of the number of trades n on cumulative return in DJIA. From top to bottom, from left to right, k takes 0.001, 0.005, 0.003 and 0.007 respectively. See text for further analysis.