Dynamic Analysis of Bevel Gear System in Main Retarder

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Dynamic Analysis of Bevel Gear System in Main Retarder

Xinbo Wang* · Xuebin Zhu · Xiaobo Song · Di Wang · Qing Liu · Kaihua Cheng

Abstract The main retarder of vehicle is gear rotor system, its vibration property is complex. The whole transfer matrix method is a new algorithm to study, relationship of the displacements and forces are established considering gyroscopic moment and coupling vibration. The meshing point between rotors is treated as coupling element, balance equations are established on deformation compatibility condition. The globe status vectors including forces and displacements of all shafts were integrated into a whole transfer matrix. So it can be programming and has high numerical stability and accuracy. The influence of the meshing effect on vibration of the system can be systematically studied. The status vectors such as displacements, deflection angles, torsion angles, shearing forces, bending moments and torques are no longer independent. The vibration property of the geared rotor system was calculated and analyzed. The numerical results revealed that many coupling critical rational speeds of the system are derived. vibration of main retarder can be improved.

Keywords Main retarder; Rotor dynamics; Meshing stiffness; Whole transfer matrix; Critical rotational speed

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1. Introduction

Vehicle power train is typical gear rotor system. Vibration characteristics of it is different from single rotor in [1]. Bending and torsional deformations are coupled due to gear meshing. There is a distance between the meshing force and the shaft, so lateral response by torsional excitation of geared rotors system and the influence of torsional-lateral coupling on the stability behavior of it were studied in [2,3]. Coupled bending torsional vibration of rotors using finite element and a modified transfer matrix method for the coupling lateral and torsional vibrations of symmetric rotor-bearing systems are visible in [4,5]. Many coupling critical rotational speeds are generated and must be considered. Resonance could occur when working speed approaching them. It's important to study the coupling critical rotational speeds of the system. General methods of vibration calculating are the finite element method and the transfer matrix method. The former is an approximate method, it is suitable to study block body. Dynamic analysis of gear rotor system by finite elements was published in [6,7,8]. The latter is an accurate algorithm and suitable to study dynamics of rotor. Now based on the whole transfer matrix method, dynamic equations of the geared rotor systems are deduced by D'Alembert principle, Lagrange's equations, machinery system dynamics, nonlinear dynamics and geared system dynamics. The research on vibration of the vehicle power transmission system is synthesized here.

2. Transfer matrix of gear-rotor system

Dynamic behaviors of geared rotor system were analyzed in [9,10]. The gear meshing link is called coupling element, it make status vectors of all rotors into a whole column vector. Status vectors of the system are coupled by meshing stiffness. Status vectors of the system are $[Z]^T = [Z^I \ Z^II \ Z^III]^T = [Z_1, Z_2, Z_3, \dots, Z_n]^T$.

Eigen value and Eigen vectors of it can be solved that has been seen in [11]. The principle of whole transfer matrix method is iteration of rotational speed for obtaining the critical rotational speeds. Dynamic equations of the system are deduced according to deformation compatibility condition. Status vectors of every shaft contain 10 parameters. $[Z]^T = [Q_x \ M_y \ Q_y \ M_x \ T \ \theta_y \ x \ \theta \ y \ \varphi]^T$.

2.1. Balance of shearing forces on the wheel

The centrifugal force is created in whirl motion. Ω is precession angular speed of the wheel, ω is rotational speed of it. Initial deflection of shaft is little at the beginning of precession, $\Omega \approx \omega$, x , y and z are displacement

components of the wheel center. External forces along forward direction of the axes are plus. Balance of shearing forces on the wheel is shown in fig. 1. Q^L is shearing force acting on left of the wheel.

Fig. 1 balance of shearing forces on the wheel

$$Q_{Ax}^R - Q_{Ax}^L = F_{Ax} - m_A \ddot{x}_A = F_{Ax} + m_A \Omega^2 x_A$$

$$Q_{Ay}^R - Q_{Ay}^L = F_{Ay} - m_A \ddot{y}_A = F_{Ay} + m_A \Omega^2 y_A$$

Fig. 2 balance of torques on the wheel

$$Q_{Bx}^R - Q_{Bx}^L = F_{Bx} - m_B \ddot{x}_B = F_{Bx} + m_B \Omega^2 x_B$$

$$Q_{By}^R - Q_{By}^L = F_{By} - m_B \ddot{y}_B = F_{By} + m_B \Omega^2 y_B$$

2.2. Balance of torques on the wheel

Balance of torques on the wheel are shown in fig. 2. $I_p \Omega^2 \varphi$ is the inertia torque, torque generated by meshing force is $T, T = F_p R_b$,

$$T_A^R - T_A^L = T - I_p^A \ddot{\varphi}_A^L = T + I_p^A \Omega^2 \varphi_A$$

$$T_B^R - T_B^L = T - I_p^B \ddot{\varphi}_B^L = T + I_p^B \Omega^2 \varphi_B$$

2.3. Balance of bending moments on the wheel

Fig. 3 inertia moments of the wheel

Fig. 4 balance of bending moments

In fig.3, $o\zeta$, $o\zeta_1$ and $o\eta_1$ are inertia central principal axes of the revolving wheel, I_p is rotating inertia of the wheel around $o\zeta$ -axis, I_d is rotating inertia of the wheel around $o\zeta_1$ -axis or $o\eta_1$ -axis considering three Euler angles. Rotation angle around Y-axis is α_p and angular speed is $\dot{\alpha}_p(t)$. Rotation angle around $o\zeta_1$ -axis is β_p and angular speed is $\dot{\beta}_p(t)$. Rotation angle around $o\zeta$ -axis is φ and angular speed is ω . θ is deflection angel of the wheel, φ is torsion angel of it. At the beginning of the precession, α_p and β_p are infinitesimal, $\cos \alpha_p \approx 1$, $\cos \beta_p \approx 1$, $\sin \alpha_p \approx \alpha_p$, $\sin \beta_p \approx \beta_p$. In accordance with the theorem of angular momentum, $I_d \dot{\alpha}_p$ is momentum along $o\eta_1$ -axis, $I_d \dot{\beta}_p$ is momentum along $o\zeta_1$ -axis, $I_p \omega$ is momentum along $o\zeta$ -axis.

$$H_{ox} = I_p \omega \sin \alpha_p + I_d \dot{\beta}_p \cos \alpha_p \approx I_p \omega \alpha_p + I_d \dot{\beta}_p \quad H_{oy} = I_p \omega \sin(-\beta_p) + I_d \dot{\alpha}_p \cos(-\beta_p) \approx -I_p \omega \beta_p + I_d \dot{\alpha}_p$$

$$\frac{dH_{ox}}{dt} = \frac{d}{dt} (I_p \omega \alpha_p + I_d \dot{\beta}_p) = I_p \omega \dot{\alpha}_p + I_d \ddot{\beta}_p = -I_d \Omega^2 \beta_p + i I_p \omega \Omega \alpha_p = -I_d \Omega^2 \theta_x + i I_p \omega \Omega \theta_y = -M_g^x$$

$$\frac{dH_{oy}}{dt} = \frac{d}{dt} (-I_p \omega \beta_p + I_d \dot{\alpha}_p) = -I_p \omega \dot{\beta}_p + I_d \ddot{\alpha}_p = -I_d \Omega^2 \alpha_p - i I_p \omega \Omega \beta_p = -I_d \Omega^2 \theta_y - i I_p \omega \Omega \theta_x = -M_g^y$$

M_g^x and M_g^y are inertia moments of the wheel, It is synchronous whirling of small deflection at the beginning of

the precession, $\Omega \approx \omega$.

Balance of bending moments on the wheel is shown in fig. 4. A is the driving wheel, its direction of rotation is minus, and the direction of revolution is minus too when it is doing synchronous forward precession,

$$M_{Ay}^R - M_{Ay}^L = I_d^A \Omega^2 \theta_{Ay} + i I_p^A \omega_A \Omega \theta_{Ax} \quad M_{Ax}^R - M_{Ax}^L = I_d^A \Omega^2 \theta_{Ax} - i I_p^A \omega_A \Omega \theta_{Ay}$$

B is driven wheel, it is rotating in the opposite direction and doing reverse backward precession.

$$M_{By}^R - M_{By}^L = I_d^B \Omega^2 \theta_{By} + i I_p^B \omega_B \Omega \theta_{Bx} \quad M_{Bx}^R - M_{Bx}^L = I_d^B \Omega^2 \theta_{Bx} - i I_p^B \omega_B \Omega \theta_{By}$$

3. Transfer matrix of the shaft

The gear rotor system is composed of gear wheels, shafts and bearings, status vectors are transferred by the shaft. According to the right-hand rule, direction of shearing force on shaft in fig. 5(a) is plus, direction of bending moment in fig. 5(b) is plus and direction of deflection angle in fig. 5(c) is plus.

fig.5 coordinate system of shaft

The relationship of status vectors are listed,

$$\begin{cases} x_E = x_S + \theta_S^y l - a_{12} M_E^y - a_{11} Q_E^x \\ \theta_E^y = \theta_S^y - a_{22} M_E^y - a_{21} Q_E^x \\ M_E^y = M_S^y - Q_S^x l \\ Q_E^x = Q_S^x \end{cases}, \quad \begin{cases} y_E = y_S - \theta_S^x l + a_{12} M_E^x - a_{11} Q_E^y \\ \theta_E^x = \theta_S^x - a_{22} M_E^x + a_{21} Q_E^y \\ M_E^x = M_S^x + Q_S^y l \\ Q_E^y = Q_S^y \end{cases}$$

$$\begin{cases} x_E = x_S + \theta_S^y l - a_{12} M_S^y + (a_{12} l - a_{11}) Q_S^x \\ \theta_E^y = \theta_S^y - a_{22} M_S^y + (a_{22} l - a_{21}) Q_S^x \\ M_E^y = M_S^y - Q_S^x l \\ Q_E^x = Q_S^x \end{cases}, \quad \begin{cases} y_E = y_S - \theta_S^x l + a_{12} M_S^x + (a_{12} l - a_{11}) Q_S^y \\ \theta_E^x = \theta_S^x - a_{22} M_S^x - (a_{22} l - a_{21}) Q_S^y \\ M_E^x = M_S^x + Q_S^y l \\ Q_E^y = Q_S^y \end{cases}$$

$a_{11}=l^3/3EJ$, $a_{12}=a_{21}=l^2/2EJ$, $a_{22}=l/EJ$. $b_1=a_{12}l-a_{11}$, $b_2=a_{22}l-a_{21}$. The whole transfer matrix of shaft is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -l & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & l & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_2 & -a_{22} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_1 & -a_{12} & 0 & 0 & 0 & l & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -b_2 & -a_{22} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_1 & a_{12} & 0 & 0 & 0 & -l & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -l & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_2 & -a_{22} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_1 & -a_{12} & 0 & 0 & 0 & l & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_2 & -a_{22} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_1 & a_{12} & 0 & 0 & 0 & -l & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The critical rotational speeds of spur gear rotor systems are listed in table 1.

Table 1		Critical rotational speeds of various length shaft				r/s
k (1.0×10^8 N/m)	l (m)	critical rotational speeds				
		1	2	3	4	
meshing pair	.127	594.0	861.4	2017.1	2447.8	
	.254	216.3	304.5	1426.4	2376.7	
	.508	76.7	107.6	1008.6	2368.2	

The lower critical rotational speeds are sensitive to length of shaft. The boldface numbers are new critical rotational speeds due to meshing effect. If transfer matrix of the shaft is false, uncoupling critical rotational speeds of the gear rotor system will not be coincidence with critical rotational speeds of single rotor. The analysis of gear shaft bearing system are available in [12].

4. Vibration of the orthogonal helical bevel gear-rotor system

The vehicle main retarder is a orthogonal helical bevel gear rotor system for power transmission shown in fig. 6. Dynamic of the spiral bevel geared rotor-bearing system has been analyzed in [13]. The coupling critical rotational speeds of the system were calculated based on the whole transfer matrix method here. The mechanical model is established in Fig. 7. Direction from intersection point to the big end of the driving wheel is Z-axes forward, direction from intersection point to the big end of the driven wheel is Y-axes forward. Direction of displacement or force along the coordinate axis forward is plus, angular displacement, moment and torque should be according to the right-hand rule.

A is the driving wheel, R_A is the base radius of it, δ_1 is the cone-apex angle of it. B is the driven wheel, R_B is the base radius of it, δ_2 is the cone-apex angle of it. $\Sigma = \delta_1 + \delta_2 = 90^\circ$, α_n is the normal pressure angle, β is the helical angle, θ is deflection angel of the rotor, φ is torsion angel of it. $R_m = (R+r)/2$ is the radius of the pitch circle at middle point. Helical angle of driving wheel is plus, it is clockwise from small end to big end looks from conical top, driven wheel is opposite.

$$s = [(x_A - x_B) - (R_A \varphi_A + R_B \varphi_B)] \cos \alpha \cos \beta + [(y_A \sin \delta_1 - R_A \theta_{Ax} \cos \delta_1) + (z_B \sin \delta_2 - R_B \theta_{Bx} \cos \delta_2)] \cos \alpha \sin \beta$$

$$+ [(y_A \cos \delta_1 - R_A \theta_{Ax} \sin \delta_1) + (z_B \cos \delta_2 - R_B \theta_{Bx} \sin \delta_2)] \sin \alpha, \quad F = k * s.$$

Fig. 6 the structure block of main retarder

Fig. 7 meshing force on helical bevel gear teeth

The relationships of forces, bending moments and torques on the wheels are

$$\left\{ \begin{array}{l} F_{Ax} = -F \cos \alpha_n \cos \beta \\ F_{Ay} = F(-\sin \alpha_n \cos \delta_1 + \cos \alpha_n \sin \beta \sin \delta_1) \\ F_{Az} = F(\sin \alpha_n \sin \delta_1 + \cos \alpha_n \sin \beta \cos \delta_1) \\ M_{Ax} = F_{Az} R_A^m \\ M_{Ay} = 0 \\ T_A = -F_{Ax} R_A^m \end{array} \right. \quad \left\{ \begin{array}{l} F_{Bx} = F \cos \alpha_n \cos \beta \\ F_{By} = F(\sin \alpha_n \sin \delta_2 - \cos \alpha_n \sin \beta \cos \delta_2) \\ F_{Bz} = F(-\sin \alpha_n \cos \delta_2 - \cos \alpha_n \sin \beta \sin \delta_2) \\ M_{Bx} = -F_{By} R_B^m \\ M_{Bz} = 0 \\ T_B = F_{Bx} R_B^m \end{array} \right.$$

the status vectors of the wheels can be listed,

$$\begin{aligned} [Z_A^L \quad Z_B^L]^T &= [Q_{Ax}^L \quad M_{Ay}^L \quad Q_{Ay}^L \quad M_{Ax}^L \quad T_A^L \quad \theta_{Ay}^L \quad x_A^L \quad \theta_{Ax}^L \quad y_A^L \quad \varphi_A^L \quad Q_{Bx}^L \quad M_{Bz}^L \\ &\quad Q_{Bz}^L \quad M_{Bx}^L \quad T_B^L \quad \theta_{Bz}^L \quad x_B^L \quad \theta_{Bx}^L \quad z_B^L \quad \varphi_B^L]^T, \\ [Z_A^R \quad Z_B^R]^T &= [Q_{Ax}^R \quad M_{Ay}^R \quad Q_{Ay}^R \quad M_{Ax}^R \quad T_A^R \quad \theta_{Ay}^R \quad x_A^R \quad \theta_{Ax}^R \quad y_A^R \quad \varphi_A^R \quad Q_{Bx}^R \quad M_{Bz}^R \\ &\quad Q_{Bz}^R \quad M_{Bx}^R \quad T_B^R \quad \theta_{Bz}^R \quad x_B^R \quad \theta_{Bx}^R \quad z_B^R \quad \varphi_B^R]^T. \end{aligned}$$

4.1. Shearing force dynamical equations of the system

Q^L is the shearing force acting on the wheel from left of it, Q^R is the shearing force acting on the wheel from right, $m\omega^2 x$ and $m\omega^2 y$ are components of centrifugal force of the wheel. F_x and F_y are components of meshing force. Shearing force transfer relationship is expressed in (4.1)–(4.4):

$$\begin{aligned} Q_{Ax}^R &= Q_{Ax}^L + m_A \omega^2 x_A^L + F_{Ax} = Q_{Ax}^L + m_A \omega^2 x_A^L - k x \cos \alpha \cos \beta = Q_{Ax}^L + m_A \omega^2 x_A^L + k [\cos \alpha \cos \beta (x_B^L - x_A^L + R_A \varphi_A^L + \\ &\quad R_B \varphi_B^L) + R_A \theta_{Ax}^L (\sin \delta_1 \sin \alpha + \cos \delta_1 \cos \alpha \sin \beta) + R_B \theta_{Bx}^L (\sin \delta_2 \sin \alpha + \cos \delta_2 \cos \alpha \sin \beta) - y_A^L (\cos \delta_1 \sin \alpha + \\ &\quad \sin \delta_1 \cos \alpha \sin \beta) - z_B^L (\cos \delta_2 \sin \alpha + \sin \delta_2 \cos \alpha \sin \beta)] \cos \alpha \cos \beta \\ &= Q_{Ax}^L + a_{11} \theta_{Ay}^L + a_{12} x_A^L + a_{13} \theta_{Ax}^L + a_{14} y_A^L + a_{15} \varphi_A^L + b_{11} \theta_{Bz}^L + b_{12} x_B^L + b_{13} \theta_{Bx}^L + b_{14} z_B^L + b_{15} \varphi_B^L, \end{aligned} \quad (4.1)$$

$$\begin{aligned} Q_{Bx}^R &= Q_{Bx}^L + m_B \omega^2 x_B^L + F_{Bx} = Q_{Bx}^L + m_B \omega^2 x_B^L + k x \cos \alpha \cos \beta = Q_{Bx}^L + m_B \omega^2 x_B^L + k [\cos \alpha \cos \beta (x_A^L - x_B^L - R_A \varphi_A^L - \\ &\quad R_B \varphi_B^L) - R_A \theta_{Ax}^L (\sin \delta_1 \sin \alpha + \cos \delta_1 \cos \alpha \sin \beta) - R_B \theta_{Bx}^L (\sin \delta_2 \sin \alpha + \cos \delta_2 \cos \alpha \sin \beta) + y_A^L (\cos \delta_1 \sin \alpha + \\ &\quad \sin \delta_1 \cos \alpha \sin \beta) + z_B^L (\cos \delta_2 \sin \alpha + \sin \delta_2 \cos \alpha \sin \beta)] \cos \alpha \cos \beta \\ &= Q_{Bx}^L + c_{11} \theta_{Ay}^L + c_{12} x_A^L + c_{13} \theta_{Ax}^L + c_{14} y_A^L + c_{15} \varphi_A^L + d_{11} \theta_{Bz}^L + d_{12} x_B^L + d_{13} \theta_{Bx}^L + d_{14} z_B^L + d_{15} \varphi_B^L, \end{aligned} \quad (4.2)$$

$$\begin{aligned} Q_{Ay}^R &= Q_{Ay}^L + m_A \omega^2 y_A^L + F_{Ay} = Q_{Ay}^L + m_A \omega^2 y_A^L + k x (\sin \delta_1 \cos \alpha \sin \beta - \cos \delta_1 \sin \alpha) = Q_{Ay}^L + m_A \omega^2 y_A^L + \\ &\quad k [(x_A^L - x_B^L - R_A \varphi_A^L - R_B \varphi_B^L) \cos \alpha \cos \beta + y_A^L (\cos \delta_1 \sin \alpha + \sin \delta_1 \cos \alpha \sin \beta) - R_A \theta_{Ax}^L (\sin \delta_1 \sin \alpha + \\ &\quad \cos \delta_1 \cos \alpha \sin \beta) - R_B \theta_{Bx}^L (\sin \delta_2 \sin \alpha + \cos \delta_2 \cos \alpha \sin \beta) + z_B^L (\cos \delta_2 \sin \alpha + \sin \delta_2 \cos \alpha \sin \beta)] \\ &\quad (\sin \delta_1 \cos \alpha \sin \beta - \cos \delta_1 \sin \alpha) \end{aligned}$$

$$=Q_{Ay}^L+a_{31}\theta_{Ay}^L+a_{32}x_A^L+a_{33}\theta_{Ax}^L+a_{34}y_A^L+a_{35}\varphi_A^L+b_{31}\theta_{Bz}^L+b_{32}x_B^L+b_{33}\theta_{Bx}^L+b_{34}z_B^L+b_{35}\varphi_B^L, \quad (4.3)$$

$$\begin{aligned} Q_{Bz}^R &= Q_{Bz}^L + m_B \mathcal{Q}^2 z_B^L + F_{Bz} = Q_{Bz}^L + m_B \mathcal{Q}^2 z_B^L - k s (\sin \delta_2 \cos \alpha \sin \beta + \cos \delta_2 \sin \alpha) = Q_{Bz}^L + m_B \mathcal{Q}^2 z_B^L + k [(x_B^L - x_A^L + \\ & R_A \varphi_A^L + R_B \varphi_B^L) \cos \alpha \cos \beta - y_A^L (\cos \delta_1 \sin \alpha + \sin \delta_1 \cos \alpha \sin \beta) + R_A \theta_{Ax}^L (\sin \delta_1 \sin \alpha + \cos \delta_1 \cos \alpha \sin \beta) + \\ & R_B \theta_{Bx}^L (\sin \delta_2 \sin \alpha + \cos \delta_2 \cos \alpha \sin \beta) - z_B^L (\cos \delta_2 \sin \alpha + \sin \delta_2 \cos \alpha \sin \beta)] (\sin \delta_2 \cos \alpha \sin \beta + \cos \delta_2 \sin \alpha) \\ &= Q_{Bz}^L + c_{31} \theta_{Ay}^L + c_{32} x_A^L + c_{33} \theta_{Ax}^L + c_{34} y_A^L + c_{35} \varphi_A^L + d_{31} \theta_{Bz}^L + d_{32} x_B^L + d_{33} \theta_{Bx}^L + d_{34} z_B^L + d_{35} \varphi_B^L. \end{aligned} \quad (4.4)$$

4.2. Bending moment dynamical equations of the system

M^L is bending moment on the wheel from left of it, M_g is inertia torque of the wheel, M is bending moment on the wheel coming from meshing force, the bending moment transfer relationship is expressed in (4.5)–(4.8):

$$\begin{aligned} M_{Ax}^R &= M_{Ax}^L + I_d^A \mathcal{Q}^2 \theta_{Ax}^L - i I_p^A \omega_A \mathcal{Q} \theta_{Ay}^L + M_{Ax} = M_{Ax}^L + I_d^A \mathcal{Q}^2 \theta_{Ax}^L - i I_p^A \omega_A \mathcal{Q} \theta_{Ay}^L + R_A k s (\sin \delta_1 \sin \alpha + \cos \delta_1 \cos \alpha \sin \beta) \\ &= M_{Ax}^L + I_d^A \mathcal{Q}^2 \theta_{Ax}^L - i I_p^A \omega_A \mathcal{Q} \theta_{Ay}^L + k [R_A (x_A^L - x_B^L - R_A \varphi_A^L - R_B \varphi_B^L) \cos \alpha \cos \beta - R_A^2 \theta_{Ax}^L (\sin \delta_1 \sin \alpha + \\ & \cos \delta_1 \cos \alpha \sin \beta) - R_A R_B \theta_{Bx}^L (\sin \delta_2 \sin \alpha + \cos \delta_2 \cos \alpha \sin \beta) + R_A y_A^L (\cos \delta_1 \sin \alpha + \sin \delta_1 \cos \alpha \sin \beta) + \\ & z_B^L (\cos \delta_2 \sin \alpha + \sin \delta_2 \cos \alpha \sin \beta)] (\sin \delta_1 \sin \alpha + \cos \delta_1 \cos \alpha \sin \beta) \\ &= M_{Ax}^L + a_{41} \theta_{Ay}^L + a_{42} x_A^L + a_{43} \theta_{Ax}^L + a_{44} y_A^L + a_{45} \varphi_A^L + b_{41} \theta_{Bz}^L + b_{42} x_B^L + b_{43} \theta_{Bx}^L + b_{44} z_B^L + b_{45} \varphi_B^L \end{aligned} \quad (4.5)$$

$$\begin{aligned} M_{Bx}^R &= M_{Bx}^L + I_d^B \mathcal{Q}^2 \theta_{Bx}^L - i I_p^B \omega_B \mathcal{Q} \theta_{Bz}^L + M_{Bx} = M_{Bx}^L + I_d^B \mathcal{Q}^2 \theta_{Bx}^L - i I_p^B \omega_B \mathcal{Q} \theta_{Bz}^L - R_B k s (\sin \delta_2 \sin \alpha - \cos \delta_2 \cos \alpha \sin \beta) \\ &= M_{Bx}^L + I_d^B \mathcal{Q}^2 \theta_{Bx}^L - i I_p^B \omega_B \mathcal{Q} \theta_{Bz}^L + k [R_A R_B \theta_{Ax}^L (\sin \delta_1 \sin \alpha + \cos \delta_1 \cos \alpha \sin \beta) + R_B^2 \theta_{Bx}^L (\sin \delta_2 \sin \alpha + \\ & \cos \delta_2 \cos \alpha \sin \beta) + R_B (x_B^L - x_A^L + R_A \varphi_A^L + R_B \varphi_B^L) \cos \alpha \cos \beta - y_A^L (\cos \delta_1 \sin \alpha + \sin \delta_1 \cos \alpha \sin \beta) - \\ & z_B^L (\cos \delta_2 \sin \alpha + \sin \delta_2 \cos \alpha \sin \beta)] (\sin \delta_2 \sin \alpha - \cos \delta_2 \cos \alpha \sin \beta) \\ &= M_{Bx}^L + c_{41} \theta_{Ay}^L + c_{42} x_A^L + c_{43} \theta_{Ax}^L + c_{44} y_A^L + c_{45} \varphi_A^L + d_{41} \theta_{Bz}^L + d_{42} x_B^L + d_{43} \theta_{Bx}^L + d_{44} z_B^L + d_{45} \varphi_B^L \end{aligned} \quad (4.6)$$

$$\begin{aligned} M_{Ay}^R &= M_{Ay}^L + I_d^A \mathcal{Q}^2 \theta_{Ax}^L + i I_p^A \omega_A \mathcal{Q} \theta_{Ay}^L \\ &= M_{Ay}^L + a_{21} \theta_{Ay}^L + a_{22} x_A^L + a_{23} \theta_{Ax}^L + a_{24} y_A^L + a_{25} \varphi_A^L + b_{21} \theta_{Bz}^L + b_{22} x_B^L + b_{23} \theta_{Bx}^L + b_{24} z_B^L + b_{25} \varphi_B^L \end{aligned} \quad (4.7)$$

$$\begin{aligned} M_{Bz}^R &= M_{Bz}^L + I_d^B \mathcal{Q}^2 \theta_{Bz}^L + i I_p^B \omega_B \mathcal{Q} \theta_{Bz}^L \\ &= M_{Bz}^L + c_{21} \theta_{Ay}^L + c_{22} x_A^L + c_{23} \theta_{Ax}^L + c_{24} y_A^L + c_{25} \varphi_A^L + d_{21} \theta_{Bz}^L + d_{22} x_B^L + d_{23} \theta_{Bx}^L + d_{24} z_B^L + d_{25} \varphi_B^L \end{aligned} \quad (4.8)$$

4.3. Torque dynamical equations of the system

T^L is torque on the wheel from left of it, $I_p \mathcal{Q}^2 \varphi$ is inertia torque of the wheel, the third is torque on the wheel from meshing force, the torque transfer relationships are expressed in (4.9)–(4.10):

$$\begin{aligned} T_A^R &= T_A^L + I_p^A \mathcal{Q}^2 \varphi_A^L + R_A k s \cos \alpha \cos \beta = T_A^L + I_p^A \mathcal{Q}^2 \varphi_A^L + k R_A [\cos \alpha \cos \beta (x_A^L - x_B^L - R_A \varphi_A^L - R_B \varphi_B^L) - \\ & R_A \theta_{Ax}^L (\sin \delta_1 \sin \alpha + \cos \delta_1 \cos \alpha \sin \beta) - R_B \theta_{Bx}^L (\sin \delta_2 \sin \alpha + \cos \delta_2 \cos \alpha \sin \beta) + \\ & y_A^L (\cos \delta_1 \sin \alpha + \sin \delta_1 \cos \alpha \sin \beta) + z_B^L (\cos \delta_2 \sin \alpha + \sin \delta_2 \cos \alpha \sin \beta)] \cos \alpha \cos \beta \\ &= T_A^L + a_{51} \theta_{Ay}^L + a_{52} x_A^L + a_{53} \theta_{Ax}^L + a_{54} y_A^L + a_{55} \varphi_A^L + b_{51} \theta_{Bz}^L + b_{52} x_B^L + b_{53} \theta_{Bx}^L + b_{54} z_B^L + b_{55} \varphi_B^L \end{aligned} \quad (4.9)$$

$$\begin{aligned} T_B^R &= T_B^L + I_p^B \mathcal{Q}^2 \varphi_B^L + R_B k s \cos \alpha \cos \beta = T_B^L + I_p^B \mathcal{Q}^2 \varphi_B^L + k R_B [\cos \alpha \cos \beta (x_A^L - x_B^L - R_A \varphi_A^L - R_B \varphi_B^L) - \\ & R_A \theta_{Ax}^L (\sin \delta_1 \sin \alpha + \cos \delta_1 \cos \alpha \sin \beta) - R_B \theta_{Bx}^L (\sin \delta_2 \sin \alpha + \cos \delta_2 \cos \alpha \sin \beta) + \\ & y_A^L (\cos \delta_1 \sin \alpha + \sin \delta_1 \cos \alpha \sin \beta) + z_B^L (\cos \delta_2 \sin \alpha + \sin \delta_2 \cos \alpha \sin \beta)] \cos \alpha \cos \beta \\ &= T_B^L + c_{51} \theta_{Ay}^L + c_{52} x_A^L + c_{53} \theta_{Ax}^L + c_{54} y_A^L + c_{55} \varphi_A^L + d_{51} \theta_{Bz}^L + d_{52} x_B^L + d_{53} \theta_{Bx}^L + d_{54} z_B^L + d_{55} \varphi_B^L \end{aligned} \quad (4.10)$$

4.4. Whole transfer matrix of the meshing orthogonal helical bevel gear meshing system

The deformation compatibility equation is established on displacement continuity condition between left and

right of the wheel, the whole transfer matrix in block form is T_C :

$$\begin{cases} \theta_{Ay}^R = \theta_{Ay}^L \\ \theta_{Ax}^R = \theta_{Ax}^L \\ x_A^R = x_A^L \\ y_A^R = y_A^L \\ \varphi_A^R = \varphi_A^L \end{cases} \quad \begin{cases} \theta_{Bz}^R = \theta_{Bz}^L \\ \theta_{Bx}^R = \theta_{Bx}^L \\ x_B^R = x_B^L \\ z_B^R = z_B^L \\ \varphi_B^R = \varphi_B^L \end{cases} \quad T_C = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix}$$

$$T_{11}=T_{22}=T_{33}=T_{44}=\mathbf{E}, T_{13}=T_{21}=T_{23}=T_{24}=T_{31}=T_{41}=T_{42}=T_{43}=\mathbf{0},$$

$$T_{12} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \quad T_{14} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix} \quad T_{32} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix} \quad T_{34} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} \\ d_{41} & d_{42} & d_{43} & d_{44} & d_{45} \\ d_{51} & d_{52} & d_{53} & d_{54} & d_{55} \end{bmatrix}.$$

elements of the partition matrices have been given in (4.1)–(4.10).

5 Computation and analysis

The computing program is part of MSDAP, Multi-rotor System Dynamic Analysis Program. The subroutine program is based on Gauss-Jordan elimination method.

program main

double precision

ar(20,20), br(20,20), cr(20,20), sh(10,10), dr(20,20),

ma, mb, k, kxx, kyy, Ipa, Ida, Ipb, Idb, omg, d1, d2,

la, lb, f, cxa, cxb, cya, det, dda, det1, bb, PI, yE,

al11, al12, al22, a14, a24, czb, sxa, sxb, wa, wb,

dd2, af, bt, zt1, zt2, qxa, qxb, qya, qzb, qza, qyb,

mxa, mxb, ra, rb, zJ, zJ2, jal11, jal12, jal22, ja14, ja24

read(*,*) yE, af, bt, zt1, zt2, ma, mb, d1, d2, la, lb,

ra, rb, Ipa, Ida, Ipb, Idb, k, kxx, kyy

PI=3.14159265

det=1.0

det1=0.0

bb=0.0

zJ=PI*d1**4*(1.0-(d2/d1)**4)/64.0

al11=la**3/(3.0*yE*zJ)

al12=la**2/(2.0*yE*zJ)

al22=la/(yE*zJ)

a14=al12*la-al11

a24=al22*la-al12

dd2=0.00D0

zJ2=PI*d1**4*(1-(dd2/d1)**4)/64.0

jal11=lb**3/(3.0*yE*zJ2)

jal12=lb**2/(2.0*yE*zJ2)

jal22=lb/(yE*zJ2)

ja14=jal12*lb-jal11

ja24=jal22*lb-jal12

cxa=k*cos(af)*cos(bt)

cxb=cxa

cya=k*(cos(zt1)*sin(af)+sin(zt1)*cos(af)*sin(bt))

czb=k*(cos(zt2)*sin(af)+sin(zt2)*cos(af)*sin(bt))

sxa=k*(sin(zt1)*sin(af)+cos(zt1)*cos(af)*sin(bt))

sxb=k*(sin(zt2)*sin(af)+cos(zt2)*cos(af)*sin(bt))

qxa=cos(af)*cos(bt)

qxb=qxa

qya=sin(zt1)*cos(af)*sin(bt)-cos(zt1)*sin(af)

qzb=cos(zt2)*sin(af)+sin(zt2)*cos(af)*sin(bt)

qza=cos(zt1)*cos(af)*sin(bt)+sin(zt1)*sin(af)

qyb=sin(zt2)*sin(af)-cos(zt2)*cos(af)*sin(bt)

mxa=sin(zt1)*sin(af)+cos(zt1)*cos(af)*sin(bt)

mxs=sin(zt2)*sin(af)-cos(zt2)*cos(af)*sin(bt)

do 600 n=1,266600

do 20 i=1,20

do 20 j=1,20

if(i.eq.j) then

br(i,j)=1.0

else

br(i,j)=0.0

end if

20 continue

br(1,7)=-kxx;


```

br(3,9)=-kyy
br(11,17)=-kxx
br(13,19)=-kyy
do 60 i=1,20
do 60 j=1,20
if(i.eq.j) then
ar(i,j)=1.0D0
dr(i,j)=1.0D0
else
ar(i,j)=0.0
dr(i,j)=0.0
end if
60 continue
ar(2,1)=-la
ar(4,3)=la
ar(6,1)=a24
ar(6,2)=-al22
ar(7,1)=a14
ar(7,2)=-al12
ar(7,6)=la
ar(8,3)=-a24
ar(8,4)=-al22
ar(9,3)=a14
ar(9,4)=al12
ar(9,8)=-la
dr(2,1)=ar(2,1)
dr(4,3)=ar(4,3)
dr(6,1)=ar(6,1)
dr(6,2)=ar(6,2)
dr(7,1)=ar(7,1)
dr(7,2)=ar(7,2)
dr(7,6)=ar(7,6)
dr(8,3)=ar(8,3)
dr(8,4)=ar(8,4)
dr(9,3)=ar(9,3)
dr(9,4)=ar(9,4)
dr(9,8)=ar(9,8)
ar(12,11)=-lb
ar(14,13)=lb
ar(16,11)=a24
ar(16,12)=-al22
ar(17,11)=a14
ar(17,12)=-al12
ar(17,16)=lb

```

```

ar(18,13)=-a24
ar(18,14)=-al22
ar(19,13)=a14
ar(19,14)=al12
ar(19,18)=-lb
cr=matmul(ar,br)
do 70 i=1,20
do 70 j=1,20
br(i,j)=cr(i,j)
70 continue
bb=bb+1.0D0
omg=-bb+1.0D0
wa=omg
wb=-wa
do 90 i=1,20
do 90 j=1,20
if(i.eq.j) then
ar(i,j)=1.0D0
else; ar(i,j)=0.0
end if
90 continue
ar(1,7)=ma*omg**2-cxa*qxa
ar(1,8)=ra*sxa*qxa
ar(1,9)=-cya*qxa
ar(1,10)=ra*cxa*qxa
ar(1,17)=cxb*qxa
ar(1,18)=rb*sxb*qxa
ar(1,19)=-czb*qxa
ar(1,20)=rb*cxb*qxa
ar(2,6)=-Ipa*(1.0+0.000)*wa*omg+Ida*omg**2
ar(3,7)=cxa*qya
ar(3,8)=-ra*sxa*qya
ar(3,9)=ma*omg**2+cya*qya
ar(3,10)=-ra*cxa*qya
ar(3,17)=-cxb*qya
ar(3,18)=-rb*sxb*qya
ar(3,19)=czb*qya
ar(3,20)=-rb*cxb*qya
ar(4,7)=ra*cxa*mx
ar(4,8)=-Ipa*wa*omg+Ida*omg**2-ra**2*sxa*mx
ar(4,9)=ra*cya*mx
ar(4,10)=-ra**2*cxa*mx
ar(4,17)=-ar(4,7)
ar(4,18)=-ra*rb*sxb*mx

```

```

ar(4,19)=ra*czb*mx
ar(4,20)=-ra*rb*cx*mx
ar(5,7)=ra*cx*qx
ar(5,8)=-ra**2*sx*qx
ar(5,9)=ra*cy*qx
ar(5,10)=Ipa*omg**2-ra**2*cx*qx
ar(5,17)=-ra*cx*qx
ar(5,18)=-ra*rb*sx*qx
ar(5,19)=ra*czb*qx
ar(5,20)=-ra*rb*cx*qx
ar(11,7)=cx*qx
ar(11,8)=-ar(1,8)
ar(11,9)=-ar(1,9)
ar(11,10)=-ar(1,10)
ar(11,17)=mb*omg**2-cx*qx
ar(11,18)=-ar(1,18)
ar(11,19)=-ar(1,19)
ar(11,20)=-ar(1,20)
ar(12,16)=-Ipb*wb*omg+Idb*(1.0+0.000)*omg**2
ar(13,7)=-cx*qx
ar(13,8)=ra*sx*qx
ar(13,9)=-cy*qx
ar(13,10)=ra*cx*qx
ar(13,17)=cx*qx
ar(13,18)=rb*sx*qx
ar(13,19)=mb*omg**2-cz*qx
ar(13,20)=rb*cx*qx
ar(14,7)=-rb*cx*mx
ar(14,8)=ra*rb*sx*mx
ar(14,9)=-rb*cy*mx
ar(14,10)=ra*rb*cx*mx
ar(14,17)=rb*cx*mx
ar(14,18)=(Idb*omg-Ipb*wb)*omg+rb**2*sx*mx
ar(14,19)=-rb*cz*mx
ar(14,20)=rb**2*cx*mx
ar(15,7)=rb*cx*qx
ar(15,8)=-ra*rb*sx*qx
ar(15,9)=rb*cy*qx
ar(15,10)=-ra*rb*cx*qx
ar(15,17)=-rb*cx*qx
ar(15,18)=-rb*rb*sx*qx
ar(15,19)=rb*cz*qx
ar(15,20)=Ipb*omg**2-rb**2*cx*qx
cr=matmul(ar,br)

do 30 i=1,20
do 30 j=1,20
br(i,j)=cr(i,j)
30 continue
do 69 i=1,20
do 69 j=1,20
if(i.eq.j) then
ar(i,j)=1.0D0
else
ar(i,j)=0.0
end if
69 continue
ar(2,1)=dr(2,1)
ar(4,3)=dr(4,3)
ar(6,1)=dr(6,1)
ar(6,2)=dr(6,2)
ar(7,1)=dr(7,1)
ar(7,2)=dr(7,2)
ar(7,6)=dr(7,6)
ar(8,3)=dr(8,3)
ar(8,4)=dr(8,4)
ar(9,3)=dr(9,3)
ar(9,4)=dr(9,4)
ar(9,8)=dr(9,8)
ar(2,1)=-lb
ar(4,3)=lb
ar(6,1)=ja24
ar(6,2)=-jal22
ar(7,1)=ja14
ar(7,2)=-jal12
ar(7,6)=lb
ar(8,3)=-ja24
ar(8,4)=-jal22
ar(9,3)=ja14
ar(9,4)=jal12
ar(9,8)=-lb
ar(12,11)=-lb
ar(14,13)=lb
ar(16,11)=ja24
ar(16,12)=-jal22
ar(17,11)=ja14
ar(17,12)=-jal12
ar(17,16)=lb
ar(18,13)=-ja24

```

```

ar(18,14)=-jal22
ar(19,13)=ja14
ar(19,14)=jal12
ar(19,18)=-lb
cr=matmul(ar,br)
do 36 i=1,20
do 36 j=1,20
br(i,j)=cr(i,j)
36  continue
do 38 i=1,20
do 38 j=1,20
if(i.eq.j) then
ar(i,j)=1.0D0
else
ar(i,j)=0.0D0
end if
38  continue
ar(1,7)=-kxx
ar(3,9)=-kyy
ar(11,17)=-kxx
ar(13,19)=-kyy
cr=matmul(ar,br)
do 10 i=1,5
do 10 j=6,10
sh(i,j-5)=cr(i,j)
10  continue
do 12 i=1,5
do 12 j=16,20
sh(i,j-10)=cr(i,j)
12  continue
do 16 i=11,15
do 16 j=6,10
sh(i-5,j-5)=cr(i,j)
16  continue
do 17 i=11,15
do 17 j=16,20
sh(i-5,j-10)=cr(i,j)
17  continue
det=1.0
f=1.0
n=10
do 220 k=1,9
call Y(sh,k,n6,f)
220  continue

```

```

do 226 i=1,10
det=det*sh(i,i)
226  continue
det=det*f
dda=det1*det
det1=det
if(dda.lt.0.0.or.det.eq.0.0) then
write(*,*) -(omg+1.0)/(2.0*PI),zJ
end if
600  continue
End
subroutine Y(sh,k,n6,f)
double precision  sh(n6,n6), f, db, db2, t
db=0.0
do 10 i=k,n6
do 10 j=k,n6
db2=abs(sh(i,j))
if(db2.gt.db) then
db=db2
js=j
is=i
end if
10  continue
if(js.ne.k) then
f=-f
do 20 i=1,n6
t=sh(i,k)
sh(i,k)=sh(i,js)
sh(i,js)=t
20  continue
end if
if(is.ne.k) then
f=-f
do 30 j=1,n6
t=sh(k,j)
sh(k,j)=sh(is,j)
sh(is,j)=t
30  continue
end if
do 38 i=k+1,n6
do 38 j=k+1,n6
sh(i,j)=sh(i,j)-sh(i,k)*sh(k,j)/sh(k,k)
38  continue
end

```

In order to highlight the meshing effect, parameters of driving and driven gears of the model are in the same here. The moment of inertia I_{dA} or I_{dB} is $0.0018\text{Kg}\cdot\text{m}^2$, the mass m_A or m_B is 1.84kg , the base radius R_A or R_B is 0.22m , the pressure angle α is 20° , the helical angle β is 35° , the length of the shaft is 0.127m , the diameter of the shaft is 0.037m , bearing stiffness is $1.0\times 10^9\text{N/m}$, the cone apex angle of A is 12° , it of B is 78° . Young's modulus of the material is $2.0\times 10^{11}\text{N/m}^2$, k_s is the meshing stiffness. The critical rotational speeds of the system are listed in table 2.

Table 2		Critical rotational speeds of helical bevel gear rotor system				$\delta_1=12^\circ, \delta_2=78^\circ$	r/s
	$K_s /$ ($\text{N} \cdot \text{m}^{-1}$)	critical rotational speeds					
		1	2	3	4	5	
single rotor			849.9	1992.6			
meshing pair	1.0e7	828.4	849.9	1992.6	2050.2	2327.3	
	1.0e8	830.6	849.9	1992.6	2009.2	7160.7	
	1.0e9	831.0	849.9	1992.6	2008.1	22290.4	

The boldface numbers are new critical rotational speeds derived from meshing effect, others are critical rotational speeds of single rotor that can be verified by calculation model. The vibration characters of geared rotors system have been changed by meshing effect, coupling critical rotational speeds must be considered. The coupling critical rotational speeds are not points but bands for the meshing stiffness be changed continuously [14,15,16].

6 conclusion

The coupling critical rotational speeds will be multiplied for varied gear types, arrangement modes and linking styles. Critical rotational speeds of the system were distributed as frequency bands for the time-varying mesh stiffness. So the safe working rotational speed range of a geared rotor system with certain structure is limited and regular. The results of accurate quantitative calculation are almost coincident with experimental data. The lower critical rotational speeds are sensitive to length of the shaft. Vibration of the gear rotor system can be undercontrol.

Declarations

Availability of data and materials. Not applicable.

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Figures

Figure 1

balance of shearing forces on the wheel

Figure 2

balance of torques on the wheel

Figure 3

inertia moments of the wheel

Figure 4

balance of bending moments

Figure 5

coordinate system of shaft

Figure 6

the structure block of main retarder

Figure 7

meshing force on helical bevel gear teeth