The Measurement of Subjective Household Poverty: Concepts and Application

Aleksandra Łuczak
University of Life Sciences in Poznań
Sławomir Kalinowski (skalinowski@irwirpan.waw.pl)
Institute of Rural and Agricultural Development

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Abstract

Poverty is a multi-dimensional phenomenon that cannot be directly measured correctly by a single indicator. Research on poverty uses objective and subjective indicators. Objective measures do not represent the true nature of poverty, because they are mainly based on income and expenses. Hence, our research focuses on subjective poverty, which shows the diversity of respondents' perceptions of poverty. The aim is to present and compare methodological approaches to the construction of a synthetic measure of subjective household poverty. The research took account of the aggregation of variables describing the past, present, and future. The approaches use the modified classical and fuzzy Hellwig's and TOPSIS methods, and were used to assess the level of subjective household poverty in Poland. The study was based on primary data from 2021. The use of fuzzy approaches to the assessment of subjective poverty allows for a more precise assessment of its level than in classical approaches.

Introduction

Poverty is a multidimensional phenomenon, the definition and measurement of which raises a lot of controversy and discussion. In research on poverty, the lines of poverty separating relatively well off (non-poor) people from poor people are most often used (Golinowska 1997, Broda-Wysocki 2012). They are criticized in many studies because they cause a dichotomous division of society. Generally, there are two approaches to determining the poverty line – economic and multidimensional (Fig. 1). The objective approach is determined both on the basis of normative and parametric lines. The first are absolute, while the second are relative. Determining the normative lines consists in determining the value of income necessary to satisfy a certain group of needs (Booth 1889, Rowntree 1901, Orshansky 1969). They are based on various types of standards (expert or political) regarding the fulfillment of needs (Kalinowski, 2015).

Relatively the least important in the measurements is the poverty threshold based on official lines. Its minor importance results, on the one hand, from a certain underestimation, and on the other from overestimation. This is due to several factors (Kalinowski, 2015):

1. lowering the statistics contributes to the apparent reduction of the poverty threshold without its actual elimination, which may lead to a lack of valorization of the number entitled to receive benefits,
2. for fear of being stigmatized some people consciously do not want receive social welfare benefits, thus they are not included in the assistance systems, and as a result they are not treated as poor, even though they cannot meet their needs
3. some people receive benefits, although they are not formally entitled to them (e.g. working illegally),
4. lack of international comparability.

The subjective measures of poverty are also important (cf. Hagenaars, van Praag 1985, Kapteyn, Van Praag, van Herwaarden 1978, Goedhart et al. 1977). These are considered the most democratic methods of defining poverty, which results from the individual setting of the limit of deprivation.

Measurement of poverty is often limited to objective, one-dimensional indicators (e.g. income or expenses). However, when assessing poverty, its subjective dimension is also important, as it shows the perceptions of the poor. The growing contrast between the rich and the poor only increases the level of feeling poverty. There are many levels of poverty, from no poverty to extreme poverty. It should be noted that poverty is not always immediately noticeable, and those that are visible are not always felt by the respondents. Hence the problem of subjective poverty measurement is important, as it identifies various degrees of poverty perception among respondents and often depends on the point of reference (on the people to whom the respondents compare themselves, e.g. family, friends, neighbors). For these reasons, research on the measurement of subjective poverty was undertaken. The study of subjective poverty allows for the identification of the diversity of the respondents' perceptions of poverty.

Existing definitions of poverty are characterized by a high degree of subjectivity and individual interpretation by individual researchers. This is why some of them have a broad scope, others are narrower. Due to this, in many cases it is difficult to make comparisons, because adopting a different understanding of the definition often means that the researcher had a research sample that was different in terms of quality. Nevertheless, in many cases one can note that despite the differences in the approach to particular definitions, the core is similar and many elements remain common (Kalinowski, 2015). Thus we defined subjective poverty as a conscious sense of the lack of sufficient resources to meet one's needs in relation to the "socioeconomic status (income and current financial situation, level of education and profession, place of residence, lifestyle and leisure activities) and one's own aspirations to achieve and maintain the desired standard of living" (Łuczak, Kalinowski, 2022).

The aim of the research is to present and compare methodological approaches to the construction of a synthetic measure of subjective household poverty. The aggregation of variables relied on modified classical and fuzzy methods based on the ideas of Hellwig (1968) as well as of Hwang and Yoon (1981). We used Hellwig's one-pattern method to construct a synthetic measurement and a two-pattern technique for order of preference by similarity to the ideal solution (TOPSIS). The procedure used the generalized distance measure (classical methods) and vertex method (fuzzy methods). Four approaches were applied to assess the level of subjective poverty of households in Poland. The research was conducted on data collected using the computer-assisted web-interview (CAWI) method in April 2021.

Apart from the introduction, the paper is composed as follows: part 2 presents the procedures for four approaches to construction for a subjective household poverty index; part 3 shows their applications to a real dataset, part 4 includes the summary with the main conclusions and part 5 presents recommendations.

Procedures for Constructing an Index of Subjective Household Poverty

We propose a procedure for the subjective assessment of household poverty as a multidimensional self-assessment by respondents using multiple-criteria methods. The process of constructing a subjective household poverty index (synthetic measure) is a multi-stage one and uses four approaches:

- two-pattern methods:
    - fuzzy TOPSIS (approach I),
    - fuzzy Hellwig's method (approach II),
- one-pattern methods:
    - TOPSIS with GDM2 (approach III),
    - Hellwig's method with GDM2 (approach IV).

Our research uses the idea of the construction of a synthetic measure introduced by Hellwig (1968, 1972) and developed by Hwang and Yoon (1982). The proposed approaches are also based on the fuzzy TOPSIS method, which was developed by Chen (2000), and Hellwig's method first proposed as a fuzzy approach by Łuczak and Wysocki (2007), and modified by Łuczak and Kalinowski (2022). We also used the vertex method Chen (2000) and generalized distance measure (Jajuga et al. 2003, Walesiak 1999, 2006, 2016) to calculate the distances from the pattern and anti-pattern. The stages of constructing a subjective household poverty index is presented in Table 1. Each step is described in detail below.

The first stage is common to all approaches. At this stage, variables (indicators) describing households' subjective poverty are selected. The variables represent households' self-assessment perceptions. We study feelings from the point of view of the three criteria: the past, present and predictions for the future. The criteria are described by variables, measured on an ordinal scale, which are related to the household's self-assessment of its standard of living, material conditions, financial and economic situation and risk of poverty. It is worth noting that the activities on the ordinal scale are limited and require special procedures.

Table 1
Stages of construction of a synthetic measure

Approaches				Stages of the procedure
I	II	III	IV	
Selection of variables describing subjective poverty and their verification in terms of content or statistics				1. Selection of variables
Determination of weights for variables				2. Determination of the weight system for variables
Replace ordinal values with fuzzy numbers		Does not require transformation of variables		3. Transformation of variable values
Determination of the direction of preference for variables in relation to the general criterion under consideration				4. Division of variables according to the direction of preference
Normalization for fuzzy numbers is required		Simplified normalization is required		5. Normalization of the variable values
Calculation of fuzzy distances from the pattern and the antipattern	Calculation of fuzzy distance from the pattern	Calculation of the GDM2 from the pattern and the antipattern	Calculation of the GDM2 from the pattern	6. Calculation of the distance from the pattern (and the antipattern)
TOPSIS	Hellwig's method	TOPSIS	Hellwig's method	7. Aggregation of variables – methods
Arbitrary manner by adopting numerical ranges of the value of the synthetic measure				8. Identification of poverty levels for household groups

Source: Own study.

Previous experience of poverty can also make the respondent feel better than they are, and vice-versa. Moreover, households' perceptions of their own poverty may have an impact on their future self-assessment, even after reducing objective poverty (Ravallion and Lokshin 2002). The assessment of the household's condition is also influenced by the actual poverty dynamics (Alem et al. 2014). The variables can be summarized in matrix:

$$\mathbf{X} = \begin{bmatrix} x_{11} & \dots & x_{1K} \\ \dots & \dots & \dots \\ x_{N1} & \dots & x_{NK} \end{bmatrix}$$

1

where x_{ij} – the value of the *j*-th variable in the *i*-th household, *i* = 1,..., *N*; *N* – the number of households; *j* = 1,..., *K*; *K* – the number of variables.

Additionally, we assume that an increase in each variable causes an increase in the level of the complex phenomenon (the level of subjective poverty), and a decrease in value indicates a decrease in the level of the complex phenomenon analyzed. We recommend this assumption. If a variable is of an opposite nature, i.e. its increase causes a decrease in the level of the phenomenon under study, it can be transformed by reversing the ordinal scale.

In the step 2 we propose adopting equal weights in this procedure, in all approaches (see Aaberge and Brandolini 2015). Generally weights can be created in a statistical, content, or integrated manner (see e.g. Ma et al. 1999, Olson 2004, Wang and Luo 2010, Łuczak and Wysocki, 2014, 2015).

Then, in stage 3 for approaches I and II, the categories of ordinal variables are transformed into triangular fuzzy numbers:

$$\widetilde{x}_{ij} = \left(a_{ij}, b_{ij}, c_{ij}\right)$$

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where $\widetilde{x}_{ij}$ – the fuzzy value of the *j*-th variable in the *i*-th household, *i* = 1,..., *N*; *N* – the number of households; *j* = 1,..., *K*; *K* – the number of variables. The formulas for replace the parameters of the triangular fuzzy numbers are presented in Table 1.

Table 2
Formulas for the parameters of triangular fuzzy numbers

Categories	Parameters of a triangular fuzzy number		
	a_{ij}	b_{ij}	c_{ij}
1	0	0	$\frac{1}{2(m-1)}$
2	$\frac{1}{2(m-1)}$	$\frac{1}{(m-1)}$	$\frac{3}{2(m-1)}$
...	...	...	...
m−1	$\frac{2m-5}{2(m-1)}$	$\frac{m-2}{(m-1)}$	$\frac{2m-3}{2(m-1)}$
m	$\frac{2m-3}{2(m-1)}$	1	1

The parameters of the triangular fuzzy numbers can be scaled by a selected constant value, freely determined by the researcher. The triangular fuzzy numbers can be summarized in fuzzy matrix $\widetilde{\mathbf{X}} = \left[\widetilde{x}_{ij}\right]$. Approaches III and IV do not require the transformation of variables.

The direction of preference for variables in relation to the general criterion under consideration should be determined in stage 4. Variables can be stimulants, destimulants, or nominants. A stimulant is a benefit variable that increases the level of subjective poverty, while a destimulant is a cost variable that decreases that level. A nominant is a special variable type that is stimulant in some variable range and a destimulant in the rest of the range. Variables defined as destimulants are converted into stimulants in the normalization stage 5.

Normalization is required for all approaches (stage 5). The values of the variables for a stimulant should be normalized in approaches I and II as follows:

$$\widetilde{z}_{ij} = \left(a_{ij}^{(z)}, b_{ij}^{(z)}, c_{ij}^{(z)}\right) = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right),$$

3

where $\widetilde{z}_{ij}$ – the fuzzy value of the *j*-th normalized variable in the *i*-th household, *i*=1,..., *N*; $j \in P_s$, P_s – a set of stimulant indexes, $c_j^+ = \max_i(c_{ij})$, $c_j^+ \neq 0$,

for a destimulant:

$$\widetilde{z}_{ij} = \left(a_{ij}^{(z)}, b_{ij}^{(z)}, c_{ij}^{(z)}\right) = \begin{cases} \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right) for a_{ij}, b_{ij}, c_{ij} \neq 0 \\ (0,0,0)\, for a_{ij}, b_{ij} = 0, c_{ij} \neq 0 \end{cases}$$

4

where $\widetilde{z}_{ij}$ – the fuzzy value of the *j*-th normalized variable in the *i*-th household, *i*=1,..., *N*, $j \in P_D$, P_D – a set of destimulant indexes, $a_j^- = \min_i(a_{ij})$.

For approach III and IV, a simplified normalization is sufficient, which can be performed using a differential transformation for the destimulants:

$$\mathrm{z}_{ij} = a - b \cdot x_{ik}$$

5

where z_{ij} – the value of the *j*-th normalized variable in the *i*-th household, *i*=1,..., *N*, $j \in P_D$, P_D – a set of stimulant indexes, *a*, *b* – constants taken arbitrarily, the most common $a = 0$ or $a = \max_i(x_{ik})$ and $b = 1$.

For approaches I and III, the fuzzy positive ideal solution (PIS) and fuzzy negative ideal solution (NIS) are calculated in stage 6:

$$\mathrm{FPIS}: \widetilde{A}^+ = \left(\max_i\left(\widetilde{z}_{i1}\right), \max_i\left(\widetilde{z}_{i2}\right), \ldots, \max_i\left(\widetilde{z}_{iK}\right)\right) = \left(\widetilde{z}_1^+, \widetilde{z}_2^+, \ldots, \widetilde{z}_K^+\right)$$

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$$\mathrm{NIS}: \widetilde{A}^- = \left(\min_i\left(\widetilde{z}_{i1}\right), \min_i\left(\widetilde{z}_{i2}\right), \ldots, \min_i\left(\widetilde{z}_{iK}\right)\right) = \left(\widetilde{z}_1^-, \widetilde{z}_2^-, \ldots, \widetilde{z}_K^-\right)$$

(7)

$$\text{where} \widetilde{z}_j^+ = \left(a_j^+, b_j^+, c_j^+\right), \widetilde{z}_j^- = \left(a_j^-, b_j^-, c_j^-\right), j = 1, \ldots, K.$$

For approaches III and IV, the positive ideal solution (PIS) and negative ideal solution (NIS) are calculated (Hwang and Yoon, 1981):

$$\mathrm{PIS}: \widetilde{A}^+ = \left(\max_i\left(z_{i1}\right), \max_i\left(\mathrm{z}_{i2}\right), \ldots, \max_i\left(\mathrm{z}_{iK}\right)\right) = \left(\mathrm{z}_1^+, \widetilde{z}_2^+, \ldots, \widetilde{z}_K^+\right)$$

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$$\mathrm{NIS}: \widetilde{A}^- = \left(\min_i\left(z_{i1}\right), \min_i\left(\mathrm{z}_{i2}\right), \ldots, \min_i\left(\mathrm{z}_{iK}\right)\right) = \left(z_1^-, z_2^-, \ldots, z_K^-\right)$$

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The fuzzy positive and negative ideal solutions are the basis for calculating the distance between them and the fuzzy values of normalized variables for the households assessed, as follows (Chen 2000):

$$d_i^+ = \sum_{j=1}^K \sqrt{\frac{1}{3}\left[\left(a_{ij} - a_j^+\right)^2 + \left(b_{ij} - b_j^+\right)^2 + \left(c_{ij} - c_j^+\right)^2\right]}$$

10

$$d_i^- = \sum_{j=1}^K \sqrt{\frac{1}{3}\left[\left(a_{ij} - a_j^-\right)^2 + \left(b_{ij} - b_j^-\right)^2 + \left(c_{ij} - c_j^-\right)^2\right]}$$

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Approaches III and IV use the generalized distance measure (GDM2) (Walesiak 1993, 1999, 2006, 2011, 2014, 2016). The operation of counting events is allowed on the ordinal scale, i.e. the number of majority, minority or equality relationships can be determined. GDM2 is based on the idea of the generalized correlation coefficient (see Kendall and Buckland, 1986, Kendall, 1955) and then it is defined by the formulas:

$$d_i^{(+)} = \frac{1}{2} - \frac{\sum_{j=1}^{K} u_{ij}^{+} v_{ij}^{+} + \sum_{k=1}^{K} \sum_{\substack{l=1 \\ l \neq i,w}}^{N} u_{ilj} v_{lj}^{+}}{2\left[\left(\sum_{j=1}^{K}\sum_{l=1}^{N} u_{ilj}^{2} \sum_{j=1}^{K}\sum_{\substack{l=1 \\ l \neq i,j}}^{N} \left(v_{lj}^{+}\right)^{2}\right)\right]^{\frac{1}{2}}}$$

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,

$$u_{ilj}\left(u_{ij}^{+}; v_{rj}^{+}\right) = \begin{cases} 1 \text{for} z_{ij} > z_{lj} \left(z_{ij} > z_j^{+}; z_j^{+} > z_{rj}\right) \\ 0 \text{for} z_{ij} = z_{lj} \left(z_{ij} = z_j^{+}; z_j^{+} = z_{rj}\right) \quad \text{dla} r = i, l \\ -1 \text{for} z_{ij} < z_{lj} \left(z_{ij} < z_j^{+}; z_j^{+} < z_{rj}\right) \end{cases}$$

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$$d_i^{(-)} = \frac{1}{2} - \frac{\sum_{j=1}^{K} u_{ij}^{-} v_{ij}^{-} + \sum_{k=1}^{K} \sum_{\substack{l=1 \\ l \neq i,w}}^{N} u_{ilj} v_{lj}^{-}}{2\left[\left(\sum_{j=1}^{K}\sum_{l=1}^{N} u_{ilj}^{2} \sum_{j=1}^{K}\sum_{\substack{l=1 \\ l \neq i,j}}^{N} \left(v_{lj}^{-}\right)^{2}\right)\right]^{\frac{1}{2}}}$$

14

,

$$u_{ilj}\left(u_{ij}^{-}; v_{rj}^{-}\right) = \begin{cases} 1 \text{for} z_{ij} > z_{lj} \left(z_{ij} > z_j^{-}; z_j^{-} > z_{rj}\right) \\ 0 \text{for} z_{ij} = z_{lj} \left(z_{ij} = z_j^{-}; z_j^{-} = z_{rj}\right) \quad \text{dla} r = i, l \\ -1 \text{for} z_{ij} < z_{lj} \left(z_{ij} < z_j^{-}; z_j^{-} < z_{rj}\right) \end{cases}$$

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where $d_i^{(+)}$ ($d_i^{(-)}$) is the generalized distance measure GDM2 *i*-th household for pattern (anti-pattern). This measure takes values from 0 to 1.

Then, in stage 7, the subjective household poverty index is calculated using formulas developed by

1. Hwang and Yoon (1981) for approaches I and III:

$$S_i^{(1)} = \frac{d_i^{-}}{d_i^{+} + d_i^{-}}$$

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1. Hellwig (1968) for approaches II and IV:

$$S_i^{(2)} = 1 - \frac{d_i^{+}}{d_0}$$

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$$where d_0 = \bar{d}_0 + 2 \cdot s_0, \bar{d}_0 = \frac{\sum_{i=1}^{N} d_i^{+}}{N}, s_0 = \sqrt{\frac{\sum_{i=1}^{N} \left(d_i^{+} - \bar{d}_0\right)^2}{N}}.$$

The higher the value S_i, the higher is the level of subjective household poverty. Values of the subjective household poverty index $S_i^{(1)}$ (approaches I and III) are normalized to the range from 0 to 1, where S_i equals 0 for an anti-pattern household and 1 for a pattern household. Most often values of the subjective household poverty index $S_i^{(2)}$ (approaches II and IV) take values from 0 to 1, but they can go beyond this range. The problem of not normalizing the value of a feature can be solved by normalizing:

$$S_i^{(\mathrm{N2})} = \frac{S_i - \min\limits_i S_i}{\max\limits_i S_i - \min\limits_i S_i}$$

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However, such a normalization of the value of the synthetic measure (index) makes it possible to determine the position of the household examined only in relation to that particular household. In the case of the TOPSIS method, the positioning is global, as the values of the features are always between 0 and 1.

Finally, types of subjective household poverty are identified (stage 8). The values of the index were averaged using the median within the criteria examined – class of locality of a household: for the country, village and city, as well as divided into: small town up to 20,000 residents, urban area with 20,000–99,000 residents, urban area with 100,000–499,000 residents, urban area with 500,000 or more residents. There was also a division according to education and gender.

The types of the subjective poverty level can be distinguished arbitrarily, e.g. by adopting numerical ranges of the value of the measureS_i(Table 2). Based on the levels of the synthetic measure, theoretical types of poverty were also proposed – poverty profiles (Table 2). The state of poverty or deprivation is not dichotomous, and households cannot be classified as poor or non-poor. There are many shades of being poor, ranging from no poverty to extreme poverty. Hence households may be characterized by a different degree of poverty (cf. Betti et al., 2008; Montrone et al. 2010).

Empirical research

The empirical materials used in the research are part of extensive five-wave primary research entitled *My Situation During the Coronavirus Pandemic* (2020–2022). The data used in the study were obtained based on the computer-assisted web-interview (CAWI) method was conducted in Poland in April 2021. A total of 1,499 respondents took part in the research.

In the first stage of assessing the level of subjective poverty, 12 variables were selected representing the feelings of households with regard to the present, past and future outlook:

1. life satisfaction,
2. degree of current fulfilment of one's own household needs through one's own income,
3. assessment of one's own household income compared to other households,
4. assessment of the change in the satisfaction of food needs over a year compared to previous years,
5. assessment of the situation of one's own household, whether it can make ends meet with the current income,
6. assessment that one's own household situation may worsen in the near future,
7. degree of possible loss of income,
8. degree of possible loss of financial stability,
9. degree of possibility of losing a job,
10. assessment of the possibility of a change in the financial situation of one's own household in the next 12 months,
11. feelings of past situations: degree of satisfaction of the needs of one's own household by income a year ago,
12. feelings of being poor in the past.

The values of the features measured on the ordinal scale are summarized in the form of a matrix (Eq. 1). In stage 2, it was assumed that the weights of variables in the process of constructing the synthetic measure are the same. Then, in stage 3, for approaches I and II variables measured on ordinal scale were transformed into triangular fuzzy numbers (stage 3, Eq. 2, Table 2). In approaches III and IV the variables did not require transformation. All variables are of a stimulant character or they had been transformed into this form at the beginning of the research (stage 4) by the reverse scale, then the values of the variables do not require normalization (stage 5). If the variables are not uniform, Eqs. 3–5 should be used. Then the pattern (approaches I and II – Eq. 6, approaches III and IV – Eq. 8) and the anti-pattern (approach I– Eq. 7, approach III – Eq. 9) are determined. In stage 6, the distances from the pattern (approaches I and II, Eq. 10) and anti-pattern (approach I, Eq. 11) were calculated using the vertex method. The generalized distance measure GDM2 was used to calculate the distances from the pattern (approaches III and IV, Eqs. 12–13) and anti-pattern (approach III, Eqs. 14–15). These were the basis for calculating the values of synthetic measures of the level of subjective household poverty (approaches I and II, Eq. 16, approaches III and IV, Eq. 17).

Table 3 presents the values of synthetic measures by approaches and their selected basic descriptive statistics, and Figs. 3–5 show the mean values of measures broken down by place of residence, sex, level of education. It should be noted that for the approaches based on Hellwig's method (approaches II and IV) negative values of the synthetic measure appeared (Table 3), which may cause some interpretation problems. One of the solutions to this problem is to standardize the value of the synthetic measure using Eq. 18, however, there will be some loss of information.

Table 3
Values of synthetic measure according to proposed approaches

Specification	Approach 1			Approach 2			Approach 3			Approach 4		
	fuzzy TOPSIS			fuzzy Hellwig's method			TOPSIS with GDM2			Hellwig's method with GDM2		
	min	mean	max	min	mean	max	min	mean	max	min	mean	max
Poland	0.000	0.436	1.000	-0.034	0.414	1.000	0.000	0.513	1.000	-0.025	0.523	1.000
females	0.000	0.433	1.000	-0.034	0.411	1.000	0.000	0.510	1.000	-0.040	0.522	1.000
males	0.020	0.439	0.980	-0.015	0.416	0.980	0.010	0.516	0.997	-0.025	0.524	0.997
village	0.000	0.498	1.000	-0.034	0.478	1.000	0.000	0.585	1.000	-0.040	0.595	1.000
city	0.020	0.391	0.980	-0.015	0.367	0.980	0.014	0.461	0.998	-0.019	0.471	0.998
small town up to 20,000 residents	0.039	0.513	0.980	0.005	0.493	0.977	0.014	0.600	0.997	-0.019	0.610	0.997
urban area with 20,000–99,000 residents	0.020	0.433	0.978	-0.015	0.410	0.977	0.017	0.524	0.998	-0.009	0.537	0.998
urban area with 100,000–499,000 residents	0.020	0.316	0.863	-0.015	0.289	0.858	0.017	0.369	0.932	-0.009	0.379	0.928
urban area with 500,000 and more residents	0.062	0.335	0.894	0.028	0.309	0.890	0.042	0.383	0.937	0.007	0.389	0.933
tertiary	0.000	0.362	0.939	-0.034	0.337	0.937	0.000	0.422	0.987	-0.040	0.432	0.987
post-secondary / upper secondary	0.039	0.445	1.000	0.005	0.422	1.000	0.010	0.533	1.000	-0.025	0.544	1.000
basic vocational / lower secondary / primary	0.084	0.473	0.980	0.051	0.452	0.980	0.069	0.550	0.997	0.055	0.560	0.997

It should be noted that the values of the synthetic measure in the approaches based on fuzzy methods (approaches I and II) were always lower than the values of the synthetic measure in the modified classical approaches III and IV (Figs. 3–5). This is due to the fact that the GDM2 measure was used, which is based on comparing ordinal numbers and counting the relations of majorities, minority and equality for pair comparisons. In fuzzy approaches, the values of the synthetic measure calculated for Hellwig's method (approach II) were always lower than for the TOPSIS method (approach I) (Figs. 3–5). The main difference between approaches I and II (and also III and IV) concerns the pattern. The TOPSIS method is a two-pattern method and Hellwig's method has one pattern. The aggregation formula is also different in both methods.

Unlike the modified classical approaches using GDM2, it is worth noting that the values of the synthetic measure were always higher for the one-pattern method (Hellwig's method, approach IV) than the values obtained in the two-pattern TOPSIS method (approach III). However, the ordering of the objects according to both methods was almost identical. The values obtained by the fuzzy Hellwig's method were even smaller than for the fuzzy TOPSIS method, contrary to the classic methods with GDM2.

Table 4
Kendall's tau coefficient and Spearman's rank-correlation coefficient for households by subjective poverty level

Compared approaches	I-II	I-III	I-IV	II-III	II-IV	III-IV
Kendall's tau coefficient	0.9953	0.9302	0.9115	0.9301	0.9103	0.9622
Spearman's rank correlation coefficient	0.9999	0.9932	0.9893	0.9932	0.9978	0.9978

Correlation coefficients are significant with $p < 0.05$.

Then the consistency of the results of synthetic measures representing levels of the subjective poverty of household in four approaches was examined (Table 4). The τ-Kendall rank-correlation coefficient was adopted as a measure of the consistency of linear ordering of households. These are high (0.9103–0.9953) for the ranking of households according to the value of synthetic measures, and similarly for the Spearman's rank-correlation coefficient (0.9932–0.9999). The four approaches provided similar linear orderings of households according to the level of subjective poverty, as evidenced by the high values of the τ-Kendall and Spearman's rank-correlation coefficient calculated for them. However, in additionally

analyzing Fig. 6, it can be noticed that the most similar are the approaches based on the fuzzy Hellwig's method and TOPSIS (approach I and II) and then the slightly less similar modified classical approaches (III and IV).

Based on the research it can be concluded that in April 2021, i.e. a year after the start of the COVID-19 pandemic, the level of subjective poverty in Poland was medium-high (Fig. 3). No significant differences between women and men were identified in the perception of the level of subjective poverty. Meanwhile, significant differences emerged in the perception of the level of subjective poverty between village and city (Fig. 3). The feeling of poverty in villages was higher than in the cities. However, it should be emphasized that cities have different character, so it is worth looking at the feelings of their residents depending town or city populations (Fig. 4).

Small towns up to 20,000 residents were in a similar situation to villages, and their level of subjective poverty can be described as medium-high. In urban areas, with 20,000–99,000 residents, the situation was better, and there was an even lower perception of poverty in cities of 100,000 and more residents. It can be concluded that the perception of poverty in these urban areas was low. It has also been identified that the level of poverty in households increased as the level of education fell (Fig. 5).

SUMMARY

The subjective poverty index is an attempt to explain poverty from the perspective of respondents who feel poverty to different degrees. It is worth noting that most methods of measuring the self-assessment of poverty only divide the respondents into poor or non-poor. The advantage of our proposals is that they determine different degrees of poverty. The research shows new possibilities for measurement of multidimensional subjective poverty. We used two methods (Hellwig's method and TOPSIS) with two distance measures (the vertex method and GDM2).

TOPSIS has an advantage over the Hellwig's method because the values of the synthetic measure are always normalized in the range from 0 to 1. In Hellwig's method, negative values can appear for individual objects, although it usually assumes values from 0 to 1. Moreover, TOPSIS takes account of the distances from pattern and anti-pattern, whereas Hellwig's method only considers the pattern. The common feature of both methods is the fact that the ordinal scale is finally strengthened into a metric scale. However, when using the GDM2 distance, it is based on the predominances (expressed by -1, 0 and 1), which weaken individual judgments.

The economic stratification of the population among classes of locality, as well as according to levels of education were identified. The greater the number of residents in a city, and the higher the level of the respondent's education the lower was the level of subjective poverty. It is worth noting that villages and small towns are similar. Their poverty level can be described as medium, and we can state these areas have an average risk of poverty. In big cities with 100,000 or more residents the subjective poverty level was low and these areas have a low risk of poverty.

Recommendations

The authors recommend using the fuzzy TOPSIS method in the assessment of the subjective poverty level described by variables with ordered categories. The proposed approach to assess the subjective poverty levels is a universal technique that can be used for households or individuals.

The quantitative measurement of subjective poverty for households is an important tool for assessing anti-poverty policy. The proposed research approach can also be the basis for the establishment of development documents, e.g. anti-poverty programs and strategies.

Declarations

ETHICS DECLARATIONS

Funding: This research received no external funding.

Conflict of Interest: The author declares no competing interests.

Ethical Conduct: No experimentation on living creatures had been carried out.

Data availability

The dataset analysed during the current study are not publicly available due the fact that they constitute and excerpt of research in progress but are available from the corresponding author on reasonable request.

*Ethical Approval

not applicable, surveys conducted with the consent of the respondents, surveys were anonymous

*Competing interests

not applicable

*Authors' contributions

Aleksandra Łuczak 50% and Sławomir Kalinowski 50%

*Funding

Without funding

*Availability of data and materials

Materials available at the request of study authors

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Figures

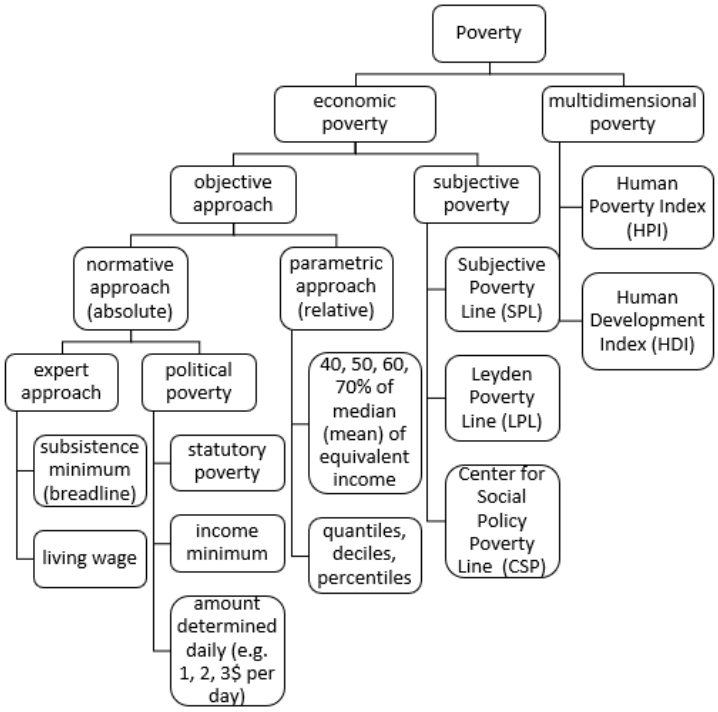

Figure 1

Methods of determining the poverty

Source: Kalinowski (2015).

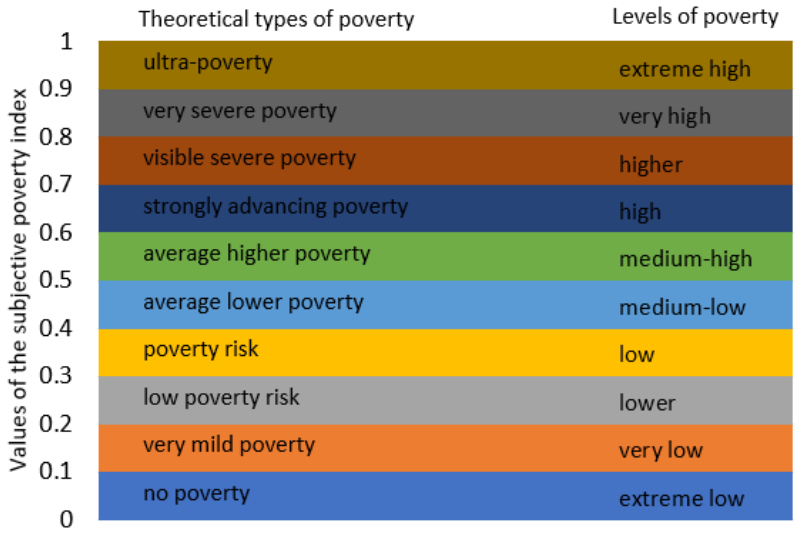

Figure 2

Values of the subjective poverty index and theoretical types and levels of poverty

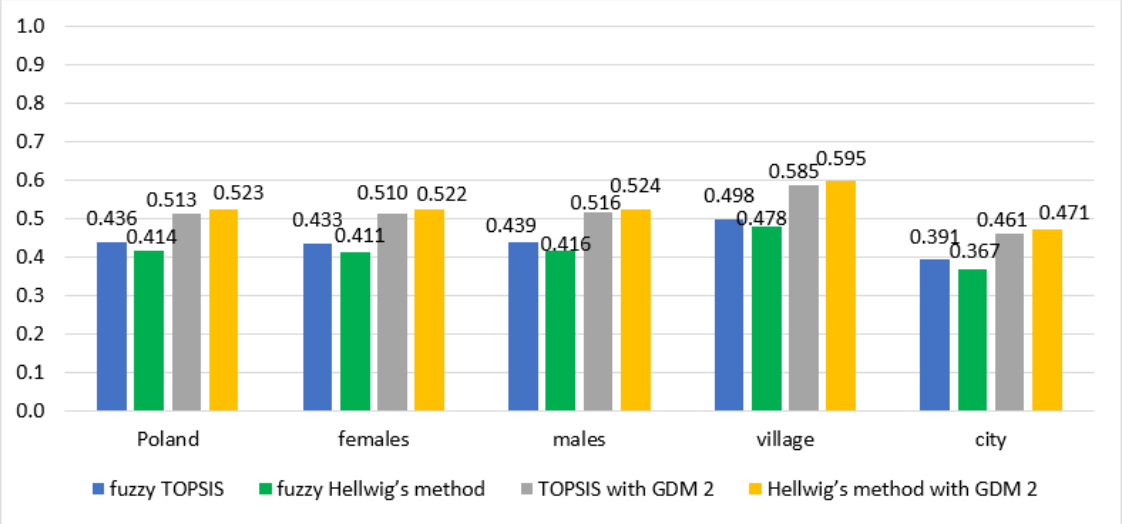

Figure 3

Mean values of the subjective poverty indexes in Poland and by sex and type of area

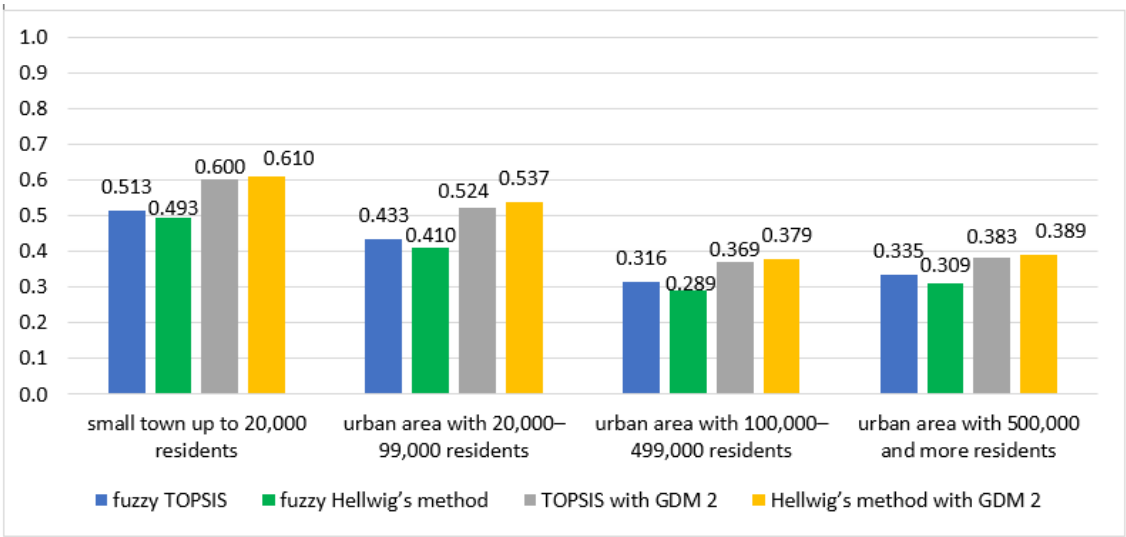

Figure 4

Mean values of the subjective poverty index by class of locality of a household

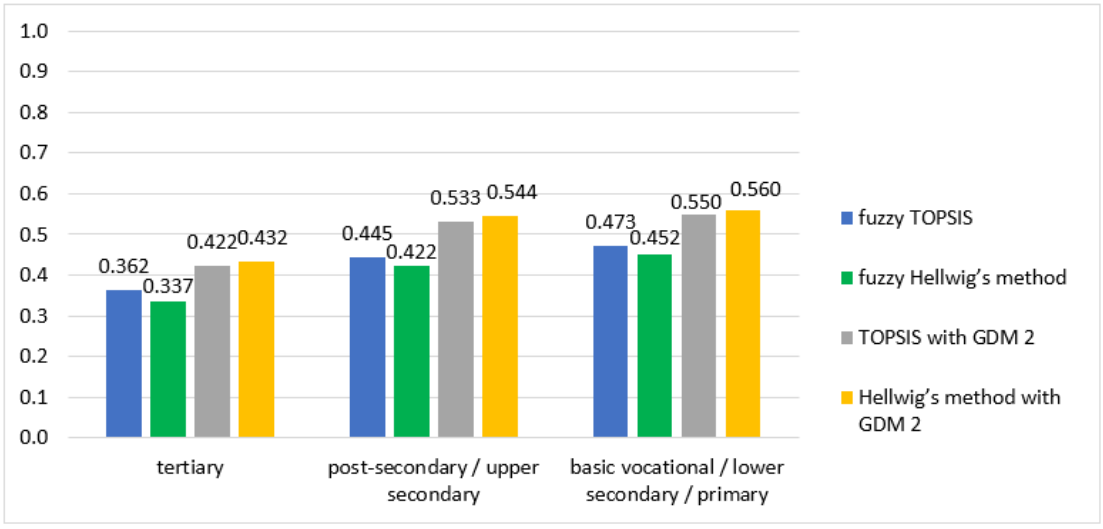

Figure 5

Mean values of the subjective household poverty index by level of education of a respondent

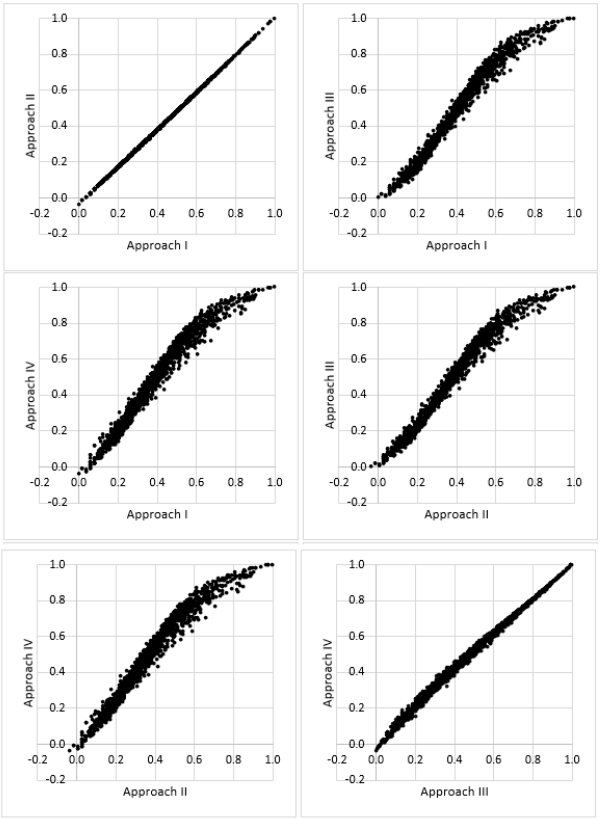

Figure 6

Graphic presentation of a pair comparison of methods according to synthetic measures of the subjective household poverty level