Real-Time Chatter Detection via Iterative Vold-Kalman Filter and Energy Entropy

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Real-Time Chatter Detection via Iterative Vold-Kalman Filter and Energy Entropy

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Abstract

Real-time chatter detection is important in improving the surface quality of workpieces in milling. Since the process from stable cutting to chatter is characterized by the progressive variation of the vibration energy distribution, entropy has been utilized to capture the decreasing randomness of vibration signals when chatter occurs. To make such an index more sensitive to transitions of the cutting state, the entropy can be computed based on signal components obtained through signal decomposition techniques. However, the classic empirical mode decomposition (EMD) is difficult to put into practice due to its weak robustness to noises. The up-to-date variational mode decomposition (VMD) has strict requirements on priori information of the signal and thus is not applicable either. In this paper, a novel method named the iterative Vold-Kalman filter (I-VKF) is proposed under the framework of the greedy algorithm, where the Vold-Kalman filter (VKF), a classic order-tracker for rotating machinery, is improved to recursively extract each signal component. In the meantime, a spectrum concentration index-based technique is developed for the instantaneous chatter frequency estimation to adaptively determine the filter parameter. Numerical examples demonstrate the superiority of the I-VKF over the original VKF, EMD, and VMD, especially in the presence of strong noises. Combined with the energy entropy of extracted components and an automatically calculated threshold, the proposed strategy greatly helps in timely chatter detection, which has been verified by dynamic simulation and experiments.

Keywords: Chatter detection; Signal decomposition; Empirical mode decomposition; Vold-Kalman filter; Energy entropy

1. Introduction

Chatter is a kind of unexpected self-excited vibration occurring in almost all machining processes, which limits productivity, damages the machined surface, and shortens the tool life [1]. With the development of automation, the flexibility of the

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machine tool leads to diverse working conditions in milling processes. Therefore, it is impossible to completely avoid chatter. Over the past decades, researchers devote their efforts to chatter prediction before milling. They focus on the stability lobe chart based on the identified dynamic model of the milling system [2]. Nonetheless, chatter may still occur when machining under stable conditions indicated by theoretical prediction, because the lobe chart is sensitive to the system parameters that are erratic in engineering.

Real-time chatter detection is an online approach necessitating the least priori information of the milling system. Combined with the subsequent controlling operation, this strategy helps to eliminate chatter in time. In real-time chatter detection, various signals are collected to provide substantial information, including the cutting force [3], spindle acceleration [4], spindle torque [5], workpiece displacement [6], and machining noise [7]. Signal processing methods are applied then to extract the crucial feature, and the corresponding cutting state indicator can be established. The effectiveness of such an indicator is the key to achieve the chatter prognosis at an early stage.

Considering the variation of the vibration energy distribution from stable cutting to chatter, entropy-based indicators are widely used such as the approximate entropy [8], multiscale entropy [9] in the time domain, and the normalized spectral entropy [10], spectral Rényi entropy [11] in the frequency domain. However, these indices fail to reflect the cutting state timely when raw data are analyzed since the complexity of milling responses reduces their sensitivity. Under such a circumstance, signal decomposition techniques are utilized as a pre-processor to decompose the original signal into a series of simple intrinsic components. Although the well-known empirical mode decomposition (EMD) [12] has been applied [13], the lack of a rigorous mathematical foundation makes this classic method extremely sensitive to noises and thus difficult to put into practice. The later developed variational mode decomposition (VMD) [14] outperforms the EMD in robustness but requires the number of signal components as a priori parameter [15], which is difficult to obtain in engineering. Moreover, nonlinearity and time-delay in milling systems result in the wideband property of milling responses [16, 17]. Therefore, when the narrowband filter-bank-

based approaches like the VMD and empirical wavelet transform (EWT) [18] are applied, the obtained signal components tend to be physically meaningless [19, 20].

In real-time state monitoring of rotating equipment, the Vold-Kalman filter (VKF) [21] is an excellent choice. Strong robustness and low computational costs are achieved thanks to the framework of the widely-used Kalman filter [22]. As a tachometer measurement-based order tracker [21], the VKF simultaneously extracts all the order-based harmonic components from measured vibration responses, which characterize tooth-passing dynamics in milling processes. However, when chatter occurs, non-harmonic modes become the main concern, and thus the order-tracking is no longer helpful [23], that is, the milling signal can only be effectively decomposed with the information of the instantaneous chatter frequency. In the meanwhile, as machining dynamics become more complex and the number of signal components increases, the joint optimization (i.e., the simultaneous extraction of all the components) in the VKF tends to be computationally unstable [24]. That being the case, the framework of the VKF needs to be improved and the accurate instantaneous frequency (IF) of each chatter mode is required to adaptively determine the filter parameter.

In this work, a novel technique based on the spectrum concentration index (SCI) [25], a metric to evaluate the degree of the signal demodulation, is developed to parametrically estimate the signal IF. This technique transforms the estimation of a continuous IF into the optimization of a few polynomial parameters, and thus works effectively even in the presence of strong noises. With this approach, only the IF of the dominant mode with the highest energy in the signal can be obtained. Therefore, under the framework of the greedy algorithm, the original VKF is modified to recursively extract each component based on the latest available IF information, just like the EMD does [12]. As a result, the number of components is automatically determined and the filter becomes more stable and more adaptive compared to the original VKF. The procedures above lead to a new multicomponent signal decomposition tool named the iterative Vold-Kalman filter (I-VKF). A numerical example is given to demonstrate the superiority of the I-VKF over the EMD, VMD, and original VKF. Then the energy entropy [26], a generalization of

Shannon's entropy [27] in the energy domain, is computed based on the extracted signal components to capture the decreasing randomness [28] in vibration responses when chatter occurs. Based on collected stable milling responses, the chatter alarm threshold of the entropy is set by the maximum likelihood estimation [29] and the three-sigma criterion [30]. Using the developed strategy, chatter can be detected accurately and timely, which has been demonstrated by dynamic simulation and experimental verification.

The remainder of this paper is organized as follows. The proposed iterative Vold-Kalman filter is detailed in Section 2. Section 3 introduces the energy entropy as the cutting state indicator. Section 4 presents dynamic simulation to demonstrate the effectiveness of the proposed method. In Section 5, the procedure of the threshold determination is given. Section 6 describes the detailed implementation of the online strategy and several milling experiments conducted. Section 7 concludes this paper.

2. Iterative Vold-Kalman filter

2.1 Chatter frequency estimation based on spectrum concentration index (SCI)

The complex dynamic response $s(t)$ measured during milling processes can be regarded as a superposition of multiple single modes as

$$s(t) = \sum_{j=1}^N s_j(t), \quad (1)$$

where the j -th mode $s_j(t)$ is parametrically modeled here as a polynomial-phase signal as

$$s_j(t) = a_j(t) \exp \left(\sqrt{-1} \left(2\pi \left(c_0 t + \sum_{i=1}^k \frac{c_i}{i+1} t^{i+1} \right) + \varphi_0 \right) \right), \quad (2)$$

where $a_j(t)$ is the amplitude, $c(t) = \sum_{i=0}^k c_i t^i$ stands for the mode IF, k is the order of the polynomial phase, c_0 represents the initial frequency, $c_i (i=1, \dots, k)$ denotes IF parameters, and φ_0 is the initial phase. In this manner, the estimation of the continuous IF is equivalent to the estimation of polynomial parameters, which greatly simplifies the problem. If the term $\sum_{i=1}^k c_i t^{i+1} / (i+1)$ is removed from the polynomial, the energy of $s_j(t)$ will concentrate on the c_0 in the frequency domain, that is, the signal mode $s_j(t)$

will be fully demodulated. That being the case, a demodulation operator can be defined as

$$\Phi(t; \tilde{\mathbf{C}}) = \exp\left(-\left(\sqrt{-1}\right)2\pi\sum_{i=1}^k\frac{\tilde{c}_i}{i+1}t^{i+1}\right), \quad (3)$$

where $\tilde{\mathbf{C}} = \{\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_k\}$ are the estimated IF parameters. Then the demodulated mode can be obtained as

$$s_{j,d}(t; \tilde{\mathbf{C}}) = s_j(t) \Phi(t; \tilde{\mathbf{C}}). \quad (4)$$

When the estimated IF is exactly the true one (i.e., $\tilde{\mathbf{C}} = \mathbf{C}$), the signal is fully demodulated as

$$s_{j,d}(t; \mathbf{C}) = a_j(t) \exp\left(\sqrt{-1}(2\pi c_0 t + \varphi_0)\right), \quad (5)$$

where the spectrum has a single peak at the constant frequency c_0 . Following this idea, the SCI is adopted to evaluate the spectrum concentration degree of the demodulated signal as [25]

$$\text{SCI}(\tilde{\mathbf{C}}) = E\left(\left|\mathcal{F}\left(s_{j,d}(t; \tilde{\mathbf{C}})\right)\right|^4\right), \quad (6)$$

where $E(\bullet)$ stands for the expectation and $\mathcal{F}(\bullet)$ denotes the Fourier transform. The optimized $\tilde{\mathbf{C}}$ can be described as

$$\tilde{\mathbf{C}}_{\text{opt}} = \{\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_k\}_{\text{opt}} = \arg \max_{\tilde{\mathbf{C}}} \text{SCI}(\tilde{\mathbf{C}}), \quad (7)$$

and then $\tilde{c}_0$ can be naturally worked out as

$$\tilde{c}_0 = \arg \max_f \left| \mathcal{F}\left(s_{j,d}(t; \tilde{\mathbf{C}}_{\text{opt}})\right) \right|, \quad (8)$$

i.e., the peak frequency in the Fourier spectrum of the demodulated mode $s_{j,d}$, and this frequency can be easily detected. The particle swarm optimization (PSO) algorithm is adopted to solve the nonlinear optimization problem in Eq. (7) [31], where the size of the population is set as 30 and the initial values are set as zero in this work. Obtaining the $\tilde{\mathbf{C}}$ and $\tilde{c}_0$ through Eq. (7) and (8), the IF of the target mode $s_j(t)$ can be accurately estimated. Since the SCI is to be maximized, the IF of the strongest mode with the highest energy in the synthetic signal will be estimated first.

2.2 Signal decomposition via iterative Vold-Kalman filter (I-VKF)

With the estimated IF, the VKF can be employed to extract the target component $s_j(t)$.

The center frequency for the VKF can be adjusted adaptively according to the estimated IF and thus makes it possible to separate closely distributed or even crossed signal modes in the time-frequency plane. However, in the original VKF the tachometer information is utilized and a joint optimization is adopted (i.e., all the components are simultaneously extracted) [21], while by using the approach in Section 2.1, only the IF of the dominant mode with the highest energy is obtained. Therefore, the VKF is modified in this work under the framework of the greedy algorithm to recursively extract each component.

The mode $s_j(t)$ in model (1) can be given in another form as

$$s_j(t) = a_j(t)\Theta_j(t), \quad (9)$$

where $a_j(t)$ represents the amplitude envelope and $\Theta_j(t)$ denotes the carrier signal as

$$\Theta_j(t) = \exp\left(\sqrt{-1}\int_0^t \omega_j(\tau)d\tau\right), \quad (10)$$

where $\int_0^t \omega_j(\tau)d\tau$ denotes the instantaneous phase with $\omega_j(\tau)$ being the instantaneous circular frequency. Different from the signal model in the original VKF [21], the assumption that $\omega_j(\tau)$ is an integer or fractional multiple of the fundamental frequency (i.e., the spindle rotational frequency) is not made here. Target components are no longer limited to the order-based harmonics.

For discrete signals, the smooth degree of the slowly-varying amplitude function $a_j(t)$ can be evaluated by

$$\nabla^s \mathbf{a}_j(m) = \boldsymbol{\varepsilon}_j(m), \quad m = 1, 2, \dots, M, \quad (11)$$

where s is the difference order, ∇ denotes the difference operator, M is the signal length, and $\boldsymbol{\varepsilon}_j$ stands for the higher-order term. Setting $s = 2$ (i.e., the second-order VKF is employed), Eq. (11) can be formulated as

$$\mathbf{a}_j(m-1) - 2\mathbf{a}_j(m) + \mathbf{a}_j(m+1) = \boldsymbol{\varepsilon}_j(m), \quad (12)$$

where $\mathbf{a}_j(0) = 0$ and $\mathbf{a}_j(M+1) = 0$ for a practical causal signal. The matrix form of Eq. (12) is given as

$$\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 \\ 1 & -2 & 1 & \cdots & 0 \\ 0 & 1 & -2 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & \cdots & 1 & -2 \end{bmatrix} \begin{bmatrix} \mathbf{a}_j(1) \\ \mathbf{a}_j(2) \\ \mathbf{a}_j(3) \\ \cdots \\ \mathbf{a}_j(M) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\varepsilon}_j(1) \\ \boldsymbol{\varepsilon}_j(2) \\ \boldsymbol{\varepsilon}_j(3) \\ \vdots \\ \boldsymbol{\varepsilon}_j(M) \end{bmatrix}. \quad (13)$$

Eq. (13) can be written as

$$\mathbf{R}\mathbf{a}_j = \boldsymbol{\varepsilon}_j, \quad (14)$$

which is the *structural equation* corresponding to the *state equation* in the Kalman filter [22]. In the meanwhile, the measured signal $s(t)$ in Eq. (1) can be re-formulated in a discrete form as

$$\mathbf{s}(m) = \mathbf{a}_j(m)\boldsymbol{\Theta}_j(m) + \boldsymbol{\xi}_j(m), \quad m = 1, 2, \dots, M. \quad (15)$$

Note that in the original VKF [21], all the components are jointly estimated and $\boldsymbol{\xi}_j(m)$ in Eq. (15) only includes the estimation error and noise, while in this work $\boldsymbol{\xi}_j(m)$ also includes the non-target components (i.e., $\mathbf{s}_p(m)$, $p = 1, 2, \dots, N$, $p \neq j$). Such an idea coincides with the greedy algorithm, that is, the non-target components are regarded as the unwanted noise, and the target component is extracted *greedily* in each local optimization. In other words, the obtained mode in each extraction takes away as much energy as possible from the current signal. This strategy is effective because it is consistent with the technique of the chatter frequency estimation developed in Section 2.1, where only the IF of the dominant mode is estimated. Finally, all the local optimization sub-problems together formulate the signal decomposition. Eq. (15) can be expressed in the form of a matrix equation as

$$\begin{bmatrix} \mathbf{s}(1) \\ \mathbf{s}(2) \\ \vdots \\ \mathbf{s}(M) \end{bmatrix} - \begin{bmatrix} \boldsymbol{\Theta}_j(1) & 0 & \cdots & 0 \\ 0 & \boldsymbol{\Theta}_j(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{\Theta}_j(M) \end{bmatrix} \begin{bmatrix} \mathbf{a}_j(1) \\ \mathbf{a}_j(2) \\ \vdots \\ \mathbf{a}_j(M) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\xi}_j(1) \\ \boldsymbol{\xi}_j(2) \\ \vdots \\ \boldsymbol{\xi}_j(M) \end{bmatrix}, \quad (16)$$

which can also be written as

$$\mathbf{s} - \mathbf{B}_j \mathbf{a}_j = \boldsymbol{\xi}_j. \quad (17)$$

Similarly, Eq. (17) is a variant of the *data equation* corresponding to the *measurement equation* in the Kalman filter [22].

Combining the structural equation (14) and the data equation (17), a weighted loss function $\mathcal{L}$ in the sense of ℓ^2 norm is given as

$$\mathcal{L} = \lambda^2 \|\boldsymbol{\varepsilon}_j\|_2^2 + \|\boldsymbol{\xi}_j\|_2^2 = \lambda^2 \boldsymbol{\varepsilon}_j^H \boldsymbol{\varepsilon}_j + \boldsymbol{\xi}_j^H \boldsymbol{\xi}_j, \quad (18)$$

where λ is a weighting factor and the superscript H denotes the conjugate transpose. To minimize the function $\mathcal{L}$, the normal equation is introduced as

$$\frac{\partial \mathcal{L}}{\partial \mathbf{a}_j} = (\lambda^2 \mathbf{R}^T \mathbf{R} + \mathbf{I}) \mathbf{a}_j - \mathbf{B}_j^H \mathbf{s} = \mathbf{0}, \quad (19)$$

where $\mathbf{I}$ denotes the identity matrix. Eq. (19) leads to the optimal solution for $\mathbf{a}_j$ as

$$\tilde{\mathbf{a}}_j = (\lambda^2 \mathbf{R}^T \mathbf{R} + \mathbf{I})^{-1} \tilde{\mathbf{B}}_j^H \mathbf{s} \triangleq \mathbf{G}^{-1} \tilde{\mathbf{B}}_j^H \mathbf{s}. \quad (20)$$

The target mode $\mathbf{s}_j$ can be finally reconstructed as

$$\tilde{\mathbf{s}}_j = \tilde{\mathbf{B}}_j \tilde{\mathbf{a}}_j, \quad (21)$$

where the square matrix $\tilde{\mathbf{B}}_j$ can be easily formulated since the $\omega_j(\tau)$ in Eq. (10) has already been worked out with the estimated IF. Considering the sparsity of matrix $\mathbf{G}$ in Eq. (20), the Cholesky factorization of matrix $\mathbf{G}$ [32] is employed here as the fastest algorithm to solve the Eq. (19). Besides, the weighting factor λ is set as 1×10^6 in this work to achieve a trade-off between the accuracy and computational cost [24].

Fig. 1. Flow chart of the developed iterative Vold-Kalman filter (I-VKF).

Thus far, the entire I-VKF method can be summarized in Fig. 1, where the extraction of signal modes is implemented recursively until the residual energy ratio is less than a

pre-set value (5% is used here), that is, the mode number N is determined automatically. Noting that the mode number (i.e., the highest order) is specified empirically in the original VKF, the improved framework makes the filter more adaptive. Besides, the recursive rather than simultaneous extraction makes the filter more stable, which will be verified by the following example.

Fig. 2. Example of a four-component signal: (a) Waveform of the noisy signal; (b) TFR of the noise-free signal; (c) TFR of the noisy signal. All the TFRs are generated by the STFT.

2.3 An example

To validate the proposed method, an example of a four-component signal is given as

$$s(t) = s_1(t) + s_2(t) + s_3(t) + s_4(t), \quad 0 \leq t \leq 15 \text{ s},$$

$$\begin{cases} s_1(t) = \exp(-0.1t) \cos(2\pi(5t)), \\ s_2(t) = \exp(-0.1t) \cos\left(2\pi\left(10t + 1.5t^2 - \frac{1}{30}t^3\right)\right), \\ s_3(t) = \exp(-0.1t) \cos\left(2\pi\left(20t + 1.8t^2 - \frac{1}{20}t^3\right)\right), \\ s_4(t) = \exp(-0.1t) \cos\left(2\pi\left(40t + 0.5t^2 - \frac{1}{15}t^3\right)\right), \end{cases} \quad (22)$$

where the IFs of modes are $f_1(t) = 5$, $f_2(t) = 10 + 3t - 0.1t^2$, $f_3(t) = 20 + 3.6t - 0.15t^2$, and $f_4(t) = 40 + t - 0.2t^2$ respectively.

The signal in Eq. (22) is contaminated by a white Gaussian noise with the signal-to-noise ratio (SNR) as 2 dB. The sampling frequency is set as 100 Hz. The waveform of the noisy signal is shown in Fig. 2 (a). Fig. 2 (c) gives the time-frequency representation (TFR) generated by the short-time Fourier transform (STFT) while Fig. 2 (b) is the noise-free version. In the parametric estimation of IFs, the order of the polynomial model

is set as three (although true IFs here are of the second-order type).

Table 1 lists the estimated IF parameters by the SCI optimization with corresponding estimation errors. Using the I-VKF, extracted four components and their TFRs are shown in Fig. 3. With accurately estimated IFs, four single components are successfully separated and embedded noises are removed. The SNRs of four reconstructed modes are 17.5516 dB, 17.0363 dB, 18.5479 dB, and 16.7213 dB, respectively.

Fig. 3. Extracted components by the I-VKF: (a) ~ (d) give $s_1 \sim s_4$ respectively. Top panels are waveforms (blue lines denote the estimated ones while red lines denote the true ones). Bottom panels are corresponding TFRs generated by the STFT.

Table 1. Estimated IF parameters based on the SCI optimization.

		c_0	c_1	c_2	c_3
$s_1(t)$	Estimated	5.0061	0.0012	0.0004	0.0025
	True	5 (0.12%)	0 (/)	0 (/)	0 (/)
$s_2(t)$	Estimated	9.9930	3.0028	-0.1003	0.0017
	True	10 (0.07%)	3 (0.09%)	-0.1 (0.30%)	0 (/)
$s_3(t)$	Estimated	19.9867	3.6047	-0.1503	0.0023
	True	20 (0.07%)	3.6 (0.13%)	-0.15 (0.20%)	0 (/)
$s_4(t)$	Estimated	40.0244	0.9978	-0.2006	0.0066
	True	40 (0.06%)	1 (0.22%)	-0.2 (0.31%)	0 (/)

Note: Values in parentheses denote the relative estimation errors.

For comparisons, decomposition results by the EMD, VMD, and original VKF are also given (note that when the original VKF is used, the number of modes and IFs of modes are taken as known information to initiate the filtering). When the EMD is applied,

the strong noise leads to severe mode aliasing [33], as Fig. 4 shows. The filter-bank property of the VMD [34] makes obtained single modes physically meaningless because four true components overlap in the time-frequency domain, as Fig. 5 shows. When using the original VKF, four chirp modes are indeed tracked but are distorted by noises (see Fig. 6), which is due to the instability of the joint-optimization [24]. Moreover, the computational time consumed by the VKF is twice that consumed by the I-VKF (all the tests are carried out with MATLAB® R2018b on a personal computer with a 3.60 GHz Intel® Core™ i7-7700 CPU).

Fig. 4. Extracted components by the EMD: (a) ~ (d) give TFRs of $s_1 \sim s_4$ generated by the STFT respectively.

Fig. 5. Extracted components by the VMD: (a) ~ (d) give TFRs of $s_1 \sim s_4$ generated by the STFT respectively.

Fig. 6. Extracted components by the original VKF: (a) ~ (d) give TFRs of $s_1 \sim s_4$ generated by the STFT respectively.

The above example demonstrates that the developed I-VKF method, which acts as an adaptive time-frequency filter, works effectively in the decomposition of multi-chirp

signals with a low SNR, and thus can be expected to help greatly in chatter detection.

3. Energy entropy-based feature extraction

It is vital to extract the fault feature from measured signals accurately and timely for chatter detection. The selected feature quantity should not only make it simple and feasible for the signal acquisition and data processing but also reflect the essence of milling processes, that is, this quantity should show close correlations with changes of the cutting state. As an indicator of the internal confusion degree of a system, entropy, such as the permutation entropy [35] and approximate entropy [36], can be described as a function of the probability distribution of signals and has been widely applied in fault diagnosis. The energy entropy [26] is utilized in this work to characterize the energy distribution of signal modes $\{s_1(t), s_2(t), \dots, s_N(t)\}$ of a milling response $s(t)$, which are extracted based on the I-VKF approach in Section 2. The energy of each mode is given by

$$E_j = \int |s_j(t)|^2 dt, \quad j = 1, 2, \dots, N. \quad (23)$$

Neglecting the residual energy, the accumulative energy of all the modes should be equal to that of the original signal. Then the energy entropy is defined based on Shannon's entropy [27] as

$$H_{\text{energy}} = -\sum_{j=1}^N p_j \log_2 p_j, \quad (24)$$

where $p_j = E_j / E$ and $E = \sum_{j=1}^N E_j$.

Energy entropy in Eq. (24) is a dimensionless index larger than zero. The larger the index, the more dispersed the energy distribution in the frequency domain [26]. Since each mode is generally located in a specific frequency band, the energy entropy can characterize the degree of randomness in milling dynamics that underlie a vibration response.

In stable milling, the vibration obeys a Gaussian-like distribution in the time domain and shows a wideband property in the frequency domain. At an early stage of chatter, the amplitudes at certain frequencies (i.e., chatter frequency and its harmonics) gradually increase, and the vibration energy tends to gather within several narrow bands. As

chatter becomes more severe, the vibration resembles a sinusoidal oscillation, which results in a rapid decrease of signal randomness in the time domain.

Fig. 7. Two simulation cases: (a) Case #1 with linearly increasing spindle speed; (b) Case #2 with linearly increasing axial cutting depth. Top panels denote the variations of the processing parameters and bottom panels denote the milling force responses in the feed direction.

According to the above mechanism, a strategy is proposed for chatter detection, which combines the I-VKF-based decomposition in Section 2 and the energy entropy-based feature extraction in Section 3. In this way, the progressive change of the cutting state can be accurately captured by the energy entropy that is computed based on signal modes extracted from milling responses. The effectiveness of this strategy will be demonstrated by dynamic simulation and experiments in Section 4 and 6, respectively.

4. Dynamic simulation

In this section, milling vibration responses are simulated using the Runge-Kutta algorithm. The classic Balachandran's two-degree-of-freedom milling model [37] is considered and the simulation parameters follow those in the original investigation [37]. We give two cases where the spindle speed and the axial cutting depth linearly increases, respectively. As Fig. 7 shows, the process from stable cutting to chatter can be observed in both cases, and the corresponding transition points A, B, and C coincide with those predicted by the stability lobe [37].

Preliminary time-frequency analysis of the response in case #1 is given in Fig. 8 (a).

The force signal consists of three types of components: the tooth-passing modes resulting from spindle rotations, random modes due to system noises, and chatter modes caused by *regenerative effects* [38], which together lead to an intricate spectrum.

Fig. 8. TFR for case #1: (a) Original simulated force signal; (b) Tooth-passing components filtered out by the VKF.

Since the spindle speed is known, the VKF is applied as a pre-processor to remove the tooth-passing components before the decomposition, with the aim to eliminate the interference of spindle rotations and intermittent cutting. The TFR of the components filtered out is shown in Fig. 8 (b). The remaining signal is then decomposed by the I-VKF into eight modes as Fig. 9 shows (note that in all the following simulated and experimental cases, only the third-order polynomial model is considered in the SCI optimization to minimize computational costs). The energy of each mode reduces as the order increases, which is consistent with the framework of the greedy algorithm in the I-VKF. Eight modes exhibit chatter-related dynamics, showing the chatter frequencies that are spindle-speed-independent [38]. Similar characteristics can be observed when the same processing is applied to the response in case #2.

The VMD and EMD are also applied to decompose the milling force signal and the results are given in Fig. 10 and Fig. 11, respectively. Mode aliasing can be observed in eight components extracted by the VMD and this problem is even exacerbated when the

EMD is used, which makes the introduction of the I-VKF technique necessary.

Fig. 9. TFRs of extracted components from the simulated milling force in case #1 by the proposed I-VKF.

Fig. 10. TFRs of extracted components from the simulated milling force in case #1 by the VMD.

As Fig. 9 shows, TFRs of extracted modes during chatter (the middle interval enclosed by two white border lines) are much more condensed compared to those during stable cutting (two end intervals), which suggests a more concentrated energy distribution. This decrease of signal randomness can be captured by the energy entropy that is computed using a sliding window with a length of 0.1 s, as Fig. 12 shows. The energy entropy is sensitive to changes of the cutting state and the variations of the entropy always precede

such changes, which demonstrates the effectiveness of the proposed strategy in timely chatter detection.

Fig. 11. TFRs of extracted components from the simulated milling force in case #1 by the EMD.

Fig. 12. Simulated milling force signals with computed energy entropies: (a) Case #1; (b) Case #2.

5. Threshold determination

Since chatter is expected to be detected automatically, a threshold of the indicator is needed. In many existing studies, the threshold is determined by manually observing the deviation between index values under chatter and stable cutting [4, 36, 39]. This empirical strategy suffers from the sensitivity to working conditions. In this work, a robust threshold is determined statistically. In many fault diagnosis problems, the fault feature is assumed to obey the Gaussian distribution, which simplifies the model construction for fault classification [40–42]. The same null hypothesis is made here in the analysis of the statistical distribution of milling signals.

At the incipient stage of a machining task, vibration signals under stable cutting can

be collected and corresponding energy entropies can be calculated as a sample. Then the normality test can be carried out for the sample, where the Lilliefors test [43] is adopted here because the population mean μ and variance σ^2 are not known. The statistic is given by

$$D_N = \max_{x \in R} |\hat{F}_N(x) - G_0(x)|, \quad (25)$$

where x is the sample, N is the sample size, $\hat{F}_N(x)$ is the empirical cumulative distribution function (CDF) of the sample, and $G_0(x)$ is the CDF of the hypothesized distribution with the estimated parameters equal to the sample parameters, that is,

$$\begin{cases} \hat{\mu} = \bar{X} = \frac{1}{N} \sum_{k=1}^N X_k, \\ \hat{\sigma}^2 = S^2 = \frac{1}{N-1} \sum_{k=1}^N (X_k - \bar{X})^2. \end{cases} \quad (26)$$

Eq. (26) is exactly the result of the maximum likelihood estimation under a Gaussian distribution [29]. The null hypothesis will be rejected when the observation of D_N is larger than the critical value at a specified significance level (5% is set here), and this critical value can be obtained by the Monte Carlo method [44].

Fig. 13. Experimental setup: (a) Picture; (b) Schematic.

Once the normality test is approved, the three-sigma criterion [30] is adopted to compute the allowable fluctuation range of the energy entropy under stable cutting,

which is given by $[\hat{\mu} - 3\hat{\sigma}, \hat{\mu} + 3\hat{\sigma}]$. Considering the decrease of randomness when chatter occurs, the alarm threshold is determined as the lower limit, i.e., $\hat{\mu} - 3\hat{\sigma}$.

6. Implementation and experimental verification

The detailed implementation of practical chatter detection is introduced in this section, and a series of milling experiments are conducted to demonstrate the performance of the proposed approach.

6.1 Implementation of chatter detection

Dynamic simulation in Section 4 shows promising results of chatter detection based on the I-VKF technique and subsequent energy entropy monitoring. Herein the practical implementation is given in Fig. 14, which consists of the following steps:

- 1) Processing parameters definition: Define the spindle speed, feed rate, and cutting depth. All these parameters can be variable in a milling task.
- 2) Vibration signals acquisition: Collect milling system-related vibration signals. The milling force is collected in this work since it suffers from the least influence of the position of transducers [45].
- 3) Online energy entropy computation: Calculate the real-time energy entropy of the collected signal based on the process given in Fig. 14 (a). A sliding window with a specified window length is used for online monitoring.
- 4) Threshold determination: Obtain the alarm threshold of the entropy by the incipient stable milling response, according to the strategy introduced in Section 5.
- 5) Cutting state monitoring: The moment when discrete data points of the computed entropy cross the given threshold three consecutive times triggers the chatter alarm. The situation where only one or two data points cross the threshold and the following point falls back will be regarded as not chatter but the accident.

6.2 Experimental setup

The experimental set-up is shown in Fig. 13. All the tests were conducted on a DMG five-axis CNC milling machine. The carbon-steel end-milling tool with four teeth has a

diameter of 4 mm, and the spindle has an overhang of 40 mm. The workpiece is an aluminum alloy block. The full-immersion cutting was adopted. Chatter vibrations were monitored with a Kistler 9123C dynamometer that was attached to the workpiece. Signals were recorded by a data acquisition card and were subsequently analyzed using a PC. The sampling frequency was set as 20 kHz.

Fig. 14. Implementation of chatter detection: (a) Flowchart of the energy entropy computation; (b) Flowchart of the online chatter detection.

Table 2. Milling tests with parameters.

Experiment	Test	Milling states	Ω (r/min)	f (mm/min)	a_p (mm)
I	#1	Stable	4200	580	6.6
	#2	Stable	3600	250	2.6
	#3	Chatter	4000	580	6.6
	#4	Severe chatter	4000	640	6.7
II	#5	Stable to chatter	4000	580	6.3

The stability of milling is closely related to the spindle speed and depth of cut. Herein the axial cutting depth varies from 2.6 mm to 6.7 mm and the spindle speed varies from 3600 rpm to 4200 rpm. Five sets of processing parameters are listed in Table 2. Test #1 ~ #4 in experiment I give four different cutting states, respectively. Test #5 in experiment II gives a dynamic process from stable cutting to chatter, and this process should be detected timely in on-line monitoring.

Fig. 15. Measured milling force signals in the offline experiment I: (a) Temporal waveforms; (b) Fourier spectra (where f denotes the tooth-passing frequency).

Fig. 16. Energy distribution of single modes extracted from milling force signals in the offline experiment I: (a) Normalized energy ratio of each mode; (b) Energy entropy of each test signal.

6.3 Results

● Offline experiment I

Four test signals are shown in Fig. 15. In test #1 and #2, the amplitudes of milling forces are relatively small and the spectra are dominated by the evenly distributed tooth-passing frequencies, which indicates a stable cutting. With an increase of the spindle speed and cutting depth, the slight and the severe chatter occur in test #3 and #4, respectively, leading to the large amplitudes and conspicuous chatter frequencies that are modulated by the spindle rotational frequency. The distribution uncertainty

decreases in these cases. Applying the I-VKF, the single modes of four test signals are obtained and the normalized energy ratios of each mode are calculated and given in Fig. 16 (a). The results verify the aforementioned Gaussian randomness in stable cutting and the harmonic concentration in chatter. Such aggregation of energy is also revealed in the computed energy entropy of four test signals, as shown in Fig. 16 (b).

The above off-line results demonstrate the effectiveness of the proposed method, and the industrial application of this strategy will be shown in the following online results further.

Fig. 17. The empirical CDF of the sample versus the CDF of the hypothesized Gaussian distribution in the Lilliefors test.

● Online experiment II

In the second experiment, the cutting state is monitored online. The length of the sliding window is set as 100 points with no overlap. The incipient milling force signals under stable cutting were collected and the corresponding energy entropy was computed. Applying the Lilliefors test, the empirical CDF of the sample versus the theoretical CDF of the hypothesized Gaussian distribution is given in Fig. 17. The linear regression shows an excellent fit between the two functions. Therefore, the entropy threshold can be determined using the strategy in Section 5. The allowable range of the fluctuation is obtained as [2.081, 2.145], and thus the threshold is set as 2.081.

The collected milling force signal with the corresponding energy entropy computed online is shown in Fig. 18. The indicator detects the premature chatter at $t = 0.58$ s,

around 0.12 s ahead of the moment when the amplitude suddenly increases (which means the destruction of the workpiece). For further verification, the machine was not shut down after the chatter alarm. Three signal samples with an equal duration of 0.3 s marked with S1, S2, and S3 in Fig. 18, respectively, are extracted and given in Fig. 19. The VKF is applied as a pre-filter to remove the tooth-passing components, and the Fourier spectra after are shown in Fig. 19 (c).

Fig. 18. The collected milling force signal in test #5 with the corresponding energy entropy computed online: (a) Overall view; (b) Local close-up view.

In the stable cutting during 0 ~ 0.3 s, the vibration energy disperses in a wide band (see the first panel in Fig. 19 (c)). With a drastic increase of the vibration amplitude at around 0.6 s, the chatter emerges, which is accompanied by the noticeable chatter frequencies (see the second panel in Fig. 19 (c)). At about 1.8 s, chatter has been fully developed. A modulated waveform appears (see the third panel in Fig. 19 (a)) and the chatter frequencies become dominant, which absorb most of the energy (see the third panel in Fig. 19 (c)).

Fig. 19. Three signal segments corresponding to the S1 (stable), S2 (transition), and S3 (chatter) in Fig. 18 respectively: (a) Temporal waveforms; (b) Original Fourier spectra; (c) Fourier spectra after the VKF filtering (where f denotes the tooth-passing frequency).

The proposed I-VKF and the classic EMD are both applied to decompose the chatter response. As Fig. 20 (a) shows, a severe mode aliasing can be observed in decomposition results by the EMD because the collected signal contains noises, which is inevitable in engineering. By using the I-VKF, the harmonic-like components have been recovered from the noisy signal, which makes the characteristic peaks of chatter conspicuous in the spectra (see Fig. 20 (b)). The superiority of the I-VKF over EMD shows further in the analyses of the energy distribution and energy entropy based on extracted single modes. As Fig. 21 and Fig. 22 show, three different cutting states cannot be distinguished with the aid of the EMD. This came as no surprise because an ineffective signal decomposition will render the subsequent computation of the chatter index ineffective.

Fig. 20. Signal decompositions of sample 3 in Fig. 19: (a) By the EMD; (b) By the I-VKF (both after the pre-filtering by the VKF).

Fig. 21. Energy distribution of single modes from test #5 signal in the online experiment II: (a) Based on the EMD; (b) Based on the I-VKF.

Fig. 22. Energy entropy of each sample in experiment II: (a) Based on the EMD; (b) Based on the I-VKF.

7. Conclusions

To protect the machine tool and workpiece from damages of chatter, it is necessary to detect chatter timely and accurately in machining processes. Since the chatter is characterized by the progressive variation of the energy distribution, when chatter occurs, the decreasing randomness in vibration responses can be captured by energy entropy, a generalization of Shannon's entropy in the energy domain. To make such an index more sensitive to transitions of the cutting state, the energy entropy should be computed based on single signal modes that are obtained by signal decomposition techniques. Considering the limited applications of the well-known EMD and VMD due to the sensitivity to noises and the need for priori information, respectively, the I-VKF is proposed in this work as an efficient multi-component signal decomposition tool.

Following the idea of the greedy algorithm, the original VKF, which adopts a joint-optimization framework, is modified to recursively extract each chatter mode (i.e., signal modes are no longer simultaneously extracted). This modification improves the stability and adaptability of the filter and reduces the computational cost. In the meanwhile, a novel technique based on the SCI is developed to parametrically estimate the instantaneous chatter frequency of the dominant mode in the current signal, and thus initiate each extraction. This technique transforms the estimation of a continuous IF into the optimization of a few polynomial parameters, which simplifies the problem and improves the robustness of the filter. As a result, the proposed I-VKF acts as an adaptive time-frequency filter and has a much better performance than the EMD, VMD, and

original VKF especially in the presence of strong noises, which has been verified by numerical examples.

By modeling the distribution of the energy entropy sample under stable cutting as a Gaussian type, the alarm threshold of the chatter indicator is determined through the maximum likelihood estimation and three-sigma criterion. With this threshold, an online chatter detection strategy is detailed. Through dynamic simulation and experimental verification, it has been demonstrated that chatter can be detected timely by the proposed strategy. The variation of the entropy can be clearly observed during the transition from stable cutting to chatter. When other similar schemes such as the EMD-based entropy monitoring are employed, the decrease of randomness of the collected signal cannot be revealed because the fundamental signal decomposition is ineffective.

Declarations

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Code availability Thanks to MATLAB® for providing software support. Code will be made available on reasonable request.

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Figures

Figure 1

Flow chart of the developed iterative Vold-Kalman filter (I-VKF).

Figure 2

Example of a four-component signal: (a) Waveform of the noisy signal; (b) TFR of the noise-free signal; (c) TFR of the noisy signal. All the TFRs are generated by

Figure 3

Extracted components by the I-VKF: (a) ~ (d) give $s_1 \sim s_4$ respectively. Top panels are waveforms (blue lines denote the estimated ones while red lines denote the true ones). Bottom panels are corresponding TFRs generated by the STFT.

Figure 4

Extracted components by the EMD: (a) ~ (d) give TFRs of $s_1 \sim s_4$ generated by the STFT respectively.

Figure 5

Extracted components by the VMD: (a) ~ (d) give TFRs of $s_1 \sim s_4$ generated by the STFT respectively.

Figure 6

Extracted components by the original VKF: (a) ~ (d) give TFRs of $s_1 \sim s_4$ generated by the STFT respectively.

Figure 7

Two simulation cases: (a) Case #1 with linearly increasing spindle speed; (b) Case #2 with linearly increasing axial cutting depth. Top panels denote the variations of the processing parameters and bottom panels denote the milling force responses in the feed direction.

Figure 8

TFR for case #1: (a) Original simulated force signal; (b) Tooth-passing components filtered out by the VKF.

Figure 9

TFRs of extracted components from the simulated milling force in case #1 by the proposed I-VKF.

Figure 10

TFRs of extracted components from the simulated milling force in case #1 by the VMD.

Figure 11

TFRs of extracted components from the simulated milling force in case #1 by the EMD.

Figure 12

Simulated milling force signals with computed energy entropies: (a) Case #1; (b) Case #2.

Figure 13

Experimental setup: (a) Picture; (b) Schematic.

Figure 14

Implementation of chatter detection: (a) Flowchart of the energy entropy computation; (b) Flowchart of the online chatter detection.

Figure 15

Measured milling force signals in the offline experiment I: (a) Temporal waveforms; (b) Fourier spectra (where f denotes the tooth-passing frequency).

Figure 16

Energy distribution of single modes extracted from milling force signals in the offline experiment I: (a) Normalized energy ratio of each mode; (b) Energy entropy of each test signal.

Figure 17

The empirical CDF of the sample versus the CDF of the hypothesized Gaussian distribution in the Lilliefors test.

Figure 18

The collected milling force signal in test #5 with the corresponding energy entropy computed online: (a) Overall view; (b) Local close-up view.

Figure 19

Three signal segments corresponding to the S1 (stable), S2 (transition), and S3 (chatter) in Fig. 18 respectively: (a) Temporal waveforms; (b) Original Fourier spectra; (c) Fourier spectra after the VKF filtering (where f denotes the tooth-passing frequency).

Figure 20

Signal decompositions of sample 3 in Fig. 19: (a) By the EMD; (b) By the I-VKF (both after the pre-filtering by the VKF).

Figure 21

Energy distribution of single modes from test #5 signal in the online experiment II: (a) Based on the EMD; (b) Based on the I-VKF.

Figure 22

Energy entropy of each sample in experiment II: (a) Based on the EMD; (b) Based on the I-VKF.