Study on Minimum Rollable Thickness in Asymmetrical Rolling

Ji Wang

State Key Laboratory of Rolling and Automation, Northeastern University, Shenyang 110819, Liaoning, PR China

Xianghua Liu (✉ liuxhres2018@163.com)

Northeastern University

Research Article

Keywords: minimum rollable thickness, asymmetrical rolling, deformation zone configuration, critical reduction rate

Posted Date: March 22nd, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-319435/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Version of Record: A version of this preprint was published at The International Journal of Advanced Manufacturing Technology on December 3rd, 2021. See the published version at <https://doi.org/10.1007/s00170-021-08364-w>.

Study on minimum rollable thickness in asymmetrical rolling

Ji Wang¹, Xianghua Liu^{1,2,*}

¹ *State Key Laboratory of Rolling and Automation, Northeastern University, Shenyang 110819, Liaoning, PR China*

² *Key Laboratory of Lightweight Structural Materials, Liaoning Province Shenyang 110819, China*

* Corresponding author: Xianghua Liu;

E-mail: liuxhres2018@163.com

Abstract: A new model for the asymmetrical rolling is proposed to calculate the minimum rollable thickness simply and fast by the slab method. The calculation formulas of the rolling pressure, the rolling force, the critical roll speed ratio and the critical front tension under different deformation zone configurations are proposed, and the deformation zone configuration - rolling parameters relationship diagram is given and analyzed. The results show that the minimum rollable thickness can be reached when the rolling parameters keep the deformation zone configuration as cross-shear zone + backward-slip zone (C+B) or all cross-shear zone (AC). The calculation formulas of the minimum rollable thickness and the required rolling parameters for different deformation zone configurations are proposed respectively. The calculated value is in good agreement with the experimental results.

Keywords: minimum rollable thickness; asymmetrical rolling; deformation zone configuration; critical reduction rate.

Symbol List

l	Contact length
P	Rolling force per unit width
$\bar{P}$	Average rolling pressure
p_1, p_2	Rolling pressures of the lower and upper rolls, respectively
p_x, σ_x	Vertical and horizontal stresses in the deformation zone, respectively
σ_f, σ_b	Front and back tension, respectively

σ_{fc}	Critical front tension for AC deformation zone configuration
μ	Friction coefficients between the work roller and the strip
i	Roll speed ratio
i_{c1}, i_{c2}, i_{c3}	Critical roll speed ratio for the C+B, AC, F+C deformation zone configuration, respectively
$\lambda_B, \lambda_C, \lambda_F$	Proportion of the backward-slip zone, cross-shear zone and forward-slip zone, respectively
K	Plane deformation resistance
h_x	Variable strip thickness in the deformation zone
H, h	Thicknesses in the entrance and exit of the deformation zone, respectively
h_1, h_2	Neutral points of the fast and slow rollers, respectively
$H_{\min}$	Minimum rollable thickness
Δh	Reduction
ε	Reduction rate
ε_0	Critical reduction rate
R, D	Radius and diameter of the work roller, respectively
E	Elastic modulus of the work roller, $E=206000\text{MPa}$
ν	Poisson ratio of the work roller, $\nu=0.3$
V_1, V_2	Peripheral speeds of the fast and slow rollers, respectively

1. Introduction

Micromanufacturing technology has attracted more and more attention with the development of the product miniaturization [1]. The ultrathin metal strip rolling is one of the hot spots in the field of micromanufacturing [2]. According to the traditional rolling theory, there is a minimum rollable thickness under the certain rolling conditions, due to the elastic deformation of the rolls and the strip. Stone [3,4] proposed the classic formula for calculating the minimum rollable thickness as following

$$H_{\min} = \frac{3.58D\mu}{E} \left(K - \frac{\sigma_f + \sigma_b}{2} \right) \quad (1)$$

The multi-roll mills that are commonly used to produce ultrathin strip are designed based on this theory. Fleck et al. [5,6] considered that the plastic deformation occurs near the entrance and exit of the deformation zone during the ultrathin strip rolling. In the middle of the deformation zone, the strip does not reduce or slip relative to the rolls. Sutcliffe and Rayner [7] verified this theory through thin strip rolling experiments. Xiao et al. [8] proposed two minimum rollable thickness models based on the Fleck theory [5,6]. The difference of the two models is whether to consider the restriction of the

---

rolling force. When the restriction of the rolling force is not considered, the theoretical minimum rollable thickness calculated by their model is approximately 22% that of the Stone model. Wu et al. [9,10] proposed a minimum rolled thickness model with considering the allowable rolling pressure and production efficiency. Their theory had been used in a 1220 five-rack cold tandem mills of china. Tateno et al. [11] considered that the minimum rollable thickness affected by the elastic deformation of the work rolls and the edge cracks of the ultrathin strip. Zhang [12] used the slab method and incremental analysis to study the deformation mechanism of the cold rolling ultrathin strip. Hwang and Kan [13] proposed a mathematical model to design the roll rape for foil rolling of a four-high mill. The above studies are researches on the minimum rollable thickness of symmetrical rolling. The production of ultrathin strip by symmetrical rolling requires relatively high equipment, and needs to perform multiple annealing processes. Using the asymmetrical rolling can improve these problems.

In the asymmetrical rolling, the roll speed, the roll radius and the lubrication conditions on the upper and lower side of the strip could be different. Asymmetrical rolling has a prominent advantage over the symmetrical rolling in the thinning capacity, which breaks through the classical Stone minimum rolling thickness theory. Tang et al. [14,15] proposed a model of the permissible minimum thickness in the asymmetrical rolling, and supposed that the midpoint of the deformation zone is the midpoint of the cross-shear zone and the length of the forward-slip and backward-slip zone is equal. Liu et al. [16] proposed a model for calculating the minimum rollable thickness in the asymmetrical rolling with the identical roll radius based on the Tselikov equation and the modified Hitchcock equation. Tzou and Huang [17] considered the minimum rollable thickness in the asymmetrical rolling occurs under the all cross-shear zone configuration of the deformation zone. However, the value of D/H proposed in their study is much smaller than the experimental results obtained by other

---

researchers. Feng et al. [18] had carried out systematic analyses and experiments of the single-roll-driven asymmetrical ultrathin strip rolling, and proposed a minimum rollable thickness model. Wang et al. [19,20] studied the influences of three asymmetrical conditions on the distribution of the rolling pressure. Wang et al. [21] proposed the relationship diagram between the deformation zone configuration and the rolling parameters, according to this diagram, the rolling parameters required for different deformation zone configurations can be determined. Sun et al. [22-24] analyzed the effects of the rolling parameters on the deformation zone configuration and the proportion of each zone in the deformation zone. Ji and Park [25] used the rigid-viscoplastic finite element method to analyze the effects of the three asymmetrical rolling conditions on the plastic deformation.

In this paper, the influences of the roll speed ratio, the back and front tension, and the critical reduction rate on the deformation zone configuration and the proportion of cross-shear zone are studied by the slab method. According to the relationship between the rolling parameters and the deformation zone configurations, the required rolling conditions for the minimum rollable thickness in the asymmetrical rolling is analyzed. Then, a new minimum rollable thickness model for the asymmetrical rolling is proposed and verified by experiments.

2. Mathematical model

In the asymmetrical rolling conditions, the roll radius asymmetry condition affects the length of the deformation zone, the friction asymmetry condition affects the friction stress. But the roll speed asymmetrical condition makes the cross-shear zone appear in the deformation zone which changes the stress state of the deformation zone [19]. This is the essential difference between the asymmetrical rolling and the synchronous rolling. In the symmetrical rolling, the thinning ability of the mill can also be improved by reducing the roll radius and improving the lubrication conditions. Obviously, the

roll speed asymmetrical condition is the main reason why the asymmetrical rolling can break through the classical minimum rollable theory. In this study, only the roll speed asymmetrical condition will be considered. The strip uses the annealed 430 stainless steel, the plane deformation resistance of the strip is

$$K = \frac{2}{\sqrt{3}} \left(205.0 + 509.6 \bar{\varepsilon}^{-0.312} \right) \quad (2)$$

where, $\bar{\varepsilon} = 0.4(H_0 - H)/H_0 + 0.6(H_0 - h)/H_0$, H_0 is the initial thickness of the strip.

2.1 Derivation of rolling pressure and rolling force

Fig. 1 shows four possible deformation zone configurations during the asymmetrical rolling, namely, F+C+B (forward-slip zone + cross-shear zone + backward-slip zone), C+B (cross-shear zone + backward-slip zone), AC (all cross-shear zone), F+C (forward-slip zone + cross-shear zone), where, C+B, AC and F+C are all critical states.

Fig. 1 Four possible deformation zone configurations during the asymmetrical rolling: (a) F+C+B, (b) C+B, (c) F+C, (d) AC; and (e) stress system acting on the differential vertical element in backward-slip zone

As shown in Fig. 1(e), the horizontal and vertical static equilibrium equations in the deformation zone can be obtained, respectively

$$\sigma_x dh_x + h_x d\sigma_x - (p_1 + p_2) \tan \alpha dx + \tau_\mu dx = 0 \quad (3)$$

$$p_x = p_1 + p_1 \mu \tan \alpha = p_2 + p_2 \mu \tan \alpha - \bar{\tau} \frac{dh_x}{dx} \quad (4)$$

In different regions of the deformation zone, the friction stress τ_μ on the upper and lower side of the strip and the average shear stress $\bar{\tau}$ on the vertical section can be expressed as

$$\begin{cases} \text{B} & \tau_\mu = \mu(p_1 + p_2) & \bar{\tau} = 0 & h_2 < h_x \leq H \\ \text{C} & \tau_\mu = \mu(p_1 - p_2) & \bar{\tau} = \mu p_x & h_1 \leq h_x \leq h_2 \\ \text{F} & \tau_\mu = -\mu(p_1 + p_2) & \bar{\tau} = 0 & h \leq h_x < h_1 \end{cases} \quad (5)$$

Substituting Eq. (4), Eq. (5) and the yield criterion $p_x - \sigma_x = K$ into Eq. (3), the following differential equation for different regions of the deformation zone can be obtained

$$\begin{cases} \text{B} & \frac{1}{\delta_1} \ln(\delta_1 p_x - K) = \ln \frac{1}{h_x} + C_1 & h_2 \leq h_x \leq H \\ \text{C} & \frac{1}{\delta_2} \ln(\delta_2 p_x - K) = \ln \frac{1}{h_x} + C_2 & h_1 \leq h_x \leq h_2 \\ \text{F} & \frac{1}{\delta_3} \ln(\delta_3 p_x + K) = \ln h_x + C_3 & h \leq h_x \leq h_1 \end{cases} \quad (6)$$

where, C_1 , C_2 and C_3 are integral constants, $\tan \alpha = \frac{\Delta h}{2l}$, $l = \sqrt{R\Delta h + \left(8R\bar{p} \frac{1-\gamma^2}{\pi E}\right)^2} + 8R\bar{p} \frac{1-\gamma^2}{\pi E}$,

$$\delta_1 = \frac{1 + \tan^2 \alpha}{1 + \mu \tan \alpha} \frac{2\mu l}{\Delta h} \approx \frac{2\mu l}{\varepsilon H}, \quad \delta_2 = \left(\frac{1 + \tan^2 \alpha}{1 + \mu \tan \alpha} - \frac{1 + \tan^2 \alpha + 2\mu \tan \alpha}{1 - \mu \tan \alpha} \right) \frac{\mu l}{\Delta h} \approx -2\mu^2,$$

$$\delta_3 = \frac{1 + \tan^2 \alpha}{1 - \mu \tan \alpha} \frac{2\mu l}{\Delta h} \approx \frac{2\mu l}{\varepsilon H}.$$

The boundary conditions on the entrance of the deformation zone are $h_x = H$ and $p_x = K - \sigma_b$; the boundary conditions on the exit of the deformation zone are $h_x = h$ and $p_x = K - \sigma_f$; the positional relationship between the neutral points is $h_2 \approx i \cdot h_1$. According to Fig. 1(a), substituting this boundary conditions into Eq. (6), the rolling pressures in different regions of the deformation zone under the F+C+B configuration can be obtained, the subscripts B, C and F indicate backward-slip zone, cross-shear zone and forward-slip zone, respectively.

$$\left\{ \begin{array}{l} p_B = \frac{K}{\delta_1} \left[(\delta_1 \xi_1 - 1) \left(\frac{H}{h_x} \right)^{\delta_1} + 1 \right] \quad h_2 \leq h_x \leq H \\ p_C = \frac{K}{\delta_2} \left[W \left(\frac{h_1}{h_x} \right)^{\delta_2} + 1 \right] \quad h_1 \leq h_x \leq h_2 \\ p_F = \frac{K}{\delta_3} \left[(\delta_3 \xi_2 + 1) \left(\frac{h_x}{h} \right)^{\delta_3} - 1 \right] \quad h \leq h_x \leq h_1 \end{array} \right. \quad (7)$$

where, $\xi_1 = 1 - \frac{\sigma_b}{K}$, $\xi_2 = 1 - \frac{\sigma_f}{K}$, $W = \frac{\delta_2}{\delta_3} \left[(\delta_3 \xi_2 + 1) \left(\frac{h_1}{h} \right)^{\delta_3} - 1 \right] - 1$.

Integrating the rolling pressures, Eq. (7), along the contact length, the rolling force per unit width under the F+C+B configuration can be obtained

$$P = \frac{IK}{\Delta h} \left\{ \begin{array}{l} \frac{1}{\delta_1} \left[\frac{(\delta_1 \xi_1 - 1) H^{\delta_1}}{\delta_1 - 1} (i^{1-\delta_1} A^{1-\delta_1} - H^{1-\delta_1}) + (H - i h_1) \right] + \\ \frac{1}{\delta_2} \left[\frac{W h_1}{\delta_2 - 1} (1 - i^{1-\delta_2}) + (i - 1) h_1 \right] + \\ \frac{1}{\delta_3} \left[\frac{\delta_3 \xi_2 + 1}{(\delta_3 + 1) h^{\delta_3}} (h_1^{\delta_3+1} - h^{\delta_3+1}) - (h_1 - h) \right] \end{array} \right. \quad (8)$$

According to Fig. 1(b), where $h_1 = h$ and $h_2 < H$, substituting the boundary conditions into Eq. (6), the rolling pressures under the C+B configuration can be obtained

$$\left\{ \begin{array}{l} \text{C} \quad p_C = \frac{K}{\delta_2} \left[(\delta_2 \xi_2 - 1) \left(\frac{h}{h_x} \right)^{\delta_2} + 1 \right] \quad h \leq h_x \leq h_2 \\ \text{B} \quad p_B = \frac{K}{\delta_1} \left[(\delta_1 \xi_1 - 1) \left(\frac{H}{h_x} \right)^{\delta_1} + 1 \right] \quad h_2 \leq h_x \leq H \end{array} \right. \quad (9)$$

To make the deformation zone configuration change from F+C+B to C+B, the roll speed ratio needs to meet

$$i \geq i_{c1} = \frac{V_1}{V_2} = \frac{h_2}{h} \left(1 - \frac{h_2 - h}{2R} \right) \approx \frac{h_2}{h} = \frac{\lambda_c \varepsilon + 1 - \varepsilon}{1 - \varepsilon} = \frac{\beta}{1 - \varepsilon} \quad (10)$$

where, $h_2 = \beta \cdot H$, $\lambda_c = \frac{\beta - 1 + \varepsilon}{\varepsilon}$.

At the point $h_x = h_2$, the rolling pressure $p_{C-h_2} = p_{B-h_2}$, λ_c can be solved by the following

formular

$$\delta_1 (\delta_2 \xi_2 - 1) (1 - \varepsilon)^{\delta_2} (1 - \varepsilon + \lambda_c \varepsilon)^{\delta_1 - \delta_2} - \delta_2 (\delta_1 \xi_1 - 1) + (\delta_1 - \delta_2) (1 - \varepsilon + \lambda_c \varepsilon)^{\delta_1} = 0 \quad (11)$$

Integrating the rolling pressures, Eq. (9), along the contact length, the rolling force per unit width under the C+B configuration can be obtained

$$P = \frac{IK}{\varepsilon} \left\{ \frac{1}{\delta_1} \left[\frac{\delta_1 \xi_1 - 1}{\delta_1 - 1} (\beta^{1 - \delta_1} - 1) + (1 - \beta) \right] + \frac{1}{\delta_2} \left\{ \frac{\delta_2 \xi_2 - 1}{\delta_2 - 1} \left[(1 - \varepsilon) - \beta \left(\frac{1 - \varepsilon}{\beta} \right)^{\delta_2} \right] + \lambda_c \varepsilon \right\} \right\} \quad (12)$$

According to Fig. 1(d), the deformation zone configuration is AC, where $h_1 = h$ and $h_2 = H$.

The rolling pressures can be obtained

$$p_c = \frac{K}{\delta_2} \left[(\delta_2 \xi_2 - 1) \left(\frac{h}{h_x} \right)^{\delta_2} + 1 \right] \quad (13)$$

To keep the deformation zone as AC configuration, the roll speed ratio and the front tension need to meet

$$\left\{ \begin{array}{l} i \geq i_{c2} = \frac{V_1}{V_2} = \frac{H}{h} \left(1 - \frac{\Delta h}{2R} \right) \approx \frac{H}{h} = \frac{1}{1 - \varepsilon} \\ \sigma_{\max} \geq \sigma_f \geq \sigma_{fc} = \left(\frac{1}{1 - \varepsilon} \right)^{\delta_2} \sigma_b + \left(\frac{1 - \delta_2}{\delta_2} \right) \left[\left(\frac{1}{1 - \varepsilon} \right)^{\delta_2} - 1 \right] K \end{array} \right. \quad (14)$$

where, $\sigma_{\max}$ is the maximum engineering allowable stress.

The rolling force per unit width under the AC configuration can be obtained

$$P = \frac{IK}{\delta_2} \left\{ \frac{(\delta_2 \xi_2 - 1)(1 - \varepsilon)}{(\delta_2 - 1)\varepsilon} \left[1 - (1 - \varepsilon)^{\delta_2 - 1} \right] + 1 \right\} \quad (15)$$

According to Fig. 1(c), the deformation zone configuration is F+C, where $h_1 > h$ and $h_2 = H$.

The rolling pressures can be obtained

$$\begin{cases} \text{F} & p_F = \frac{K}{\delta_3} \left[(\delta_3 \xi_2 + 1) \left(\frac{h_x}{h} \right)^{\delta_3} - 1 \right] & h \leq h_x \leq h_1 \\ \text{C} & p_C = \frac{K}{\delta_2} \left[(\delta_2 \xi_1 - 1) \left(\frac{H}{h_x} \right)^{\delta_2} + 1 \right] & h_1 \leq h_x \leq H \end{cases} \quad (16)$$

To keep the deformation zone configuration as F+C, the roll speed ratio and the front tension need to meet

$$\begin{cases} i_{c2} > i \geq i_{c3} = \frac{V_1}{V_2} = \frac{H}{h_1} \frac{2R - \Delta h}{2R - (h_1 - h)} \approx \frac{H}{h_1} \\ \sigma_{\max} \geq \sigma_f > \sigma_{fc} \end{cases} \quad (17)$$

According to Eqs. (10), (14) and (17), the deformation zone configuration - rolling parameters relationship diagram can be obtained as Fig. 2. In the normal asymmetrical rolling process, in order to balance the production efficiency and the energy saving, the rolling technological parameters will be as close as possible to the point (σ_{fc}, i_{c2}) when the max tension $\sigma_{\max} \geq \sigma_{fc}$, or the point $(\sigma_{\max}, i_{c1})$ when the max tension $\sigma_{\max} < \sigma_{fc}$ [21]. When discussing the minimum rollable thickness of asymmetrical cold rolling, the above issues do not need to be considered.

Fig. 2. Deformation zone configuration - rolling parameters relationship diagram. ($H=0.01\text{mm}$, $\varepsilon=10\%$, $R=44\text{mm}$, $K=481.3\text{MPa}$, $f=0.1$, $\sigma_b=50\text{MPa}$).

As shown in Fig. 2, when the rolling parameters are fixed and the front tension $\sigma_f < \sigma_{fc}$, only need to increase the roll speed ratio to make it is greater than i_{c1} to ensure that the deformation zone configuration remains C+B. With the increase of the roll speed ratio, the proportion of the forward-

slip zone decreases until it is 0, the rolling force and the proportion of the backward-slip zone decrease and that of the cross-shear zone increases, as shown in Fig. 3(a). When the roll speed ratio $i \geq i_{c1}$, the above parameters no longer change with the increase of the roll speed ratio. When the front tension $\sigma_f \geq \sigma_{fc}$, with the increase of the roll speed ratio i , the deformation zone configuration changes from F+C+B to F+C and AC in turn, as shown in Fig. 2. When the roll speed ratio increases to i_{c3} , the backward-slip zone disappears, when the roll speed ratio continues to increase to i_{c2} , the forward-slip zone also disappears, the deformation zone is all the cross-shear zone, As shown in Fig. 3(b). The rolling force decreases with the increases of the roll speed ratio, until the roll speed ratio $i \geq i_{c2}$.

Fig. 3 Variations of the proportions of each zone in the deformation zone and the rolling force with the roll speed ratio: (a) $\sigma_f = 0 \text{ MPa}$; (b) $\sigma_f = 100 \text{ MPa}$. ($R = 44 \text{ mm}$, $K = 481.3 \text{ MPa}$, $f = 0.1$, $H = 0.1 \text{ mm}$, $\varepsilon = 10\%$, $\sigma_b = 0 \text{ MPa}$, $\sigma_{fc} = 50.7 \text{ MPa}$)

The rolling parameters need to exceed line i_{c1} or line i_{c2} to keep the deformation zone configuration as C+B or AC and maximize the thinning capacity of the rolling mill. According to Eq. (14), the critical roll speed ratio i_{c2} and critical front tension σ_{fc} decrease with the decrease of the reduction rate ε , as shown in Fig. 4. Therefore, it can be considered that the upper limit of the adjustable range of the roll speed ratio and tension of the asymmetrical rolling mill are greater than i_{c2} and σ_{fc} , respectively. In this paper, only the C+B and AC configuration are needed to study the minimum rollable thickness of the asymmetrical rolling.

Fig. 4 Variations of the critical roll speed ratio i_{c2} and the critical front tension σ_{fc} with the reduction rate. ($H=0.01\text{mm}$, $R=44\text{mm}$, $f=0.1$, $\sigma_b=50\text{MPa}$).

2.2 Solution of minimum rollable thickness

2.2.1 Derivation of $H_{\min}$

As shown in Fig. 2, with increasing the front tension to σ_{fc} , the critical roll speed ratio i_{c1} increases to i_{c2} , and the deformation zone configuration changes from C+B to AC. Therefore, it is only necessary to study the minimum rollable thickness of the asymmetrical rolling under the C+B configuration. The AC configuration can be approached by C+B configuration through increasing the front tension.

The average rolling pressure under the C+B configuration can be expressed as

$$\bar{p} = \frac{K}{\epsilon} \left\{ \frac{1}{\delta_1} \left[\frac{\delta_1 \xi_1 - 1}{\delta_1 - 1} (\beta^{1-\delta_1} - 1) + (1 - \beta) \right] + \bar{p}_c \right\} \quad (18)$$

$$\text{where, } \bar{p}_c = \frac{1}{-2\mu^2} \left\{ \frac{2\mu^2 \xi_2 + 1}{2\mu^2 + 1} \left[(1 - \epsilon) - \beta \left(\frac{1 - \epsilon}{\beta} \right)^{-2\mu^2} \right] + (\beta - 1 + \epsilon) \right\}.$$

When the reduction ϵH is small enough, the contact length can be expressed as

$$l = 2\bar{p} \cdot 8R \frac{1 - \nu^2}{\pi E} \quad (19)$$

Substituting Eq. (18) into Eq. (19), the following formula can be obtained

$$H = \frac{2\mu l}{\varepsilon \delta_1} = \frac{16\mu DK(1-\nu^2)}{\pi \varepsilon^2 E} \left\{ \frac{1}{\delta_1^2} \left[\frac{\delta_1 \xi_1 - 1}{\delta_1 - 1} (\beta^{1-\delta_1} - 1) + (1 - \beta) \right] + \frac{\overline{p_c}}{\delta_1} \right\} \quad (20)$$

When the back and front tension is zero, Eq. (20) can be simplified to

$$H = \frac{16\mu DK(1-\nu^2)}{\pi \varepsilon^2 E} \frac{\beta^{1-\delta_1} - \beta + \delta_1 \overline{p_c}}{\delta_1^2} = \frac{16\mu DK(1-\nu^2)}{\pi \varepsilon^2 E} f(\delta_1) \quad (21)$$

When the reduction rate ε is determined, the minimum rollable thickness appears at the minimum value of the function $f(\delta_1)$. Derivative of $f(\delta_1)$ can be obtained

$$f'(\delta_1) = \frac{-\delta_1 \overline{p_c} - \delta_1 \beta^{1-\delta_1} \ln \beta - 2\beta^{1-\delta_1} + 2\beta}{\delta_1^4} \quad (22)$$

where $\overline{p_c} = \frac{\beta}{-2\mu^2} \left[1 - \left(\frac{1-\varepsilon}{\beta} \right)^{-2\mu^2} \right]$.

Mathematically, it can be easily obtained that $f'(\delta_1)=0$ has a unique solution in the interval $(0, +\infty)$, and $f(\delta_1)$ reaches the minimum value at this solution. Iteratively solve Eq. (11) and Eq. (22), if the calculation results converge, the minimum rollable thickness is obtained by Eq. (21). Because of the positions of the neutral points in the deformation zone is determined by the velocity relationship between the strip and the upper and lower rolls, the reduction rate ε should be greater than 0. This causes the result to fail to converge when calculating the value of the minimum rollable thickness $H_{\min}$ by using the above formulas. If $\varepsilon \rightarrow 0^+$ is assumed when strip reaches the minimum rollable thickness, the proportion of each part in the deformation zone is obviously uncertain. Therefore, after confirming that the minimum rollable thickness appears in the C+B or AC configuration, the deformation zone is further assumed to be a flat plate compression process to obtain a convergence result of $H_{\min}$.

2.2.2 Solution of $H_{\min}$

When the reduction rate reaches a critical value ε_0 , the reduction $\varepsilon_0 H$ is small enough, the rolling

process is regarded as an asymmetrical flat plate compression process, as shown in Fig. 5.

Fig. 5 Schematic of the asymmetrical flat plate compression process.

According to Fig. 5, the rolling force model is remodeled. In the backward-slip zone, the following differential equation can be obtained from the force balance equation and yield criterion

$$dp_x = -\frac{2\mu}{H}dx \quad (23)$$

The boundary conditions on the entrance of the deformation zone are $x=l$ and $p_x = K - \sigma_b$. Substituting the boundary conditions into the differential equation, the rolling pressure in the backward-slip zone can be expressed as

$$p_x = (K - \sigma_b) \exp\left(\delta - \frac{2\mu x}{H}\right) \quad (24)$$

where, $\delta = \frac{2\mu l}{H}$

At the neutral point of the slow roll, $x_2 = \lambda_c l$, where $\lambda_c \in [0, 1]$, the rolling pressure p_{x_2} can be expressed as

$$p_{x_2} = (K - \sigma_b) \exp[\delta(1 - \lambda_c)] \quad (25)$$

The distribution of the rolling pressure in the cross-shear zone can be expressed as

$$\frac{dp_x}{dx} = C_0 \quad (26)$$

The boundary conditions on the exit of the deformation zone are $x=0$ and $p_x = K - \sigma_f$. Substituting the Eq. (25) and the boundary conditions into Eq. (26), the rolling pressure in the cross-shear zone can be obtained

$$p_x = C_0 x + K - \sigma_f \quad (27)$$

---

where, $C_0 = \frac{(K - \sigma_b) \exp[\delta(1 - \lambda_c)] - (K - \sigma_f)}{\lambda_c l}$.

Integrating the rolling pressure along the contact length, the average rolling pressure can be obtained

$$\bar{p} = \frac{\lambda K}{2} (\xi_1 e^m + \xi_2) + (1 - \lambda_c) K \xi_1 \frac{e^m - 1}{m} \quad (28)$$

where, $m = \delta(1 - \lambda_c)$.

Similar to Eq. (19), the thickness H can be expressed as

$$H = \frac{16\mu DK(1 - \nu^2)}{\pi E} g(m) = \frac{16\mu DK(1 - \nu^2)}{\pi E} \left[\frac{\lambda_c(1 - \lambda_c)}{2} \frac{\xi_1 e^m + \xi_2}{m} + (1 - \lambda_c)^2 \xi_1 \frac{e^m - 1}{m^2} \right] \quad (29)$$

The minimum rollable thickness can be obtained by solving the minimum value of the function $g(m)$. Derivation of $g(m)$ can be obtained

$$g'(m) = \frac{1 - \lambda_c}{m^4} \left[\frac{\lambda_c}{2} m^2 e^m + \left(1 - \frac{3\lambda_c}{2}\right) m e^m - 2(1 - \lambda_c)(e^m - 1) - \frac{\lambda_c \xi_2}{2\xi_1} m \right] \quad (30)$$

When the critical reduction rate ε_0 , the proportion of the cross-shear zone λ_c and the back and front tensions are determined, m can be obtained by the following formula

$$j(m) = \frac{\lambda_c}{2} m^2 e^m + \left(1 - \frac{3\lambda_c}{2}\right) m e^m - 2(1 - \lambda_c)(e^m - 1) - \frac{\lambda_c \xi_2}{2\xi_1} m = 0 \quad (31)$$

where, $m \in [0, +\infty)$, $\lambda_c \in [0, 1]$, $\frac{\xi_2}{\xi_1} \in \left[\frac{2 - \sqrt{3}}{2}, \frac{2}{2 - \sqrt{3}} \right]$.

Derivation of function $j(m)$ can be obtained

$$\begin{cases} j'(m) = e^m (m - 1) \left(1 - \frac{\lambda_c}{2}\right) + \frac{\lambda_c}{2} \left(m^2 e^m - \frac{\xi_2}{\xi_1}\right) \\ j''(m) = m e^m \left(1 + \frac{\lambda_c}{2}\right) + \frac{\lambda_c}{2} m^2 e^m \end{cases} \quad (32)$$

In the interval $[0, +\infty)$, $j''(m) \geq 0$, function $j'(m)$ monotonous increases. Because of $j'(0) < 0$ and $j'(m \rightarrow +\infty) > 0$, there must be $j'(m_1) = 0$, where $m_1 > 0$. When $m \in [m_1, +\infty)$,

the function $j'(m) \geq 0$. Because of $j(m_1) < j(0) = 0$ and $j(m \rightarrow +\infty) > 0$, there must be $j(m_2) = 0$, where $m_2 > m_1$. Therefore, function $g(m)$ has a minimum value in the interval $(0, +\infty)$, that is m_2 .

Substituting m into Eq. (29), the minimum rollable thickness $H_{\min}$ can be obtained. But the value of λ_c needs to be calculated iteratively through Eq. (11) and Eq. (29), the calculated result still does not converge. λ_c needs to be confirmed by other means.

2.2.3 Solution of proportion of cross-shear zone λ_c

According to the locations of the neutral points, h_1 and h_2 , the proportions of each part of the deformation zone can be obtained, when the reduction rate is determined and the back and front tension is not considered. Fig. 6(a) shows the variations of the proportions of each part of the deformation zone with the roll speed ratio. With the increase of the roll speed ratio, the proportions of the backward-slip zone and the forward-slip zone decrease almost equally, as shown in Fig. 6(b). When the proportion of the forward-slip zone decreases to zero, the deformation zone configuration changes from F+C+B to C+B.

Fig. 6 (a) Variations of proportions of each part in the deformation zone with the roll speed ratio; (b) variations of reductions of λ_F and λ_B with the roll speed ratio

Therefore, the proportion of the cross-shear zone under the C+B configuration can be obtained by calculating the proportion of the forward-slip zone in the symmetrical rolling. When other rolling

parameters are the same, the proportion of the cross-shear zone under the C+B configuration is twice as large as that of the forward-slip zone in the symmetrical rolling and can be expressed as

$$\lambda_c = \frac{2(1-\varepsilon_0)}{\varepsilon_0} \left\{ \left[\frac{1 + \sqrt{1 + (\delta_\gamma \xi_1 - 1)(\delta_\gamma \xi_2 + 1)(1-\varepsilon_0)^{-\delta_\gamma}}}{\delta_\gamma \xi_2 + 1} \right]^{\frac{1}{\delta_\gamma}} - 1 \right\} \quad (33)$$

where, $\delta_\gamma = \frac{\delta}{\varepsilon_0}$.

3. Results and discussion

Fig. 7 shows the comparison of the proportions of the cross-shear zone under C+B configuration calculated by Eq. (11) and Eq. (33), marked as λ_{c1} and λ_{c2} , respectively. With the decrease of the critical reduction rate ε_0 , the error between λ_{c1} and λ_{c2} decreases. When the critical reduction rate $\varepsilon_0 = 10\%$, the error is 2.48%. In fact, when $\varepsilon_0 = 20\%$ the error between λ_{c1} and λ_{c2} does not exceed 5%. Since the minimum rollable thickness is sensitive to changes in the proportion of the cross-shear zone, the large the critical reduction rate ε_0 , the larger the error of the minimum rollable thickness calculated by Eq. (33). At the same time, when the value of ε_0 is large, the value of $\varepsilon_0 H$ is not small enough and does not meet the previous assumptions. Therefore, it is appropriate to set the value range as $10\% \geq \varepsilon_0 > 0$.

Fig. 7 Comparison of the proportions of the cross-shear zone calculated by Eq. (11) and Eq. (33)

When the back and front tensions are zero and the critical reduction rate ε_0 is determined, m and λ_c can be iteratively calculated by Eq. (31) and Eq. (33) to obtain a unique solution, then the minimum rollable thickness $H_{\min}$ can be obtained by Eq. (29). Therefore, $H_{\min}$ can be expressed as a function of the critical reduction rate ε_0 as following

$$\begin{cases} H_{\min} = (2.1949\varepsilon_0^2 + 1.9033\varepsilon_0) \frac{\mu DK}{E} & 0 < \varepsilon_0 \leq 0.1 \\ i \geq i_{c1} = 2.0233\varepsilon_0^2 + 0.8361\varepsilon_0 + 1 \end{cases} \quad (34)$$

The effects of the back and front tensions on the minimum rollable thickness are reflected in two aspects. At first, the average rolling pressure decreases with the increases of the tensions, it is conducive to the decreases of $H_{\min}$. On the other hand, when the deformation zone configuration is C+B, the proportion of cross-shear zone λ_c and the critical roll speed ratio i_{c1} decrease with the increase of the back tension, and increase with the increase of the front tension [21]. As long as the coilers on both sides of the rolling mill are the same, it can ensure that $\sigma_f \geq \sigma_b$, so that the influence of the tensions is always conducive to the reduction of the minimum rollable thickness $H_{\min}$. When the front tension $\min\{\sigma_{fc}, \sigma_{\max}\} \geq \sigma_f \geq \sigma_b$, $H_{\min}$ can be obtained by Eq. (29), Eq. (31), Eq. (33) and the following formular.

$$\begin{cases} \lambda'_c = \lambda_c + \frac{\sigma_f - \sigma_b}{\sigma_{fc} - \sigma_b} (1 - \lambda_c) \\ i \geq i_{c1} + \frac{\sigma_f - \sigma_b}{\sigma_{fc} - \sigma_b} (i_{c2} - i_{c1}) \end{cases} \quad \min\{\sigma_{fc}, \sigma_{\max}\} \geq \sigma_f \geq \sigma_b \quad (35)$$

Fig. 8 shows the comparison of the critical roll speed ratio i_{c1} calculated by Eq. (10) and Eq. (35). The maximum error between the calculated values is 0.012%. The critical roll speed ratio i_{c1} varies almost linearly with the front tension. When the front tension $\sigma_f \geq \sigma_{fc}$, the critical roll speed ratio i_{c2} can also be approximated by this linear relationship.

Fig. 8 Comparison of the critical roll speed ratio i_{c1} calculated by Eq. (9) and Eq. (33). ($H=0.01\text{mm}$, $\varepsilon=10\%$, $R=44\text{mm}$, $K=481.3\text{MPa}$, $f=0.1$, $\sigma_b=50\text{MPa}$)

When the maximum engineering allowable stress $\sigma_{\max} \geq \sigma_{fc}$, the AC configuration can be obtained by adjusting the roll speed ratio and tensions, and the proportion of cross-shear zone $\lambda_c = 1$. According to Eq. (29), $H_{\min}$ can be expressed as

$$\begin{cases} H_{\min} = 0^+ \\ \sigma_{\max} \geq \sigma_f \geq \sigma_{fc} \text{ \& } i \geq i_{c2} \end{cases} \quad (36)$$

Fig. 9 shows the comparison between the contact length ignoring the plastic deformation (calculated by Eq. 19) and that considering the plastic deformation (calculated by Hitchcock formula), denoted by l_1 and l_2 respectively. With the increase of the critical reduction rate, the error between l_1 and l_2 increases. According to Eq. (34), the minimum rollable thickness increases with increasing the critical reduction rate. When the critical reduction rate is larger, the reduction εH cannot be ignored. When the critical reduction rate $\varepsilon_0 = 10\%$, the error between l_1 and l_2 is 1.98%, it is reasonable to set the value range of ε_0 as $(0, 10\%]$.

Fig. 9 Comparison between the contact length ignoring the plastic deformation and that considering the plastic deformation under different critical reduction rates.

Fig. 10 shows the comparison of the theoretical value with the experiment result. The experimental platform is a four-high reversing asymmetrical cold mill. The diameter of the work roll is 88mm. Lubricants use the mineral emulsifiers, the friction coefficient $\mu = 0.1$. The strip uses the annealed 430 stainless steel, and has been rolled from 0.5mm to 0.01mm in 18 rolling passes without annealing. The critical reduction rate $\varepsilon_0 = 10\%$, according to Eq. (34), the theoretical minimum rollable thickness $H_{\min} = 5.9\mu\text{m}$, the theoretical value is in good agreement with the experimental result. In fact, the reduction rate of the last two passes is very close to 10%, and the rolled piece has the potential to continue to thin, as long as the rolling force is slightly increased. The plasticity of the strip is very poor after multiple passes, since it has not been annealed. This is a problem that needs attention.

Fig.10 (a) Comparison of the theoretical value with the experimental result; (b) comparison of the critical roll speed ratio of each pass with the used value.

Fig. 11(a) shows the variation of the exit thickness of deformation zone with the rolling pass under different roll speed ratios, when the rolling force is constant. When the symmetrical rolling is used, the strip is rolled from 0.5mm to 0.046mm through 25 passes rolling and multiple annealing. After the third annealing, annealing is required for each rolling pass, but the reduction rate still decreases with the increase of the rolling pass, as shown in Fig. 11(b). When the asymmetrical rolling is used, the strip can be thinned from 0.5mm to about 1 μ m without annealing, but the required number of rolling pass decreases with the increase of the roll speed ratio. Obviously, choosing an appropriate roll speed ratio and critical reduction rate can improve the production efficiency of ultrathin strip and reduce the production costs.

Fig. 11 Variations of the (a) exit thickness and (b) reduction rate with the rolling pass under different roll speed ratios.

$$(R=44\text{mm}, f=0.1, \sigma_f=\sigma_b=100\text{MPa}, P=150\text{kN})$$

4. Conclusion

A new minimum rollable thickness model for the asymmetrical rolling is proposed based on the slab method. The calculation formulas of rolling pressure, rolling force, required critical roll speed ratio and critical front tension for F+C+B, C+B, AC and F+C deformation zone configurations are proposed respectively. Thus, the deformation zone configuration - rolling parameters relationship diagram is obtained. Based on the analysis of this diagram, when the rolling parameters keep the deformation zone configuration as C+B or AC, the rolling mill has the strongest thinning ability. The

---

minimum rollable thickness can be obtained as following

(1) When $\min\{\sigma_{fc}, \sigma_{max}\} > \sigma_f \geq \sigma_b \geq 0$ and the deformation zone is C+B configuration, the minimum rollable thickness and the required critical roll speed ratio can be calculated by Eqs. (29), (31), (33) and (35). Specially, When the tension $\sigma_f = \sigma_b = 0\text{MPa}$, the minimum rollable thickness and the required critical roll speed ratio can be calculated by Eq. (34);

(2) When the tension $\sigma_{max} \geq \sigma_f \geq \sigma_{fc}$ and the deformation zone is AC configuration, the minimum rollable thickness is close to 0 under ideal rolling conditions. The required critical roll speed ratio can be calculated by Eq. (14).

The efficiency of using asymmetrical rolling for ultrathin strip is very significant. After 18 rolling passes, the 430 stainless steel strip is rolled from 0.5mm to 10 μm without annealing. The calculated minimum rollable thickness is 5.9 μm when the critical reduction rate $\varepsilon_0 = 10\%$ and agree with the experimental result well. This model is simple and easy to use, and has high application value for the design of asymmetrical rolling mills and the formulation of the asymmetrical rolling ultrathin strip technology.

Declarations

Author contribution

Ji Wang: Conceptualization, Methodology, Writing - Original Draft, Formal analysis, Investigation, Validation, Visualization.

Xianghua Liu: Resources, Writing - Review & Editing, Supervision.

Funding This research received no funding.

Availability of data and material Data will be made available on request.

Code availability Not applicable

Ethical approval Not applicable

Consent to participate All authors have consented to participate in this study

---

Consent to publish All authors have consented to the publication of this work.

Competing interests The authors declare no competing interests.

Reference

- [1] Fu MW, Chan WL (2012) A review on the state-of-the-art microforming technologies. *Int J Adv Manuf Technol* 67(9-12):2411-2437. doi:10.1007/s00170-012-4661-7.
- [2] Liu XH, Song M, Sun XK, Zhao QL, Yu QB (2017) Advances in research and application of foil rolling. *Journal of Mechanical Engineering* 53(10). doi:10.3901/jme.2017.10.001.
- [3] Stone MD (1953) Rolling of thin strip. *Iron Steel Eng* 30(2):61.
- [4] Stone MD (1956) Rolling of thin strip partII. *Iron Steel Eng* 33(12):55.
- [5] Fleck NA, Johnson KL (1987) Towards a new theory of cold rolling thin foil.pdf. *Int J Mech Sci* 29:507-524. doi:10.1016/0020-7403(87)90012-9.
- [6] Fleck NA, Johnson KL, Mear ME, Zhang LC (1992) Cold rolling of foil. *Proc Instn Mech Engrs* 206:119-131. doi:10.1243/PIME_PROC_1992_206_064_02.
- [7] Sutcliffe MPF, Rayner PJ (1998) Experimental measurements of load and strip profile in thin strip rolling. *Int J Mech Sci* 40(9):887-899. doi:10.1016/S0020-7403(97)00138-0.
- [8] Xiao H, Ren ZK, Liu X (2017) New mechanism describing the limiting producible thickness in ultra thin strip rolling. *Int J Mech Sci* 133:788-793. doi:10.1016/j.ijmecsci.2017.09.046.
- [9] Wu SM, Wei LS, Huang G (2014) Research on the minimum rolled thickness of strip steel in cold tandem rolling. *Key Eng Mater* 622-623:993-999. doi:10.4028/www.scientific.net/KEM.622-623.993.
- [10] Wu SM, Zhou LL (2007) Research of smallest permissive-rolling thickness during strip rolling. *China Metallurgy* 17(3):10-12. doi:10.3969/j.issn.1006-9356.2007.03.003.
- [11] Tateno J, Hiruta T, Katsura S, Honda A, Miyata T, Kamimaru A (2011) Experimental analysis of thickness reduction limits in ultra thin stainless steel foil rolling. *ISIJ Int* 51(5):788-792. doi:10.2355/tetsutohagane.98.89.
- [12] Zhang LC (1995) On the mechanism of cold rolling thin foil. *Int J Mach Tools Manufact* 35(3):363-372. doi:10.1016/0890-6955(94)E0028-H.
- [13] Hwang YM, Kan CC (2016) Roll shape design for foil rolling of a four-high mill. *Int J Adv Manuf Technol* 91(5-8):1587-1597. doi:10.1007/s00170-016-9843-2.
- [14] Tang D, Liu X, Song M, Yu H (2014) Experimental and theoretical study on minimum achievable foil thickness during asymmetric rolling. *PLoS One* 9(9):e106637. doi:10.1371/journal.pone.0106637.
- [15] Tang DL, Liu XH, Li XY, Peng LG (2013) Permissible minimum thickness in asymmetrical cold rolling. *J Iron Steel Res Int* 20(11):21-26. doi:10.1016/S1006-706X(13)60191-0.
- [16] Liu X, Liu XH, Song M, Sun XK, Liu LZ (2016) Theoretical analysis of minimum metal foil thickness achievable by asymmetric rolling with fixed identical roll diameters. *Trans. Nonferrous Met. Soc. China* 26(2):501-507. doi:10.1016/s1003-6326(16)64138-9.
- [17] Tzou GY, Huang MN (2001) Study on minimum thickness for asymmetrical hot-and-cold PV rolling of sheet considering constant shear friction. *J Mater Proc Technol* 119:229-233. doi:10.1016/S0924-0136(01)00965-7.
- [18] Feng YF, Liu WW, Yang TS, Du FS, Sun JN (2019) Theoretical and experimental analysis of the deformation zone and minimum thickness in single-roll-driven asymmetric ultrathin strip rolling. *Int J Adv Manuf Technol* 104(5-8):2925-2937. doi:10.1007/s00170-019-04027-z.
- [19] Wang J, Liu XH, Guo WP (2018) Analysis of mechanical parameters for asymmetrical strip rolling by slab method. *Int J Adv Manuf Technol* 98(9-12):2297-2309. doi:10.1007/s00170-018-2368-0.

-
- [20] Wang J, Liu XH, Sun XK (2020) Study on asymmetrical cold rolling considered sticking friction. *J Mater Res Technol* 9(6):14131-14141. doi:10.1016/j.jmrt.2020.10.027.
- [21] Wang J, Liu XH, Sun XK (2020) Study on the relationship between asymmetrical rolling deformation zone configuration and rolling parameters. *Int J Mech Sci* 187. doi:10.1016/j.ijmecsci.2020.105905.
- [22] Sun XK, Liu XH, Wang J, Qi JL (2020) Analysis of asymmetrical rolling of strip considering two deformation region types. *Int J Adv Manuf Technol* 110(9-10):2767-2785. doi:10.1007/s00170-020-06022-1.
- [23] Sun XK, Liu XH, Qi JL, Wang J (2020) Experimental study on deformation region types and percentages of each region in asymmetrical rolling of strip. *IOP Conference Series: Materials Science and Engineering* 892. doi:10.1088/1757-899x/892/1/012021.
- [24] Sun X, Liu X, Wang J, Qi J (2020) Analysis of asymmetrical rolling of strip considering percentages of three regions in deformation zone. *Int J Adv Manuf Technol*. doi:10.1007/s00170-020-05690-3.
- [25] Ji YH, Park JJ (2009) Development of severe plastic deformation by various asymmetric rolling processes. *Mater Sci Eng, A* 499(1-2):14-17. doi:10.1016/j.msea.2007.11.099.

Deformation zone configuration - rolling parameters relationship diagram. ($H=0.01\text{mm}$, $\varepsilon=10\%$, $R=44\text{mm}$, $K=481.3\text{MPa}$, $f=0.1$, $\sigma_b=50\text{MPa}$).

Figure 3

Variations of the proportions of each zone in the deformation zone and the rolling force with the roll speed ratio: (a) $\sigma_f=0\text{MPa}$; (b) $\sigma_f=100\text{MPa}$. ($R=44\text{mm}$, $K=481.3\text{MPa}$, $f=0.1$, $H=0.1\text{mm}$, $\varepsilon=10\%$, $\sigma_b=0\text{MPa}$, $\sigma_{fc}=50.7\text{MPa}$)

Figure 4

Variations of the critical roll speed ratio i_{c2} and the critical front tension σ_c with the reduction rate. ($H=0.01\text{mm}$, $R=44\text{mm}$, $f=0.1$, $\sigma_b=50\text{MPa}$).

Figure 5

Schematic of the asymmetrical flat plate compression process.

Figure 6

(a) Variations of proportions of each part in the deformation zone with the roll speed ratio; (b) variations of reductions of λ_F and λ_B with the roll speed ratio

Figure 7

Comparison of the proportions of the cross-shear zone calculated by Eq. (11) and Eq. (33)

Figure 8

Comparison of the critical roll speed ratio i_{c1} calculated by Eq. (9) and Eq. (33). ($H=0.01\text{mm}$, $\varepsilon=10\%$, $R=44\text{mm}$, $K=481.3\text{MPa}$, $f=0.1$, $\sigma_b=50\text{MPa}$)

Figure 9

Comparison between the contact length ignoring the plastic deformation and that considering the plastic deformation under different critical reduction rates.

Figure 10

(a) Comparison of the theoretical value with the experimental result; (b) comparison of the critical roll speed ratio of each pass with the used value.

Figure 11

Variations of the (a) exit thickness and (b) reduction rate with the rolling pass under different roll speed ratios. ($R=44\text{mm}$, $f=0.1$, $\sigma_f=\sigma_b=100\text{MPa}$, $P=150\text{kN}$)