Five-axis Tri-NURBS Spline Interpolation Method Considering Nonlinear Error Compensation and Correction of CC Paths

Liangji Chen
Tiangong University
Zisen Wei (zisenwei@163.com)
Tiangong University https://orcid.org/0000-0002-7243-0697
Longfei Ma
Tiangong University

Research Article

Keywords: Five-axis machining, Tri-NURBS interpolation, CC paths' nonlinear error, Error compensation and correction

Posted Date: March 19th, 2021

DOI: https://doi.org/10.21203/rs.3.rs-319873/v1

Title Page

Title

Five-axis Tri-NURBS Spline Interpolation Method Considering Nonlinear Error Compensation and Correction of CC Paths

Author information

First author：professor Liangji Chen, (email: chenliangji@tiangong.edu.cn), affiliation: School of Mechanical Engineering, Tiangong University. Tianjin 300387, China.

Corresponding author and second author : Zisen Wei (email: zisenwei@163.com), affiliation :School of Mechanical Engineering, Tiangong University. Tianjin 300387, China.

Third author: Longfei Ma(email: longfordma@163.com), affiliation: School of Mechanical Engineering, Tiangong University.Tianjin 300387, China.

Abstract

In order to improve the accuracy of tool axis vector position and direction in traditional five-axis NURBS interpolation methods and the controlling accuracy of cutter contacting(CC) paths between cutter and work-piece, a five-axis Tri-NURBS spline interpolation method is presented in this article. Firstly, the spline interpolation instruction format is proposed, which includes three spline curves, such as CC point spline, tool center point spline and tool axis point spline. The next interpolation parameter is calculated based on the tool center point spline combined with the conventional parametric interpolation idea. Different from the traditional spline interpolation using the same interpolation parameter for all spline curves, the idea of equal ratio configuration of parameters is proposed in this paper to obtain the next interpolation parameter of each spline curve. The next interpolation tool center point, tool axis point and CC point on the above three spline curves can be obtained by using different interpolation parameters, so as to improve the accuracy of tool axis vector position and direction. Secondly, the producing mechanism of CC paths' nonlinear error of the traditional spline interpolation is analyzed and the mathematical calculation model of the nonlinear error is established. And then, the nonlinear error compensation and correction method is also put forward to improve the controlling accuracy of CC paths. In this method, the next CC point on the cutter can be firstly obtained according to the next interpolation tool center point, tool axis point and CC point on the three spline curves. And then, the error compensation vector is determined with the two next CC points. To correct the nonlinear error between the next CC point on the cutter and the CC point spline curve, the cutter is translated so that the two next CC points can be coincided. In the end, the new tool center point and tool axis point after translation can be calculated to obtain the motion control coordinates of each axis of machine tool. The MATLAB software is used as simulation of the real machining data. The results show that the proposed method can effectively reduce the CC paths' nonlinear error. It has high practical value for five-axis machining in effectively controlling the accuracy of CC paths and im-proving the machining accuracy of complex surfaces.

Key Words

Five-axis machining; Tri-NURBS interpolation; CC paths' nonlinear error; Error compensation and correction

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Funding

National natural science foundation of china

Award Number:51905377

Grant Recipient:Professor Liangji Chen

Conflicts of interest

Liangji Chen , Zisen Wei and Longfei Ma declare that they have no conflict of interest.

Availability of data and material

The datasets used or analysed during the current study are available from the corresponding author on reasonable request.

Code availability

The codes used or analysed during the current study are available from the corresponding author on reasonable request.

Contributors

The overarching research goals were developed by Liangji Chen , Zisen Wei and Longfei Ma. Liangji Chen , Zisen Wei and Longfei Ma established the models and calculated the predicted consequence. Liangji Chen and Zisen Wei analyzed the calculated results. The initial draft of the manuscript was written by Liangji Chen , Zisen Wei and Longfei Ma.

Consent to participate

Consent for publication

On behalf of my co-authors, I submit our manuscript entitled "Five-axis Tri-NURBS Spline Interpolation Method Considering Nonlinear Error Compensation and Correction of CC Paths" to *"The International Journal of Advanced Manufacturing Technology"* for publishing.

Five-axis Tri-NURBS Spline Interpolation Method Considering Nonlinear Error Compensation and Correction of CC Paths

Liangji Chen, Zisen Wei, Longfei Ma
(School of Mechanical Engineering, Tiangong University, Tianjin 300387, China)
Corresponding author: Zisen Wei (zisenwei@163.com)

Abstract: In order to improve the accuracy of tool axis vector position and direction in traditional five-axis NURBS interpolation methods and the controlling accuracy of cutter contacting(CC) paths between cutter and work-piece, a five-axis Tri-NURBS spline interpolation method is presented in this article. Firstly, the spline interpolation instruction format is proposed, which includes three spline curves, such as CC point spline, tool center point spline and tool axis point spline. The next interpolation parameter is calculated based on the tool center point spline combined with the conventional parametric interpolation idea. Different from the traditional spline interpolation using the same interpolation parameter for all spline curves, the idea of equal ratio configuration of parameters is proposed in this paper to obtain the next interpolation parameter of each spline curve. The next interpolation tool center point, tool axis point and CC point on the above three spline curves can be obtained by using different interpolation parameters, so as to improve the accuracy of tool axis vector position and direction. Secondly, the producing mechanism of CC paths' nonlinear error of the traditional spline interpolation is analyzed and the mathematical calculation model of the nonlinear error is established. And then, the nonlinear error compensation and correction method is also put forward to improve the controlling accuracy of CC paths. In this method, the next CC point on the cutter can be firstly obtained according to the next interpolation tool center point, tool axis point and CC point on the three spline curves. And then, the error compensation vector is determined with the two next CC points. To correct the nonlinear error between the next CC point on the cutter and the CC point spline curve, the cutter is translated so that the two next CC points can be coincided. In the end, the new tool center point and tool axis point after translation can be calculated to obtain the motion control coordinates of each axis of machine tool. The MATLAB software is used as simulation of the real machining data. The results show that the proposed method can effectively reduce the CC paths' nonlinear error. It has high practical value for five-axis machining in effectively controlling the accuracy of CC paths and im-proving the machining

accuracy of complex surfaces.

Key Words: Five-axis machining; Tri-NURBS interpolation; CC paths' nonlinear error; Error compensation and correction.

Introduction

Five-axis CNC machines are widely used for machining precision devices such as impellers, blades, and precision optical parts due to their high speed, high precision, and the ability to machine complex free-form surfaces. Compared with the three-axis CNC machine, it has two extra axes endowing it with a higher degree of machining freedom. However, a new principle error, the nonlinear error [1-4], is caused by the rotary motion of the cutter, which has become an urgent problem to be solved in the field of five-axis machining technology.

Scholars around the world have proposed various solutions to the nonlinear error control problem of the five-axis machine. Wu Jichun et al. [5] established a nonlinear error model based on harmonic functions according to the error distribution in the classical post-processing to achieve the real-time error compensation at the middle interpolation points. Yang Xujing et al. [6] and Zheng Fengmo et al. [7] controlled the nonlinear error by interpolation at the endpoint of the tool axis between adjacent interpolation points, but the method would significantly increase the number of CNC programs. To reduce the nonlinear error, Wu Zhiqing et al. [8] interpolated the tool axis vector according to the error constraints and obtained the new tool axis vector by a projection method. Ming-Che Ho et al. [9] and LIANG et al. [10] reduced the nonlinear error caused by cutter oscillation through controlling the cutter attitude to achieve the smooth transition between cutter vectors. Dong Chaojie et al. [11] gave an interpolation algorithm with the integrated RTCP function and fixed the nonlinear error. Aiming to ensure the continuity of the interpolation path, Hong et al. [12] proposed an interpolation method that can reduce the rotation error angle by analyzing the nonlinear error between cutter positions during machining. Xu et al. [13] proposed a fairing method for five-axis machining cutter path, which ensures the smooth transition between cutter orientations so as to reduce nonlinear errors. Liu et al. [14] proposed a dual NURBS cutter path interpolation method based on the improved co-evolutionary genetic algorithm and third-order derived Newton-type parameter calculation to reduce the interpolation error and improve the real-time interpolation performance. To obtain a smoother machined surface, Zhou [15] et al. explained the nonlinear error in terms of the cutter path and real machining cutter motion and gave a mathematical model of the error. Li et al. [16] densified the cutter path and data to control the nonlinear error. Zhang et al. [17] proposed a single spherical linear interpolation method and established an optimized algorithm to avoid the nonlinear error.

All of the above methods can effectively reduce the non-linear error of the cutter center point path during the five-axis machining, but they are limited in precisely controlling the accuracy of the cutter-part CC point path. To address this problem, this paper investigates a five-axis Tri-NURBS interpolation method based on the synchronization method of spline interpolation recorded in the literature [18-20] to realize the interpolation control of the tool center point, axis points, and contact spline path considering the nonlinear error of the CC path. When the current three points are obtained, the real CC points can be found according to their geometric relationships. Using the position relationship between the real CC point and the theoretical interpolated CC point, and considering the nonlinear error compensation repair model, we obtained the new tool center point position information, and ultimately improved the control accuracy of the CC path.

1 NURBS curves and data interpolation densification

1.1 NURBS curve expressions

In the 3D Cartesian space, a pth NURBS curve with respect to the parameter ξ can be expressed as:

$$\boldsymbol{Q}(\xi)=\frac{\sum_{i=0}^{n} N_{i,p}(\xi) r_i \boldsymbol{P}_i}{\sum_{i=0}^{n} N_{i,p}(\xi) r_i}=\left[x(\xi) \;\; y(\xi) \;\; z(\xi)\right]' \tag{1}$$

where $\{\boldsymbol{P}_i\}$ is the n+1 3D control points of the NURBS curve; $\{r_i\}$ is the corresponding weight value of each control point; $\{N_{i,p}(\xi)\}$ is the pth B-spline basis function defined on the node vector $\boldsymbol{X}$, and $\boldsymbol{X}=\{\xi_0,\xi_1,\mathrm{L}\;\; \xi_{n+p+1}\}$.

The expression to calculate the pth B-spline basis function is as follows

$$\begin{cases} N_{i,0}(\xi)=\begin{cases}1 & \xi_i<\xi<\xi_{i+1} \\ 0 & \text{else}\end{cases} \\ N_{i,p}(\xi)=\dfrac{\xi-\xi_i}{\xi_{i+p}-\xi_i}N_{i,p-1}(\xi)+\dfrac{\xi_{i+p+1}-\xi}{\xi_{i+p+1}-\xi_{i+1}}N_{i+1,p-1}(\xi) \\ specify\,\dfrac{0}{0}=0 \end{cases} \tag{2}$$

1.2 Five-axis Tri-NURBS spline interpolation format

In the five-axis double spline interpolation method, the tool center point spline is usually used to describe the tool center point path, and the tool axis point spline curve is used to describe the change of the cutter axis vector, i.e., for the determination of the swing angle of the cutter. Although this method can effectively control the accuracy of the tool center point path, it fails to consider the influence of the rotary motion of the cutter on the accuracy of the CC point path after two extra rotary axes are involved in the interpolation. In the real machining of complex curved parts, the key factor in determining the machining quality of the part surface is the accuracy of the CC path between the cutter and the part surface. It can be seen that in order to improve the surface machining quality of the part, the nonlinear error of the CC path generated during the spline interpolation process has to be compensated and repaired in real-time. In this paper, we attempt to introduce a CC curve based on the existing double NURBS spline interpolation method to describe the ideal contact path between the cutter and the part surface (Figure 1). In the proposed Tri-NURBS interpolation method, the tool center point spline curve and tool axis point soline curve are used to control the cutter position and attitude angle, respectively; and the CC point spline curve is used as an important reference for the compensation and repair of the nonlinear error of the CC path.

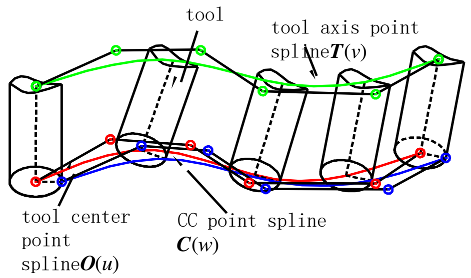

Figure.1 The spline curves in Tri-NURBS interpolation

At the same time, to avoid the increase of the cutter axis vector velocity fluctuations when the same interpolation parameters are used for the conventional double spline interpolation, this paper proposes a five-axis Tri-NURBS interpolation method based on the parameter position synchronization of the NURBS spline curve constructed with differential node vectors.

In summary, in order to improve the control accuracy of cutter axis vector and provide a basis for the repair of CC nonlinear error and improve the control accuracy of CC path, this paper proposes a Tri-NURBS spline interpolation method for five-axis machining and defines its interpolation instruction format as shown in Figure 2.

In the Tri-NURBS spline interpolation format, NURBSON and NURBSOFF indicate the interpolation start and end commands, respectively; P is the number of times the three NURBS spline curves are constructed; F is the feed rate of the tool center point; X/Y/Z stores the control point coordinates of the tool center point spline curve, TX/TY/TZ stores the control point coordinates of the tool axis point spline curve, CX/CY/CZ stores the control point coordinates of the CC point spline curve, and R is the weight value of the control point; K, TK and CK indicate the node values for constructing the tool center point spline curve, tool axis point spline curve and CC point spline curve respectively.

```
NURBSON P_F_;
X_Y_Z_K_TX_TY_TZ_TK_CX_CY_CZ_CK_R_;
X_Y_Z_K_TX_TY_TZ_TK_CX_CY_CZ_CK_R_;
......
X_Y_Z_K_TX_TY_TZ_TK_CX_CY_CZ_CK_R_;
K_TK_CK_;
......
K_TK_CK_;
NURBSOFF;
```

Fig.2 Five-axis Tri-NURBS interpolation format

1.3 The implementation of Tri-NURBS spline interpolation

As mentioned before, according to the interpolation format defined in Figure 2, three NURBS curves can be determined according to the equation (1), i.e., the tool center point spline curve $\boldsymbol{O}(u)$, tool axis point spline curve $\boldsymbol{T}(v)$ and CC point spline curve $\boldsymbol{C}(w)$ (Figure 1). The tool center point is a special position point, which can be used for the precise location of the cutter during the machining. Therefore, during Tri-NURBS interpolation, the tool center point spline curve $\boldsymbol{O}(u)$ still needs to be regarded as the reference position of the cutter in the CNC system. According to the principle of CNC parametric interpolation, the goal of Tri-NURBS interpolation is to calculate the parameters u_{i+1} corresponding to the tool center point at the next interpolation time ($t = t_{i+1}$) based on the tool center point spline curve $\boldsymbol{O}(u)$, and the parameter u_i corresponding to the tool center point at the current interpolation time ($t = t_i$).

In the parameter space, the parameters u_{i+1} can be expanded as the second-order Taylor expressions on the time parameters u_i when $t = t_i$

$$u_{i+1} = u_i + \tau \left.\frac{du}{dt}\right|_{t=t_i} + \frac{\tau^2}{2} \left.\frac{d^2u}{dt^2}\right|_{t=t_i} + \Delta(u_i^2) \tag{3}$$

where: $\Delta(u_i^2)$ is the higher-order infinitesimal residual term; τ is the interpolation period of the CNC, and $\tau = t_{i+1} - t_i$.

According to the velocity f of the feed direction along the tool center point spline curve $\boldsymbol{O}(u)$, and interpolation period τ, and ignoring the higher-order infinitesimal remainder term, the equation (3) can be transformed into

$$u_{i+1} = u_i + \frac{f\tau}{\left.\left\| \frac{d\boldsymbol{O}(u)}{du} \right\|\right|_{u=u_i}} - \frac{f^2\tau^2 \left.\left(\frac{d\boldsymbol{O}(u)}{du} g \frac{d^2\boldsymbol{O}(u)}{du^2} \right)\right|_{u=u_i}}{2\left.\left\| \frac{d\boldsymbol{O}(u)}{du} \right\|^2\right|_{u=u_i}} \tag{4}$$

At this point, the obtained parameter u_{i+1} is brought into the spline curve expression of the tool center point $\boldsymbol{O}(u)$ to obtain the coordinates of the next interpolation tool center point $\boldsymbol{O}(u_{i+1})$.

If the interpolation parameters of the three spline curves are taken as u_{i+1}, the same as those of double NURBS interpolation in the Tri-NURBS interpolation, the interpolation calculation process can be simplified, and the interpolation calculation time shortens, but a bigger problem will occur, which is the unfixed distance between the tool center point, tool axis points and CC point of each cutter position after interpolation. Moreover, the spatial position relationship is not consistent between the corresponding three points given by the programming software. The consequences of these problems are: assuming the interpolated tool center point is accurate, the deviation of interpolated tool axis point (CC point) from the programmed tool axis point (CC point) will result in the error of the cutter axis vector (theoretical CC position error), which will increase the inaccuracy of the cutter attitude angle or the inaccurate reference position of the CC non-linear error compensation repair, as shown in Figure 3.

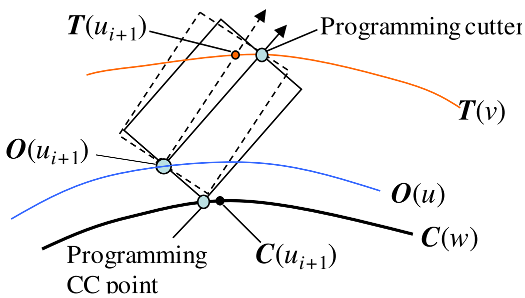

Fig.3 Tri-NURBS interpolation with same parameters

In order to avoid the above situations, the spatial positions of the three points calculated by interpolation need to coincide with those of the three points given by programming. To achieve this, it is necessary to consider the different parameters of spline curves for the tool center point, tool axis point, and CC point, respectively. During programming, the sequence sets of tool center point, tool axis point, and CC point are obtained, respectively. In order to make the spatial positions of three points obtained by interpolation coincide with those of the three points required during programming at the same time, the parameters of each curve need to be changed in their respective parameter space in a cooperative manner. In this way, the positions of the three points calculated are exactly the same as those of the three points at the time of programming after the interpolated parameter values are substituted into the respective NURBS curve expressions. Accordingly, this paper proposes a method for parameter position synchronization for Tri-NURBS spline curve interpolation to solve the problem.

From equation (4), the parameters of the interpolation tool center-point u_{i+1} can be calculated. Based on this, we can determine the nodal interval $(u_j, u_{j+1}]$ (*j*=*p*, *p*+1,…,*n*+*p*−3), in which u_{i+1} is located in the nodal vector ***U*** of the center point spline ***O***(*u*). The parameter interpolation speed ratio of the interpolated tool center point in the nodal interval $(u_j, u_{j+1}]$ is

$$Vel = \frac{u_{i+1} - u_j}{u_{j+1} - u_j} \tag{5}$$

From Fig. 2, it can be seen that the three spline curves ***O***(*u*)、***T***(*v*), and ***C***(*w*) in the Tri-NURBS interpolation method in this paper have the same number of curves and control points, thus leading to the same number of nodes in the corresponding node vectors ***U, V***, and ***W***. Accordingly, the *j*th nodes in intervals $(u_j, u_{j+1}]$、$(v_j, v_{j+1}]$ and $(w_j, w_{j+1}]$ in ***U, V***, and ***W*** can be corresponded one-to-one. More specifically, if the parameters of the interpolation tool center point u_{i+1} are located in the nodal interval $(u_j, u_{j+1}]$, when calculating the parameters of the interpolation tool axis point v_{i+1} and the parameters of the interpolation CC w_{i+1}, the nodal intervals $(v_j, v_{j+i}]$ and $(w_j, w_{j+1}]$ in which the two parameters locate can be found in the nodal vectors V and W, respectively.

In order to ensure that the positions of three points obtained by interpolation coincide with those of the three points obtained by programming at the same time, it is necessary to ensure the equal ratio of the positions for parameters at each point in the respective nodal intervals, which should be expressed in the following equation:

$$\frac{v_{i+1} - v_j}{v_{j+1} - v_j} = \frac{w_{i+1} - w_j}{w_{j+1} - w_j} = Vel \tag{6}$$

The parameters of the interpolation tool axis point v_{i+1} and the parameters of the interpolation CC point w_{i+1} are

$$\begin{cases} v_{i+1} = Vel g(v_{j+1} - v_j) + v_j \\ w_{i+1} = Vel g(w_{j+1} - w_j) + w_j \end{cases} \tag{7}$$

The interpolation parameters are calculated by the Equation. (6) and the interpolation parameters are brought into the expressions of the tool axis point spline curve T(v) and the CC point spline curve C(w), respectively, to obtain the corresponding coordinates of the interpolation points, $\boldsymbol{T}(v_{i+1})$ and $\boldsymbol{C}(w_{i+1})$.

2 Generation of Tri-NURBS spline curves

In the traditional five-axis CNC system, the linear interpolation method is often used to process complex free-form surfaces. The CAM software first generates the cutter cutting path, and then creates the discrete cutter path to generate the cutter position file containing the tool center point, cutter axis vector and CC point. The cutter position file contains a large number of tiny linear segments, and the format of the cutting statement is as follows:

$$GOTO / x, y, z, i, j, k \$\$ x', y', z'$$

In the above format: (x, y, z) is $\boldsymbol{O}_i$, the coordinate data of tool center point position in workpiece coordinate system; (i, j, k) is $\boldsymbol{A}_i$, the unit vector data of cutter axis in workpiece coordinate system; (x', y', z') is $\boldsymbol{C}_i$, the coordinate data of CC point position in workpiece coordinate system.

The machining of the complex free-form surface by linear approximation will lead to low machining accuracy; and the interpolation control of a large number of tiny linear segments will result in the high-frequency accelerated and decelerated motions by the machine, which will eventually affect the machining efficiency and quality. In this regard, the discrete cutter position data $\boldsymbol{O}_i$, $\boldsymbol{A}_i$, and $\boldsymbol{C}_i$ generated by the CAM software are fitted to three NURBS spline curves using the chord length parameterization method, and then the Tri-NURBS interpolation method proposed in Subsection 1 is used to achieve the efficient and high precision machining of complex free-form surfaces. It should be specially noted that in order to fit the tool axis point spline curve $\boldsymbol{T}(v)$, the fitted data points Ti of the tool axis point spline curve need to be calculated using Oi, the coordinates of the tool center point position and Ai, the cutter axial unit vector in the cutter position file, which are calculated as follows

$$\boldsymbol{T}_i = \boldsymbol{O}_i + L g \boldsymbol{A}_i \tag{8}$$

L in the equation (8) is the distance between the tool center point and a fixed point on the cutter axis.

3. Compensation and repair of nonlinearerror for CC point

In the five-axis machining of complex free-form parts, the CC point's spline curve $\boldsymbol{C}(w)$ can better reflect the real geometry of the part surface better than the micro-line segment , its control accuracy directly determines the surface machining quality of the part. Therefore, $\boldsymbol{C}(w)$ is considered as the ideal machining path in our study when a cutter is utilized to cut the part surface. In the current five-axis double NURBS spline curve interpolation method, the CNC system implements motion control with axis interpolation for the tool center's spline $\boldsymbol{O}(u)$ and the orientation spline $\boldsymbol{T}(v)$, and the two rotary axes of the five-axis machine cutter participating in interpolation will lead to the rotary motion of the cutter, which in turn affects the path accuracy of the CC point. As shown in Fig. 4, if the spline interpolation is performed directly without intervention, the cutting edge of the cutter at the i+1st interpolation position at the next interpolation moment may deviate from the ideal interpolation point $\boldsymbol{C}(w_{i+1})$ on the CC point's spline curve, directly bringing about nonlinear errors in the CC point path during the double NURBS spline interpolation.

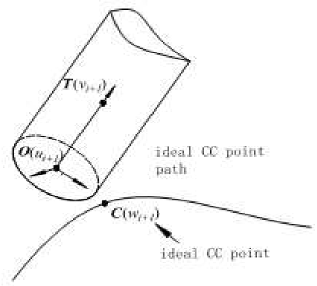

Fig.4 Schematic diagram of nonlinear error for CC point

3.2 Approximation of nonlinear error for CC point

Following the aforementioned method, after obtaining the Tri-NURBS CNC machining code for the surface machining of the part, the corresponding data in the code are read; the target parameters u_{i+1} v_{i+1} and w_{i+1} for the splines of tool center point, tool axis point and CC point at the next interpolation moment can be obtained from Eq. (4), (5), (6) and (7), respectively, and then the coordinates of three points $\boldsymbol{O}(u_{i+1})$, $\boldsymbol{T}(v_{i+1})$ and $\boldsymbol{C}(w_{i+1})$ for the i+1st interpolated position can be calculated. The possible spatial positions of the above three points are shown in Fig. 5 where the circle represents the cutting edge of the cutter.

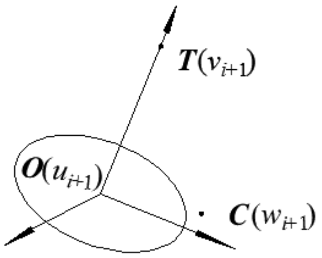

Fig.5 position relationship between three points

Theoretically, the contact between the cutter and the part should be point contact, namely, CC point. However, in the real machining, subjected to the cutting force, the cutter and the part contact more closely, and line contact is often identified between the cutting edge of the cutter and the surface of the part, making it more difficult to determine the position of the real CC point $\boldsymbol{C}_{\text{real}}(w_{i+1})$ when calculating the compensation and repair at such position. To solve the problem and shorten the compensation and repair distance, the point on the cutting edge at the cutter bottom with the shortest distance to the ideal CC point $\boldsymbol{C}(w_{i+1})$ is seen as the real CC point in this paper.

As shown in Fig. 6, the ideal CC point $\boldsymbol{C}(w_{i+1})$ is projected to the space plane where the cutting edge of the cutter is located. Assuming that the projection point is named as B. The unit vector b in the vector direction from the interpolated tool center point $\boldsymbol{O}(u_{i+1})$ to B can be calculated as

$$\boldsymbol{b} = \frac{\left[\boldsymbol{A}_{i+1} \times \left(\boldsymbol{C}\left(w_{i+1}\right) - \boldsymbol{O}\left(u_{i+1}\right)\right)\right] \times \boldsymbol{A}_{i+1}}{\left|\left[\boldsymbol{A}_{i+1} \times \left(\boldsymbol{C}\left(w_{i+1}\right) - \boldsymbol{O}\left(u_{i+1}\right)\right)\right] \times \boldsymbol{A}_{i+1}\right|} \tag{9}$$

where, $\boldsymbol{A}_{i+1}$ is the unit vector of the cutter axis at the i+1st interpolation moment. $\boldsymbol{A}_{i+1}$ can be calculated with the following equation:

$$\boldsymbol{A}_{i+1} = \frac{\boldsymbol{T}\left(v_{i+1}\right) - \boldsymbol{O}\left(u_{i+1}\right)}{\left|\boldsymbol{T}\left(v_{i+1}\right) - \boldsymbol{O}\left(u_{i+1}\right)\right|} \tag{10}$$

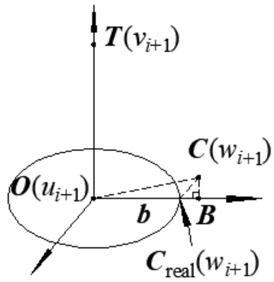

Figure.6 Schematic diagram of the real CC point

The line connecting the interpolated tool center point $\boldsymbol{O}(u_{i+1})$ and B will intersect with the cutting edge circle of the cutter at one point; and this point is the real CC point $\boldsymbol{C}_{\text{real}}(w_{i+1})$ defined before. Since B is the projection point of the ideal CC point $\boldsymbol{C}(w_{i+1})$ in the plane where the cutter's cutting edge lies in, the real CC point $\boldsymbol{C}_{\text{real}}(w_{i+1})$ can be obtained by the following equation

$$C_{\text{real}}(w_{i+1}) = O(u_{i+1}) + r \cdot b \quad (11)$$

where r is the radius of the cutter.

As shown in Fig. 7, the nonlinear error ε of the CC point path can be defined as the minimum distance that the real CC point $C_{\text{real}}(w_{i+1})$ deviates from the spline curve C(w) of CC point, and ε can be expressed as

$$\varepsilon = \min_{0 \le w \le 1} \left\{ \left| C_{\text{real}}(w_{i+1}) - C(w) \right| \right\} \quad (12)$$

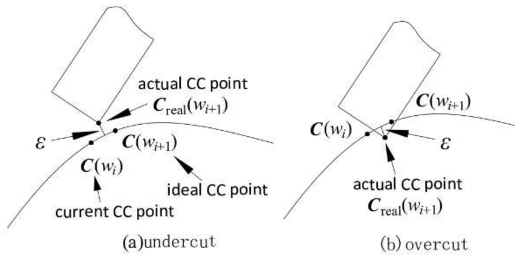

Fig.7 Schematic diagram of nonlinear error for CC point path

When solving ε, performing a global search for all parameter values w of the CC point's spline curve $C(w)$ would be laborious and difficult. For this reason, the search interval of the parameter w is quickly determined by using the position relationship between the real CC point $C_{\text{real}}(w_{i+1})$, the current CC point $C(w_{i+1})$, and the ideal CC point $C(w_{i+1})$.

The machining undercutting shown in Fig. 7a is partially enlarged to facilitate the drawing of the three spatial position relations shown in Fig. 8, respectively. In the triangle formed by the real CC point $C_{\text{real}}(w_{i+1})$, the current CC point $C(w_i)$, and the ideal CC point $C(w_{i+1})$, when the coordinates of three vertexes are known, and the angles α and β can be calculated based on the sine and cosine theorem of the triangle. The parameter increment $d_1 = |w_{i+1} - w_i|$ is used as the length of the search interval of the parameter w.

(1) When $\alpha > 90°$ and $\beta < 90°$, the search interval of the parameter w can be initially determined as $[w_i - d_1, w_i]$ (as shown in Fig. 8a).

(2) When $\alpha \le 90°$ and $\beta < 90°$ or $\alpha < 90°$ and $\beta \le 90°$, the search interval of parameter w can be determined as $[w_i, w_{i+1}]$ (as shown in Fig. 8b).

(3) When $\alpha < 90°$ and $\beta > 90°$, the search interval of parameter w can be determined initially as $[w_{i+1}, w_{i+1} + d_1]$ (as shown in Fig. 8c).

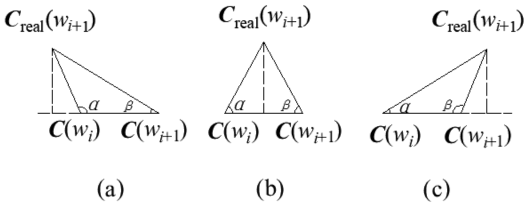

Fig8. Three different cases for the search interval

For Cases (1) and (3), the search interval can be initially determined as $[w_-, w_+]$; w_- and w_+ are substituted into the expression of CC point spline curve C(w), respectively, thus obtaining $C(w_-)$ and $C(w_+)$. In the triangle formed by $C_{\text{real}}(w_{i+1})$, $C(w_-)$ and $C(w_+)$, the edge and angle relations are utilized again to solve angles α and β. Further, the above method is adopted to initially or ultimately redetermine the search interval of the parameter w. The above determination process is repeated until the ultimate search interval of Case (2) is finalized.

After the search interval of w is finalized, the approximation of the minimum distance ε can be carried out for search solving. The core idea is to divide the search interval of the parameter w into several parameters at equal spacing. After that, the relatively small distance and its corresponding parameter can be obtained by using Eq. (12). With the parameter as the central parameter, the search interval is narrowed and then divided again at equal spacing until the difference between the smaller distances obtained before and after is less than the preset solution error. At this time, the search calculation stops. The specific search solving process is described as follows.

Supposing that the finalized search interval of the parameter w is set as $[w_{\text{left}}, w_{right}]$, which is divided into X parameters at equal spacing, then the spacing between parameters should be

$$\Delta w = \frac{1}{X+1}(w_{\text{right}} - w_{\text{left}}) \qquad (13)$$

Therefore, each parameter value is calculated as

$$w_j = w_{\text{left}} + j\Delta w \quad (j = 1, 2, \ldots, X) \quad (14)$$

The smaller distance ε_1 for the first search can be obtained by substituting w_{left}, w_{right} and all w_j into Eq. (12).

In the second search, the length of the search interval is reduced to half that of the previous search interval by using the corresponding parameter ε_1 as the central parameter w_{middle}, i.e. $d_2 = d_1/2$, and w_{left} and w_{right} are recalculated as

$$w_{\text{left}} = w_{\text{middle}} - d_2/2 \qquad (15)$$

$$w_{\text{right}} = w_{\text{middle}} + d_2/2 \qquad (16)$$

After another division of the search interval $[w_{\text{left}}, w_{right}]$ into X parameters at equal spacing, the smaller distance ε_2 of the second search is found.

At this time, it is determined whether $|\varepsilon_2 - \varepsilon_1|$ is less than or equal to the preset solution error σ. If yes, the minimum distance ε is the smaller one of ε_1 and ε_2; if not, we continue to search for the 3rd, 4th, and mth time until $|\varepsilon_m - \varepsilon_{m-1}| \leq \sigma$ is established. The minimum distance ε is the smaller one of ε_m and ε_{m-1}.

The above approximation of the minimum distance for undercutting is also applicable to the minimum distance ε for overcutting shown in Fig. 7b.

3. 3 Nonlinear error control for CC point

In this paper, the machining tolerance is $[\varepsilon]=2\mu m$. If the interpolated cutter position is found with $\varepsilon > [\varepsilon]$, then position compensation and repair shall be made at the tool center point and tool axis point. The basic principle is to compensate and repair the nonlinear error ε without changing the axial unit vector of the cutter. The direction vector of the error compensation and repair can be determined by Eq. (17) in combination with the real and ideal CC points.

$$\boldsymbol{E}_{\text{cr}} = \frac{\boldsymbol{C}(w_{i+1}) - \boldsymbol{C}_{\text{real}}(w_{i+1})}{\left|\boldsymbol{C}(w_{i+1}) - \boldsymbol{C}_{\text{real}}(w_{i+1})\right|} \qquad (17)$$

The new tool center point $\boldsymbol{O}_{\text{new}}(u_{i+1})$ and new tool axis point $\boldsymbol{T}_{\text{new}}(w_{i+1})$ are obtained after compensating and repairing the current interpolated tool center point and tool axis point with the position compensation amount ε, as shown below

$$\begin{cases} \boldsymbol{O}_{\text{new}}\left(u_{i+1}\right)=\boldsymbol{O}\left(u_{i+1}\right)+\varepsilon g\boldsymbol{E}_{\text{cr}} \\ \boldsymbol{T}_{\text{new}}\left(w_{i+1}\right)=\boldsymbol{T}\left(w_{i+1}\right)+\varepsilon g\boldsymbol{E}_{\text{cr}} \end{cases} \tag{18}$$

It can be seen from Eq. (18) that the cutter axis vector is constant because the tool center point and the tool axis point move in the same direction and for the same distance during compensate. The technical flow of the above five-axis Tri-NURBS spline interpolation method containing nonlinear error compensation and repair of the CC point path is shown in Fig. 9.

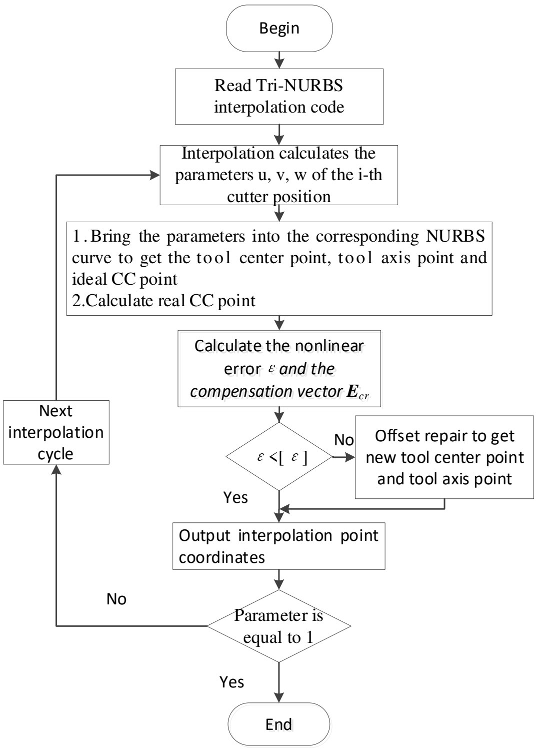

Figure.9 Tri-NURBS interpolation and nonlinear error compensation

4. Simulation Instance and Result Analysis

In order to verify the effectiveness of the proposed method, the CAD module of UG/NX10.0 is adopted to construct a three-dimensional model of an integral impeller, and the discrete cutter position data (cutter radius r=5mm) for the five-axis machining of the impeller runner are obtained, and a machining cutter position file is formed through the calculations by the CAM module of UG/NX10.0. The cutting and machining statements mainly consist of a series of GOTO statements. The GOTO statements for one of the runner machining cutter position data are extracted and listed in Table 1. The distance between the tool axis point and the tool center point is set to be a constant L=7mm. By using Eq. (8), the unit vector (i,j,k) along the cutter axis direction is transformed into the tool axis point, respectively. The spline fitting of the tool center point, tool axis point, and CC point is made based on NURBS spline method introduced in Section 2, forming three parallel NURBS spline curves, whose spatial position relationship is shown in Fig. 10.

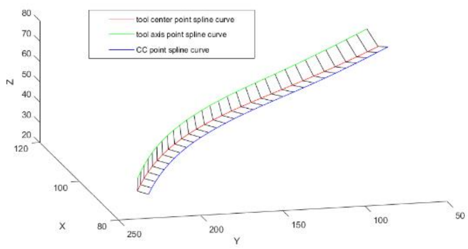

Fig.10 Tri-NURBS spline after fitting

Following the interpolation format shown in Fig. 2, a piece of the five-axis Tri-NURBS spline interpolation CNC machining program for impeller runner machining is obtained as follows.

Table 1 NC machining cutter datasheet

序号	x/mm，y/mm，z/mm	i，j，k	x'/mm，y'/mm，z'w/mm
1	105.5358，61.4483,72.4432	0.4982，0.3784，0.7801	101.9968，59.7376，75.5333
2	106.6209，64.6477,69.0715	0.4707，0.3807，0.7959	103.0860，64.7633，72.0637
......	……	……	……

29	90.6741, 223.9203, 26.0905	-0.0461, 0.0115,0.9989	88.7990, 219.2853, 26.0573
30	87.6732, 229.4377, 26.0308	-0.0499, 0.0002,0.9988	85.8925, 224.7664, 25.9426

```
NURBSON P3 F333;
X105.5358 Y61.4483 Z72.4432 K0 TX109.
X106.2678 Y64.8924 Z70.1670 K0 TX109.
6343 TY67.5438 TZ75.7045 TK0 CX102.7
362 CY63.0712 CZ73.1958 CK0 R1;
X107.3417 Y70.1190 Z66.8380 K0 TX110.
4995 TY72.7734 TZ72.4987 TK0 CX103.8
084 CY68.1282 CZ69.7530 CK0 R1;
X108.6938 Y77.2286 Z62.6019 K0 TX111.
5336 TY79.8778 TZ68.4281 TK0 CX105.1
495 CY75.0118 CZ65.3399 CK0 R1;
X105.6542 Y82.6347 Z59.5376 K0.0690 T
X112.2365 TY85.2647 TZ65.4911 TK0.069
2 CX106.1015 CY80.2559 CZ62.1285 CK0.
0689 R1;
……
X87.6732 Y229.4377 Z26.0308 K0.9310 T
X87.3240 TY229.4388 TZ33.0221 TK0.930
6 CX85.8925 CY224.7664 CZ25.9426 CK0.
9311 R1;
K1 TK1 CK1;
K1 TK1 CK1;
K1 TK1 CK1;
K1 TK1 CK1;
NURBSOFF;
```

After reading the Tri-NURBS program by the MATLAB program, the interpolation data densification of each spline curve is completed after the five-axis Tri-NURBS interpolation calculation in this paper.

Figs. 11-13 compare the three components of vector $\boldsymbol{A}$, i.e. i, j, k. The unit vector $\boldsymbol{A}$ along the cutter axis direction in the workpiece coordinate system. is obtained using the same parameters for interpolation, different parameters for interpolation based on the

```
0234 TY64.0972 TZ77.9039 TK0 CX101.9
968 CY59.7376 CZ75.5333 CK0 R1;
```

equipartition method of the parameters in this paper, and the original data at the node, respectively. In each of the above cases, the calculation model of the unit vector A along the cutter axis direction is as follows

$$\begin{cases} \boldsymbol{A}_{\text{Same parameters}} = \dfrac{\boldsymbol{T}(u_{i+1}) - \boldsymbol{O}(u_{i+1})}{\left|\boldsymbol{T}(u_{i+1}) - \boldsymbol{O}(u_{i+1})\right|} \\ \boldsymbol{A}_{\text{Different parameters}} = \dfrac{\boldsymbol{T}(v_{i+1}) - \boldsymbol{O}(u_{i+1})}{\left|\boldsymbol{T}(v_{i+1}) - \boldsymbol{O}(u_{i+1})\right|} \\ \boldsymbol{A}_{\text{Node}} = \dfrac{\boldsymbol{T}(v_{\text{node}}) - \boldsymbol{O}(u_{\text{node}})}{\left|\boldsymbol{T}(v_{\text{node}}) - \boldsymbol{O}(u_{\text{node}})\right|} \end{cases} \tag{19}$$

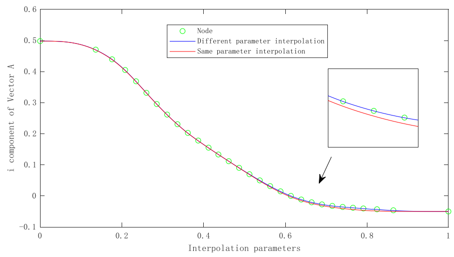

Fig.11 The i component of Vector $\boldsymbol{A}$

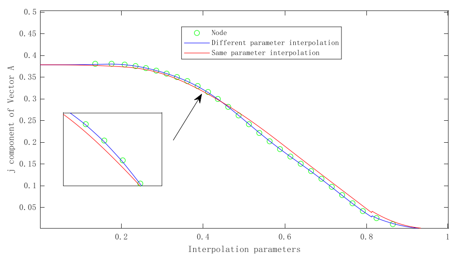

Fig.12 The j component of Vector $\boldsymbol{A}$

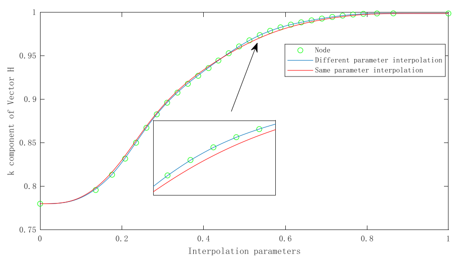

Fig.13 The k component of Vector $\boldsymbol{A}$

From the data extraction process mentioned above, it can be seen that the data at the node is the original data of the machining cutter position before NURBS spline fitting, so the unit vector along the cutter axis direction at the node should be an ideal value after spline interpolation, and it should be taken as the reference standard for determining the accuracy of the unit vector along the cutter axis direction after spline interpolation. According to Figs. 11-13, it is concluded that the unit vector along the cutter axis direction obtained by different parameter interpolation methods proposed is strictly consistent with that of the cutter axis direction at the node, while the unit vector along the cutter axis direction obtained by the same parameter method is identified with deviations. This also indicates that if spline interpolation is carried out for all the interpolation parameters of the tool center point, it will cause deviation between the interpolated data and the programmed data, which will reduce the control accuracy of the cutter attitude angle. On the contrary, it is proposed that the cutter attitude angle obtained by using different parameters for the parallel interpolation with three NURBS spline curves can more accurately represent the original data.

Taking advantage of the same parameters for interpolation, different parameters for interpolation using the parameter synchronization method proposed herein, and the original data at the nodes, the directional unit vector H from the tool center point to the CC point in the workpiece coordinate system is calculated. The H vector is calculated as follows

$$\begin{cases} \boldsymbol{H}_{\text{Same parameters}} = \dfrac{\boldsymbol{C}(u_{i+1}) - \boldsymbol{O}(u_{i+1})}{\left|\boldsymbol{C}(u_{i+1}) - \boldsymbol{O}(u_{i+1})\right|} \\ \boldsymbol{H}_{\text{Different parameters}} = \dfrac{\boldsymbol{C}(v_{i+1}) - \boldsymbol{O}(u_{i+1})}{\left|\boldsymbol{C}(v_{i+1}) - \boldsymbol{O}(u_{i+1})\right|} \\ \boldsymbol{H}_{Node} = \dfrac{\boldsymbol{C}(v_{node}) - \boldsymbol{O}(u_{node})}{\left|\boldsymbol{C}(v_{node}) - \boldsymbol{O}(u_{node})\right|} \end{cases} \tag{20}$$

The components of the H vector along the three-axis directions of the workpiece coordinate system are compared, and the comparison results are shown in Figs. 14-16. Likewise, the raw data of the tool center point and CC point at the node are used as the reference standard for the accuracy of the CC point position after NURBS spline interpolation.

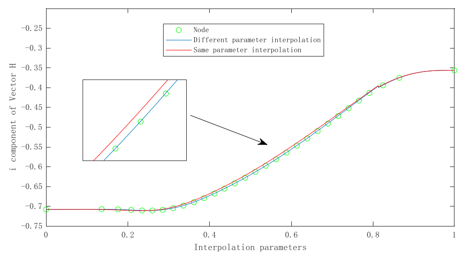

Fig.14 The *i* component of Vector ***H***

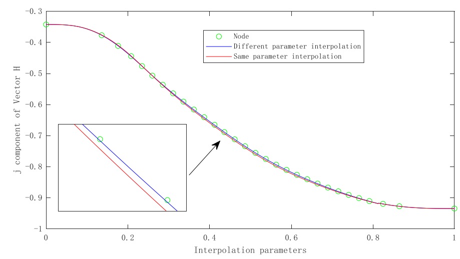

Fig.15 The *j* component of Vector ***H***

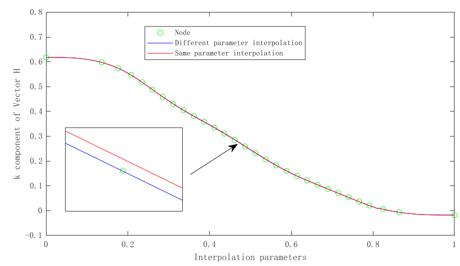

Fig.16 The *k* component of Vector ***H***

It can be seen that the H-vectors at the nodes are in good agreement with the H-vectors obtained when interpolating with different parameters, and on the contrary, the H-vector deviation occurs when interpolating with the same parameters, suggesting a low accuracy of the CC point position obtained when interpolating with the same parameters in the conventional spline interpolation method using the tool center point interpolation parameters. It can be seen that it is appropriate to use the interpolation method with different parameters after equipartition of the interpolation parameters of the three spline curves proposed in this paper when performing Tri-NURBS spline interpolation so as to obtain more accurate theoretical CC point position and provide an accurate reference for future error repair of the CC point.

The interpolated cutter position data after Tri-NURBS spline interpolation are brought into the CC point's nonlinearity error equation to calculate such error before the compensation and repair (i.e., the CC point's nonlinearity error that cannot be compensated by the traditional double NURBS spline interpolation), and the error curve is plotted as shown in Fig. 17. It can be seen that if the CC point's nonlinearity error is not compensated, a nearly $8\mu m$ nonlinearity error will be generated in the CC path by using the conventional double NURBS spline interpolation method.

In order to reduce the above error and improve the control accuracy of the CC point position, such position is recalculated by using the Tri-NURBS spline interpolation method and considering the compensation and repair of the CC point's nonlinearity error when calculating the interpolation parameters. The upper limit of the error compensation and repair is set to $[\varepsilon]=2\mu\text{m}$, and the CC point's nonlinear error curve is redrawn, as shown in Fig. 18. Comparing with Fig. 17, it can be seen that after the error compensation and repair, the CC point's nonlinear error is effectively reduced in the area where the previous error exceeds 2 μm, which greatly improves the control accuracy of the CC point position.

The CC point's nonlinear error compensation and repair process changes not only the position of the CC point but also the position of the tool center point, thus obtaining a new interpolated tool center point. The new interpolated tool center point and unit vector along the cutter axis direction are converted into the machine cutter's feed axis motion coordinates to effectively improve the surface machining quality and machining accuracy of the part. The position deviation of the interpolated tool center point after the error compensation and repair is shown in Fig. 19. Comparing with Fig. 17, it can be seen that the areas with more than 2 μm error in Fig. 17 are found with deviation and change of the tool center point position and that the nonlinear error in these areas is also reduced. Moreover, the tool center point position experiences greater deviation in the area with larger error.

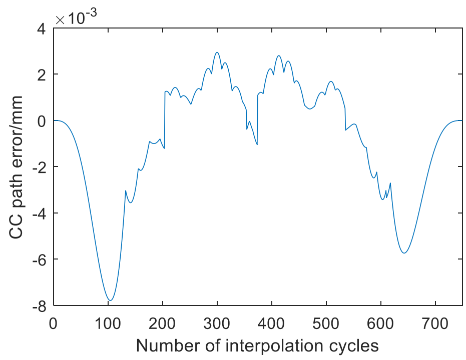

Fig.17 Nonlinear error of the path of CC point before compensation and repair

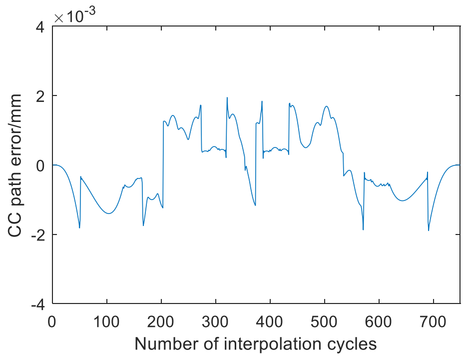

Fig.18 Nonlinear error of the path of CC point after compensation and repair

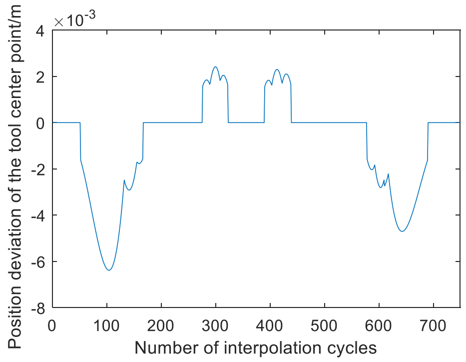

Fig.19 Position deviation of tool center point after error compensation and repair

5. Conclusions

(1) To address the problem that the five-axis double NURBS spline interpolation method can only be adopted to control the accuracy of tool center point path but not reduce the nonlinear error in the CC point path, a five-axis Tri-NURBS spline interpolation method is proposed, and the corresponding interpolation format is given, providing a feasible solution to the repair of the CC point's nonlinear error.

(2) The reasons for the inaccuracy of the cutter axis vector deviation and CC point's error repair reference frame caused by the traditional five-axis dual-NURBS interpolation with the same parameters are analyzed. In the Tri-NURBS algorithm, we propose to use the parameter synchronization method based on the interpolation parameters of the tool center point to solve the problem, thus improving the accuracy of interpolation. Comparative analyses reveal that the interpolated cutter can reach the ideal position.

(3) In order to reduce the nonlinear error of the CC point path, a mathematical model is firstly established for the approximation of the nonlinear error of the CC point path during the five-axis double NURBS spline interpolation. After that, it can be seen from the simulation instance that before the compensation and repair of the nonlinear error of the CC point, the nonlinear error can reach up to $8\mu m$. The machining tolerance is set to $2\mu m$ and for all the CC points exceeding the designated tolerance, the method of compensation and repair of the nonlinear error proposed is applied herein, which verifies that the compensation and repair effect can achieve the expectation of improving the machining accuracy of the CC point path

References

[1] Li Y Q , CHEN Y X , CHEN Q . Research on Nonlinear Error Analysis and Overproof Processing Method in Five-Axis NC Machining with Dual Turntable[J]. Applied Mechanics and Materials, 2011, 121-126:3662-3666.

[2] HWANG H C , JONG-DAE, KI-BEOM P , et al, Development of Practical Postprocessor for Five-axis machine Cutter with Non-orthogonal Rotary Axes[J].Journal of Central South University of Technology,2011,18(01):159-164.

[3] ZHOU Y H, ZHOU J, ZHOU Y F, et al. Theoretical error analysis and control of five-axis NC machining[J]. Journal of Mechanical Engineering,1999(05):54-57. (in Chinese)

[4]WU D Z, Nonlinear error control and pos

t-processing of five-axis NC machining [D]. Shanghai Jiaotong University,2007. (in Chinese)

[5] WU J C, WANG X J, ZHOU H C, et al, Nonlinear error compensation of five-axis CNC machine with double turntable[J/OL]. Computer Integrated Manufacturing Systems: :1-12[2020-02-28].http://kns.cnki.net/kcms/detail/11.5946.TP.20190125.1151.002.html. (in Chinese)

[6] YANG X J, ZHOU Y S, CHEN Z Z, et al, Analysis and control of nonlinear errors caused by the movement of rotary axis in five-axis NC machining[J]. Journal of Mechanical Enginee ring,2012,48(03):140-146. (in Chinese)

[7] ZHENG L M, LIN H, ZHANG X H, et al. Nonlinear error control of five-axis machining based on real-time interpolation [J]. Journal of Chinese Computer Systems, 2010, 31(7):1389-1392. (in Chinese)

[8] WU Z Q. Optimal strategy of cutter nonlinear error in five-axis machining[J]. Manufacturing Automation,2019,41(05):63-65+101. (in Chinese)

[9] LIANG H , HONG H , SVOBODA J . A Cutter Orientation Modification Method for the Reduction of Non___inearity Errors in Five||xis CNC Machining[J]. Machiningence & Technology, 2003, 7(1):1-18.

[10]HO M C , HWANG Y R , HH C H . Five-axis Cutter Orientation Smoothing Using Quaternion Interpolation Algorithm[J]. International Journal of Machine Cutters and Manufacture, 2003, 43(12):1259-1267.

[11]DONG C J, FAN S T. Research on nonlinear errors of five-Axis CNC machine cutters [J].Modular Machine Cutter & Automatic Manufacturing Technique,2012(07):85-88. (in Chinese)

[12]HONG, X., HONG, R. & LIN, X. Cutter Orientations' Generation and Nonlinear Error Control Based on Complex Surface Meshing[J]. Int J Adv Manuf Technol, 2019, 105:4279-4288.

[13]XU R , CHENG X , ZHENG G , et al. A Cutter OrIentation Smoothing Method Based on Machine Rotary Axes for Five-axis Machining with Ball end Cutters[J]. International Journal of Advanced Manufacturing Technology, 2017, 92(7):3615-3625.

[14]LIU X J, ZHANG S Y, WEI D, et al. Efficient interpolation method for complicated surface machining with double NURBS cutter path[J]. Computer Integrated Manufacturing Systems,2017,23(06):1286-1295. (in Chinese)

[15]ZHOU Y , CHEN Z C , FANG Z . Nonlinearity Error Analysis with the Application of the Envelope Surface in Five-axis CNC Machining[J]. Ifac Papersonline, 2015, 48(3):675-679.

[16] Li J , ZHOU Y H , LOU Y . Cutter Path OptiMization in Postprocessor of Five-axis Machine Cutters[J]. International Journal of Advanced Manufacturing Technology, 2013, 68(9-12):2683-2691.

[17]ZHANG K , ZHANG L , YAN Y . Single Spherical Angle Linear Interpolation for the Control of Non-linearity Errors in Five-axis Flank Milling[J]. International Journal of Advanced Manufacturing Technology, 2016, 87(9-12):3289-3299.

[18]Li D , ZHANG W , ZHOU W , et al. Dual NURBS Path Smoothing for Five-axis Linear Path of Flank Milling[J]. International Journal of Precision Engineering and Manufacturing, 2018, 19(12):1811-1820.

[19]ZHANG J , ZHANG L , ZHANG K , e

t al. Double NURBS Path Generation and Synchronous Interpolation for Five-axis Machining Based on dual Quaternion Algorithm[J]. The International Journal of Advanced Manufacturing Technology, 2016, 83(9-12):2015-2025.

[20]JANG B Ci, WANG J B, SU X M. An iterative interpolation algorithm for parameters of double NURBS curves[J]. Mechanical Science and Technology for Aerospace Engineering,2019,38(05):754-760. (in Chinese)

Figures

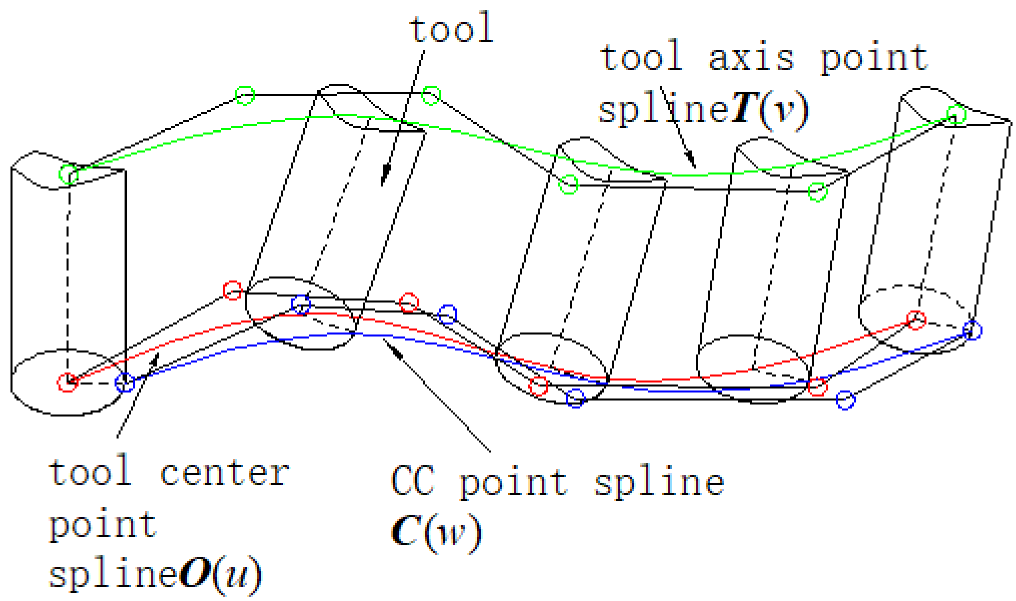

Figure 1

The spline curves in Tri-NURBS interpolation

NURBSON P_ F_;
X_Y_Z_K_TX_TY_TZ_TK_CX_CY_CZ_CK_R_;
X_Y_Z_K_TX_TY_TZ_TK_CX_CY_CZ_CK_R_;
……
X_Y_Z_K_TX_TY_TZ_TK_CX_CY_CZ_CK_R_;
K_TK_CK_;
……
K_TK_CK_;
NURBSOFF;

Figure 2

Five-axis Tri-NURBS interpolation format

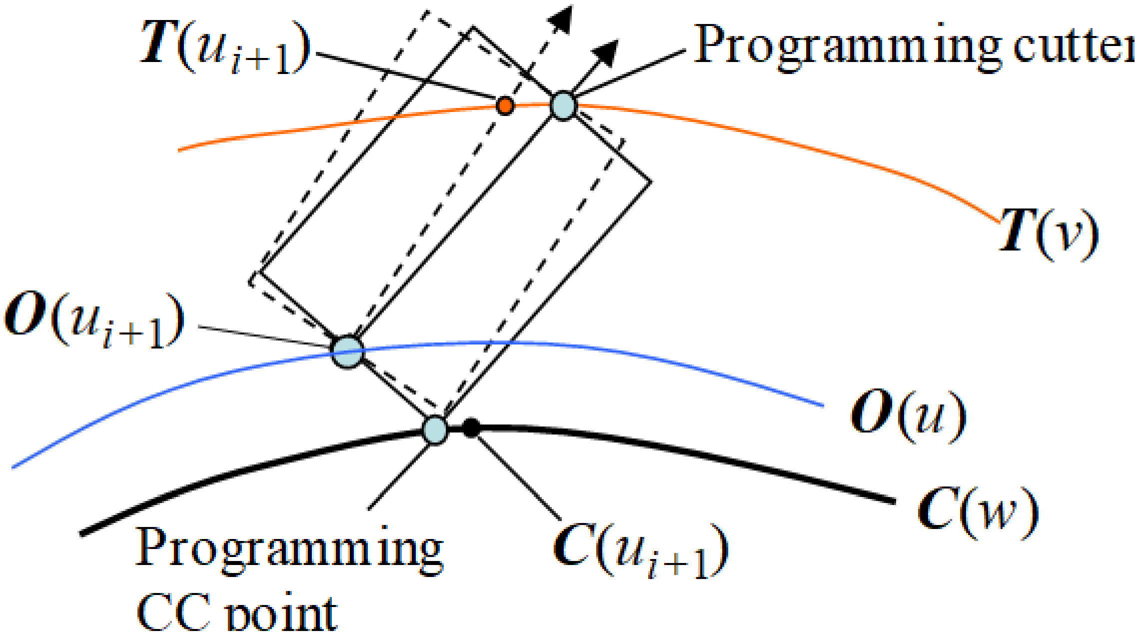

Figure 3

Tri-NURBS interpolation with same parameters

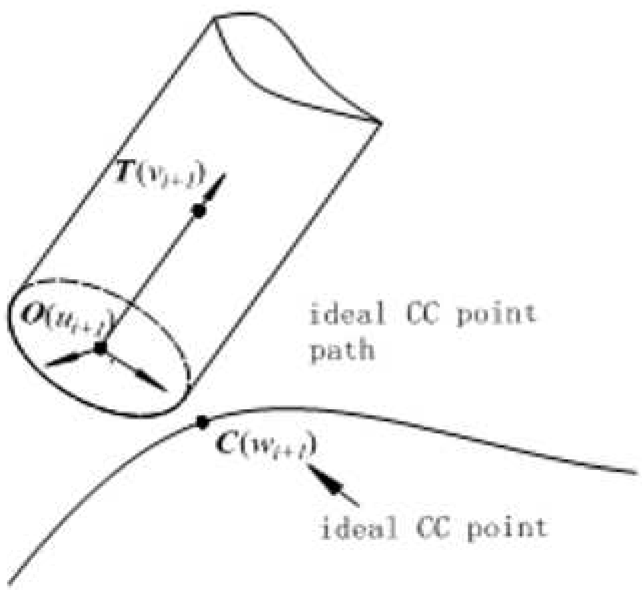

Figure 4

Schematic diagram of nonlinear error for CC point

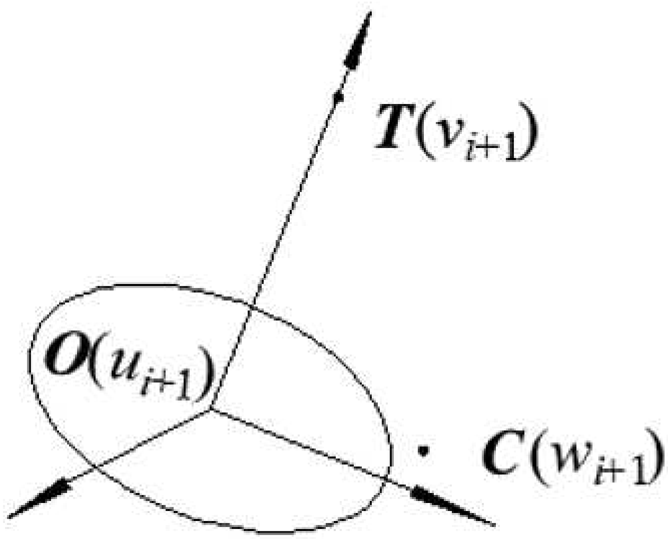

Figure 5

position relationship between three points

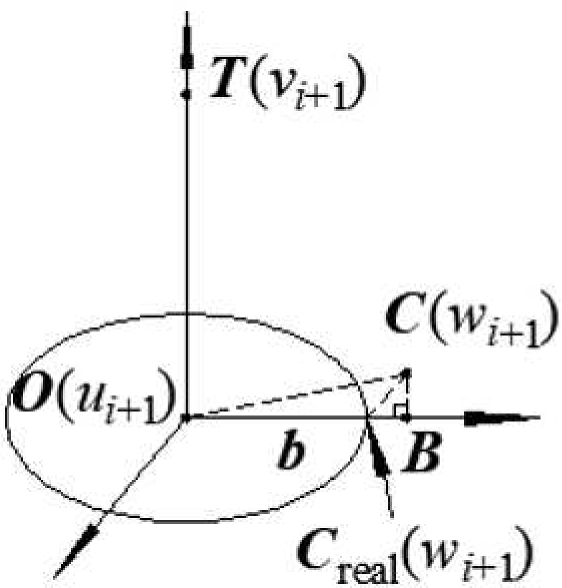

Figure 6

Schematic diagram of the real CC point

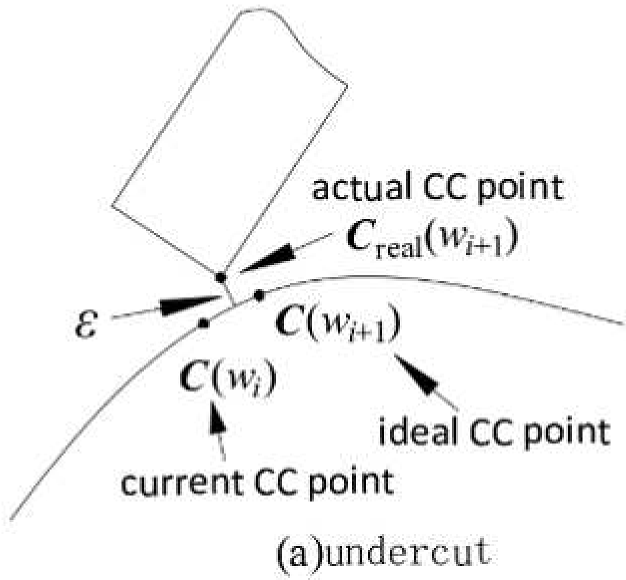

Figure 7

Schematic diagram of nonlinear error for CC point path

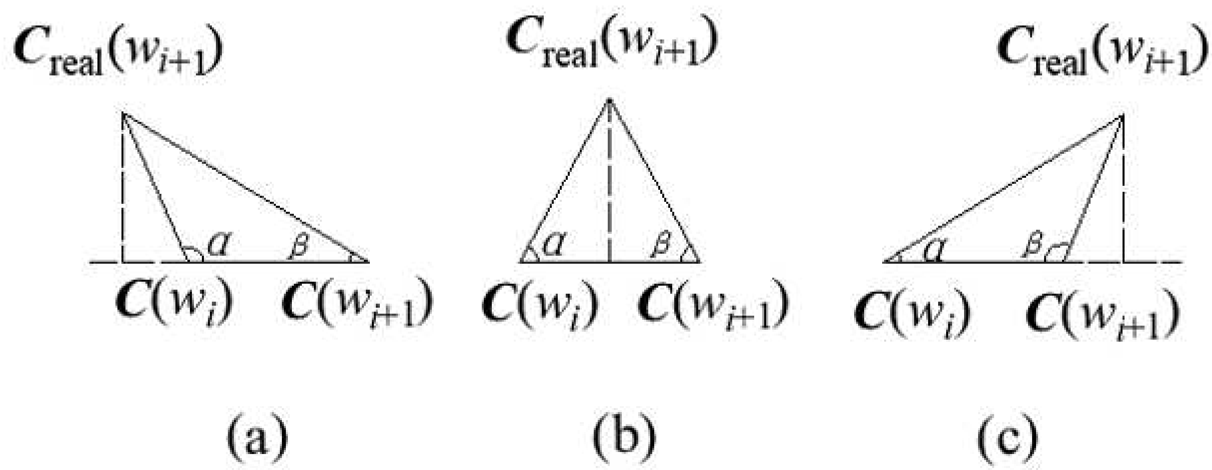

Figure 8

Three different cases for the search interval

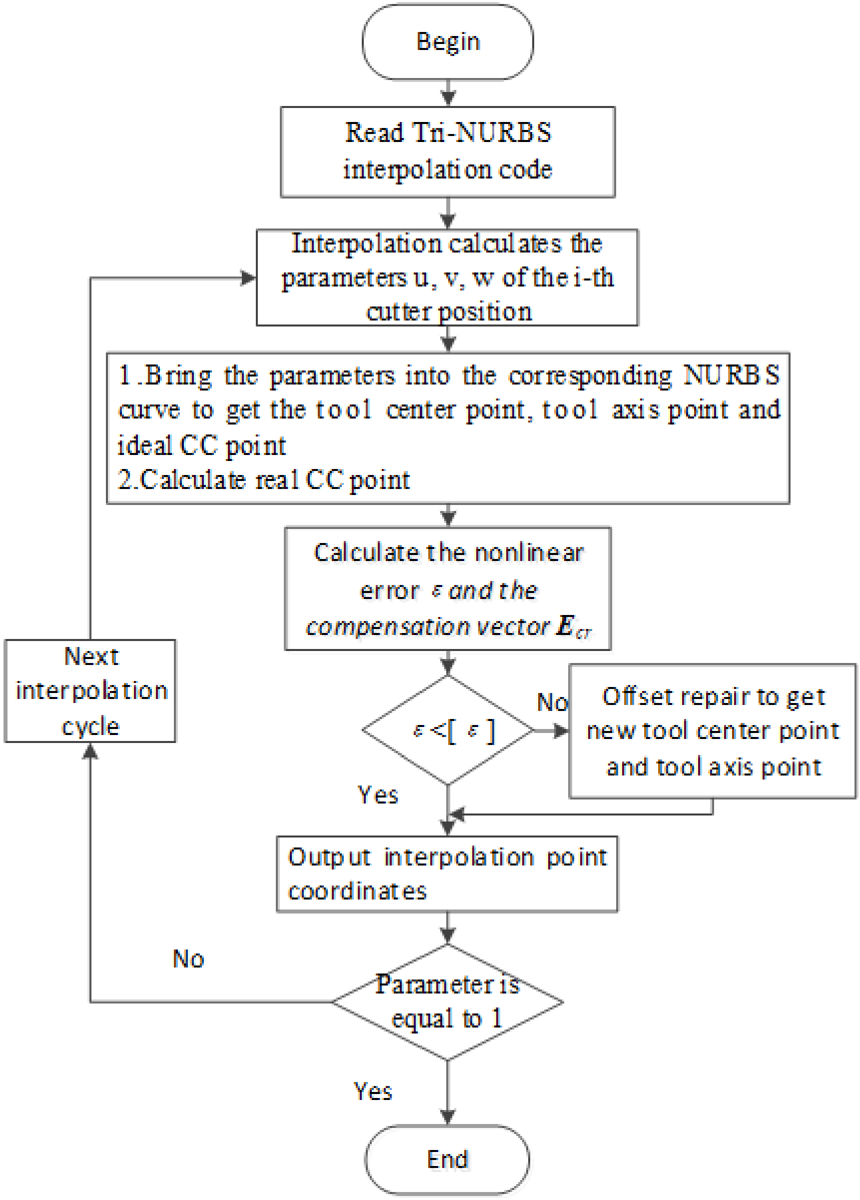

Figure 9

Tri-NURBS interpolation and nonlinear error compensation

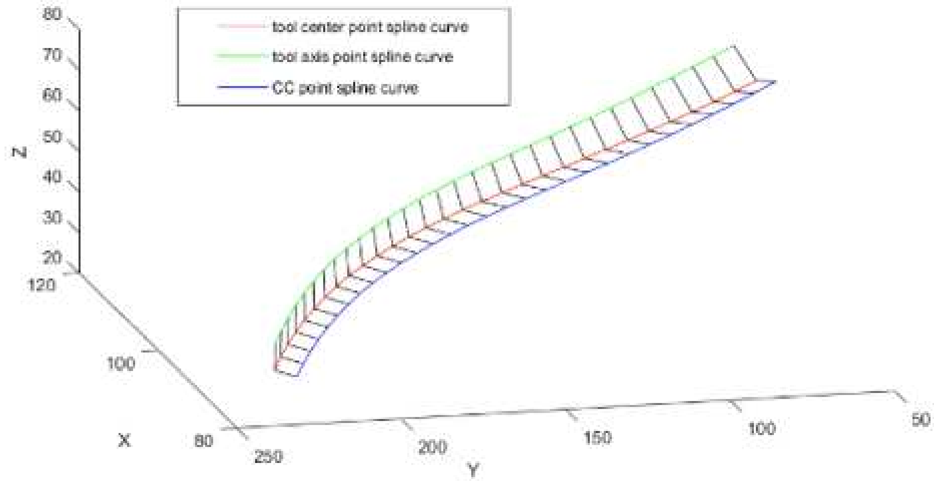

Figure 10

Tri-NURBS spline after fitting

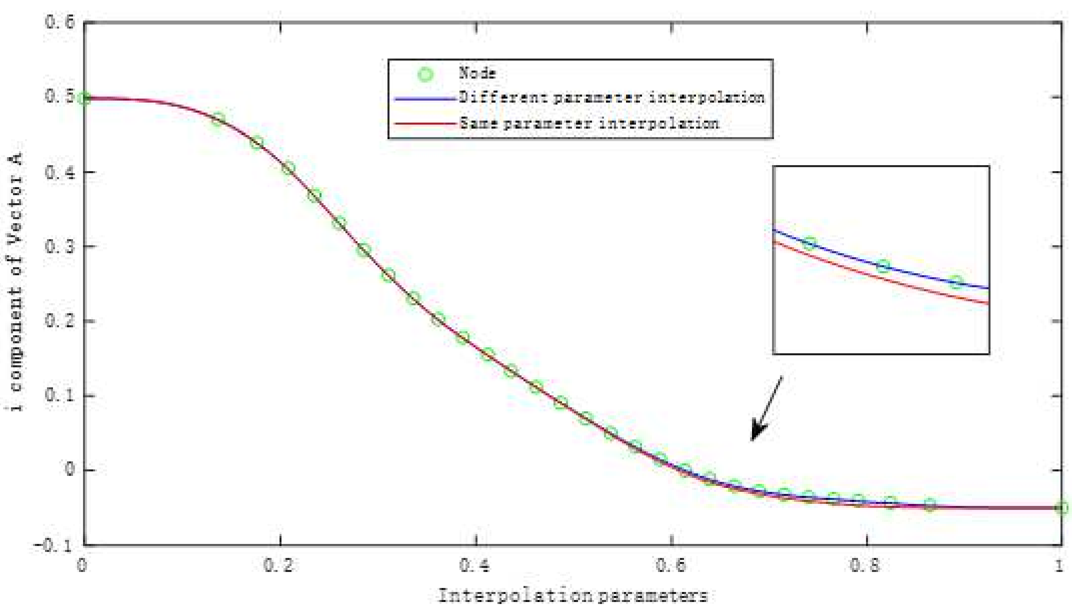

Figure 11

The i component of Vector A

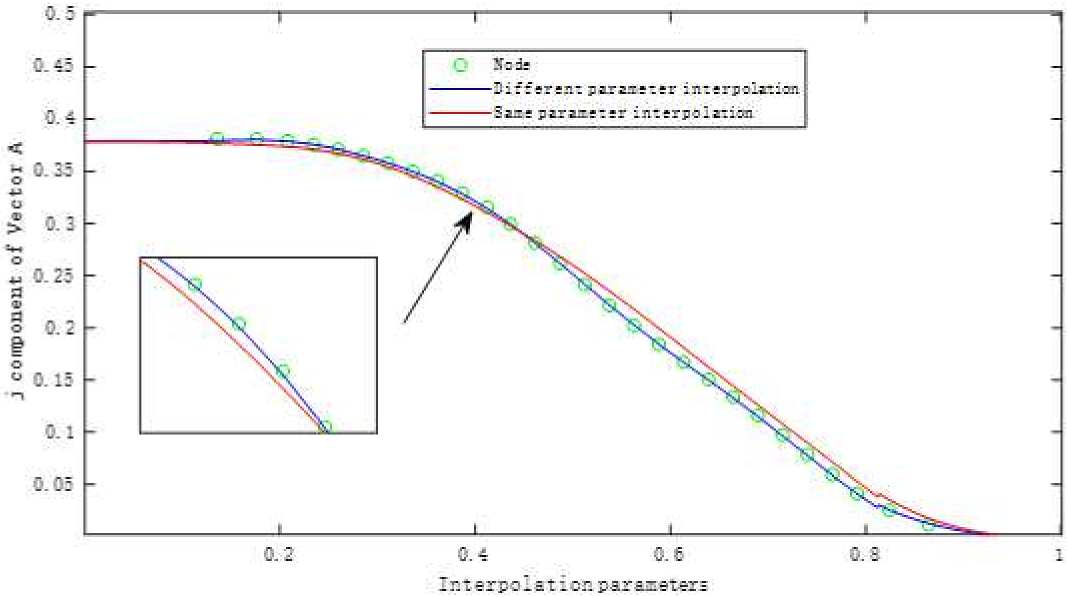

Figure 12

The j component of Vector A

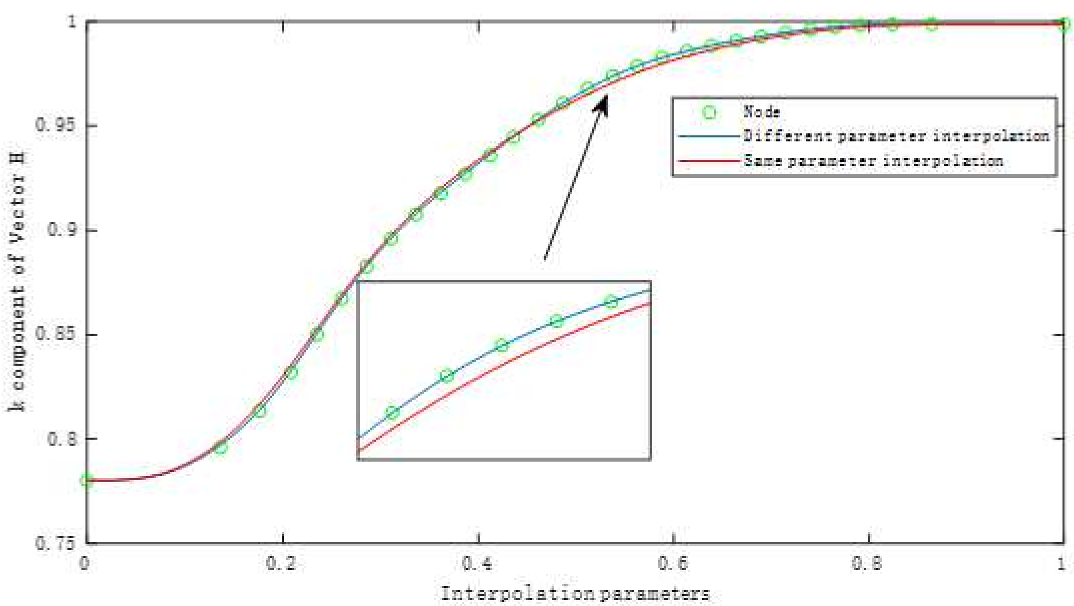

Figure 13

The k component of Vector A

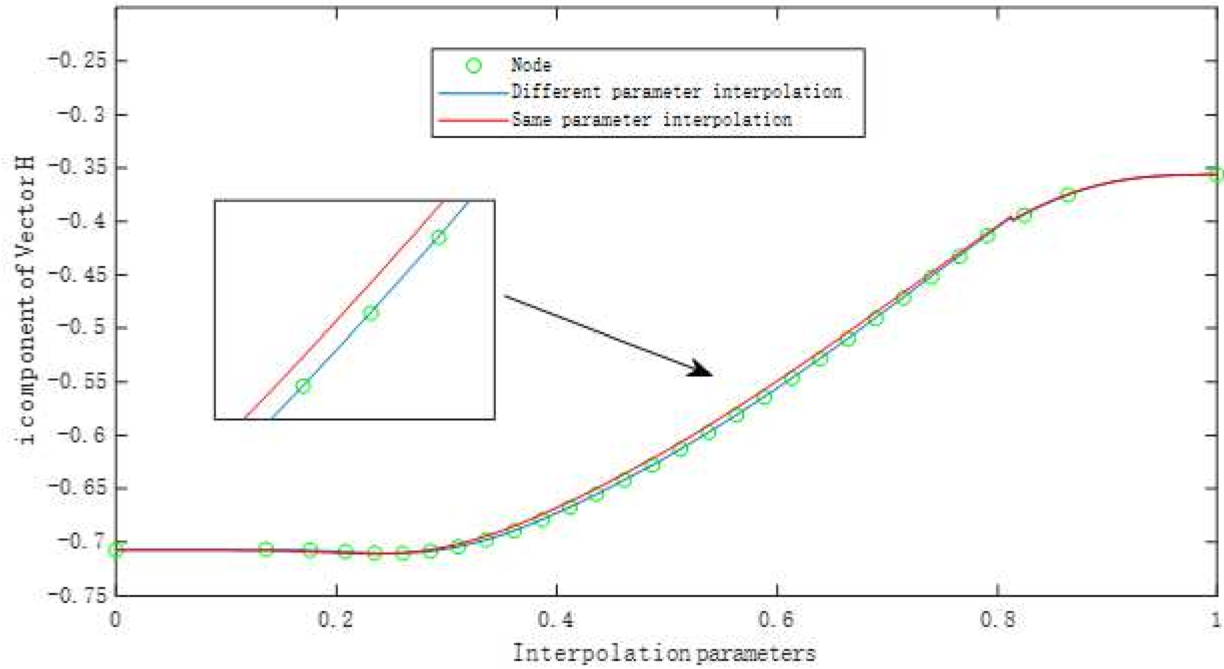

Figure 14

The i component of Vector H

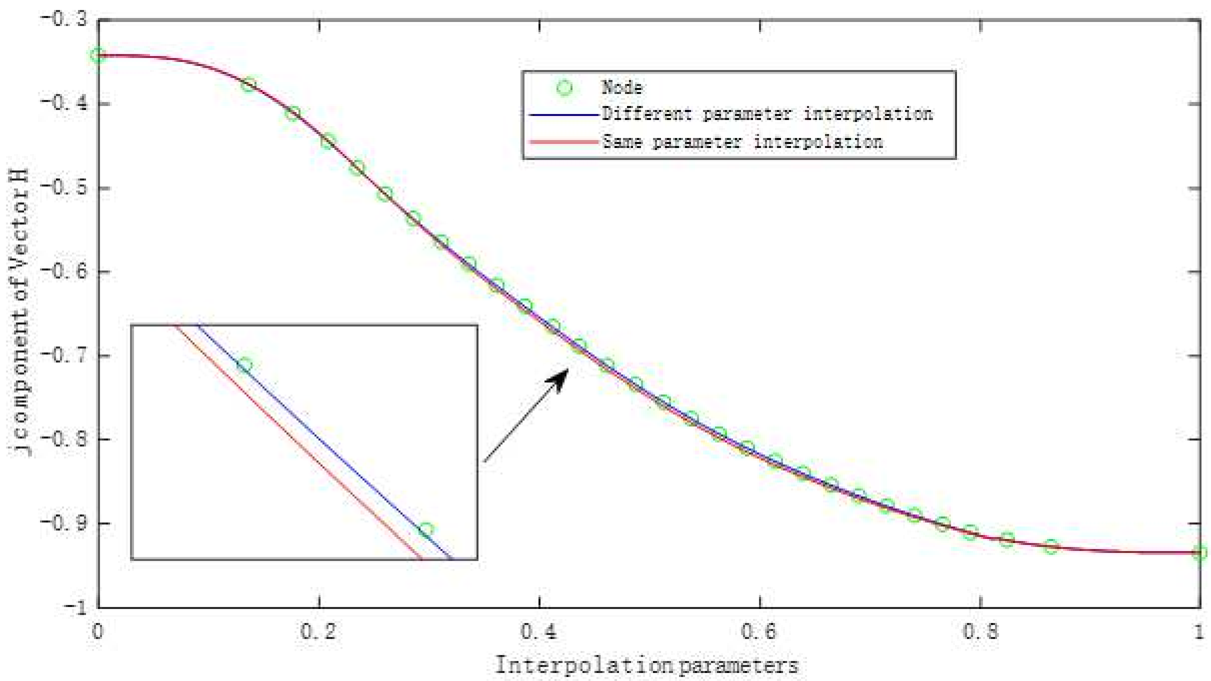

Figure 15

The j component of Vector H

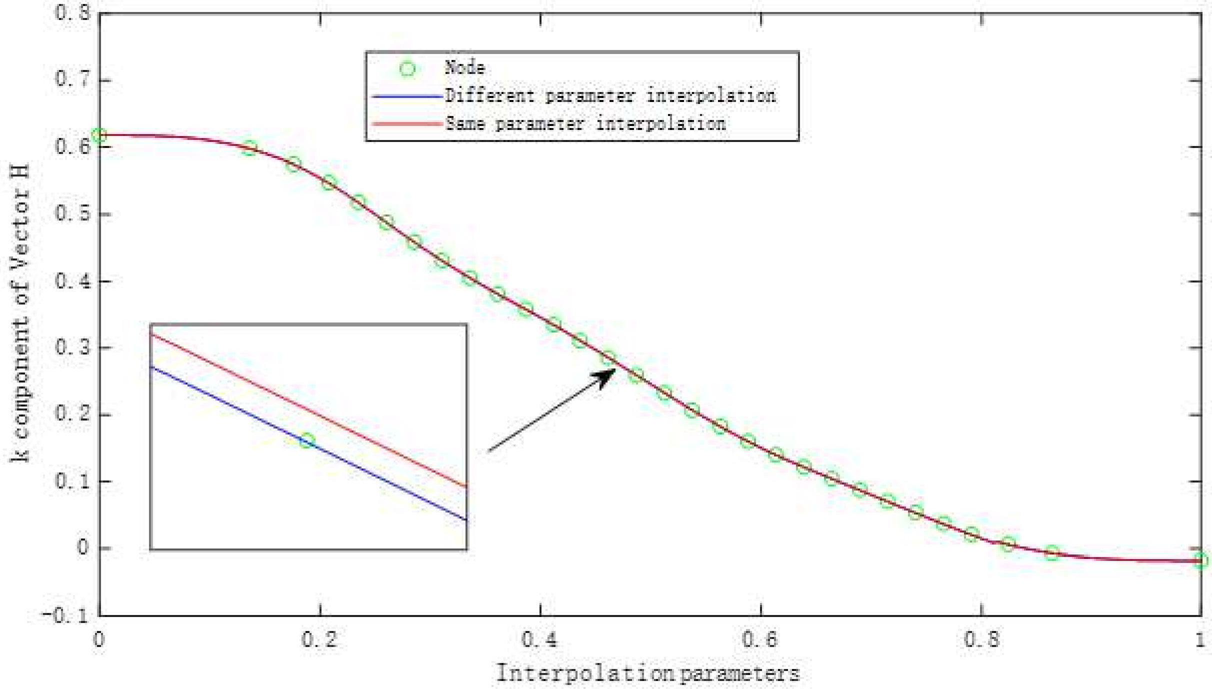

Figure 16

The k component of Vector H

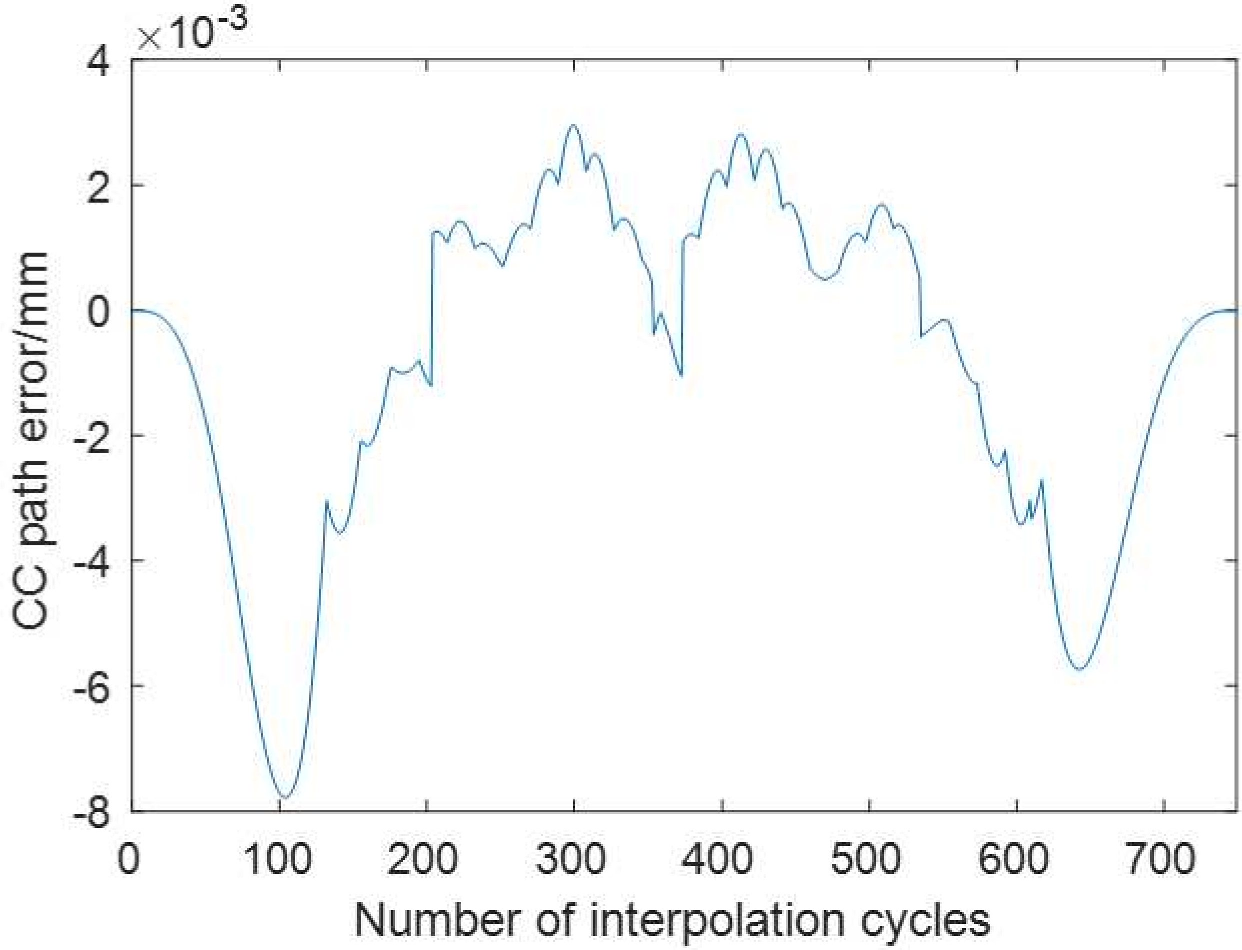

Figure 17

Nonlinear error of the path of CC point before compensation and repair

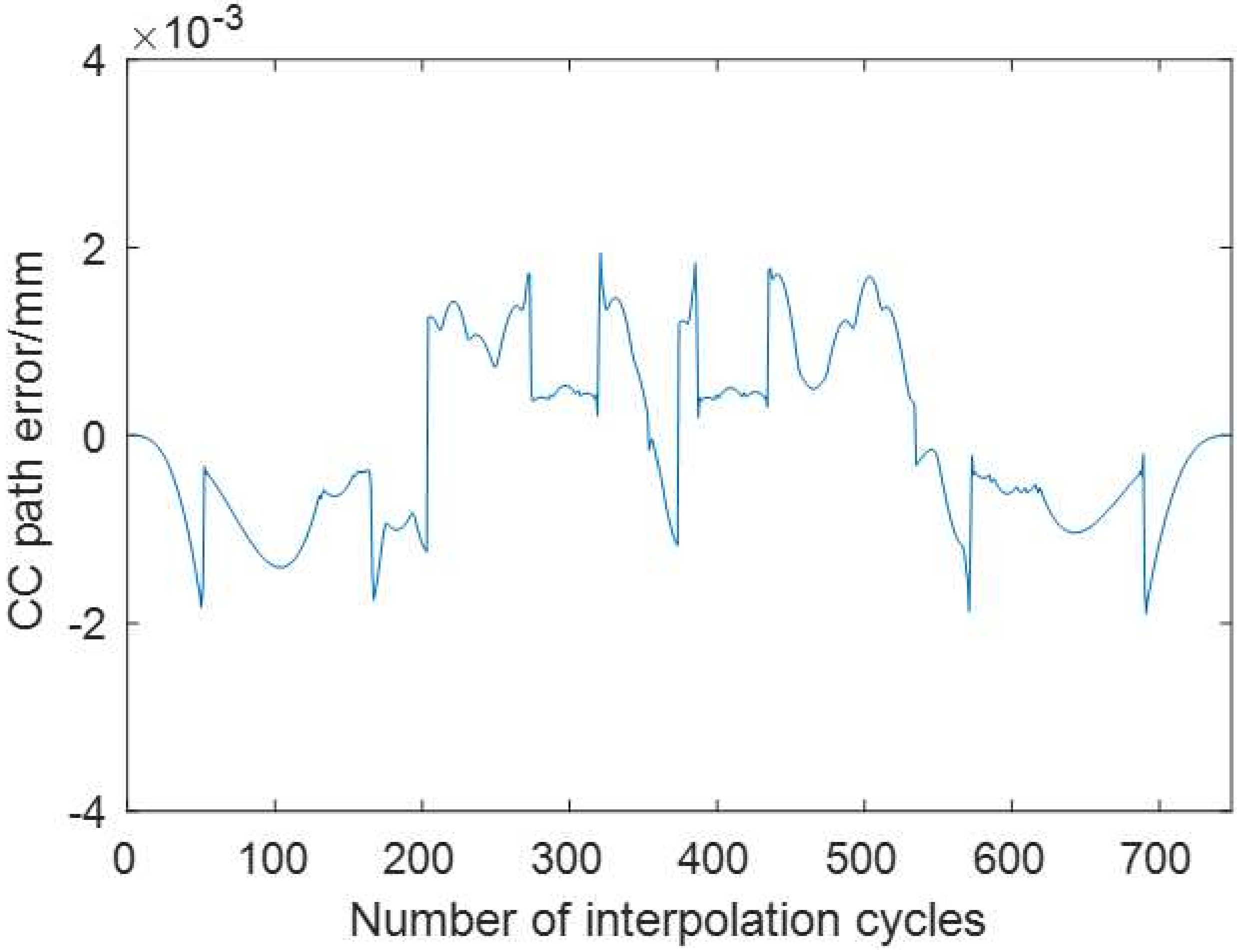

Figure 18

Nonlinear error of the path of CC point after compensation and repair

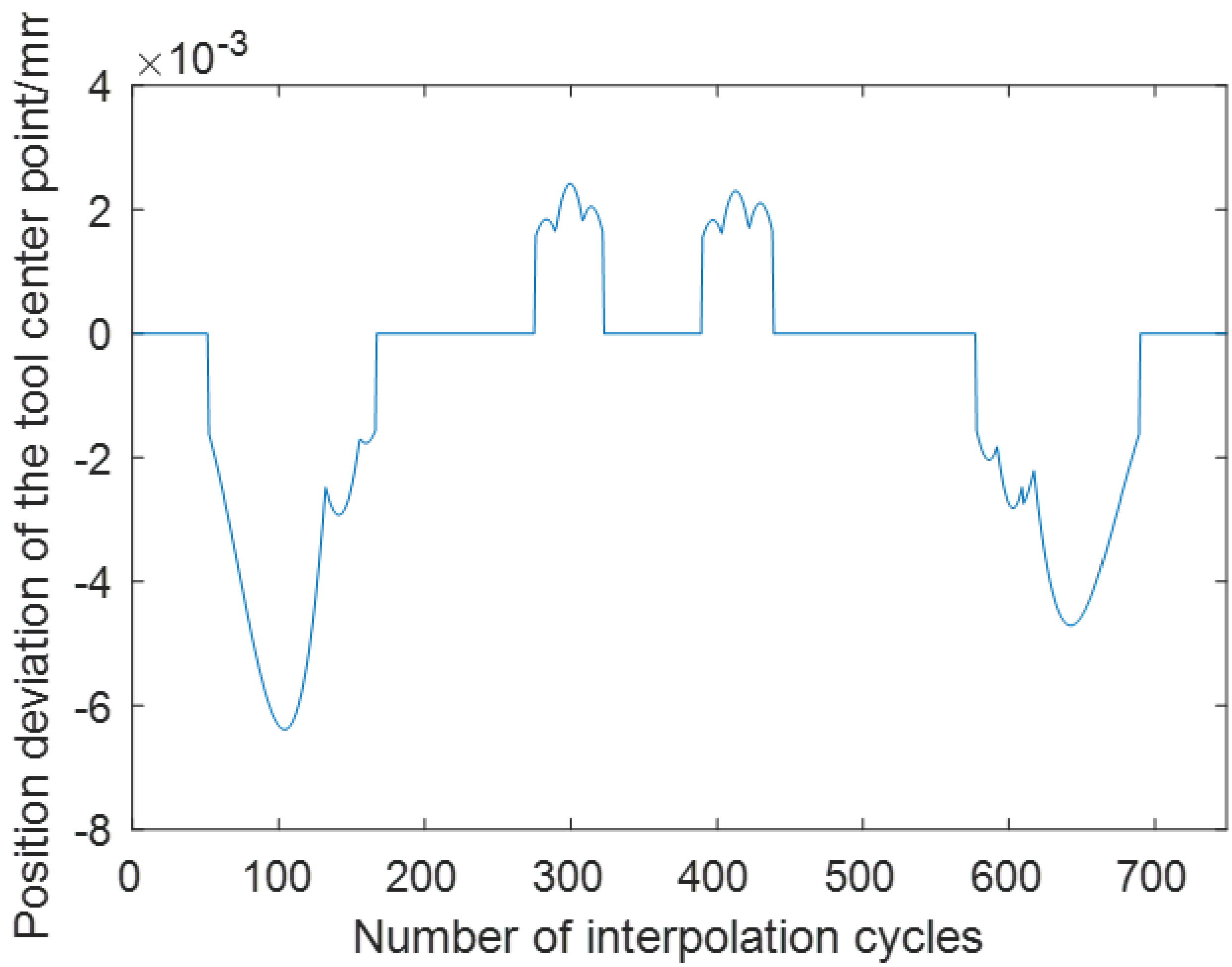

Figure 19

Position deviation of tool center point after error compensation and repair