

Study on Complex Dynamics for the Waste Electrical and Electronic Equipment Recycling Activities Oligarchs Closed-loop Supply Chain

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1 Study on Complex Dynamics for the Waste Electrical and
2 Electronic Equipment Recycling Activities Oligarchs
3 Closed-loop Supply Chain

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28

29 **Abstract**

30 Nowadays, with more and more WEEE (Waste Electrical and Electronic
31 Equipment) being abandoned, WEEE recycling activities are increasingly popular. In
32 this paper, we build a closed-loop supply chain model and focus on the recycling
33 behaviors of the members in this supply chain, which contains two manufacturers, the
34 retailer and the consumer. In the reverse chain, we set up dual channels and design
35 two recycling methods: sell-back method and trade-in method. We use classical
36 backward induction to run the model. And then we analyze the stability of the system
37 and the impacts of some essential parameters by numerical simulation. The speed of

38 the manufacture's decision adjustment has a significant effect on the stability of the
39 model. When the decision variables of manufacturers go into instability, the profits of
40 both manufacturers and the retailer have a profit loss in multiple cycles on average.
41 The lower production cost of manufacturer 1 will increase the retailer's profit and
42 self-profit, while manufacturer 2's profit will decrease. And the smaller price
43 coefficient of the retailer can bring higher profits but aggravate the chaotic condition,
44 and the stability of the whole system will decrease. In the end, we adopt the parameter
45 control method and the decision-making method to control chaos, and they both have
46 a good control effect.

47

48 Keywords: Reverse chain, WEEE recycling, Game theory, Oligarchs ,Complex
49 system, Closed-loop Supply Chain

50 **1. Introduction**

51 An increasing number of states pay more attention to environmental issues
52 including climate change, air pollution, and breakage of land, etc. Many corporations
53 need a more stable and sustainable business model. It is the requirement for
54 environmental consequences, social factors, and economic constraints.
55 The generally accepted concept of "e-waste" is discarded electrical or electronic
56 devices, such as common household appliances the recovery of mobile phones,
57 washing machines, televisions, air conditioners computers (Guo et al. 2018).
58 Recovery of WEEE (waste electrical and electronic equipment) products has become
59 a hot issue in the economic field. Compared with the recycling of traditional materials,
60 WEEE products are composed of more complicated material and components (Gu et
61 al. 2017).

62 With the increasingly frequent update and upgrade of electrical and electronic
63 equipment, the trade of related products rises and a great deal of electrical and
64 electronic equipment is abandoned every day. Regulatory control of re-using,
65 recycling, and recovery of electrical and electronic waste materials has been
66 promulgated. (Ministry of Environment and Urbanization, 2012). In the EU, the
67 amount of WEEE has been increasing by 3-5% per year. In 2012 the electrical waste
68 sums up to 3.6 billion kg, of which over 70% was recycled (Zlamparet et al., 2017). In
69 China, according to the calculated theoretical scrap amount of waste electrical and
70 electronic products, the theoretical scrap amount of 14 products in "The Disposal

71 Catalogue of Waste Electrical and Electronic Products” in 2017 is shown in Table
 72 1(White paper WEEE recycling industry in China 2017.). Among them, the
 73 theoretical scrap quantity of the first batch of products is about 100 million, including
 74 3.216 million TV sets, 2.439 million refrigerators, 16.2 million washing machines,
 75 and 27.23 million room air conditioners. Compared with 2016, the theoretical scrap
 76 volume of the first batch of products presents an overall upward trend.¹ For this reason,
 77 WEEE recycling activities have become an extensively researched issue in recent
 78 years.

79 Table 1. The theoretical scrap amount of China Electric & Electronic Products in
 80 2017¹

Electric & Electronic Products	Scrap amount (million)	Electric & Electronic Products	scrap amount (million)
TV	32.16	Gas water heater	9.33
Refrigerator	24.39	Printer	30.99
washing machine	16.20	Copier	6.69
Air conditioner	27.23	Fax machine	5.06
Micro-computer	25.24	Fixed telephone	23.80
Oil absorption	39.87	Mobile phone	232.75
Electric water heater	25.85	Monitors	0.48

81 To investigate how WEEE recycling activities affect the oligarch's closed-loop
 82 supply chain, a method that has dual channels sales and two recycling is designed.

83 Based on observations from current literature and actual situation, our model's
 84 innovation can be summarized as follows.

85 (1) We build a closed-loop supply chain including two manufacturers and one
 86 retailer. Two different recovery modes are taken into account. It can be expanded by
 87 applying other models from literature in the future.

88 (2) We consider competition between two different recycling methods in the
 89 waste recycling market. Besides, based on Complex System Theory, we adopt a
 90 dynamic decision-making method instead of a single period of static decision-making.

91 (3) Chaos is controlled by adjusting the means of relevant parameters and
 92 decision-making methods.

93 The contribution of this paper is to enrich the analytical analysis on CLSC,
 94 investigate the complexity of the decision-making process, and inspire the operations
 95 method in recycling activities.

96 Based on the above background, this paper tries to discuss the following research
 97 questions:

98 (1) How is the complex chaotic dynamical process in the reverse channel
99 structure?

100 (2) How to control and coordinate the WEEE recycling activities under the
101 closed-loop supply chain structures?

102 The rest of this paper is organized as follows. In section 2, we describe the
103 problem, show symbols and assumptions, and a closed-loop supply chain network
104 which is composed of two manufacturers and one retailer. We solve the model and
105 analyze the local stability in section 3. And then we further analyze the influence of
106 parameters on the model's stability in section 4. Based on what we have stated, we
107 introduce the method to control the chaos system in section 5. Finally, we make
108 conclusions in section 6.

109 **2. Literature Review**

110 WEEE is composed of lots of complex devices and components. Proper disposal
111 of WEEE not only reduces environmental pollution but also enhances resource
112 utilization efficiency which benefits manufacturers as well as the environment. For
113 this reason, product recycling activities have been paid attention to around the world
114 (Lu et al., 2018; Polat et al., 2018). We take China as an example since the Chinese
115 government puts great emphasis on WEEE recycling activities. A model of green
116 design, green production, green sales, green recycling, and disposal led by production
117 enterprises is emerging now.¹ Under great pressure from the government, other firms
118 and consumers are responsible for environmental and social as well, manufacturers
119 would choose to recycle used products. For example, HUAWEI is collecting used
120 products through VWALL; Changhong and Haier are also actively building green
121 recycling systems for used products. Meanwhile, retailers such as SuNing, JingDong,
122 also participate in collecting used products activities.

123 The recycling process leads to better economic viability and environmental
124 sustainability which makes the closed-loop supply chain more attractive. (Hong et al.,
125 2017). The closed-loop supply chain contains the forward logistics and reverses
126 logistics. In forward logistics, the supply chain process includes purchasing,
127 manufacturing, and selling; in reverse logistics, the supply chain differs from
128 traditional supply chain realizes recycle and remanufacture, is designed to increase the
129 utilization ratio of raw materials, reduce pollution emissions to protect the
130 environment and lower costs at the same time. Therefore, constructing a closed-loop

131 supply chain can promote sustainable development. The recycling activity has
132 become a new trend in logistics and supply chain management. Based on related
133 research in the closed-loop supply chain and reverse supply chain, the collector
134 recycling used products in the closed-loop supply chain can be classified into three
135 categories: manufacturer, retailer, and the third-party (Savaskan et al, 2004).

136 A great number of researches about WEEE recycling has been done in recent
137 years. Xu et al. (2017) investigated the design of a global reverse supply chain. The
138 results show the effect of uncertainties and carbon constraints on decisions to reduce
139 costs and emissions. Bal and Satoglu (2018) investigated a multi-facility,
140 multi-product, and multi-period model which collects WEEE products and recover the
141 waste materials. Bing (2015) redesigned a reverse supply chain to provide useful
142 insights for managing waste as a resource in the view of a global perspective. Liu et al.
143 (2017) studied a manufacturer who owes a WEEE recycling qualification under fund
144 policy and analyzed the effect of fund policy on the manufacturer. Liu et al. (2006)
145 built a dual-channel WEEE recycling model that contains a formal section and
146 informal section. They considered price competition based on quality and government
147 subsidy, obtained the equilibrium prices as well as the influence of government
148 subsidy under four competitive scenarios. Ma et al. (2020) constructed a closed-loop
149 supply chain and studied how different channel power affect the dynamic
150 characteristic of the system. In terms of competition in the reverse supply chain, most
151 papers focus on competitions between two retailers, between two third parties, or
152 between one retailer and one-third party. For example, Savaskan et al. (2006) studied
153 two competitive retailers in 2006. Therefore, we consider the fact that some
154 manufacturers recycle used products by themselves and build a dual-channel supply
155 chain in which one manufacturer collects used products from consumers directly and
156 the other manufacturer collects used products through retailers. Yin et al. (2010)
157 discussed the effect of the retail used goods market and the electronic peer-to-peer
158 (P2P) market on the primary market. Inspired by Yin et al. (2010), we set a trade-in
159 recycling method where the retailer buys back a used product and sells a new one to
160 consumers. Zhou et al. (2017) explored a two-period model in which a manufacturer
161 sells products and two competitive recyclers recycle used products under Extended
162 Producer Responsibility (EPR) and further discussed the decision about setting
163 subsidy from the government. Based on current research, we further consider a
164 three-period model while putting more emphasis on the reverse supply chain.

165 Considering the situations that the buyers promise to buy back the unsold products
166 from the seller, Xie et al (2020) studied the effect of income uncertainty and relative
167 bargaining power on the performance of repurchase contracts in supply chain.

168 Analyzing product recycling activities, most literature focus on the closed-loop
169 supply chain. Based on different strategies of collecting used products, Savaskan et al.
170 (2004) analyzed three different reverse loops of the supply chain and analyzed the
171 results. Östlin et al. (2008) focused on remanufacturing. They expanded seven
172 different types of closed-loop supply chains and analyzed the advantages and
173 disadvantages of each model. Qiang et al. (2013) built a closed-loop supply chain
174 which contains raw material suppliers, retail outlets, and manufacturers, and gave the
175 optimality conditions of the various decision-makers. Huang et al. (2013) explored the
176 optimal strategies in the closed-loop supply chain with a dual recycling channel in
177 which retailers and the third party collect used products. Giovanni et al. (2014)
178 introduced a two-period closed-loop supply chain and analyzed whether to collect
179 end-of-life products by remanufacturer exclusively or introduce a retailer or a
180 third-service provider. Wei et al. (2015) analyzed a closed-loop supply chain under
181 symmetric and asymmetric information. Gao et al. (2016) focused on exploring power
182 structures' effect on optimal strategies in a closed-loop supply chain. In a word, for
183 the reverse channel in the closed-loop supply chain, most literature focus on the
184 method of collection, power structure, and so on. At present, competition between two
185 different recycling methods in the waste recycling market is less mentioned.
186 Consequently, based on Guo and Ma (2013) and Ma et al. (2013) and associated with
187 reality, we enter into discussion in the two different recycling methods in a
188 closed-loop supply chain and expect to make some useful supplements on this.
189 Besides, most literature base on two hypotheses: complete information and
190 "completely rational person" hypothesis. Actually, in the long run, players would
191 adjust their decisions according to the information they received. Despite that, there is
192 a lot of literature with WEEE recycling activities, the study of complexity theory is
193 not widely in the current literature. Thus, we introduce dynamic game when analyzing
194 the closed-loop supply chain in the light of Ma and Guo (2014) in which they
195 introduced dynamic system concepts to simulate the evolution of the strategic
196 decisions over multiple periods. Tian et al. (2020) analyze the complexity of a
197 multi-channel supply chain and revealed that a larger adjustment speed of the sale
198 effort causes the unstable state of the system. Based on bounded rationality, Ma et al.

199 (2020) explored the impact of decision sequence on the profits of the enterprise and
200 how the adjustment speed affect the stability of the system. Considering a supply
201 chain with two different manufacturers who produce fuel vehicle and battery vehicle
202 respectively, Bao et al. (2020) derived the optimal decisions and make comparative
203 analysis between three different games, finding that the adjustment speed of the
204 ecofriendly level has the greatest impact on the market.

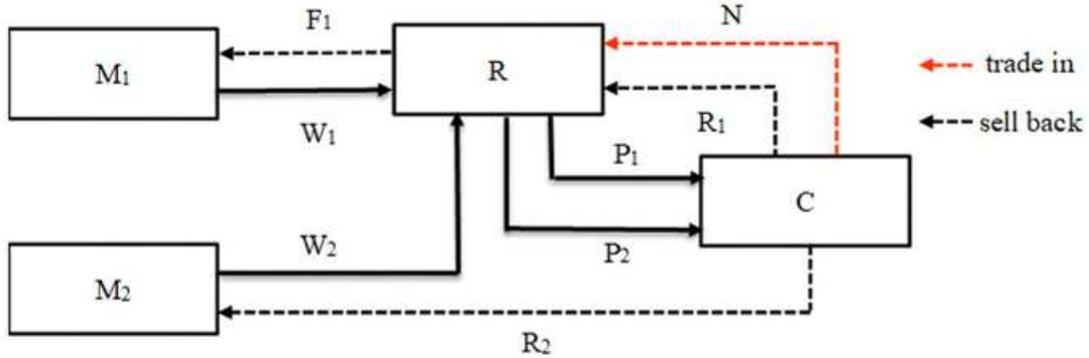
205 **3. Methodology**

206 During the last four decades, nonlinear dynamics have made great progress and
207 have attracted increasing attention from different fields including supply chain
208 management. Ma and Wu (2014) explored a discrete triopoly dynamical model and
209 analyzed the complexity of this system. Elsadany (2010) proposed and analyzed a
210 duopoly delayed bounded rationality game and the analysis proved that delayed
211 bounded rationality contributed to the higher possibility of Nash equilibrium point.
212 Du et al (2013) analyzed a dynamic duopoly game with heterogeneous players. They
213 found the chaos was harmful to at least one player of the two, and an appropriate
214 limiter would be conducive to the players' performance. As an essential tool, the
215 dynamic game also has been applied to the reverse supply chain. Xie and Ma (2016)
216 researched a Duopoly market of color TV recycling, analyzed the influence of
217 coefficients changing on the system's stability, and introduced the method of chaos
218 control. Ma and Wang (2014) first built a closed-loop supply chain in which the
219 retailer sells and collects products and the manufacturer produces and recovers
220 products and then presents the dynamic phenomena.

221 **3.1. Model Framework**

222 In this paper, we consider a two-period model, but we focus on the recycling
223 behavior of the members in a typical supply chain including two manufacturers, one
224 retailer and consumers. In the forward chain, two manufacturers sell their products to
225 the retailer and the retailer sells products to consumers. In the reverse chain, we set up
226 dual channels and design two recycling methods: sell-back method and trade-in
227 method. Consumers sell the used product back to the retailer utilizing the sell-back
228 recycling or trade the used product in for a new one through trade-in recycling. When
229 using the method of sell-back, consumers can get money by selling waste products.
230 When using the method of trade-in, consumers can sell back waste products to the
231 retailer in exchange for lower prices of new products. And then the retailer sells the
232 used products back to manufacturer 1. Meanwhile, consumers are allowed to sell used

233 products to manufacturer 2 directly. In the next stage, manufacturer 1 and
 234 manufacturer 2 remanufacture used products and sell new products to consumers. Our
 235 model in this paper is shown in Fig. 1.
 236



237

238 Fig. 1. Recycling Supply Chain Model

239

240 In Fig.1, M1 represents manufacturer 1 and in the same way, M2 shows
 241 manufacturer 2. R represents the retailer. And C is the consumer.

242

3.2 Symbols and Assumptions

243

The relevant symbols are as follows.

244

Q_i demand for the product i .

245

a_i basic market size of product i .

246

b_i sensitivity of consumers to the retail price of product i .

247

d_i sensitivity of consumers to the rival products' retail price of product i .

248

p_i unit retail price of product i .

249

w_i unit wholesale price of product i .

250

Q_{ri} recycling quantity of product i .

251

N difference between the sell-back price and the trade-in price.

252

g_i sensitivity of consumers to the sell-back price of product i .

253

h_i sensitivity of consumers to the rival products' sell-back price of product i .

254

t_1 sensitivity of the difference between the sell-back price and the trade-in price.

255

F_1 sell-back price of manufacturer 1.

256

R_1 sell-back price of the retailer.

257

R_2 sell-back price of manufacturer 2.

258

C_i cost of product i .

259

Q_e trade-in quantity.

260

C_{ri} cost of product i made from the used product.

261

Π_i the profit of manufacturer i .

262

Π_R retailer's profit

263

λ_3 price coefficient of retailer

264

k_i potential maximum waste quantity of product i for recycling in the market

265

v_i ($i=1,2,3$) indicate the adjusting speed of F_1 , R_2 , and N respectively.

266

To simplify the analysis, we made those assumptions as follows.

- 267 (a) The manufacturer and retailer are bounded rational.
 268 (b) Remanufactured products are exactly the same as new products.
 269 (c) The transportation cost and other expenses are not taken into account.
 270 (d) The demand is linearly dependent on price.
 271 (e) The recycling quantity is linearly dependent on the recycling price.

272 3.3 Model Construction

273 In our model, manufacturer 1 produces product1, and manufacturer 2 produces
 274 product2. Those two products are made of the same raw materials, possess the same
 275 function, quality, and price et cetera. We assumed that the demand is linearly
 276 dependent on price. Meanwhile, the rival product's retail price also affects demand.
 277 Thus, the demand for new product 1 and new product 2 can be given as follows.

$$278 \begin{cases} Q_1 = a_1 - b_1 P_1 + d_1 P_2 = a_1 - b_1(1 + \lambda_3)W_1 + d_1(1 + \lambda_3)W_2 \\ Q_2 = a_2 - b_2 P_2 + d_2 P_1 = a_2 - b_2(1 + \lambda_3)W_2 + d_2(1 + \lambda_3)W_1 \end{cases} \quad (1)$$

279 Comparably, we set the recycling quantity linearly depending on the recycling
 280 price. So the sell-back quantity of used products for the retailer and the sell-back
 281 quantity of used products for manufacturer 2 can be given as follows.

$$282 \begin{cases} Q_{r1} = k_1 + g_1 R_1 - h_1 R_2 - t_1 N = k_1 + g_1 F_1(1 - \lambda_3) - h_1 R_2 - t_1 N \\ Q_{r2} = k_2 + g_2 R_2 - h_2 R_1 - t_1 N = k_2 + g_2 R_2 - h_2 F_1(1 - \lambda_3) - t_1 N \end{cases} \quad (2)$$

283 Similarly, the trade-in quantity can be given as follows.

$$284 Q_e = \beta + \gamma N - t_2(R_1 + R_2) = \beta + \gamma N - t_2 F_1(1 - \lambda_3) - t_2 R_2 \quad (3)$$

285 In the set recycling supply chain, the profits of manufacturers include two parts:
 286 the profits of new products and the profits of used products. And the profits of the
 287 retailer come from selling and recycling products 1 and 2. The profit model can be
 288 given as follows.

$$289 \begin{cases} \Pi_1 = (W_1 - C_1)(Q_1 - Q_e - Q_{r1}) + (W_1 - C_{r1} - F_1)(Q_e + Q_{r1}) \\ \Pi_2 = (W_2 - C_2)(Q_2 - Q_{r2}) + (W_2 - C_{r2} - R_2)Q_{r2} \\ \Pi_r = (P_1 - W_1)Q_1 + (F_1 - R_1)Q_{r1} + (F_1 - R_1 - N)Q_e + (P_2 - W_2)Q_2 \end{cases} \quad (4)$$

290 To maximize the profit, the three parties (manufacturer 1, manufacturer 2, and
 291 the retailer) have to make appropriate decisions. According to the profit model (4), the
 292 decision variable of manufacturer 1 is the sell-back price of manufacturer 1, F_1 , the
 293 decision variable of manufacturer 2 is the sell-back price of manufacturer 1, R_2 , and
 294 the decision variable of the retailer is the difference between the sell-back price and
 295 the trade-in price, N .

296 As to manufacturer 1, its profit can be written in the form

$$297 \begin{aligned} \Pi_1 &= (W_1 - C_1)(Q_1 - Q_e - Q_{r1}) + (W_1 - C_{r1} - F_1)(Q_e + Q_{r1}) \\ &= (W_1 - C_1)Q_1 + (C_1 - C_r - F_1)(Q_e + Q_{r1}) \end{aligned} \quad (5)$$

298 where

$$300 \quad \frac{\partial(W_1 - C_1)Q_1}{\partial F_1} = \frac{\partial(W_1 - C_1)Q_1}{\partial R_2} = \frac{\partial(W_1 - C_1)Q_1}{\partial N} = 0 \quad (6)$$

301 Thus, $(W_1 - C_1)Q_1$ it is independent of the three decision variables. To simplify
302 the analysis, model (5) can be simplified as

$$303 \quad \Pi_1' = (C_1 - C_r - F_1)(Q_e + Q_{r1}) \quad (7)$$

304 As to manufacturer 2, its profit can be written in the form

$$305 \quad \begin{aligned} \Pi_2 &= (W_2 - C_2)(Q_2 - Q_{r2}) + (W_2 - C_{r2} - R_2)Q_{r2} \\ &= (W_2 - C_2)Q_2 + (C_2 - C_{r2} - R_2)Q_{r2} \end{aligned} \quad (8)$$

306 where

$$307 \quad \frac{\partial(W_2 - C_2)Q_2}{\partial F_1} = \frac{\partial(W_2 - C_2)Q_2}{\partial R_2} = \frac{\partial(W_2 - C_2)Q_2}{\partial N} = 0 \quad (9)$$

308 Comparably, $(W_2 - C_2)Q_2$ is independent of the three decision variables. To
309 simplify the analysis, the model (8) can be simplified as

$$310 \quad \Pi_2' = (C_2 - C_{r2} - R_2)Q_{r2} \quad (10)$$

311 As to the retailer, its profit is given as

$$312 \quad \Pi_r = (P_1 - W_1)Q_1 + (F_1 - R_1)Q_{r1} + (F_1 - R_1 - N)Q_e + (P_2 - W_2)Q_2 \quad (11)$$

313 Similarly, $(P_1 - W_1)Q_1$, $(P_2 - W_2)Q_2$ is independent of the three decision
314 variables. To simplify the analysis, the model (11) can be simplified as

$$315 \quad \Pi_r' = (F_1 - R_1)Q_{r1} + (F_1 - R_1 - N)Q_e \quad (12)$$

316 So the simplified profit model can be given as

$$317 \quad \begin{cases} \Pi_1' = (C_1 - C_{r1} - F_1)(Q_e + Q_{r1}) \\ \Pi_2' = (C_2 - C_{r2} - R_2)Q_{r2} \\ \Pi_r' = (F_1 - R_1)Q_{r1} + (F_1 - R_1 - N)Q_e \end{cases} \quad (13)$$

318 For further simplification, if we set

$$319 \quad \begin{cases} C_1 - C_{r1} = dC_1 \\ C_2 - C_{r2} = dC_2 \end{cases} \quad (14)$$

320 then the model (13) can be written in the form

$$321 \quad \begin{cases} \Pi_1' = (dC_1 - F_1)(Q_e + Q_{r1}) \\ \Pi_2' = (dC_2 - R_2)Q_{r2} \\ \Pi_r' = (F_1 - R_1)Q_{r1} + (F_1 - R_1 - N)Q_e \end{cases} \quad (15)$$

322 4. Results and Discussion

323 4.1. Model Analysis

324 Our model mainly involves three parties, and they all have their own decision
 325 variable. To maximize the profit, they will adjust the value of decision variables in the
 326 game. We first consider the retailer's decision. The derivative of the retailer's profit
 327 model with respect to N is given as

$$328 \quad \frac{\partial \Pi_r'}{\partial N} = R_2 t_2 - t_1 (F_1 + F_1 (\lambda_3 - 1)) - \gamma n - \beta + \gamma (F_1 - n + F_1 (\lambda_3 - 1)) - F_1 t_2 (\lambda_3 - 1) \quad (16)$$

329 We set equation (16) equals to zero. Then we can get the optimal decision of
 330 retailer

$$331 \quad N^* = \frac{-(\beta - F_1 t_2 - R_2 t_2 - F_1 \gamma \lambda_3 + F_1 \lambda_3 t_1 + F_1 \lambda_3 t_2)}{2\gamma} \quad (17)$$

332 Substituting the optimal decision of retailer N^* into manufacturers profit models,
 333 we get the manufacturers profit models to the retailer's decision

$$334 \quad \left\{ \begin{array}{l} \Pi_1' = (dC_1 - F_1) \left(\frac{\beta}{2} + k_1 + \frac{F_1 t_2}{2} - h_1 R_2 - \frac{R_2 t_2}{2} + F_1 t_2 (\lambda_3 - 1) + \frac{F_1 \gamma \lambda_3}{2} - \frac{F_1 \lambda_3 t_1}{2} \right. \\ \quad \left. - \frac{F_1 \lambda_3 t_1}{2} + \frac{t_1 (b - F_1 t_2 - R_2 t_2 - F_1 \gamma \lambda_3 + F_1 \lambda_3 t_1 + F_1 \lambda_3 t_2)}{2\gamma} - F_1 g_1 (\lambda_3 - 1) \right) \\ \Pi_2' = (dC_2 - R_2) \left(k_2 + g_2 R_2 + \frac{t_1 (\beta - F_1 t_2 - R_2 t_2 - F_1 \gamma \lambda_3 + F_1 \lambda_3 t_1 + F_1 \lambda_3 t_2)}{2\gamma} \right. \\ \quad \left. + F_1 h_2 (\lambda_3 - 1) \right) \end{array} \right. \quad (18)$$

335 The derivatives of the manufacturers' profit models are given as

$$336 \quad \left\{ \begin{array}{l} \frac{\partial \Pi_1'}{\partial F_1} = h_1 R_2 - k_1 - (dC_1 - F_1) \left(\frac{\lambda_3 t_1}{2} - \frac{\gamma \lambda_3}{2} - \frac{t_2}{2} + \frac{\lambda_3 t_2}{2} + g_1 (\lambda_3 - 1) \right. \\ \quad \left. - t_2 (\lambda_3 - 1) + \frac{t_1 (t_2 + \gamma \lambda_3 - \lambda_3 t_1 - \lambda_3 t_2)}{2\gamma} \right) - \frac{F_1 t_2}{2} - \frac{\beta}{2} + \frac{R_2 t_2}{2} - F_1 t_2 (\lambda_3 - 1) \\ \quad - \frac{F_1 \gamma \lambda_3}{2} + \frac{F_1 \lambda_3 t_1}{2} + \frac{F_1 \lambda_3 t_2}{2} \\ \quad \left. - \frac{t_1 (t_1 (b - F_1 t_2 - R_2 t_2 - F_1 \gamma \lambda_3 + F_1 \lambda_3 t_1 + F_1 \lambda_3 t_2))}{2\gamma} + F_1 g_1 (\lambda_3 - 1) \right) \\ \frac{\partial \Pi_2'}{\partial R_2} = (dC_2 - R_2) \left(g_2 - \frac{t_1 t_2}{2\gamma} \right) - g_2 R_2 - k_2 - \\ \quad \frac{t_1 (\beta - F_1 t_2 - R_2 t_2 - F_1 \gamma \lambda_3 + F_1 \lambda_3 t_1 + F_1 \lambda_3 t_2)}{2\gamma} - F_1 h_2 (\lambda_3 - 1) \end{array} \right. \quad (19)$$

337 In our model, manufacturers are of bounded rationality and they play the leading
 338 roles. And the retailer makes decisions following the manufacturers. So the evolution
 339 game model for the three parties is given as

$$\begin{cases}
F_1(t+1) = F_1(t) + v_1 F_1(t) \frac{\partial \Pi_1'}{\partial F_1} \\
R_2(t+1) = R_2(t) + v_2 R_2(t) \frac{\partial \Pi_2'}{\partial R_2} \\
N(t+1) = N(t) + v_3 (N(t)^* - N(t))
\end{cases} \quad (20)$$

341 **Proposition 1**

342 The Nash equilibrium of the game, (F_1^*, R_2^*, N^*) , could be derived by solving

343 the equations of $\frac{\partial \Pi_1'}{\partial F_1} = 0, \frac{\partial \Pi_2'}{\partial R_2} = 0$.

344 **Proposition 2**

345

346 For the optimal trade in strategy, we can observe that:

- 347 1. The trade in price will ramp up with the increasing of sell-back price of
348 manufacturer 1 when $t_2 + \lambda_3(\gamma - t_1 - t_2) > 0$;
- 349 2. The trade in price will ramp up with the increasing of sell-back price of
350 manufacturer 1 when $t_2 + \lambda_3(\gamma - t_1 - t_2) < 0$;
- 351 3. The trade in price will definitely ramp up with the increasing of sell-back price
352 of manufacturer 2.

353 *Proof:* It can be obtained that $\frac{\partial N^*}{\partial F_1} = \frac{t_2 + \lambda_3(\gamma - t_1 - t_2)}{2\gamma}$. Hence, the first and the

354 second lemma are easily proved by figuring out the inequation of $\frac{\partial N^*}{\partial F_1} > 0$ and

355 $\frac{\partial N^*}{\partial F_1} < 0$. Additionally, we can obtain that $\frac{\partial N^*}{\partial R_2} = \frac{t_2}{2\gamma}$, which can generate the third

356 lemma of Proposition 2.

357 From Proposition 1, we can know that that if manufacturer 2 sets a higher
358 sell-back price, the trade in price will be increased. This finding reveals the
359 competition of recycling between the retailer and the manufacture 2 – a player will
360 increase the recycling price if his competitor set a more higher recycling price. That
361 make sense in practice.

362

363 **4.2. Inherent Complexity Analysis**

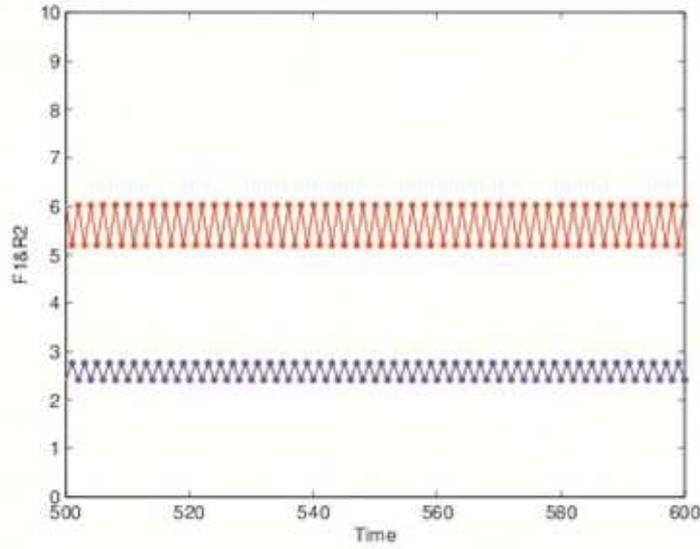
364 First, a numerical simulation has been done to do a pre-analysis for our model.

365 We set parameters as

366
$$k_1 = 0.5, k_2 = 0.5, g_1 = 1, g_2 = 1, h_1 = 0.3, h_2 = 0.3,$$

$$\beta = 0.3, \gamma = 0.3, t_2 = 1, dC_1 = 2.5, dC_2 = 2.5, \lambda_3 = 0.2$$

367 The dynamic changes of decision variables F_1 and R_2 from cycle 100 to cycle
 368 200 are shown in Fig.2.



369

370 Fig. 2. Time Sequence Diagram

371

372 As shown in Fig.2, under the set parameters, the changes of decision variables
 373 are cyclical. The whole game is in an unstable state. Thus, we take a further analysis
 374 of the stability of the model.

375 The Jacobi matrix of the system can be given as

376
$$Jac = \begin{bmatrix} J_{11} & v_1 F_1 \left(h_1 + \frac{t_2}{2} + \frac{t_1 t_2}{2\gamma} \right) & 0 \\ -v_2 R_2 \left(h_2 (\lambda_3 - 1) + \frac{(t_1 (\lambda_3 t_1 - \gamma \lambda_3 + t_2 (\lambda_3 - 1)))}{2\gamma} \right) & J_{22} & 0 \\ \frac{-(v_3 (\lambda_3 t_1 - \gamma \lambda_3 + t_2 (\lambda_3 - 1)))}{2\gamma} & \frac{v_3 t_2}{2\gamma} & 1 - v_3 \end{bmatrix} \quad (21)$$

377 According to the Jury criterion, the stability of the discrete dynamic system
 378 relates to whether the eigenvalues are in the unit circle. The eigenvalues of the model
 379 can be written as

$$\begin{aligned}
& |Jac - \lambda E| \\
& = \begin{vmatrix} J_{11} - \lambda_1 & v_1 F_1(h_1 + \frac{t_2}{2} + \frac{t_1 t_2}{2\gamma}) \\ -v_2 R_2(h_2(\lambda_3 - 1) + \frac{(t_1(\lambda_3 t_1 - \gamma \lambda_3 + t_2(\lambda_3 - 1)))}{2\gamma}) & J_{22} - \lambda_2 \end{vmatrix} |1 - v_3 - \lambda_3| \\
& \Rightarrow \lambda_3 = 1 - v_3
\end{aligned}
\tag{22}$$

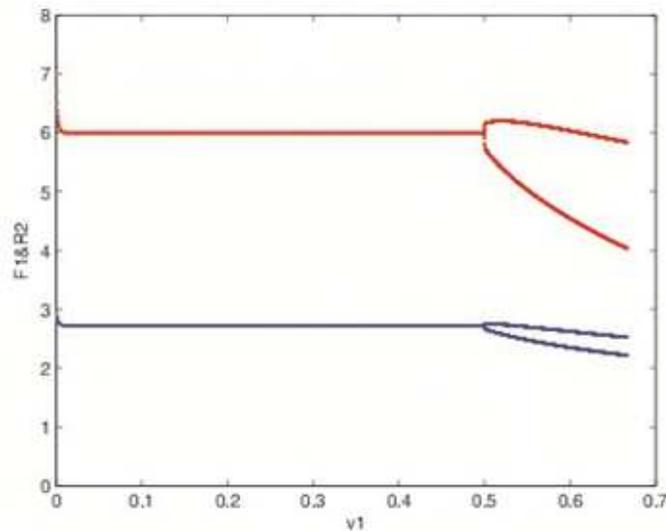
382 **Proposition 3**

383 The dynamic system will be stable if the adjustment speed of the
384 retailer is in the range of $v_3 \in [0, 2]$.

385 *Proof* According to equation (22), if $v_3 \in [0, 2]$, then $\lambda_3 \in [-1, 1]$. In such a
386 condition, the third eigenvalue is in the unit circle.

387 Proposition 3 indicates that if the retailer adjusts his trade in speed in
388 a not so high speed, the dynamic system will not trapped into unstable
389 state accordingly. Hence, the stability of the model is related to
390 manufacturers' decisions.

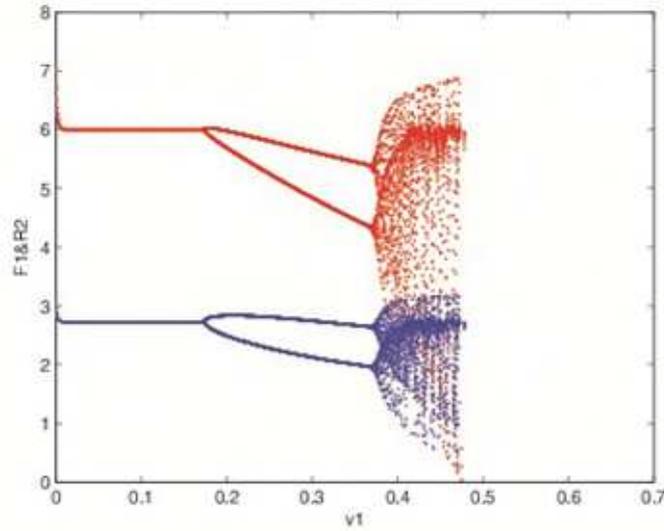
391 Then, we further analyze the influence of the decision adjustment speed of
392 manufacturer 1 and manufacturer 2 on the stability of the game. We set the decision
393 adjustment speed of manufacturer 1 v_1 as a variable and the decision adjustment
394 speed of manufacturer 2 as $v_2 = 0.1$ and $v_2 = 0.3$. The results of numerical
395 simulation are shown in Fig.3 and Fig.4.



396

397

Fig. 3. Bifurcation diagram when $v_2 = 0.1$



398

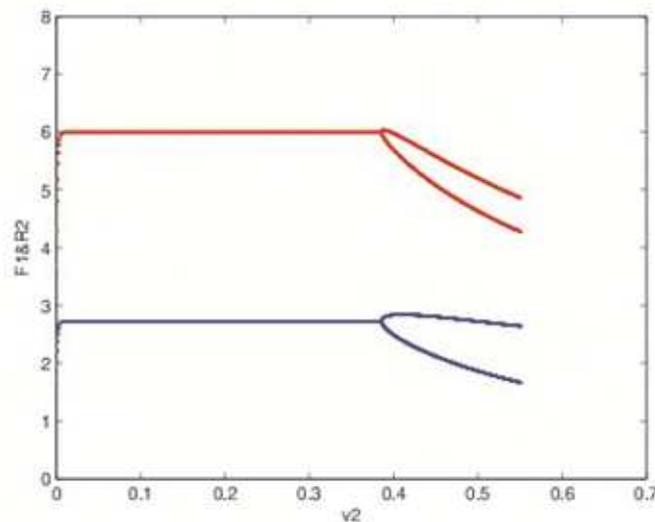
399

Fig. 4. Bifurcation diagram when $v_2 = 0.3$

400 The results show that with the increase of the decision adjustment speed of
 401 manufacturer 1, the decision variables of manufacturer 1 and manufacturer 2 go from
 402 a stable state into chaos. Besides, the decision adjustment speed of manufacturer 2 has
 403 a significant impact on the decision variables.

404 Next, we set the decision adjustment speed of manufacturer 2 v_2 as a variable
 405 and the decision adjustment speed of manufacturer 1 as $v_1 = 0.1$ and $v_1 = 0.5$. The
 406 results of numerical simulation are shown as Fig.5 and Fig.6.

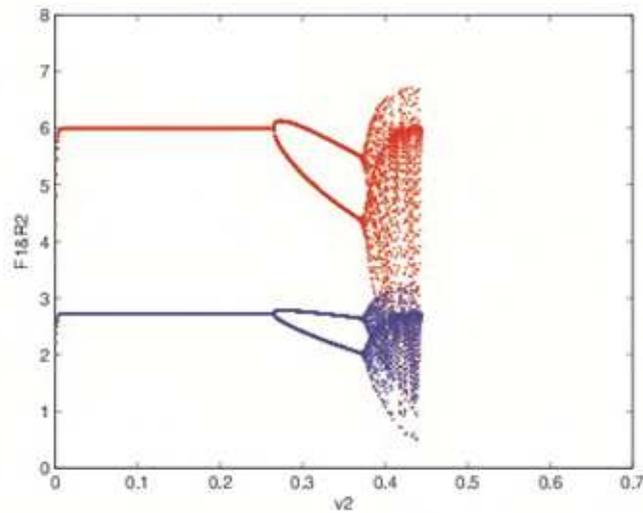
407



408

409

Fig. 5. Bifurcation diagram when $v_1 = 0.1$



410

411

Fig. 6. Bifurcation diagram when $v_1 = 0.5$

412

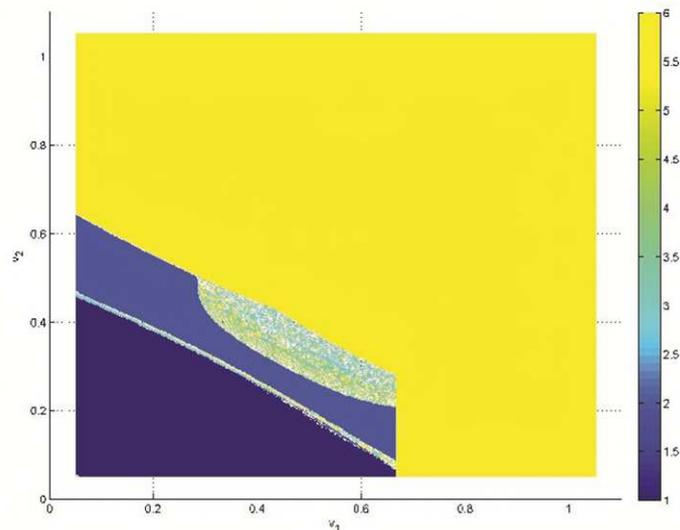
According to the results, the trends of the change of the decision variables are similar to those of the previous numerical simulation, which also go from a stable state into chaos. Hence, we can get the conclusion that the stability of the game is related to both v_1 and v_2 . To make a better illustration, we draw the stability domain diagram concerning v_1 and v_2 .

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418

Fig. 7. The stable region of v_1 and v_2

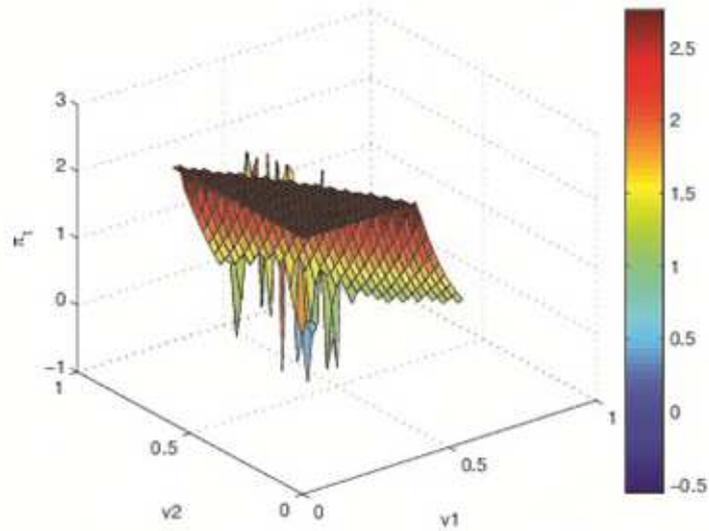
419

In Fig.7, different color represents a different stability state. The Dark blue represents the model in a stable state, the light blue represents the model in the bifurcation phase, and the color of scattering represents the model in a chaotic state. According to this figure, the stability of the model decreases with the increase of

422

423 adjustment speed v_1 and v_2 , In other words, the swifter response indicates weaker
424 market stability.

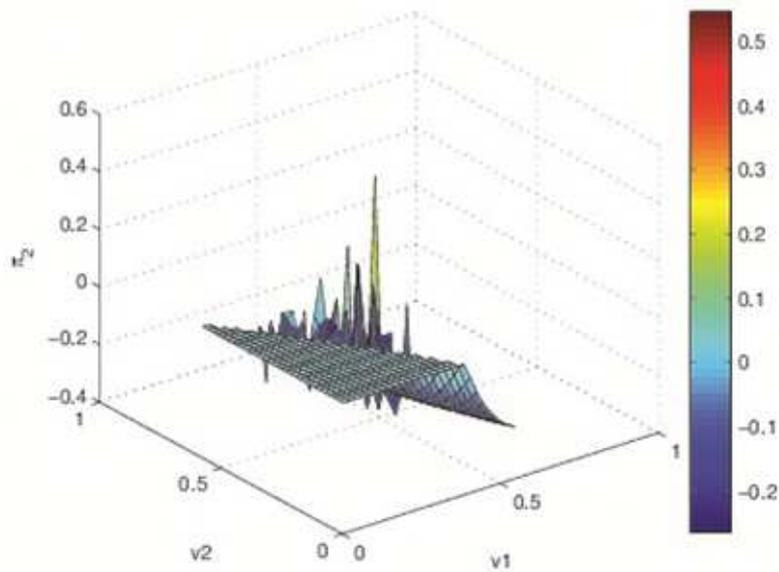
425 Next, we will further analyze the impact of the instability on the profits of
426 manufacturers and retailers. The three-dimensional figures from Fig. 8 to Fig. 10
427 reveal the trend of the profits of the manufacturers and the retailer.



428

429

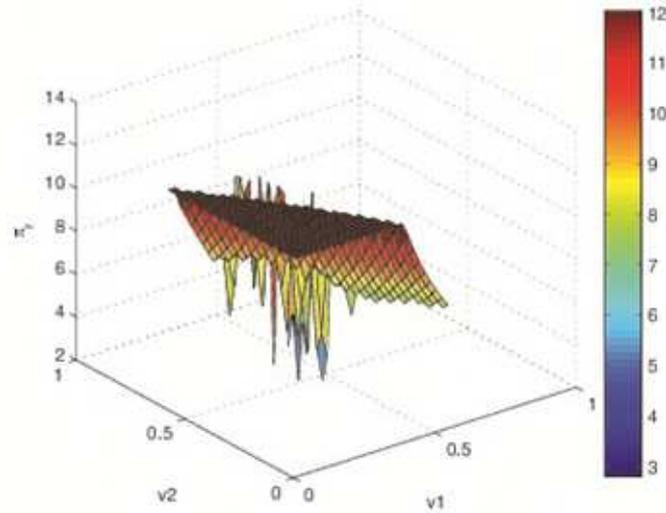
Fig. 8. The profit of the manufacturer 1 for v_1 and v_2



430

431

Fig. 9. The profit of the manufacturer 2 with respect to v_1 and v_2



432

433

Fig. 10. The profit of the retailer with respect to v_1 and v_2

434

As expected, with the increase of the decision adjustment speed v_1 and v_2 , the decision variables go unstable, resulting in the instability of the profits of manufacturers and retailers. And on average, both manufacturers and retailers have a profit loss in multiple cycles. Thus, the chaos is unfavorable for the entire market, which seriously affects the normal operation of the parties in the game.

435

436 4.3. Impact from Parameters

437

In addition to analyzing the speed of adjustment v_1 and v_2 , how to impact the market of manufacturers and retailers. We also analyze the impact of several key parameters in the game on the evolution process from the perspective of profit, which is the concern of all parties.

438

439 4.3.1 Cost reduction of manufacturer 1

440

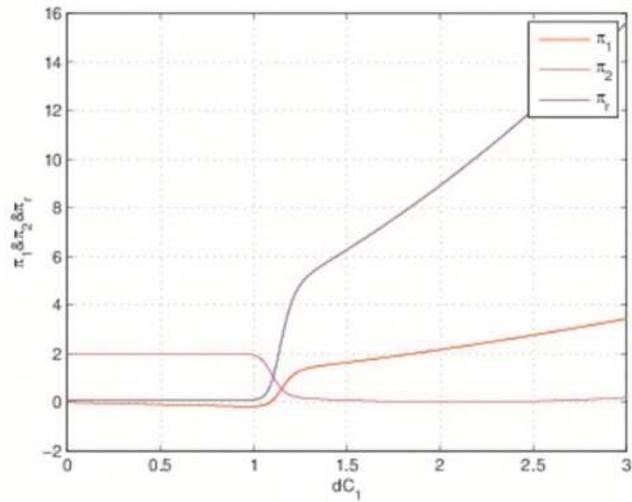
According to the above results, the stability of the system is related to the manufacturer's adjustment speed v_1 and v_2 . With the increase of adjustment speed v_1 and v_2 , the system goes from a stable state into an unpredictable instability. Here, we analyze the impact of cost reduction of manufacturer 1 dC_1 and the profits of the three parties in the game under the stable state and the unstable state.

441

442

Based on the above analysis, we set $v_1 = 0.2$ and $v_2 = 0.2$ for the stable state while $v_1 = 0.3$ and $v_2 = 0.5$ for the unstable state.

443



452

453

Fig. 11. Profit with respect to dC_1 when $v_1 = 0.2, v_2 = 0.2$

454

455

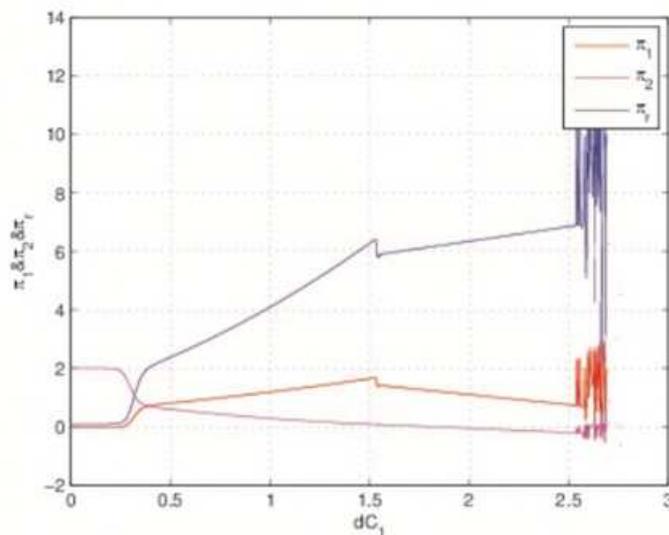
456

457

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459

As shown in Fig.11, with the increase in cost reduction of manufacturer 1, the profit of manufacturer 1 and the retailer have a similar trend in the stable state. The change is indistinctive at the beginning and then goes significant. The profit trend of manufacturer 2 stays at first, then different from those of manufacturer 1 and the retailer, which decreases with a cost reduction of manufacturer 1 increasing. The profit with respect to the diagram in the unstable state can be seen in Fig.12



460

461

Fig. 12. Profit with respect to dC_1 when $v_1 = 0.3, v_2 = 0.5$

462

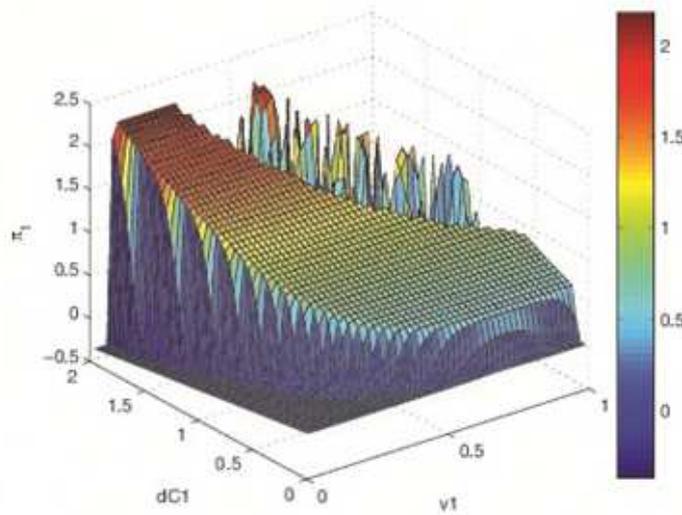
463

464

465

However, in the unstable state, the profit of manufacturer 1 and that of the retailer have a similar trend. They both have little change with the cost reduction dC_1 then go up and fall off a cliff finally. After that, the profit of manufacturer 1 keeps increasing and becomes fluctuant; meanwhile, the profit of the retailer keeps

466 decreasing and becomes fluctuant. The explanation is that in the recycling stage, the
 467 retailer is responsible for recycling used products and sells them back to
 468 manufacturer 1, and a cooperative relationship forms between manufacturer 1 and
 469 the retailer. The similar trend of the profit of manufacturer 1 and that of the retailer is
 470 shown in Fig. 12. Manufacturer 2 also collects used products from the market, so he
 471 competes with manufacturer 1 and the retailer. In Fig. 12, with the cost reduction dC_1
 472 increasing, the profit of manufacturer 2 remains stable first and then goes decreasing
 473 which is different from the trend of manufacturer 1 and the retailer. In the end, the
 474 profit of manufacturer 2 also becomes fluctuant.



475
 476

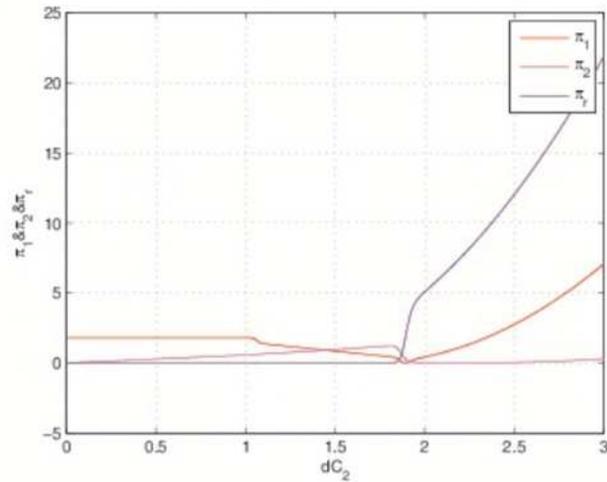
Fig. 13. Profit of manufacturer 1 with respect to v_1 and dC_1

477 For further analysis, we make a numerical simulation of the profit of
 478 manufacturer 1 with respect to the adjustment speed v_1 and cost reduction dC_1 .
 479 Fig.13 shows the result. From Fig.13, we can get a similar conclusion that with the
 480 increase of cost reduction, the profit of manufacturer 1 is increasing. However, when
 481 the adjustment speed v_1 is far too high, the system will step into chaos and the
 482 manufacture will suffer profit losses.

483 4.3.2 Cost reduction of manufacturer 2

484 Similarly, we analyze the impact of cost reduction of manufacturer 2 dC_2 on
 485 the profits of the three parties in the game in the stable state and the unstable state.
 486 And again we set $v_1 = 0.2$ and $v_2 = 0.2$ for the stable state while $v_1 = 0.3$ and
 487 $v_2 = 0.5$ for the unstable state.

488 The profit with respect to dC_2 when $v_1 = 0.2, v_2 = 0.2$ a diagram can be seen in
 489 Fig.14.



490

491

Fig. 14. Profit with respect to dC_2 when $v_1 = 0.2, v_2 = 0.2$

492

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497

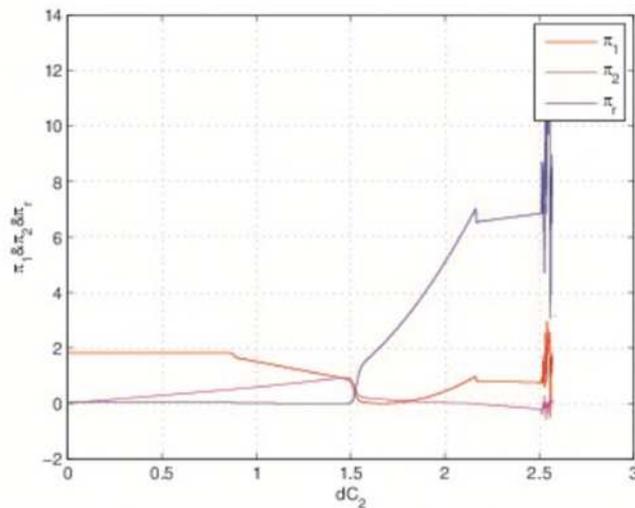
498

It's easy to find out that in the stable state, with the increase of its cost reduction dC_2 , the profit of manufacturer 2 rises first and then decreases suddenly, after that, the profit of manufacturer 2 stays approximate to zero. While the profit of manufacturer 1 stays the same first and then drops, the profit of the retailer has no change at the same time. Then the profit of manufacturer 1 and that of the retailer increase simultaneously. There is a minimum value for the total profit of the three parties in the game. In general, the profit of each party is stable.

499

500

The profit with respect to dC_2 when $v_1 = 0.3, v_2 = 0.5$ the diagram can be seen in Fig.15



501

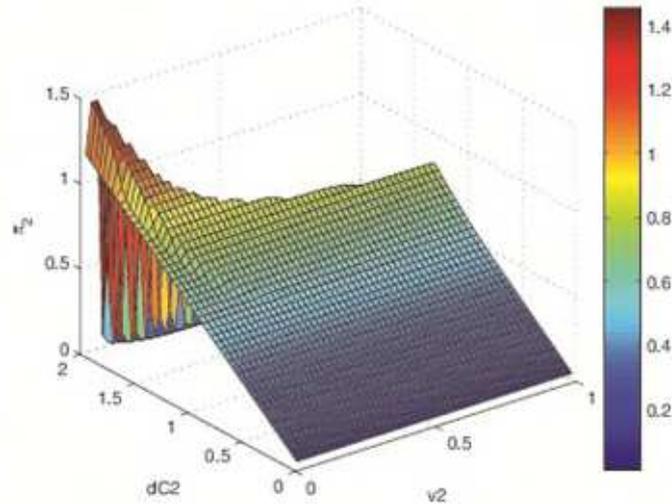
502

Fig. 15. Profit with respect to dC_2 when $v_1 = 0.3, v_2 = 0.5$

503

In the unstable state, it goes similarly to Fig.14 when the cost reduction dC_2 is

504 small. Then the profit of manufacturer 1 and that of the retailer fall off a cliff. The
 505 profit of manufacturer 1 keeps decreasing; At the same time, the profit of
 506 manufacturer 2 decreases continuously. After that, with the increase in its cost
 507 reduction, the profit of the three parties fluctuate and becomes unstable, which is bad
 508 for all parties in the game.



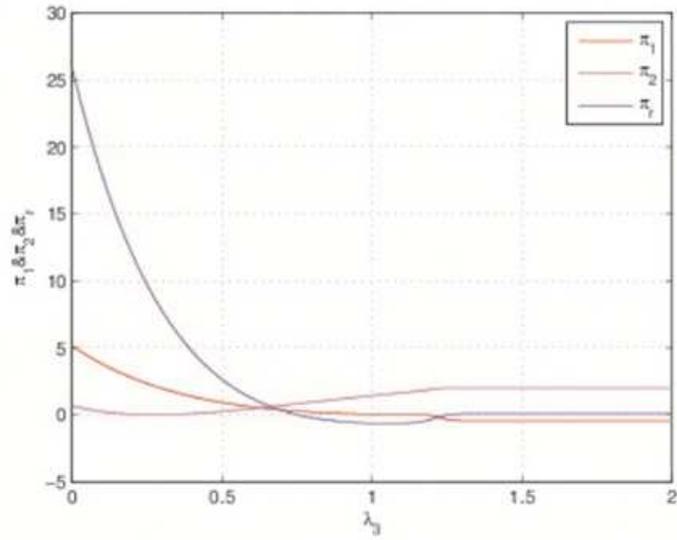
509

510 Fig. 16. Profit of manufacturer 2 with respect to v_2 and dC_2

511 As before, we analyze the profit of manufacturer 2 with respect to adjustment
 512 speed v_2 and cost reduction dC_2 and Fig.16 presents the result. From Fig.16, we
 513 can get a similar conclusion that with the increase of cost reduction dC_2 , the profit of
 514 the manufacturer reaches the maximum and then drops off sharply. However, when
 515 the adjustment speed v_2 is too high, the system will step into chaos and it will take a
 516 toll on the profit.

517 4.3.3 The price coefficient of the retailer

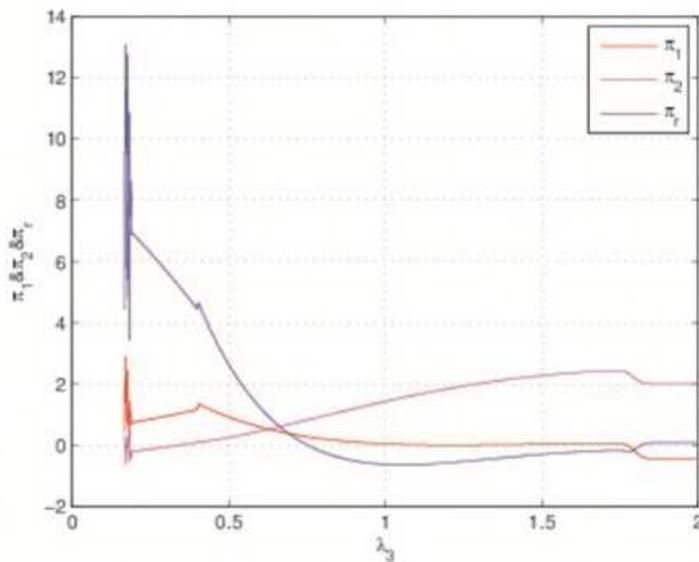
518 Next, we move on to the price coefficient of the retailer λ_3 and explore its
 519 impact on the profit of the parties in the game. And we also set $v_1 = 0.2$ and $v_2 = 0.2$
 520 for the stable state and $v_1 = 0.3$ $v_2 = 0.5$ for the unstable state.



521

522

Fig. 17. Profit with respect to λ_3 when $v_1 = 0.2, v_2 = 0.2$



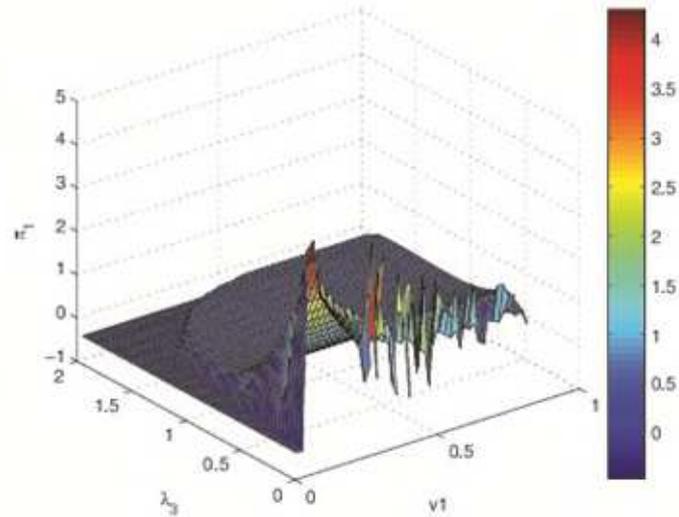
523

524

Fig. 18. Profit with respect to λ_3 when $v_1 = 0.3, v_2 = 0.5$

525 From the comparison of Fig.17 and Fig.18, it can be concluded that as for
 526 manufacturer 1 and the retailer, the smaller price coefficient of the retailer can bring
 527 higher profits, but also aggravates the chaotic condition in which the stability of the
 528 whole system will decrease. Thus, there is a contradiction between the profitability
 529 and the stability of the whole system. With the increasing price coefficient of the
 530 retailer, the profit of manufacturer 2 also increases in general, which is different from
 531 the trend of manufacturer 1 and the retailer.

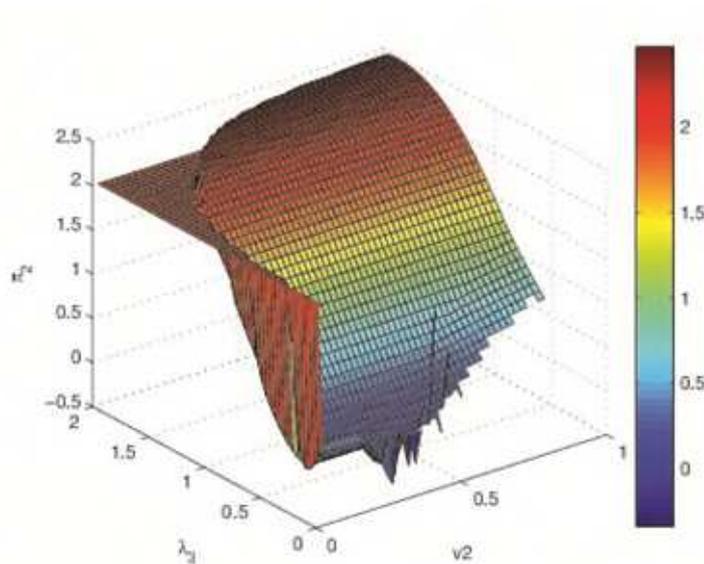
532



533

534

Fig. 19. Profit of manufacturer 1 with respect to v_1 and λ_3



535

536

Fig. 20. Profit of manufacturer 1 with respect to v_2 and λ_3

537 From the above three-dimensional Fig.19 and Fig.20 we can also see that with
 538 the increase of the price coefficient of the retailer, the profit trend of manufacturer 1
 539 and that of manufacturer 2 are opposite. As for manufacturer 1, the profit goes
 540 decreasing with the price coefficient of the retailer increasing; as for manufacturer 2,
 541 the profit goes increasing with the price coefficient of the retailer increasing.

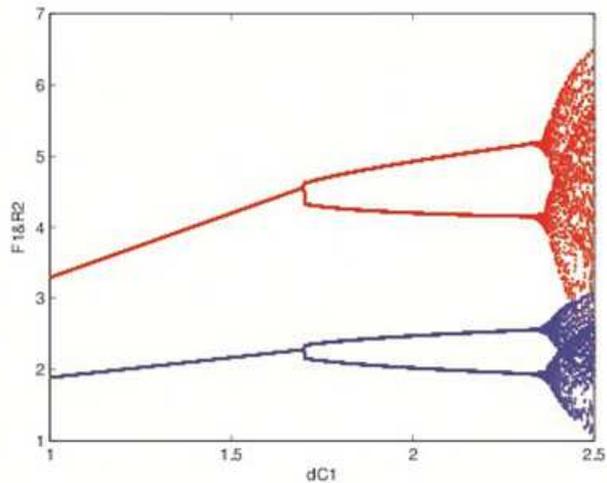
542 4.3.4 Chaos Control

543 As we stated, when the state of the system becomes chaotic, that is, the market is
 544 in a chaotic state, the profit of the system will decrease and the decision-making
 545 process of supply chain's members will become more complicated than before, which
 546 is harmful to economic development. Therefore, we need to control the chaotic state
 547 to benefit the whole system. In this section, we control chaos from two aspects:

548 adjusting the related parameters and the decision method.

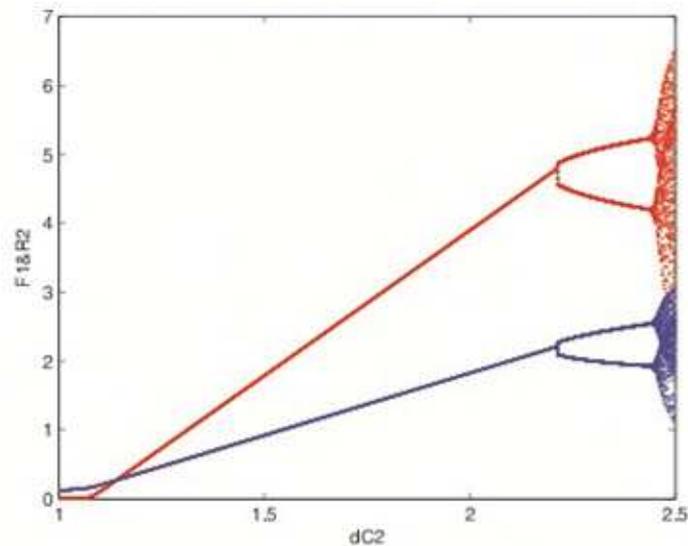
549 (1) Parameter control method

550 As for manufacturer 1 and manufacturer 2, according to the above discussion and
551 conclusion, dC_1 and dC_2 are two important parameters that can be improved. Then,
552 we set parameters $v_1 = 0.4, v_2 = 0.4$ and study the change of the system stability with
553 the change of different manufacturer cost variables. The results of numerical
554 simulation are shown in Fig.21 and Fig.22.



555

556 Fig.21. Bifurcation diagram with the change of dC_1 when $v_1 = 0.4, v_2 = 0.4$



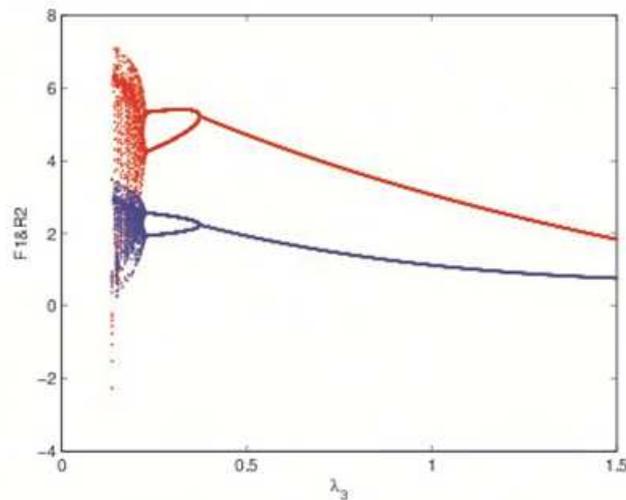
557

558 Fig.22. Bifurcation diagram with the change of dC_2 when $v_1 = 0.4, v_2 = 0.4$

559 In Fig.21 and Fig.22, we draw the trend of the change of decision variables with

560 the change of dC_1 and dC_2 . As can be seen in Fig.21 and Fig.22, with the increase
 561 of dC_1 and dC_2 , the stability of the system declines, bifurcation, and chaos form. As
 562 for manufacturer 1 and manufacturer 2, dC_1 and dC_2 represent profit margin
 563 between remanufacturing and manufacturing, and according to the conclusion we got,
 564 the profit of manufacturer 1 will increase with dC_1 increasing and the profit of
 565 manufacturer 2 will increase with dC_2 increasing. So profit and system stability
 566 need to be balanced, that is, manufacturers need to further optimize the manufacturing
 567 process of new products, and higher stability can be obtained through reducing C_1
 568 and C_2 .

569 As for the retailer, according to the above analysis, the retailer price coefficient
 570 λ_3 , which represents the increase in retailer price, can be improved. The trend of the
 571 change of decision variables with the change λ_3 is shown in Fig.23.



572

573 Fig.23. Bifurcation diagram with the change of λ_3 when $v_1 = 0.4$, $v_2 = 0.4$

574 In Fig.23, we can see that with the decrease of λ_3 , the stability of the system
 575 declines, bifurcation and chaos form, which means the higher λ_3 is, the higher
 576 retailer price is, and the stronger system's stability is. Therefore, retailers can
 577 enhance their earnings stability by increasing the price of corresponding products.

578 (2) Decision-making method

579 Based on the characteristics of the whole decision-making process, we adopt an
 580 adaptive method to control the chaotic state. That is, when making decisions next
 581 period, we take not only decisions in this period into consideration, but also earlier
 582 decisions. Thus, the system can be summarized as follows.

583

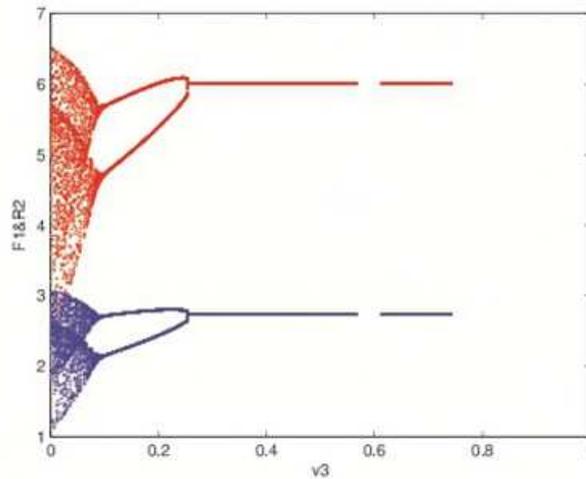
$$\begin{cases} F_1(t+1) = F_1(t) + v_1 F_1(t) \frac{\partial \Pi_1'}{\partial F_1} + v'(F_1(t) - F_1(t-1)) \\ R_2(t+1) = R_2(t) + v_2 R_2(t) \frac{\partial \Pi_p}{\partial R_2} + v'(R_2(t) - R_2(t-1)) \end{cases} \quad (23)$$

584

We simplify equation (23) based on topological equivalence and then adopt $v_1 = 0.5, v_2 = 0.35$ and make Fig. 24 as follows to show the effect of the control parameter's control of chaos.

585

586



587

Fig. 24. Bifurcation diagram of decision variables with v' increasing

588

589

It can be seen from Fig. 24 that with the increasing control parameter v' , the state of the system goes into period-doubling bifurcation from chaos and then becomes stable. When the system is in a stable state, players who make decisions in this game will obtain higher profits and make their own decisions more controllable.

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In summary, we propose two methods of chaos control in the game process of two manufacturers and the retailer. In the parameter control method, we keep the enterprise decision-making adjustment speed and control system from the perspective of the actual production situation. As a result, we obtain final equilibrium. In the decision-making method, we maintain the original game situation and change the decision plan of the enterprises, which makes the response of the enterprise to the market delay. Manufacturers and the retailer can consider two methods based on the actual situation in the actual decision-making process.

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5. Conclusion and Policy Implications

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Studying the WEEE recycling activities has important ramifications for corporations, the whole supply chain system, and the market. In an electronics CLSC, more and more large retailers (eg: Mediaworld, Fnac, and Darty) offer

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605 recycling services to increase their attractiveness. Recycling services are extremely
606 relevant for the key manufacturer, retailer, and customers in the supply chain.
607 Especially, some kinds of electronic products are very difficult to transport and handle,
608 which is beneficial to customers (De Giovanni, 2018).

609 In this paper, a closed-loop supply chain game model is constructed based on
610 two recycling methods for product 1. The influence of decision variables on the
611 stability of the model is analyzed, and the stability control of the unstable system by
612 two control methods is explored. The main findings of the paper get the following
613 results.

614 (1) The results show that the stability of the model is related to the decision
615 adjustment speed of the manufacturers. Excessive decision adjustment speed can
616 easily lead to the unstable state of the system.

617 (2) At this time, the profits of the manufacturer and the retailer will become
618 unstable, seriously affecting the normal operation of both sides of the game. If the
619 adjustment speed of the manufacturers keeps in a rational value,
620 players would achieve great benefits in the real market.

621 (3) The manufacturer's cost reduction is beneficial to itself as well as the retailer,
622 but not to its competitor; the higher the retailer's price coefficient the higher the profit
623 is, the more unstable the system will be.

624 (4) There are two kinds of methods are analyzed in this paper. They can be used
625 to controlled chaotic. The chaotic control of the unstable system is carried out by
626 adjusting the relevant parameters and adjusting the decision method. Both methods
627 have a good control effect.

628 Besides, given the model studied in this paper, further research may be done
629 from following two aspects: one is to consider the environmental benefits brought by
630 waste recycling process; the other is to consider government intervention in this
631 process, such as government subsidies, preferential tax on remanufactured products,
632 etc. Our model can be extended to account for these aspects.

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634 **Competing Interests**

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636 The authors declare that they have no competing interests.

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638 **Availability of data and materials**

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640 Not applicable.

641

642 **References**

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Figures

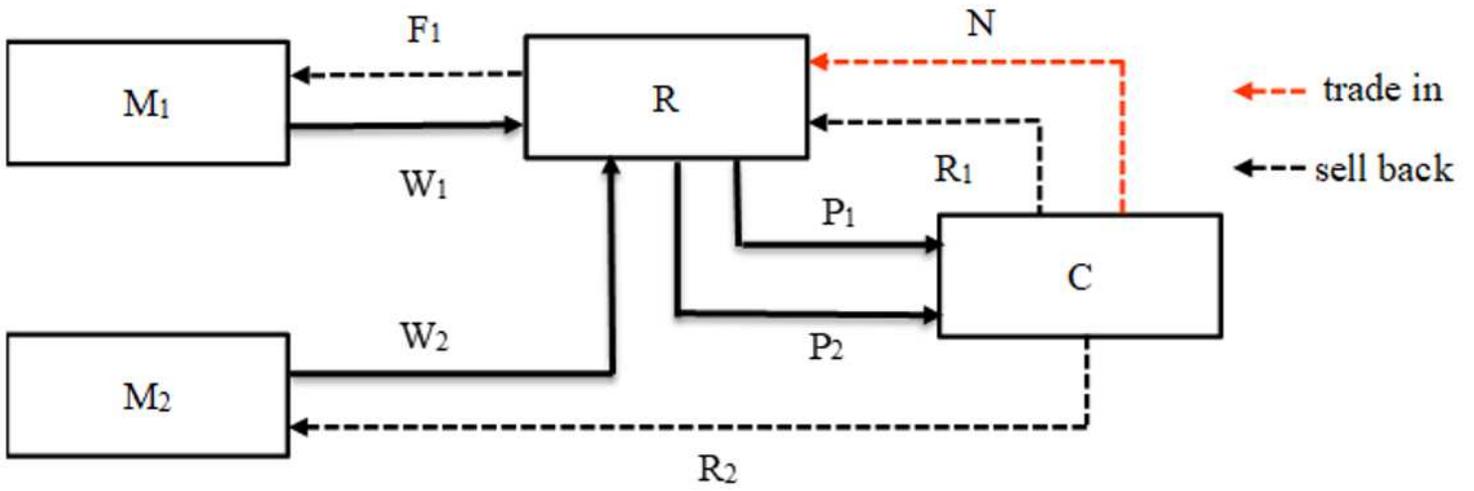


Figure 1

Recycling Supply Chain Model

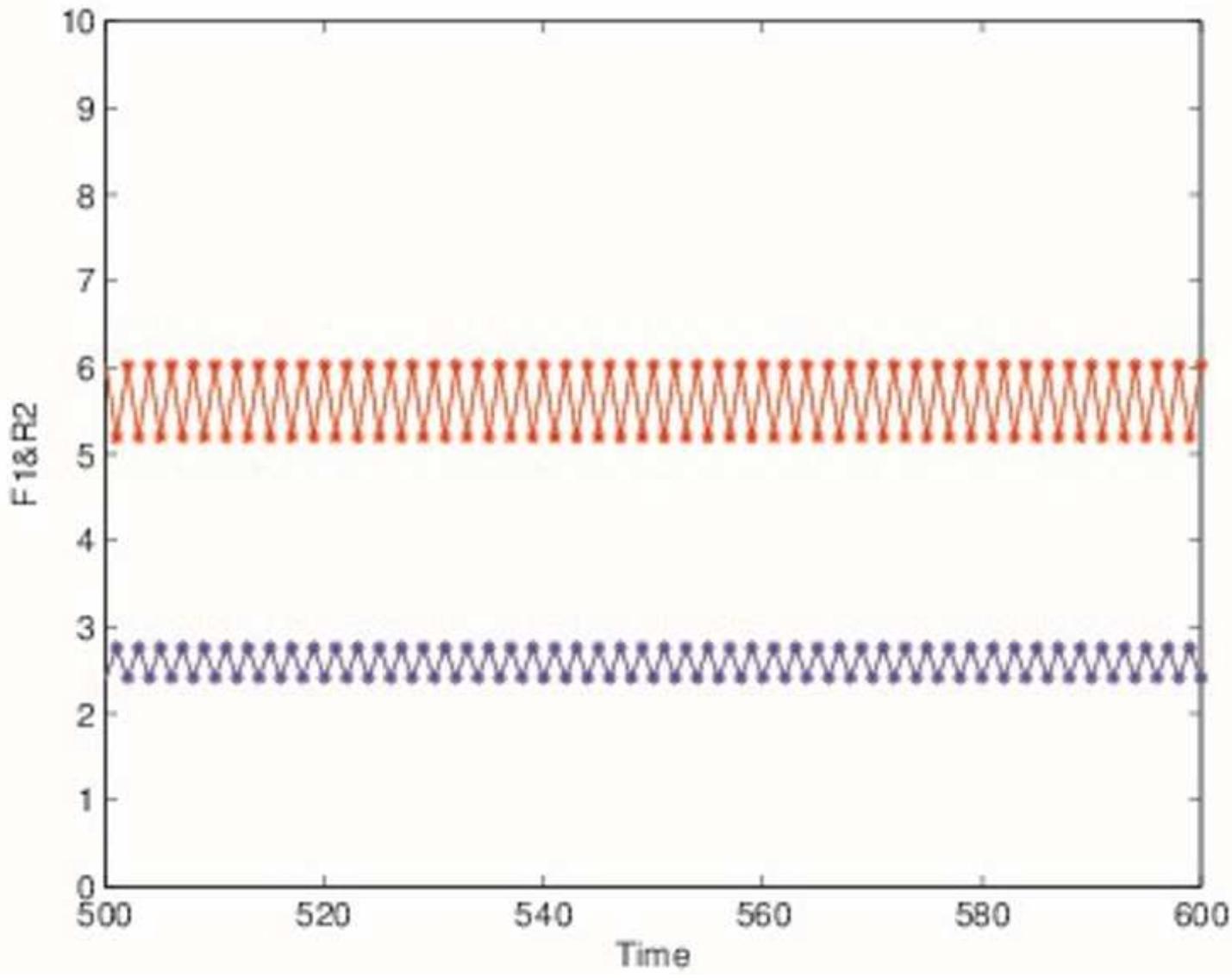


Figure 2

Time Sequence Diagram

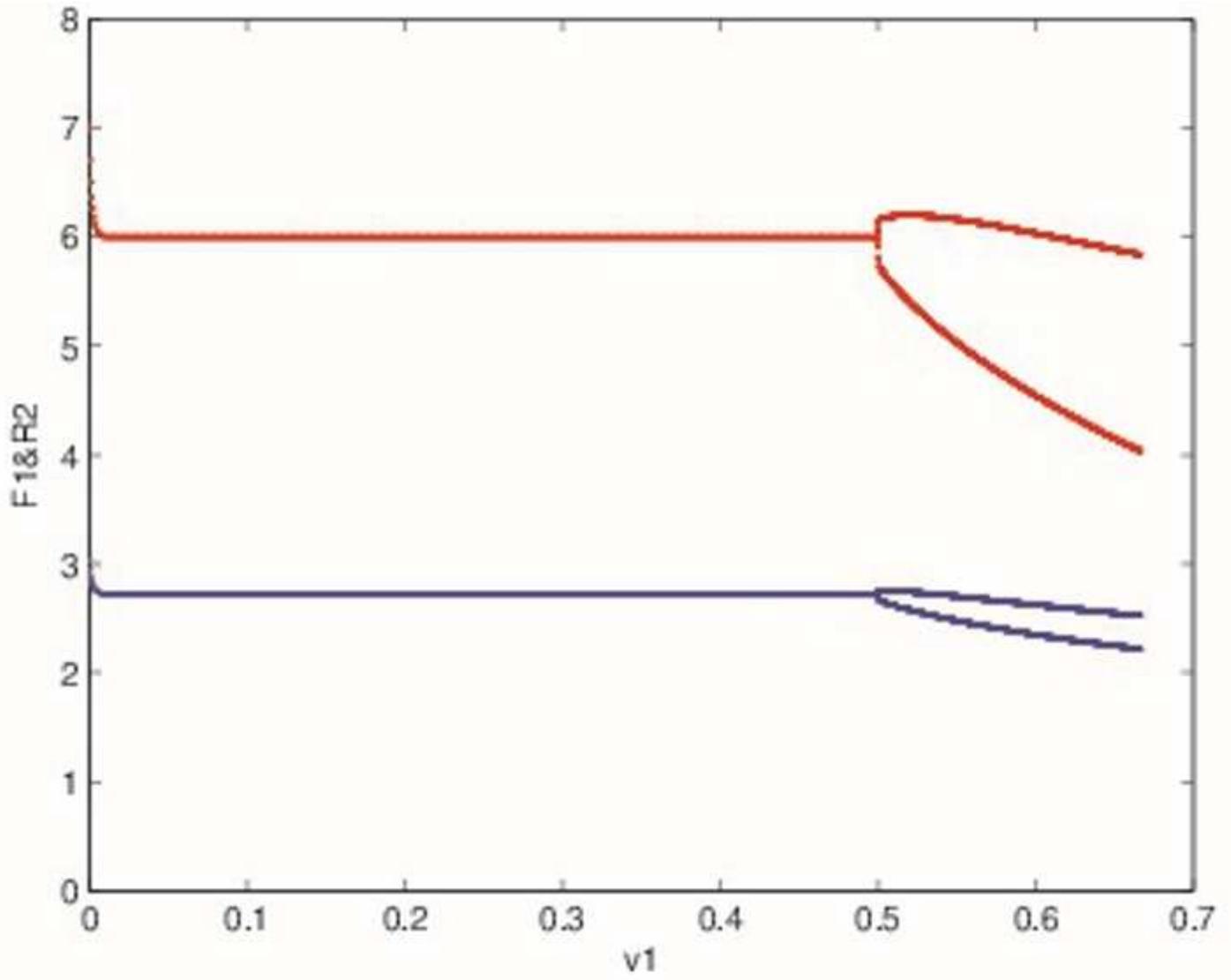


Figure 3

Bifurcation diagram when $v_2=0.1$

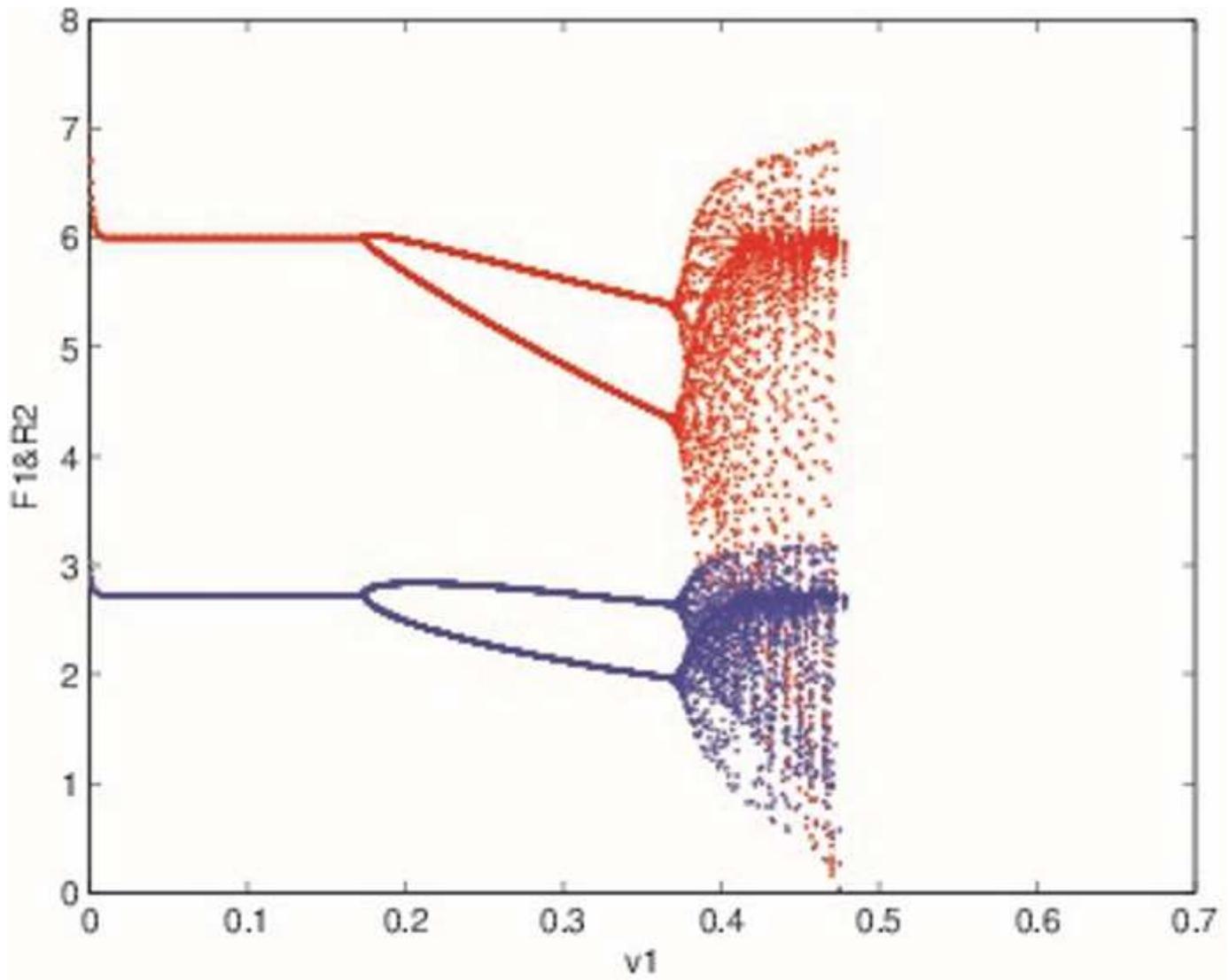


Figure 4

Bifurcation diagram when $v_2 = 0.3$

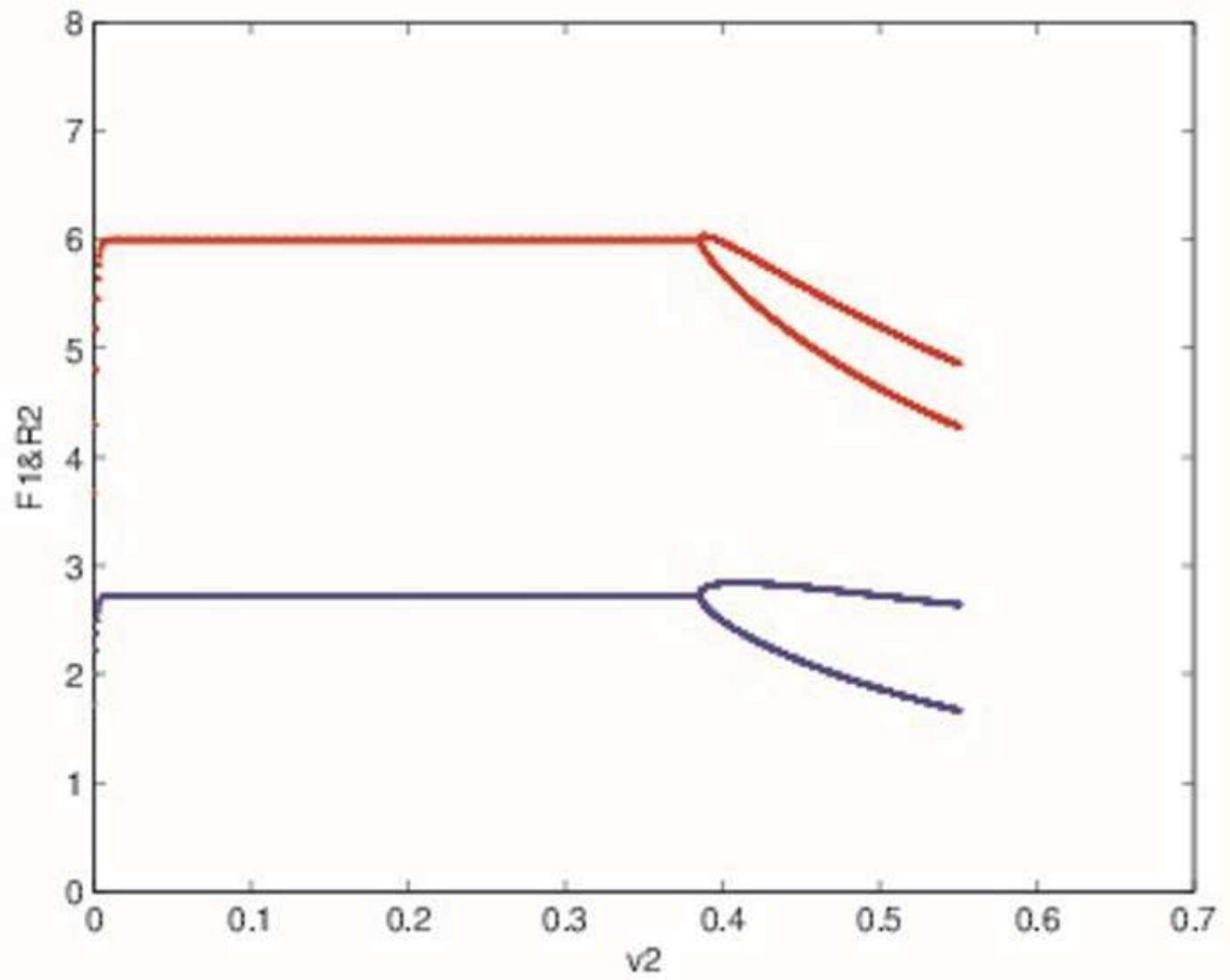


Figure 5

Bifurcation diagram when $v_1 = 0.1$

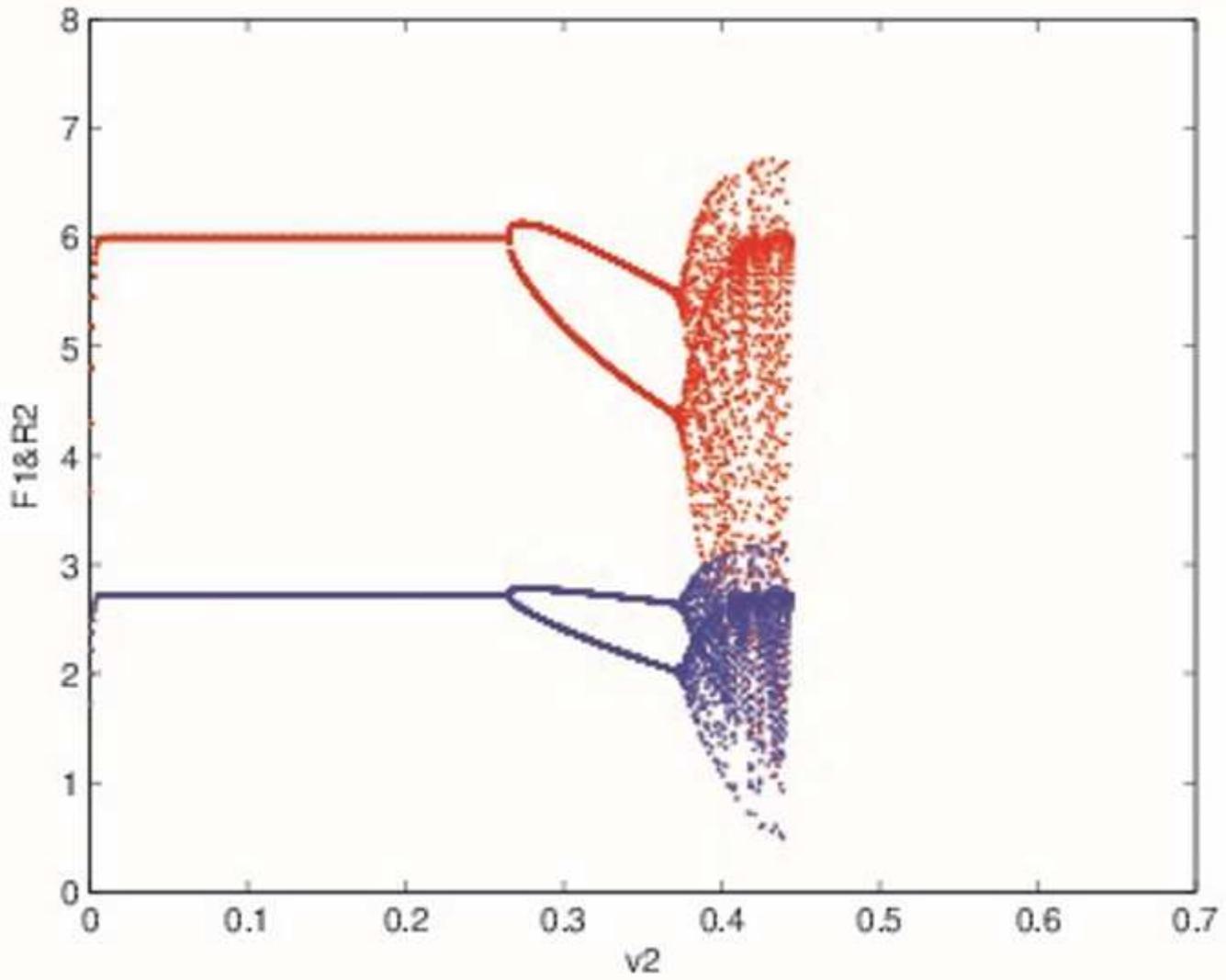


Figure 6

Bifurcation diagram when $v_1 = 0.5$

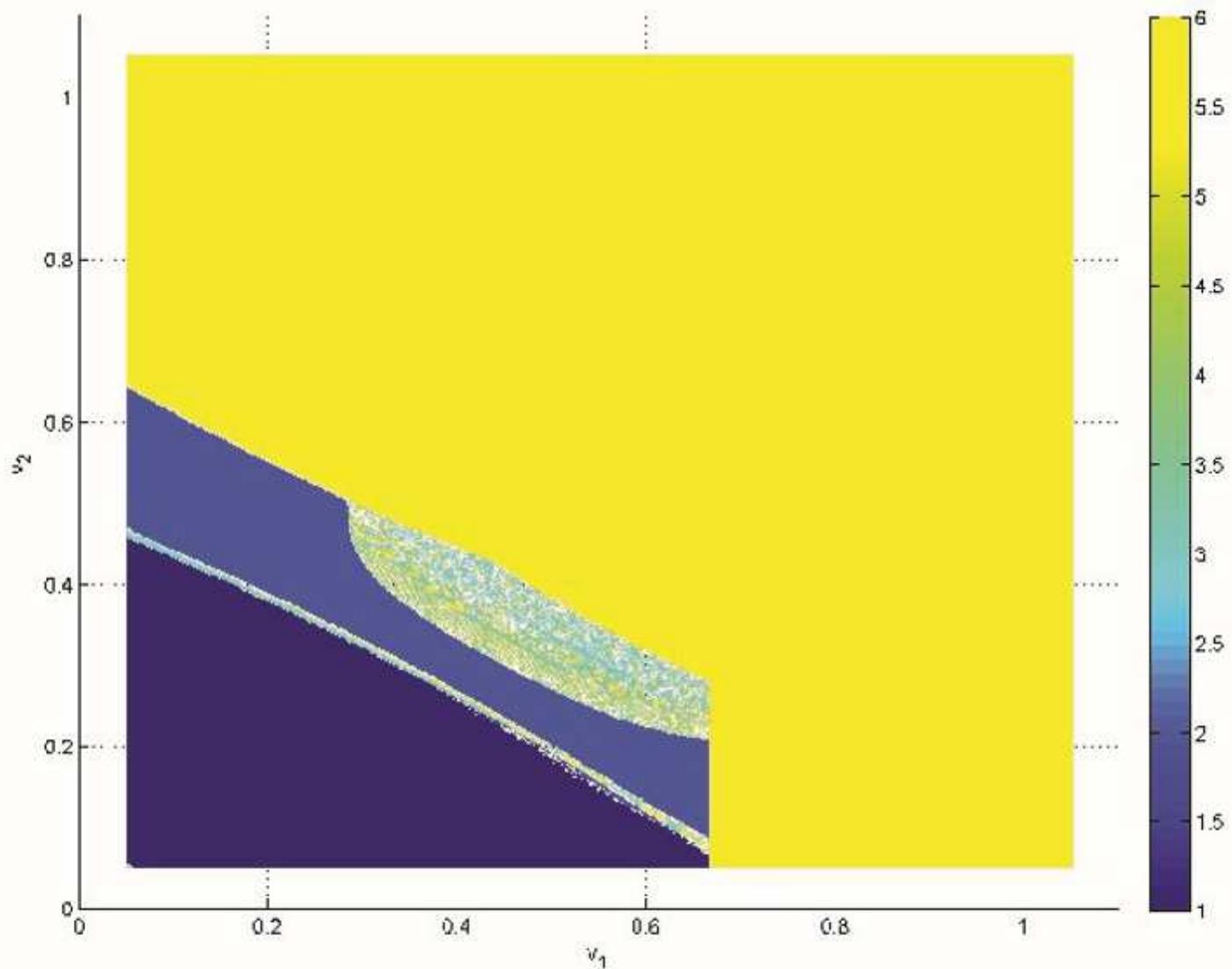


Figure 7

The stable region of v_1 and v_2

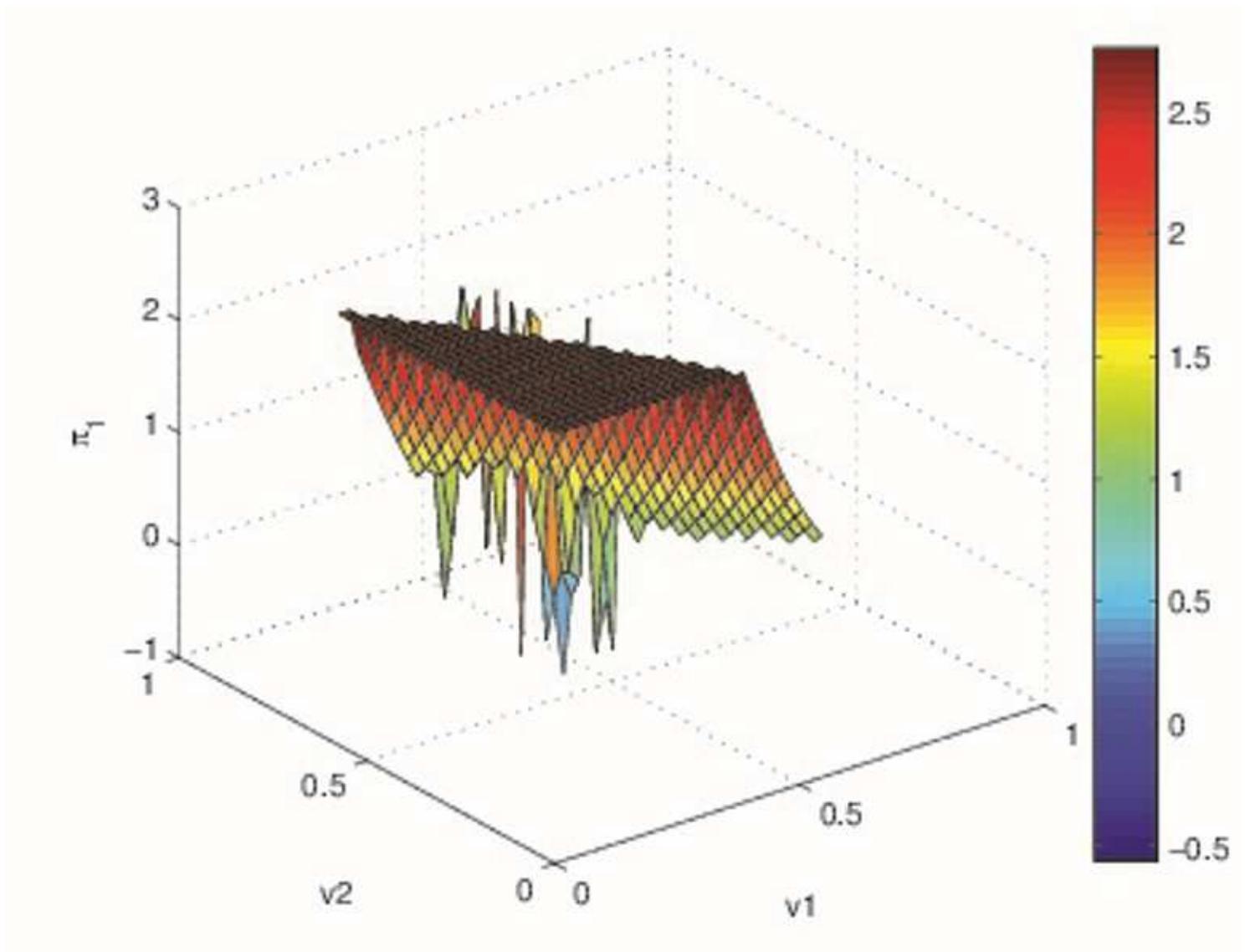


Figure 8

The profit of the manufacturer 1 for v_1 and v_2

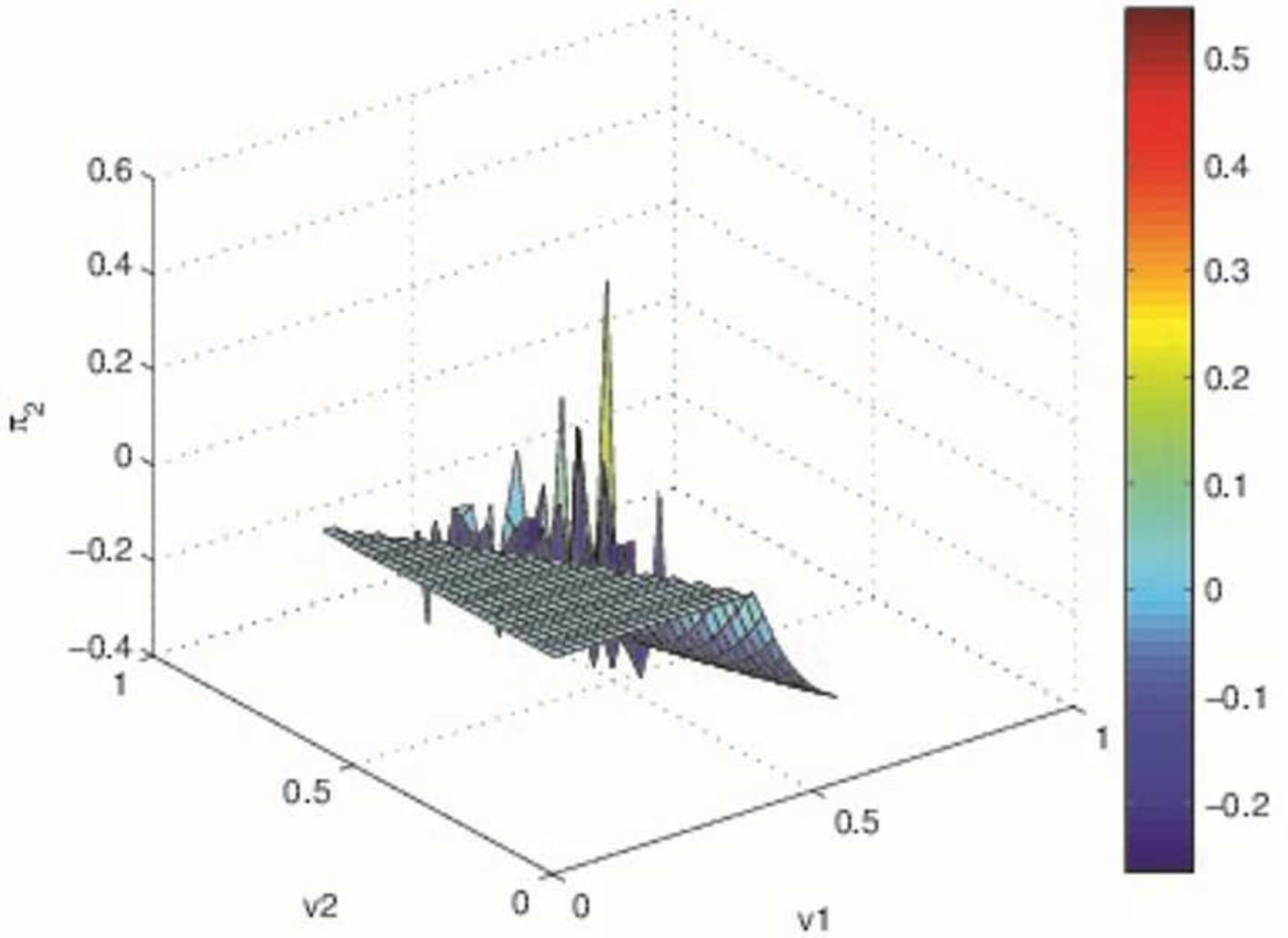


Figure 9

The profit of the manufacturer 2 with respect to v_1 and v_2

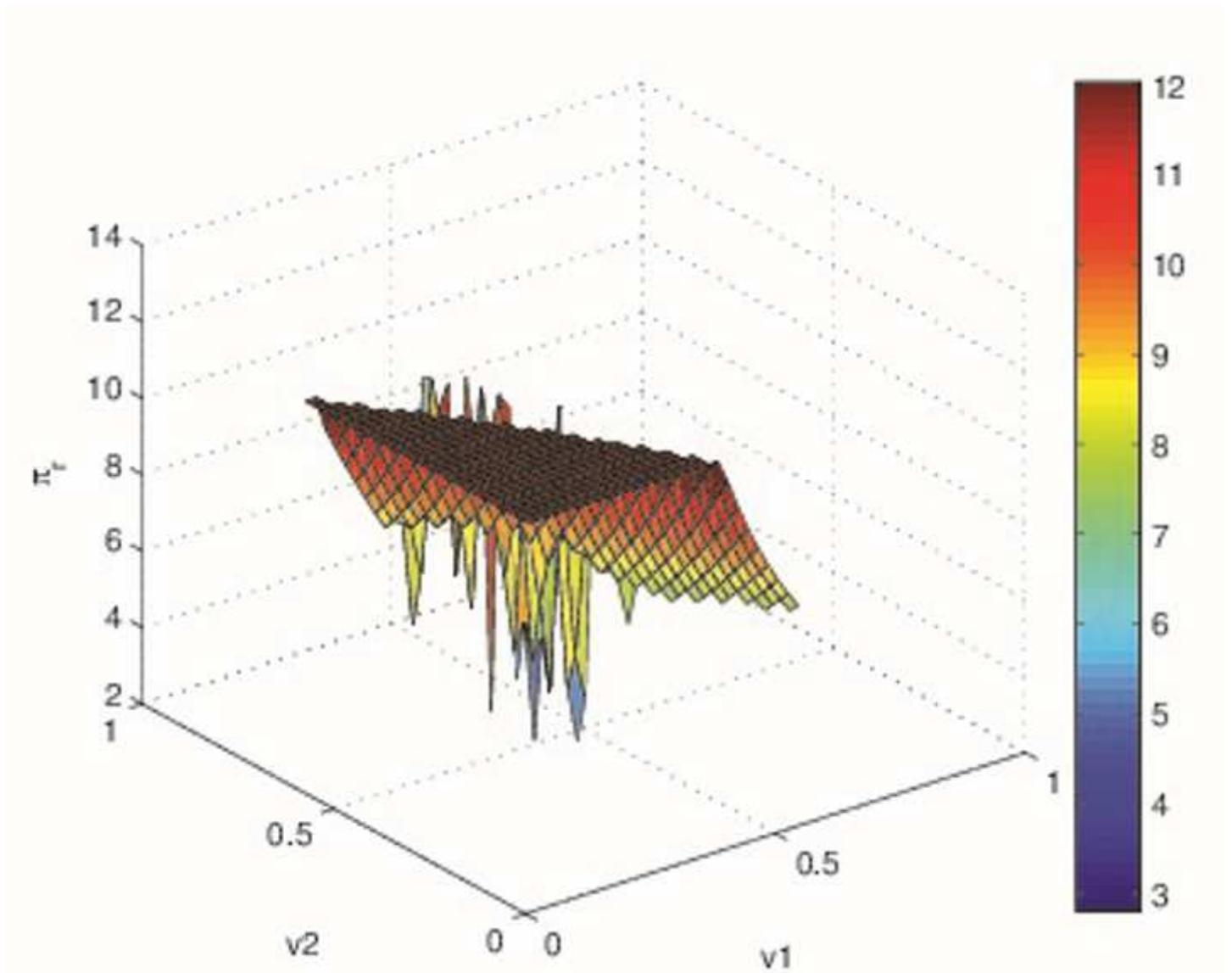


Figure 10

The profit of the retailer with respect to v_1 and v_2

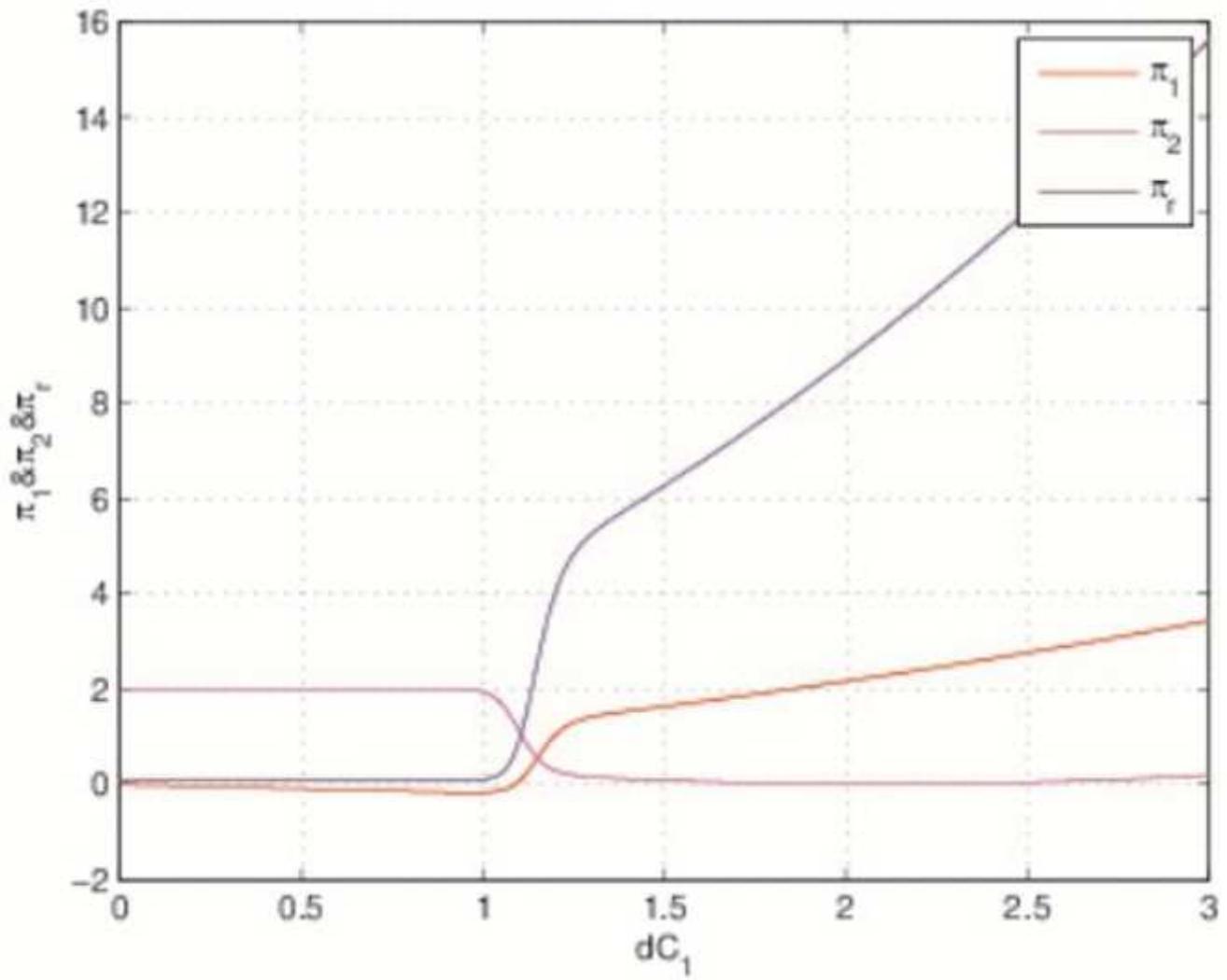


Figure 11

Profit with respect to dC_1 when $v_1 = 0.2, v_2 = 0.2$

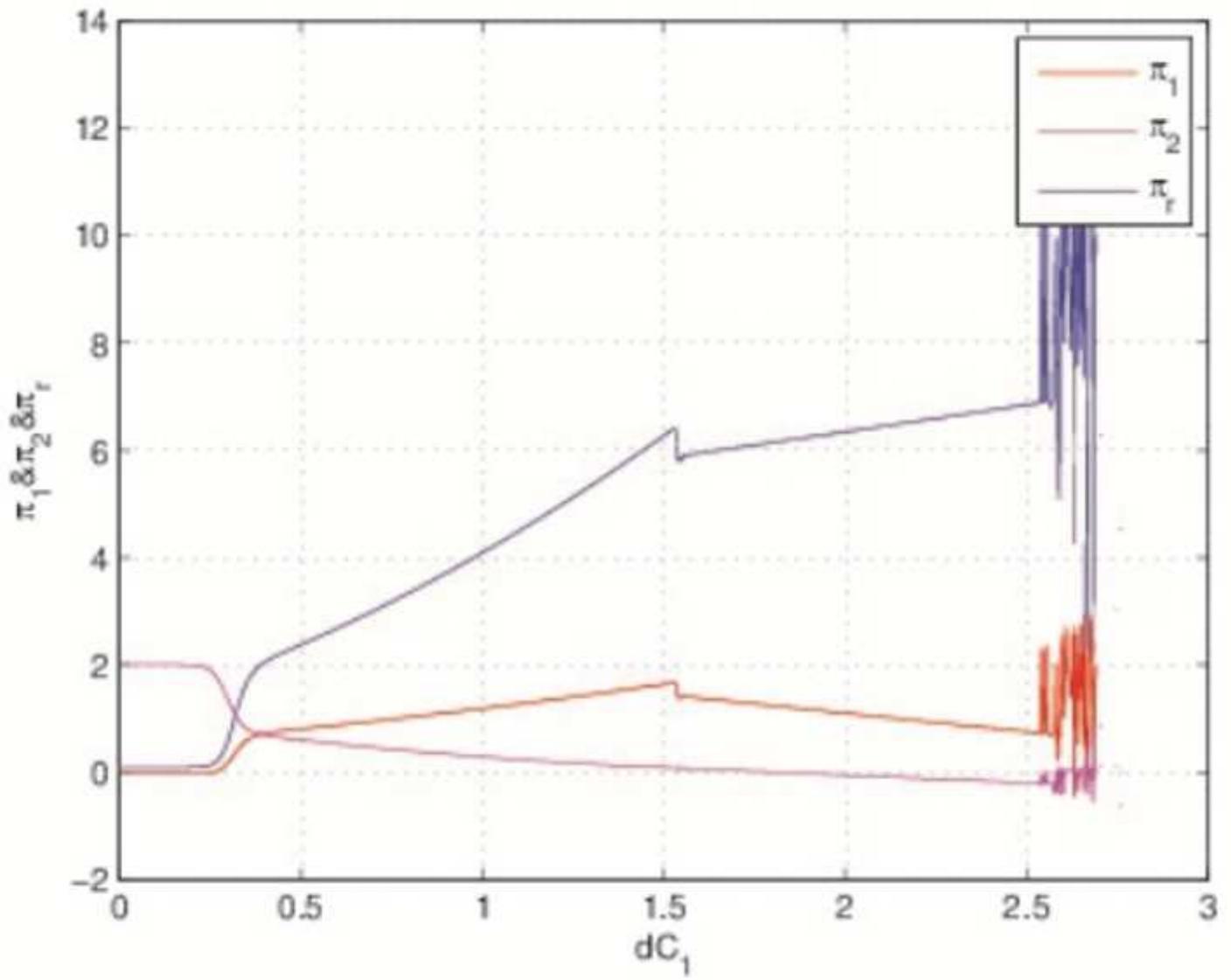


Figure 12

Profit with respect to dC_1 when $v_1 = 0.3, v_2 = 0.5$

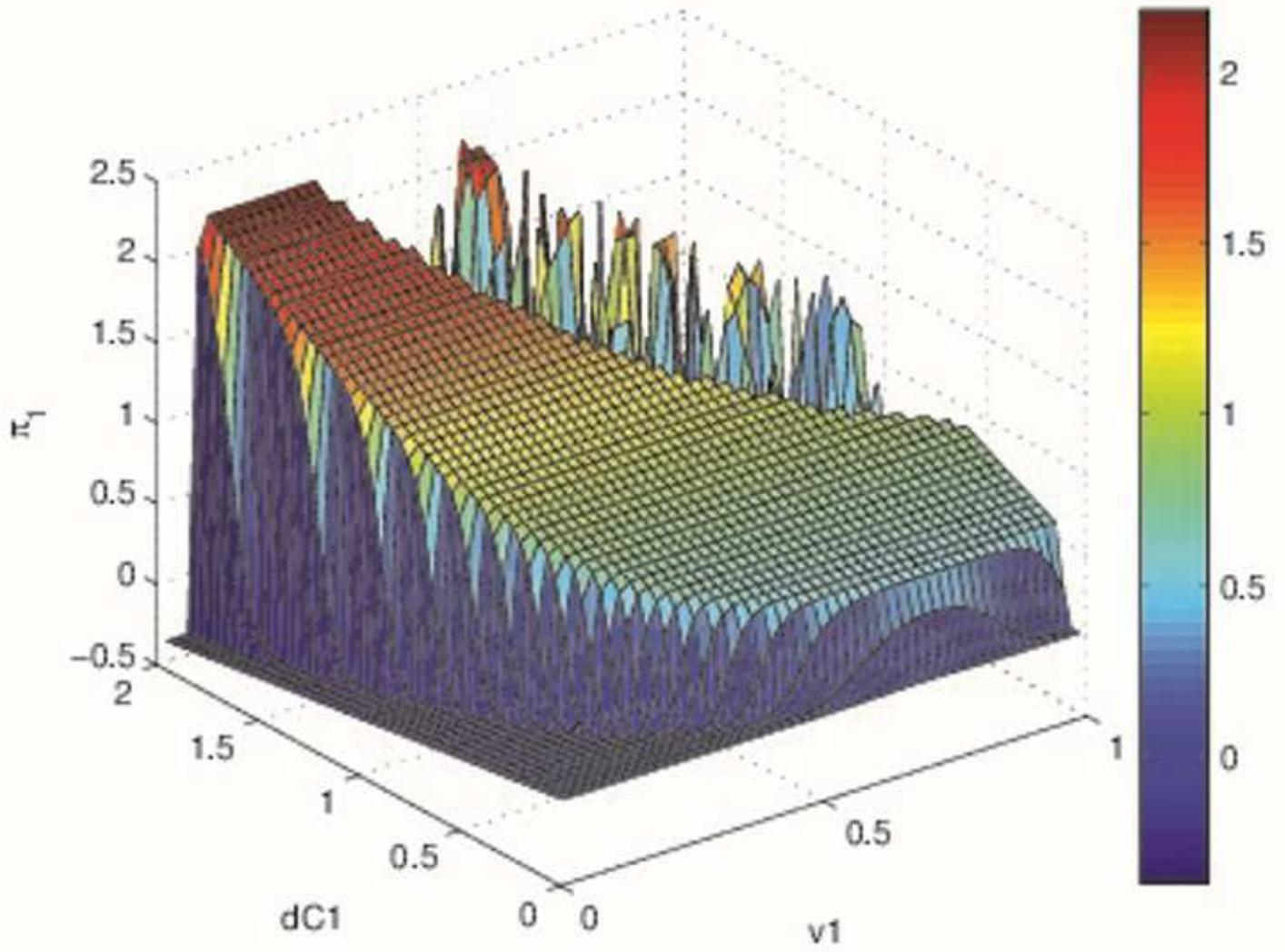


Figure 13

Profit of manufacturer 1 with respect to v_1 and dC_1

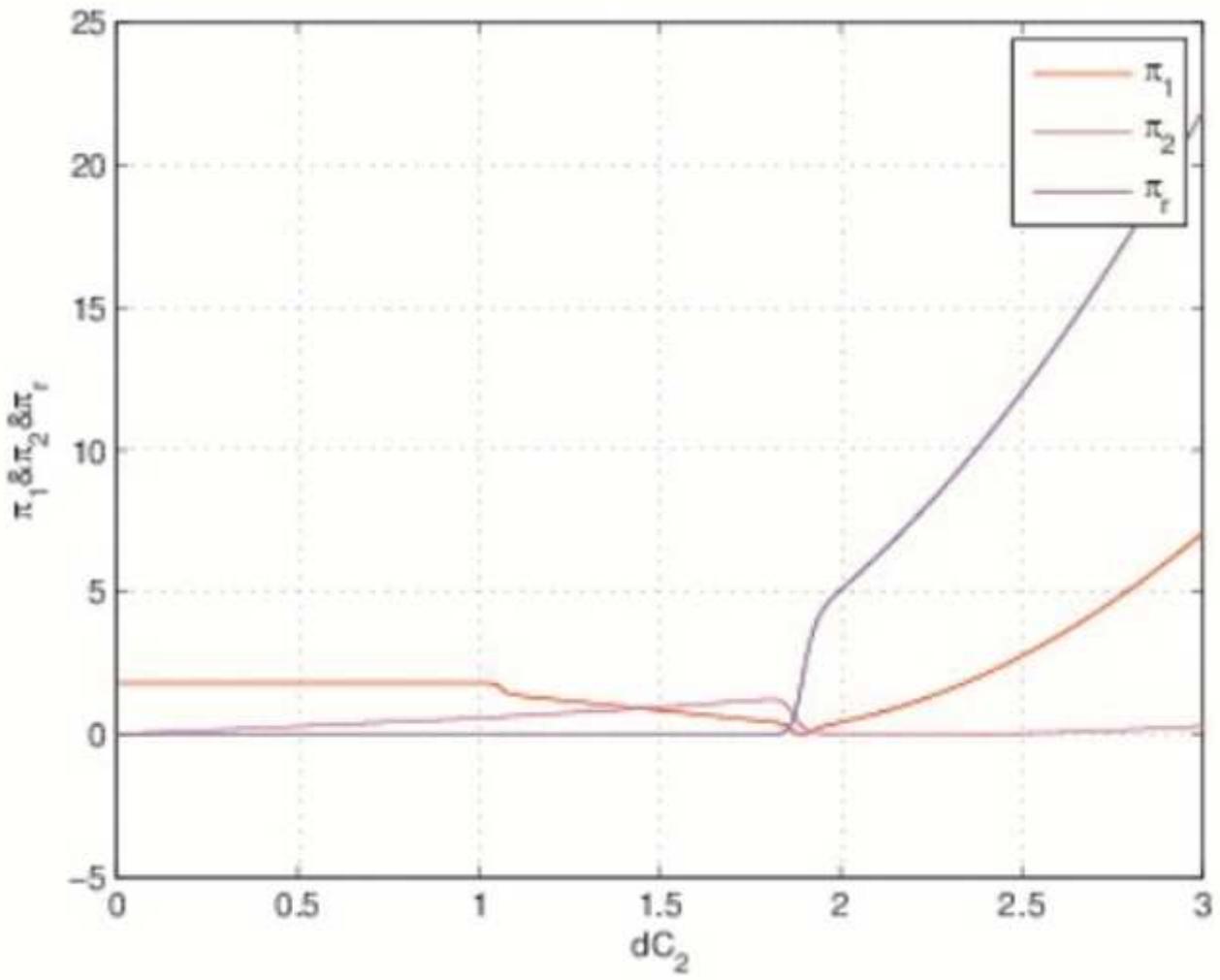


Figure 14

Profit with respect to dC_2 when $v_1 = 0.2, v_2 = 0.2$

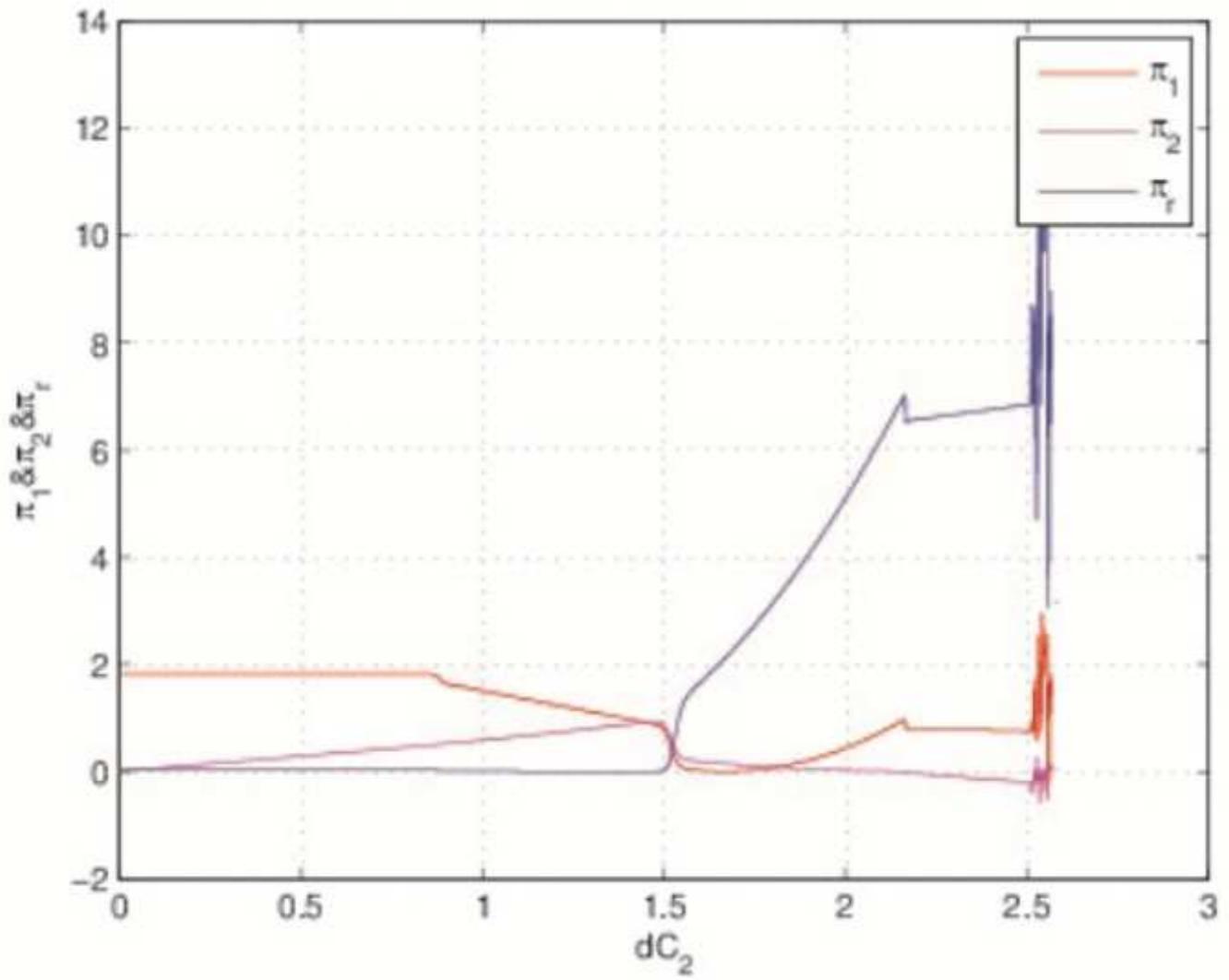


Figure 15

Profit with respect to dC_2 when $v_1 = 0.3, v_2 = 0.5$

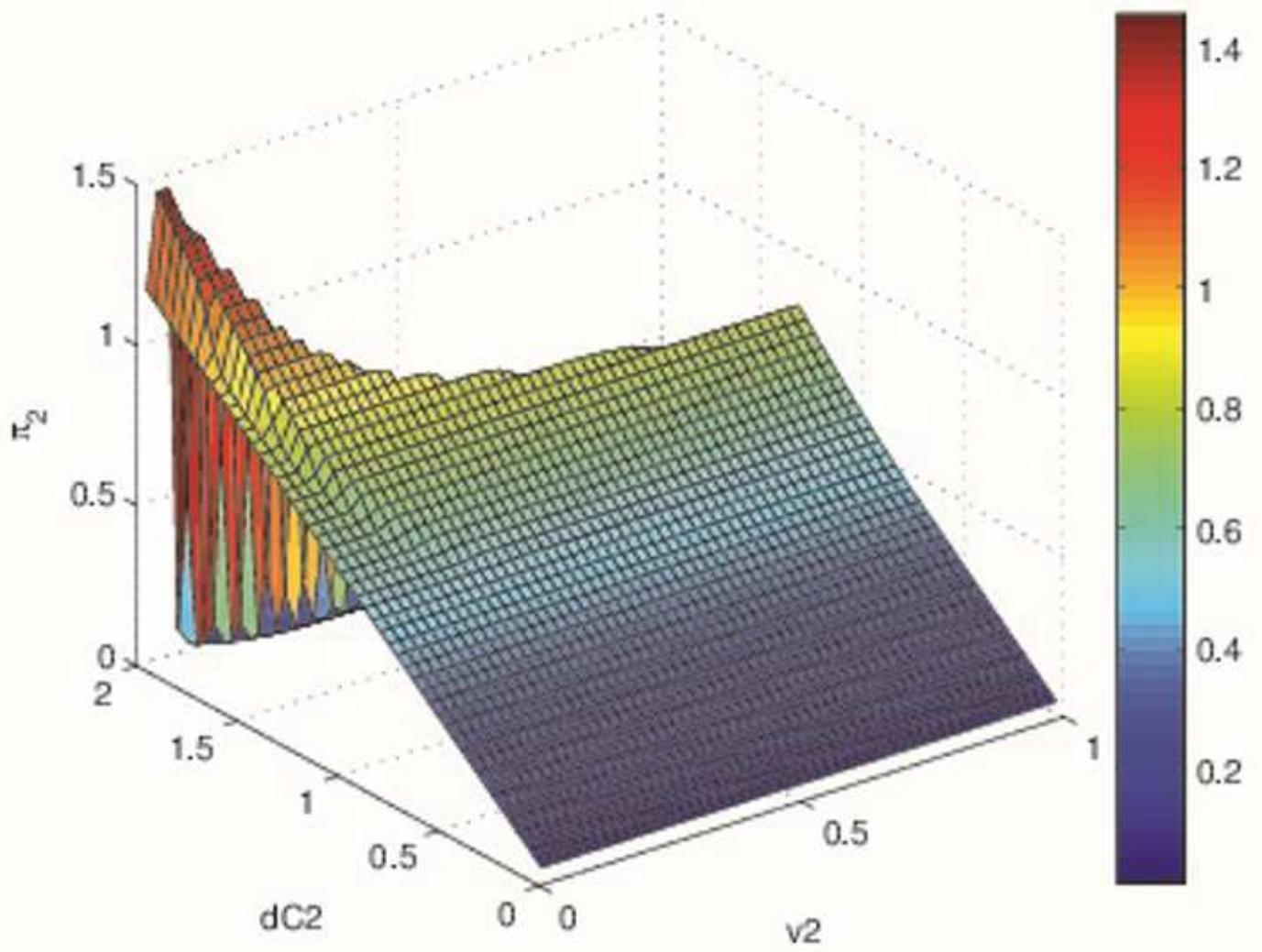


Figure 16

Profit of manufacturer 2 with respect to v_2 and dC_2

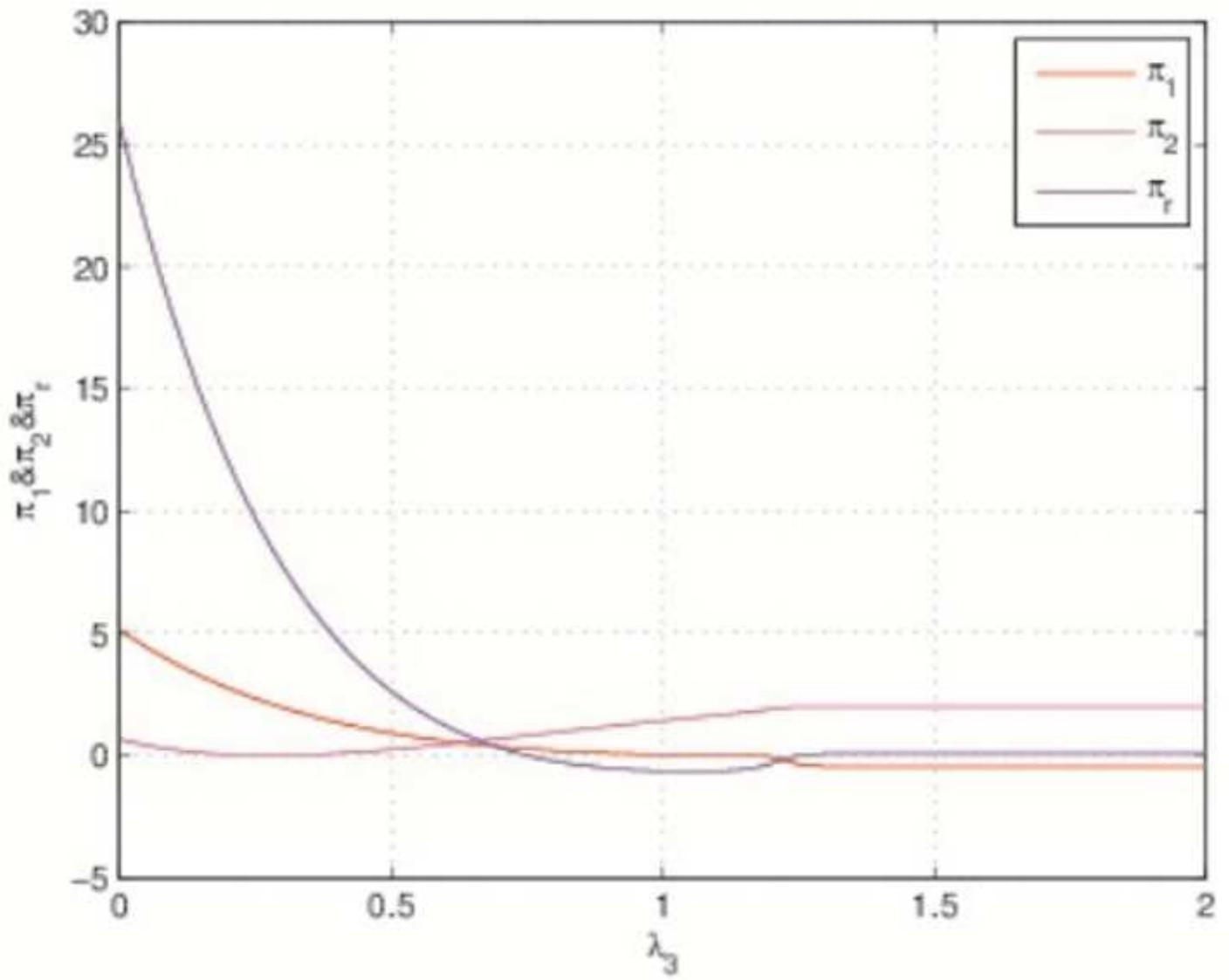


Figure 17

Profit with respect to λ_3 when $v_1 = 0.2, v_2 = 0.2$

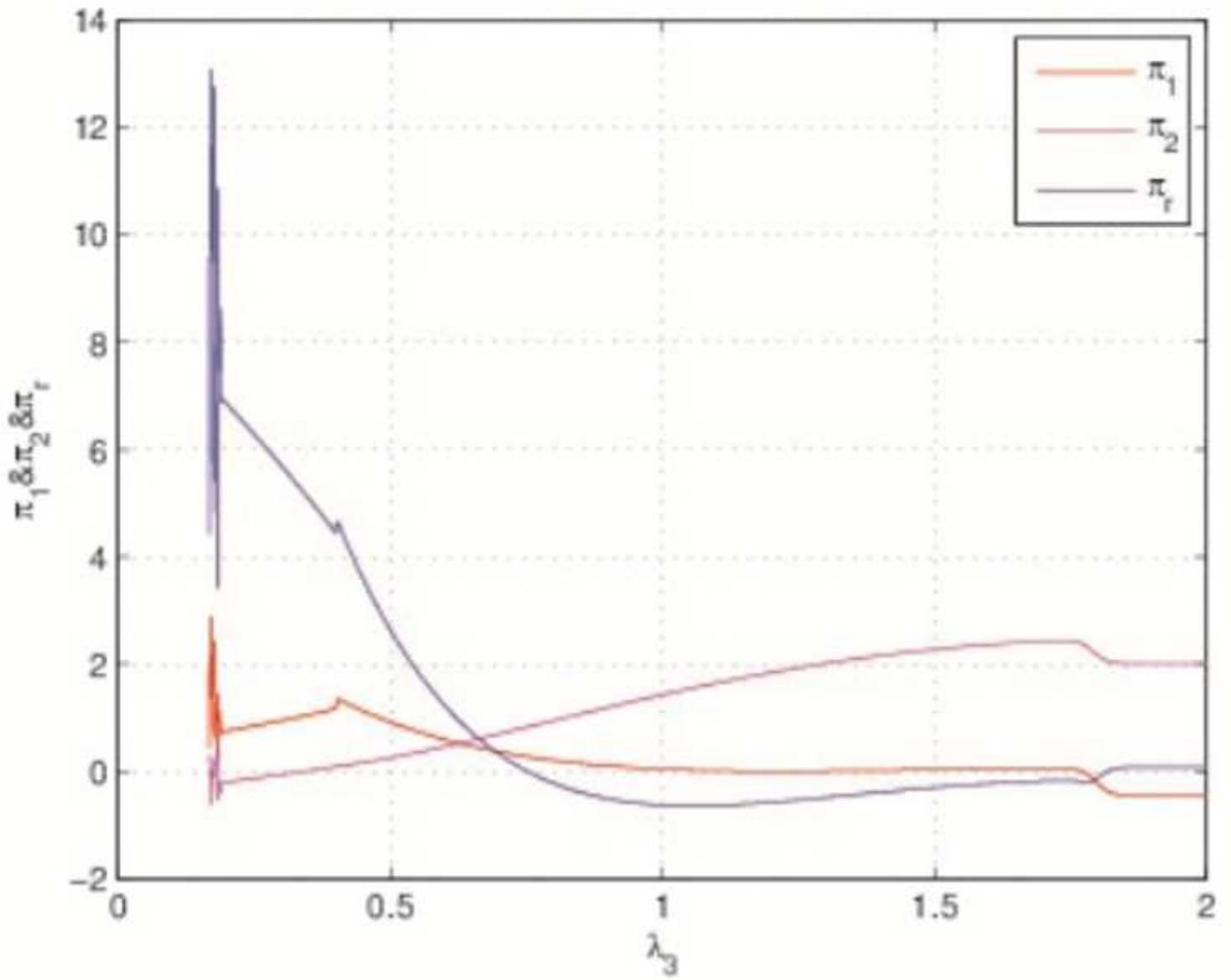


Figure 18

Profit with respect to λ_3 when $v_1 = 0.3, v_2 = 0.5$

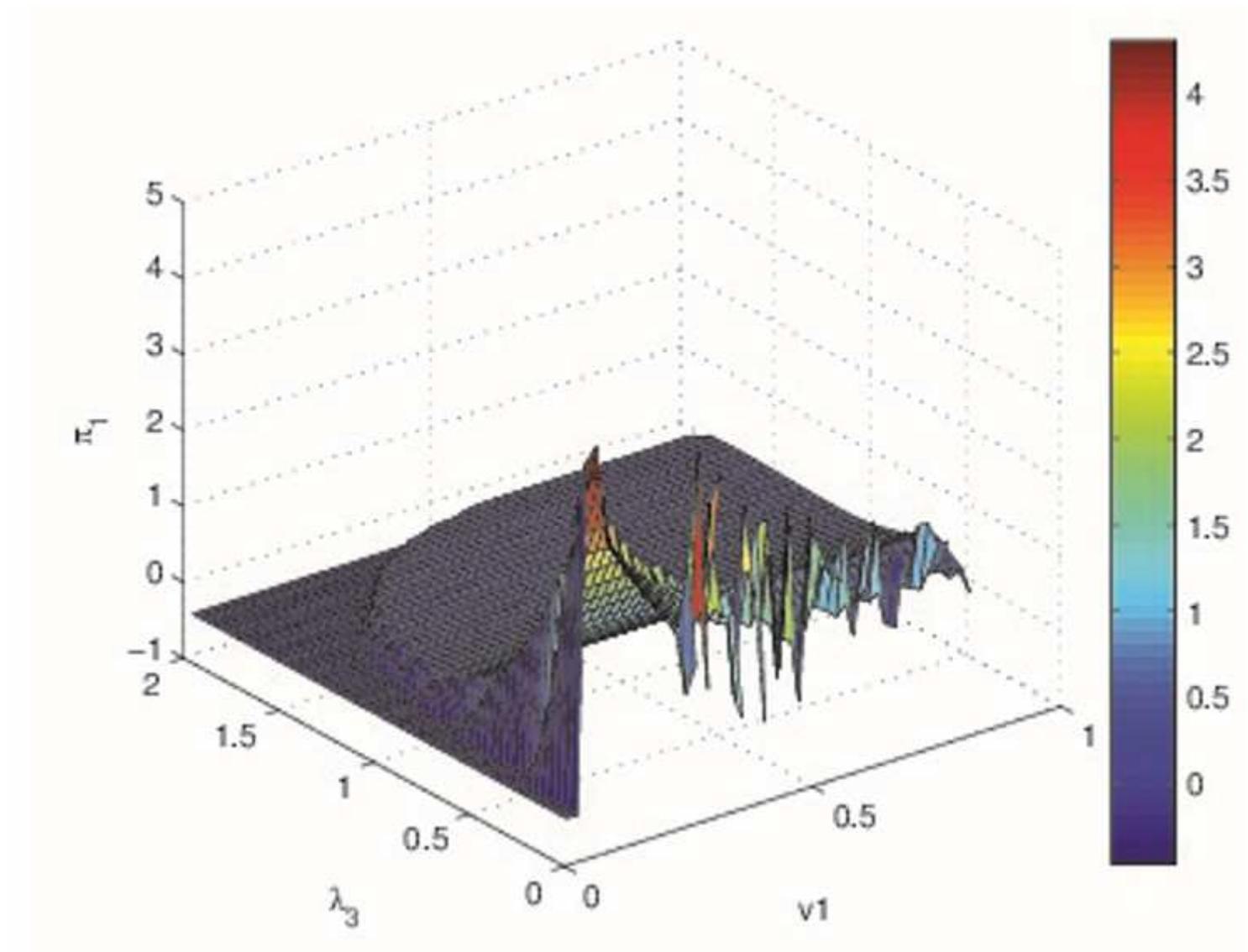


Figure 19

Profit of manufacturer 1 with respect to v_1 and λ_3

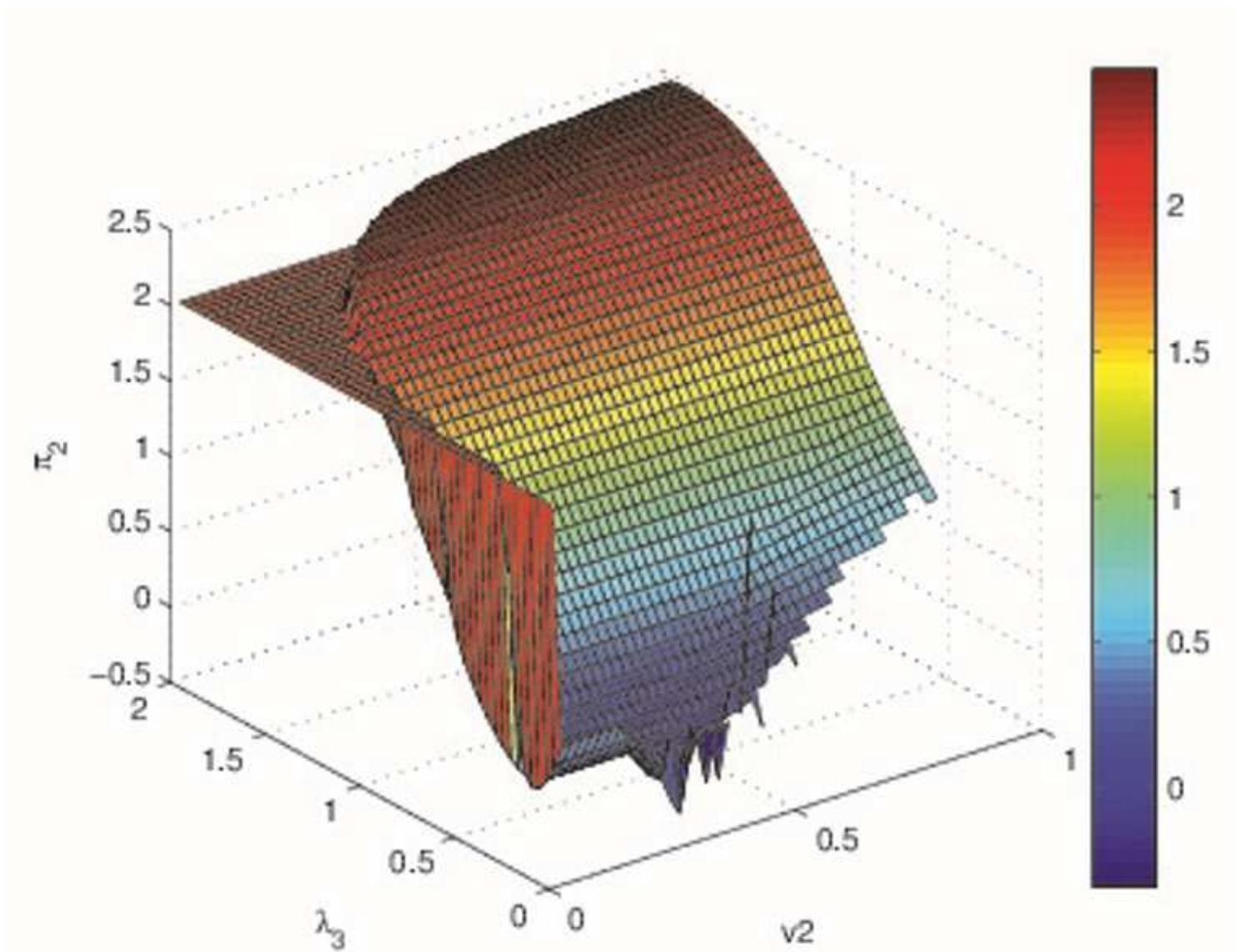


Figure 20

Profit of manufacturer 1 with respect to v_2 and λ_3

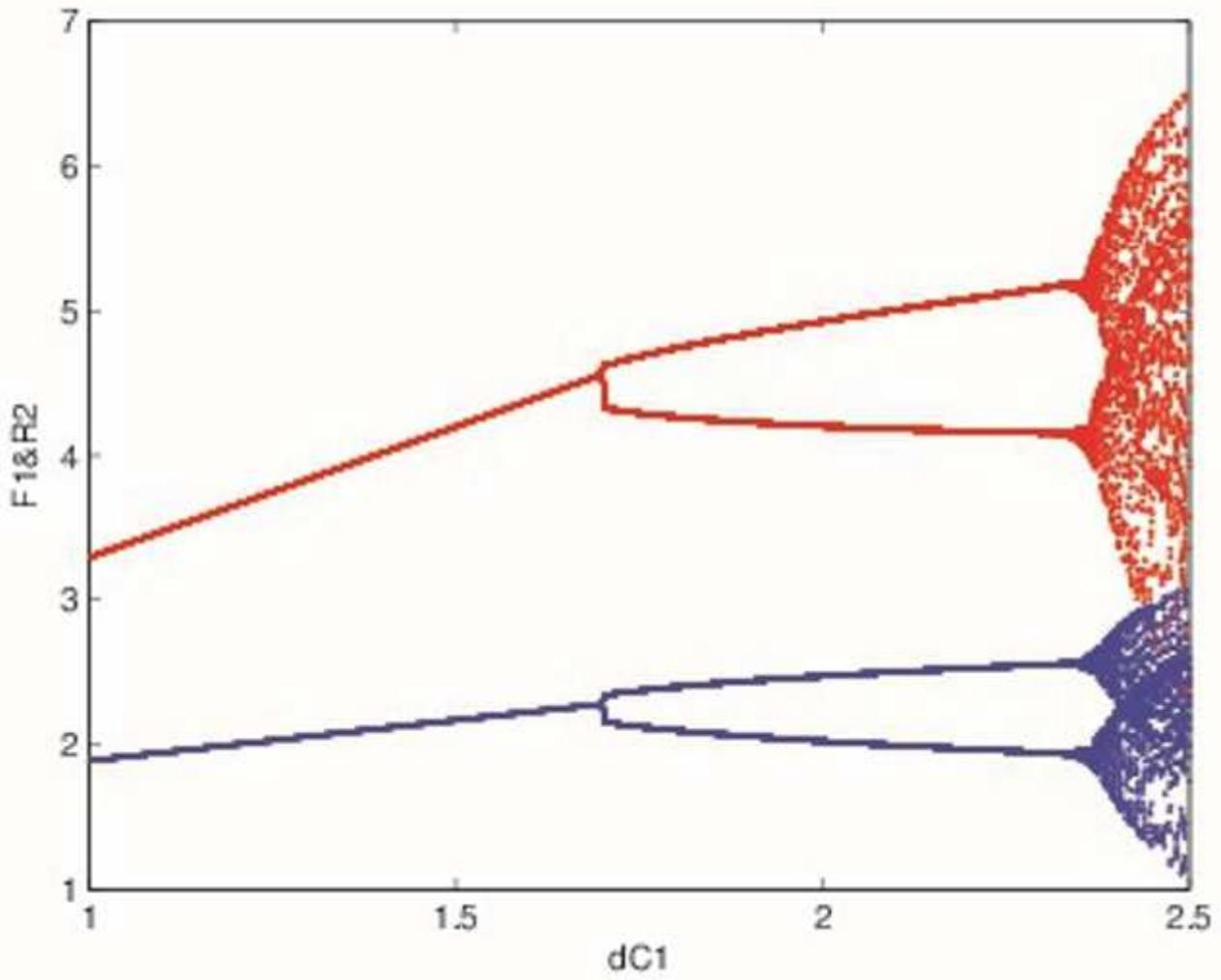


Figure 21

Bifurcation diagram with the change of dC1 when $v_1 = 0.4$, $v_2 = 0.4$

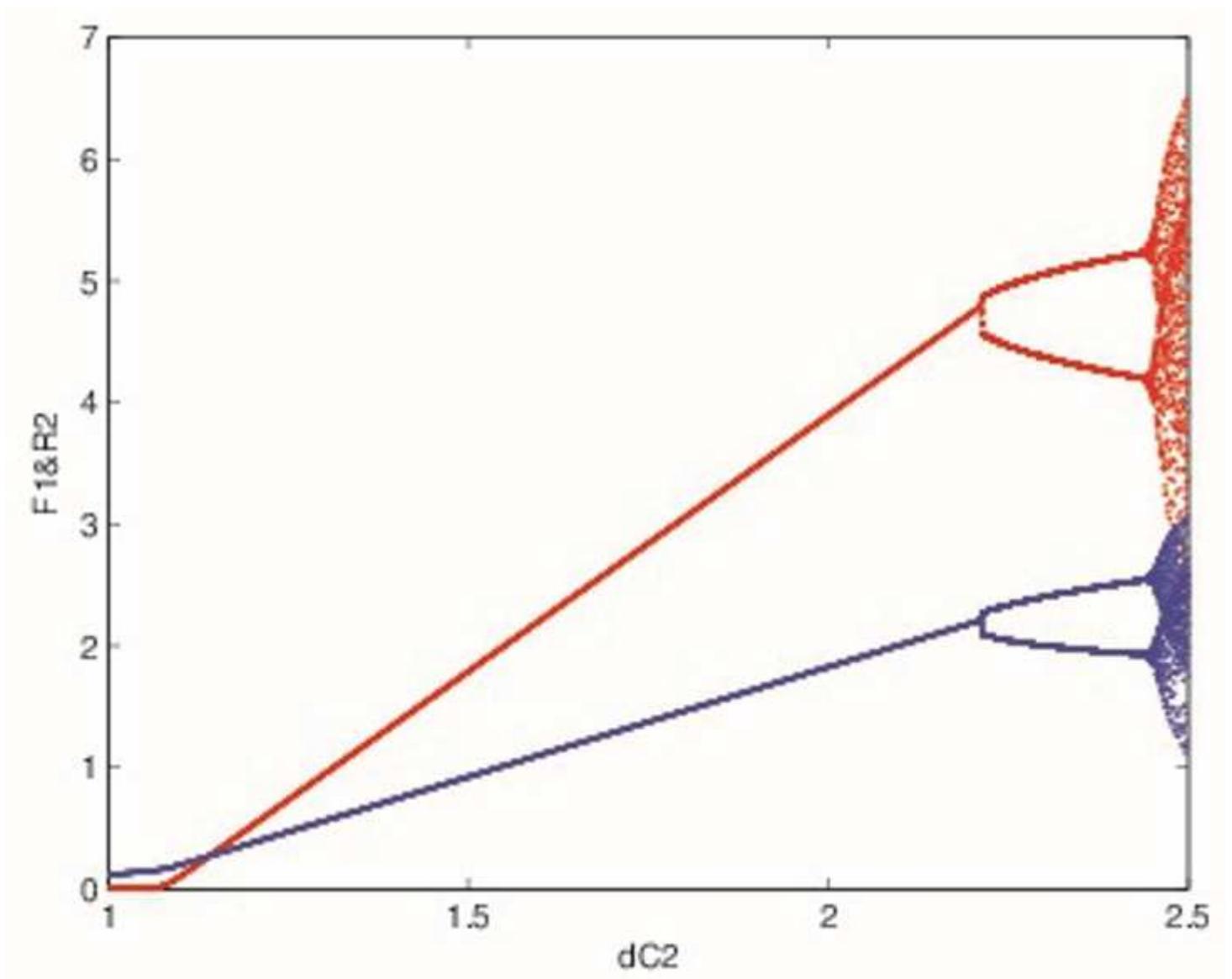


Figure 22

Bifurcation diagram with the change of dC_2 when $v_1 = 0.4, v_2 = 0.4$

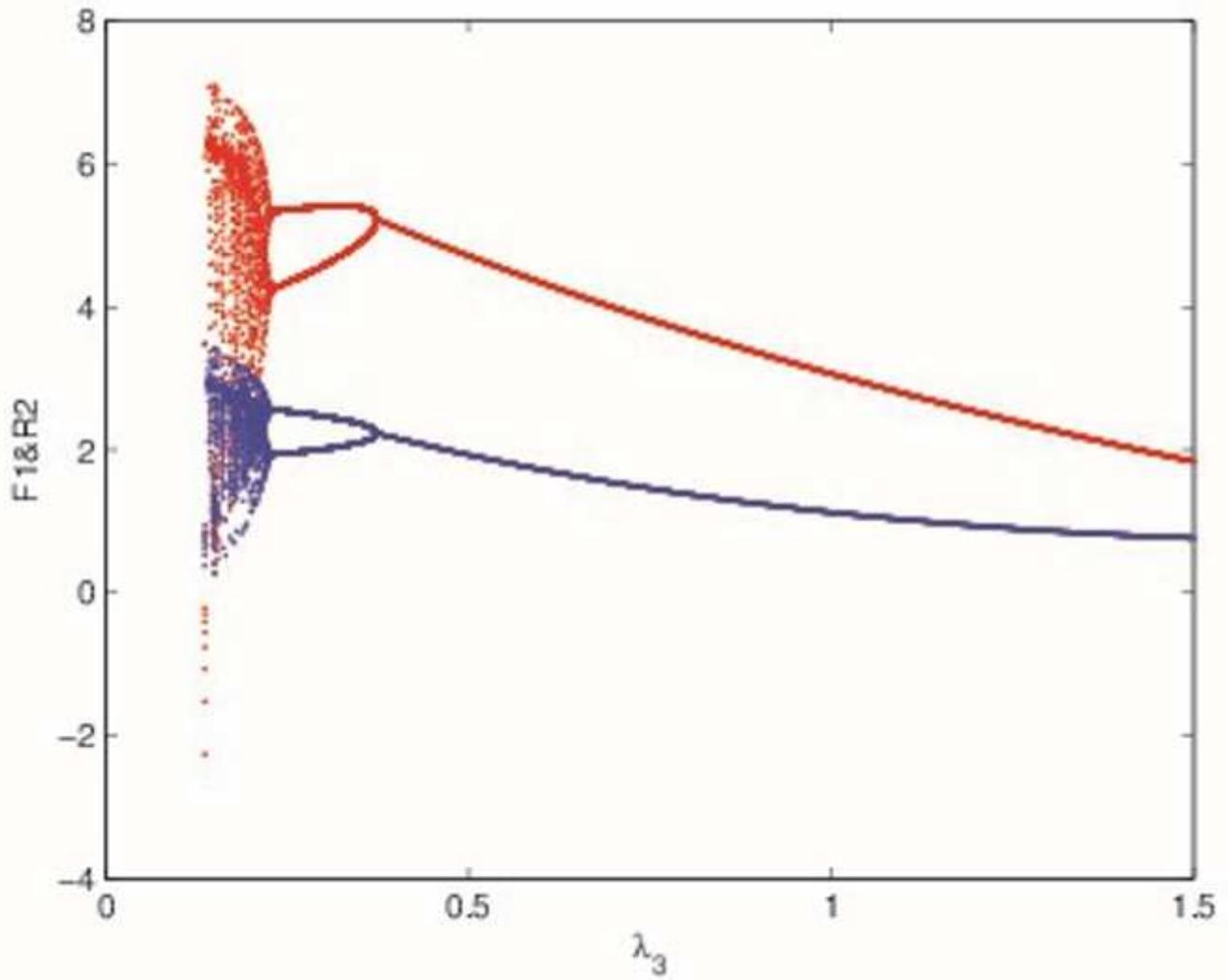


Figure 23

Bifurcation diagram with the change of λ_3 when $v_1 = 0.4, v_2 = 0.4$

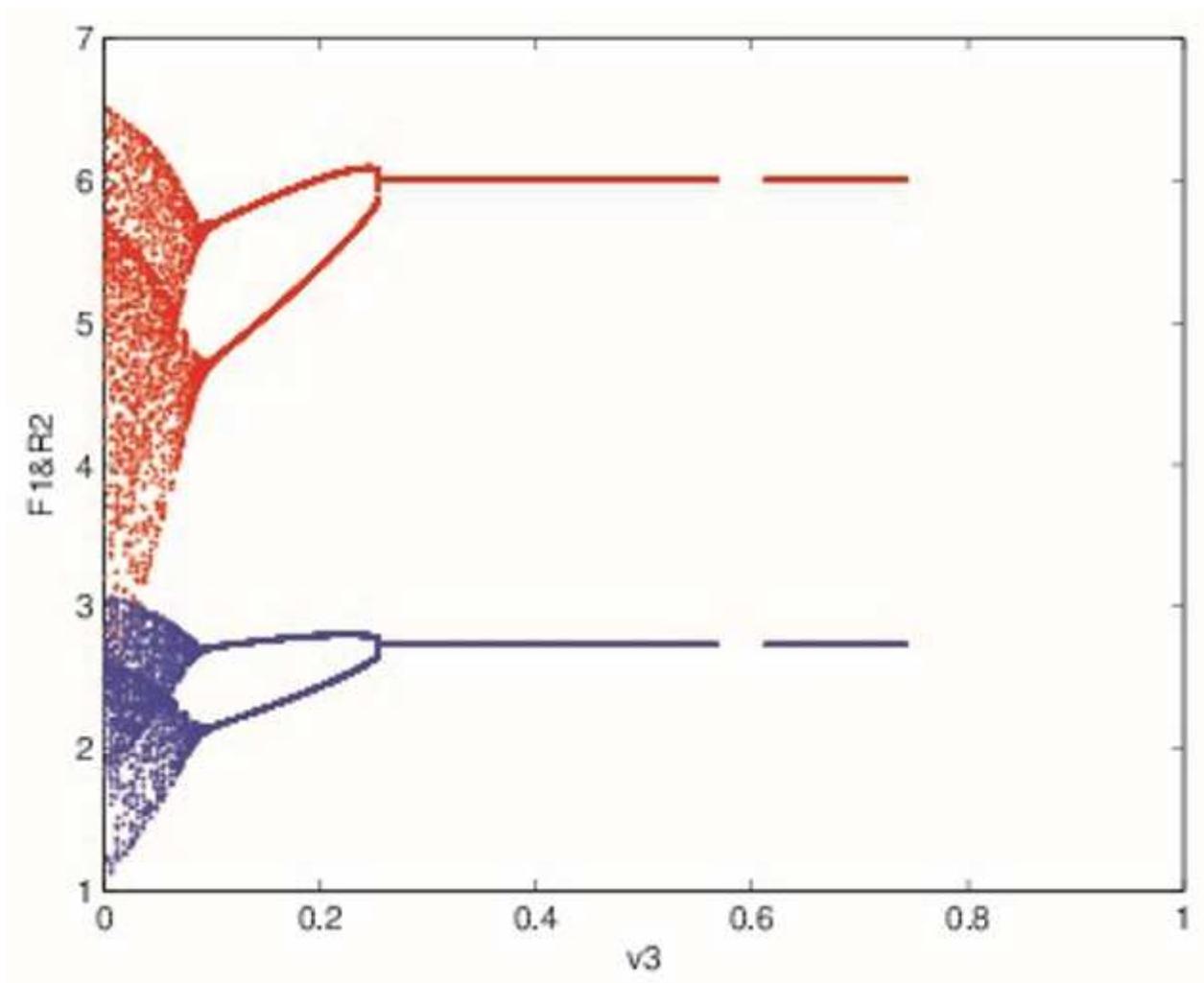


Figure 24

Bifurcation diagram of decision variables with v' increasing