

A Mathematical Model for Soil-Water Characteristic Curve by Weibull Distribution

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12 **Abstract**

13 The soil-water characteristic curve (SWCC) plays a crucial role in unsaturated soil behavior.
14 However, none of the models are fully applicable to all soil classes. Therefore, it is necessary to
15 come up with more different models to best-fit the measured SWCC data. In this paper, a
16 mathematical model (that is, Weibull model) for the soil-water characteristic curve was proposed
17 based on two-parameters Weibull distribution. It only contains two parameters a and n , the effects
18 of which on the SWCC are independent. The Brutsaert, van Genuchten, Boltzman and Weibull
19 models were fitted to 24 SWCC data sets from UNSODA 2.0. The quality of fit for these models
20 was compared. Results showed that Weibull model was desirably accurate to fit data from a variety
21 of soil classes with 0.999 for R^2 and 0.010 for RMSE. Taking into account the $\overline{R^2}$, RMSE and $\sum R_i$
22 criteria, it is therefore suggested that the exponential-based Weibull model had a higher fitting
23 accuracy and performed marginally better than the Brutsaert and VG models. As respect to the
24 criteria of AICc, the Weibull and Brutsaert models performed almost equally well but both had a
25 better performance than VG model. The VG model had the largest average number of iterations, as
26 such, it was relatively difficult to fit. However, the Boltzman model had a lower fitting accuracy
27 and less flexibility in comparison with the other models. Consequently, the Weibull model could be
28 used as an alternative to the soil-water characteristic curve models.

29 **Keywords:** soil-water characteristic curve; mathematical SWCC models; fitting; VG model

30 **Introduction**

31 The soil-water characteristic curve is defined as the relationship between soil suction and the amount
32 of water (volumetric or gravimetric water content or degree of saturation) contained in the pores.
33 Different aspects of unsaturated soil behavior such as shear strength, volume change, chemical
34 diffusivity, chemical adsorption water volume storage, specific heat and thermal conductivity are
35 related to the SWCC(Sillers and Fredlund 2001), commonly estimated by the soil-characteristic
36 curve and saturated soil properties(van Genuchten 1980, Vanapalli et al. 1996). For example, it has
37 become an acceptable procedure to derive an equation to obtain the unsaturated permeability
38 function usually by combining the SWCC with a flow equation(Rahimi et al. 2015). Therefore,
39 knowledge of the SWCC is crucial in theoretical analysis and engineering application of unsaturated
40 soil.

41 Currently, there are various methods available for measuring the SWCC for a given soil. One way
42 is by conducting laboratory tests, including direct methods using pressure plate, Buchner funnel,
43 tensiometers, pressure membranes, and indirect methods utilizing the filter paper, porous blocks,
44 heat dissipation sensors. Almost all the methods are based on the equilibrium of water content in the
45 soil. However, a reliable measurement of soil suction is challenging and cumbersome since it is
46 time-consuming and costly, even not accurate.

47 However, because of its importance and usefulness, a number of alternative analytical methods have
48 been proposed to determine SWCC for unsaturated soils. Ahangar-asr et al.(Ahangar-Asr et al.
49 2012)classified these methods into five major categories; 1) fitting type equations for
50 SWCC(BrooksR.H. and CoreyA.T. 1964, Fredlund and Xing 1994); 2) a curve-fitting method

51 correlating the soil properties to the matric suction at the different water contents(Reddi and Poduri
52 1997, Zapata 2000);3) prediction of SWCC model parameters(Torres Hernandez 2011, Sahin et al.
53 2014);4) physico-empirical model(G. Fredlund and Pham 2006); 5) artificial intelligence
54 methods(Johari and Javadi 2011, Saha et al. 2018).

55 Among these approaches, category 1 usually is an empirical model for making mathematical
56 representations or predictions of SWCC. To date, the more commonly used soil-water characteristic
57 curve models are those by Gardner(1956)(Gardner 1958), Brooks and Corey (1964)(BrooksR.H.
58 and CoreyA.T. 1964), Brutsaert (1966)(Brutsaert 1967), van Genuchten (1980)(van Genuchten
59 1980), Williams(1983)(Williams et al. 1983), McKee and Bumb 1984 (a Boltzman exponential
60 form)(McKee and Bumb 1984), McKee and Bumb 1987 (Fermi)(McKee and Bumb 1987), Fredlund
61 and Xing (1994)(Fredlund and Xing 1994). Most of these models have been developed by
62 agricultural researchers since soil-water relationship relates to water supply for plant growth. These
63 models are also of considerable significance in geotechnical engineering. Previous studies showed
64 that empirical models can be used to represent the variation of water content with soil suction
65 changing, however, none of the models are fully applicable to all soil classes. With this in view,
66 therefore, it is necessary to come up with more different models to best-fit the measured SWCC
67 data.

68 The objective of this study is to propose a mathematical model that can be used to fit the measured
69 SWCC data. The development of SWCC model based on Weibull function was presented in detail.
70 A database of 24 undisturbed soil samples covering all soil textural classes, selected from the
71 UNSONA 2.0, was used to be fitted with the Weibull model. After that, the Weibull SWCC model
72 was compared with other previously proposed models in terms of fitting accuracy. It can be drawn

73 from the comparison that the Weibull SWCC model performed better than other studied models and
 74 exhibited a better ability of fitting accuracy.

75 **Weibull model for SWCC**

76 *Proposal for Weibull model*

77 The Weibull distribution, a continuous distribution in probability theory and statistics, is named after
 78 Swedish mathematician Waloddi Weibull in 1951. As is well-known, the Weibull distribution is
 79 considered the most fundamental lifetime distribution and has been used successfully in many fields
 80 such as survival analysis, reliability engineering and failure analysis, industrial engineering, extreme
 81 value theory, and weather forecasting. For instance, Lu et al.(Lu et al. 2002) fitted the fracture
 82 strength data of brittle material to the Weibull distribution; Martínez-Antúnez et al.(Martínez-
 83 Antúnez et al. 2015)used the Weibull function to define the bioclimatic niche of some trees.

84 The equation for the probability density function (PDF) and the cumulative distribution function
 85 (CDF) of basic two-parameter Weibull distribution is

$$86 \quad f(x) = \frac{n}{a^n} x^{n-1} e^{-(x/a)^n}; a, n > 0 \quad (1)$$

87 Whose cumulative function is

$$88 \quad F(x) = \int_{-\infty}^{+\infty} f(x) dx = 1 - e^{-(x/a)^n}; a, n > 0 \quad (2)$$

89 where a and n are the scale and shape parameters of the distribution, respectively.

90 In Fig. 1, the black curve shows the cumulative distribution function, and the blue curve is the
 91 derivative of the black one. It can be seen that the black curve in Fig. 1 (CDF) is a s-shaped curve.

92 And it is acknowledged that the SWCC poses as a reversed S-shaped curve in semi-logarithmic axis,

93 so that one may use the CDF of Weibull distribution to fit SWCC data. In addition, we can see that
 94 the CDF becomes zero when x equals zero, i.e., $F(x)|_{x=0}=0$; while the water content, θ , is equal to
 95 the value of θ_s at a suction of zero kPa, i.e., $\theta(\psi)|_{\psi=0}=\theta_s$. Therefore, the CDF of Weibull distribution
 96 needs to be modified to show characteristics of SWCC. With this in view, the modified CDF and its
 97 differential equation used to describe the relationship between water content and suction for the soil
 98 corresponding to Eq. (2) and Eq. (1) are presented as follows.

$$99 \quad \theta(\psi) = \theta_r + \frac{(\theta_s - \theta_r)}{e^{(a\psi)^n}} \quad (3)$$

$$100 \quad \theta'(\psi) = -\frac{(\theta_s - \theta_r)}{e^{(a\psi)^n}} an(a\psi)^{n-1} \quad (4)$$

101 The Eq. (3) can be written in terms of the normalized volumetric water content, Θ , defined as the
 102 amount of water in soils between the residual and saturated volumetric water contents (see Eq. (5),
 103 called Weibull model).

$$104 \quad \Theta = \frac{\theta(\psi) - \theta_r}{\theta_s - \theta_r} = \frac{1}{e^{(a\psi)^n}} \quad (5)$$

105 where Θ is normalized volumetric water content; $\theta(\psi)$ is volumetric water content at the suction, ψ ;
 106 θ_s is volumetric water content at saturated state; θ_r is volumetric water content at residual state; a
 107 and n are fitting parameters.

108 ***The effects of two parameters***

109 Fig. 2(a) shows a plot of Weibull model (Eq. (5)) with n parameter constant (n equal to 0.5) and a
 110 varying. The a parameter has a unit of reciprocal suction (kPa^{-1}) and is equal to the inverse of soil
 111 suction where normalized volumetric water content, Θ , is equal to $1/e$ (namely 0.3679). The a
 112 parameter does not affect the shape of the curve, but provides a shift in the curve towards the higher

113 suction region as a is decreased. Thus, it is indicated that the inverse of parameter a is related to the
114 air-entry value.

115 Fig. 2(b) shows a plot of Weibull model with a parameter constant (a equal to 0.01 kPa^{-1}) and n
116 varying. All the curves pass through the same point ($1/a, 1/e$). As can be seen from the curves, the
117 larger the value of n , the steeper the curve in the transition zone. So the n parameter is related to the
118 pore size distribution index. the more uniform the pore size distribution in the soil, the greater the
119 value of n .

120 From the above analysis, the primary merits of the Weibull model can be summarized as follows:
121 the two fitting parameters are physically meaningful; the effect of one parameter can be
122 differentiated from that of the other parameter; The form of the model is relatively simple and
123 contains only two parameters.

124 **Data**

125 24 soil samples with soil-water characteristic data selected from the unsaturated soil hydraulic
126 database (UNSODA 2.0)(Nemes et al. 2001) were used to demonstrate the performance of the
127 Weibull model and make comparisons with the three widely used models: Brutsaert model, van
128 Genuchten model, Boltzman model.

129 Several reasons for choosing the dataset from UNSODA in this study: the selected dataset covers
130 all soil textural classes in which each soil class is represented by 2 samples with experimental data
131 (see table 1). In addition to soil-water characteristic data, other information, such as, particle size
132 distribution, mineralogy, hydraulic conductivity and water diffusivity, can also be found in
133 UNSODA. Last but not least, other researchers can repeat your work based on the UNSODA code

134 and test the results. Therefore, it could be a very suitable dataset to evaluate the performance of the
135 SWCC model. The soil-water characteristic curves in this paper are presented in terms of volumetric
136 water content, θ , plotted on an arithmetical scale, and soil suction ψ drawn on a logarithmic scale.

137 **Results**

138 *Fitting results*

139 24 sets of experimentally θ - ψ data points were fitted with the Weibull model so that the values of
140 model parameters (a , n , θ_r , θ_s) could be determined for each soil, which were then used to calculate
141 the volumetric water contents using the Weibull model at given various suction values for each soil.
142 The predicted volumetric water contents were compared against the measured ones at identical
143 suction level to investigate the Weibull model prediction accuracy.

144 Fig. 3 shows the plot of measured versus predicted volumetric water content of 330 SWCC
145 experimental data for 24 samples. A statistical analysis was performed to determine the root mean
146 squared error (RMSE) and coefficient of determination (R^2) associated with the predicted
147 volumetric water content. The high R^2 value and the small RMSE value represent a high prediction
148 accuracy for the developed model(Gu Fan et al. 2016a). As shown in Fig. 3, a good agreement was
149 found between two sets of volumetric water contents with a high R^2 value (0.999) and a low RMSE
150 value (0.010). This indicates that the Weibull model is desirably accurate to fit data from a variety
151 of soil classes.

152 ***Comparison of the fitting capacity with other models***

153 *Model selection*

154 The soil-water characteristic models compared with the Weibull model in this study are parametric
155 models based on a pore size function and the capillary theory. The equations representing each
156 model are given in table 2. Each equation was written in the normalized water content form so as to
157 can be applied between the saturated water content and residual water content.

158 The Brutseart model was chosen in the present study because the Brutsaert model performs better
159 than the other two-parameter models among nine SWCC models including both two-parameters and
160 three-parameters models in a study by SILLERS(Sillers et al. 2001). The van Genutchen model was
161 included due to its a wide range of flexibility allows it to better fit data from a great diversity of soil
162 classes. The Boltzman model was included since it is similar to the Weibull model, with only two
163 parameters and taking on an exponential form, Although the Boltzman model is less commonly used.

164 *Fitting procedure*

165 The optimum fitting parameters of all 24 soil samples' water characteristic curve data sets were
166 obtained by optimization technique. the parameters were determined by nonlinear curve fitting of
167 custom functions in ORIGIN software and the model functions were as close as possible to the
168 experimental data points without necessarily passing through any points. This is an iterative method
169 that starts with some initial values of the parameters.

170 To compare fairly the number of iterations converging to the best fitting parameters (that is, the
171 difficult degree of fitting) of each model, the initial values of each parameter were set to start with

172 1. As the setting range of the parameters has a great influence on the fitting results, the upper and
 173 lower bounds of θ_s and θ_r were set to 1 and 0, and the other parameters were greater than 0, except
 174 for the parameter a of the Boltzman model because it could sometimes be less than zero.

175 *Evaluation criteria for models performance*

176 There were two measures used to compare the fitting accuracy of the SWCC models in this study.
 177 One was including the root-mean-square error (RMSE) and the adjusted coefficient of determination
 178 ($\overline{R^2}$), which were used as relative measure of the goodness of fit of SWCC models to the measured
 179 data of soil SWCC. RMSE statistic is an indicator to evaluate the total error of the model function.
 180 the closer its value is to zero, the better the fit is. R^2 (coefficient of determination) statistic is
 181 generally an evaluation index of fitting quality. Mathematically speaking, R^2 (computed by Eq. (7))
 182 will rise with number of parameters increasing. In this paper, the model involved in the comparison
 183 has different number of parameters. Thus, to avoid this effect, adjusted R^2 ($\overline{R^2}$) is accepted to
 184 overcome the rise in R^2 . A value of $\overline{R^2}$ close to 1 indicates that the fit is a good one. The RMSE
 185 and $\overline{R^2}$ are computed by using Eq. (6) and (8).

$$186 \quad RMSE = \sqrt{\frac{\sum (\theta_m - \theta_f)^2}{n}} \quad (6)$$

$$187 \quad R^2 = \frac{SS_{res}}{SS_{tot}} = 1 - \frac{\sum (\theta_m - \theta_f)^2}{\sum (\theta_m - \overline{\theta_m})^2} \quad (7)$$

$$188 \quad \overline{R^2} = R^2 - \frac{k-1}{n-k} (1-R^2) \quad (8)$$

189 Where θ_m and θ_f denote measured and fitted water content, respectively; n is number of soil-water
 190 characteristic data points for each soil sample. k is number of parameters.

191 The other was the corrected Akaike Information Criterion (AICc) imposing penalties for additional
192 fitting parameters. It's based on information theory, but a heuristic way to think about it is as a
193 criterion that seeks a model that has a good fit to the data but contains the least parameters. In the
194 case of small sample size, AICc is defined as follows:

$$195 \quad AICc = AIC + \frac{2k(k+1)}{n-k-1} \quad (9)$$

196 The definitions of n and k are the same as above. AIC is shorted for the Akaike information criterion.
197 As n increases, AICc converges to AIC so that AICc can be applied to any sample size (P. Burnham
198 and R. Anderson 2004). The lower the AICc value, the better the fit of model to the data. So the
199 priority model should be the one with the lowest AICc value.

200 *Comparing the accuracy of the SWCC models*

201 Figures 4(a) to (d) show the best-fit curves to the experimental data from samples S2, S5, S10, S24
202 using the four models, representing 4 types of results. As can be seen in Fig. 4(a), the Brutsaert and
203 van Genuchten models almost have identical fitted curves and show much closeness to each other.
204 In addition, it is can be observed in Fig. 4(b) that the fitting curves of van Genuchten and Weibull
205 models are completely coincident. In Figures 4(c) and (d), the Brutsaert, van Genuchten and Weibull
206 models were able to match all the measured data accurately without much deviations between each
207 other. Nevertheless, the fitting curves of the extrapolation regions are different in the two figures.
208 That is, the suction extends to infinity at constant water content with fitting curves close and parallel
209 to each other in the high soil suction region (see Fig. 4(c)), but some extrapolated regions have
210 significant differences in the same range (see Fig. 4(d)). Looking at the best-fitting curves of 24 soil
211 samples, most of them belong to the type of figure 4 (c). The Boltzman model typically have

212 problems matching data in the junctions of three zones (that is, saturated zone, transition zone and
213 residual zone) in almost all the soil samples, decreasing the flexibility of the curve.

214 Overall, figures of best-fitting curves for 24 soil samples indicates that all models performed very
215 well for data sets except for the Boltzman model with its low accuracy and inflexibility character.

216 As we can see in best-fit curves, out of 24 soil samples, the differences in fitting the observed data
217 between the Brutsaert, van Genuchten and Weibull models are merely marginal.

218 The fitted parameters and evaluation indicators for fitting quality of the models for various soil
219 textural classes are summarized in table 3. The higher R^2 value and the smaller RMSE value
220 represent a higher prediction accuracy for the developed model(Gu Fan et al. 2016b). For each soil
221 sample, 4 models were ranked from 1 to 4 according to the RMSE value. Model with the smallest
222 RMSE comes first (i.e., $R_i=1$), meaning the best fit. The ranking results have been shown in Table
223 3. The ranks of each model were added up to compare the fitting performance of different models.

224 The overall ranking (i.e., $\sum R_i$) of each model is shown in Table 4. The smaller the $\sum R_i$, the better
225 the fit. As such, these four models can be orderly ranked as Weibull-Brutsaert-VG-Boltzman.

226 Compared with Brutsaert and VG model in terms of RMSE and $\overline{R^2}$ criteria, Weibull model has the
227 largest average and the smallest standard deviation, although there is little difference between them;

228 while Boltzman model is significantly different from the other three models with the lowest average
229 value of $\overline{R^2}$ and the highest RMSE. Thus it can be seen that the performance of Weibull model is

230 slightly better than Brutsaert and VG model according to RMSE and $\overline{R^2}$ criteria as well as the $\sum R_i$.
231 No matter which evaluation criterion is adopted, Boltzman shows the worst performance.

232 From the results presented in Table 4, the Brutsaert, van Genuchten, Boltzman, Weibull models had

233 the corrected Akaike Information Criterion of -131, -122, -115 and -130, respectively. Fig. 5 is a
234 plot of the average corrected Akaike Information Criterion versus the model type. The lowest AICc
235 indicates the best fit of the given data sets. Although the Brutsaert model had the lowest AICc, i.e.,
236 -131, the AICc of Weibull model was just a little bigger than that of Brutsaert model. In other words,
237 there was little difference in the calculated corrected Akaike Information Criterion between the
238 Brutsaert and Weibull model. As mentioned above, Boltzman model provided the poorest fit to the
239 soil-water characteristic data according to the largest average AICc. The van Genutchen model
240 didn't perform well in terms of AICc criterion which may be due to the penalties imposed by the
241 AICc for an extra parameter comparing with the Brutsaert and Weibull models. Because, there is no
242 significant difference between these models from two aspects of RMSE and $\overline{R^2}$ criteria.

243 Table 4 also shows the average number of iterations required for the parameters to converge for
244 every model. When the initial values for the parameters are the same, the number of iterations
245 represents the degree of ease of fitting. It can be observed that the average number of iterations for
246 Boltzman model is indeed the smallest, though it has the largest AICc value, meaning that Boltzman
247 model tends to easiness in finding best-fit parameters. On the contrary, van Genutchen model is the
248 most difficult to fit since it requires relatively quite large number of iterations. The difference in
249 average number of iterations between Brutsaert and Weibull models is very small.

250 (Sillers and Fredlund 2001) conducted analyses on statistical assessment for nine models and
251 reported that the model with the lowest Akaike information Criterion tended to require the least
252 effort to find best-fit parameters and the exponential based models were the most difficult to fit.
253 Then, their result was different from the result of this study. More specifically, Boltzman model, the
254 exponential based model, has the smallest average number of iterations with the largest average

255 AICc value. The initial values of all parameters in this study were set to 1, while that of parameters
256 were not necessarily equal in study of Sillers and Fredlund (2001) (at least not mentioned in the
257 paper). Therefore, different initial values of parameters may be a reason for this disagreement.

258 **Conclusions**

259 Soil-water characteristic curve (SWCC) is one of the most important components for describing
260 unsaturated soil behavior. A better representation of unsaturated soil behavior could be gained by
261 describing the SWCC using mathematical models. Numerous different options have been proposed
262 to choose a model to characterize SWCC. In this paper, we used the modified cumulative function
263 of Weibull distribution to give a description of the relationship between the water content and soil
264 suction. The following conclusions of this paper can be summarized:

- 265 1. A mathematical model for SWCC based on Weibull distribution was proposed, called Weibull
266 model. It only contains two parameters a and n , the effects of which on the SWCC are
267 independent. A total of 330 SWCC experimental data from UNSODA 2.0 covering all
268 textural classes (two soil samples for one soil class) were used to test the performance of the
269 Weibull model and conduct comparisons with the 3 other widely used models, namely,
270 Brutsaert model, van Genuchten model and Boltzman model.
- 271 2. The Weibull model with R^2 value equal to 0.999, and RMSE value equal to 0.010 can
272 accurately estimate the volumetric water content of soil at any given matric suction for the
273 selected data sets.
- 274 3. Taking into account the $\overline{R^2}$, RMSE and $\sum R_i$ criteria, it is therefore suggested that the
275 exponential-based Weibull model had a higher fitting accuracy and performed marginally

276 better than the Brutsaert and VG models. As respect to the criteria of AICc, the Weibull and
277 Brutsaert models performed almost equally well but both had a better performance than VG
278 model. The VG model had the largest average number of iterations, as such, it was relatively
279 difficult to fit. However, the Boltzman model had a lower fitting accuracy and less flexibility
280 in comparison with the other models. Consequently, the Weibull model could be used as an
281 alternative to the soil-water characteristic curve models.

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285

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Figures

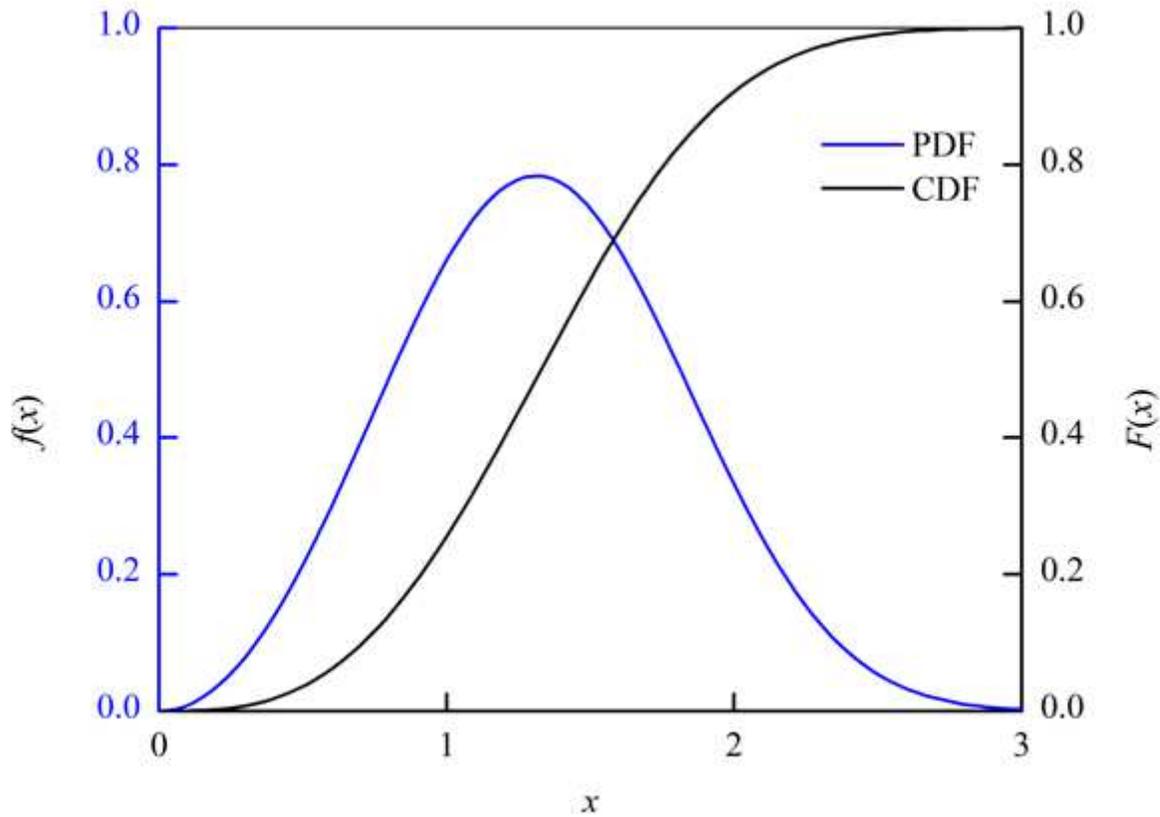


Figure 1

Black and blue curves are the cumulative function and its derivative of Weibull distribution, respectively, with $a=1.5$, $n=3$.

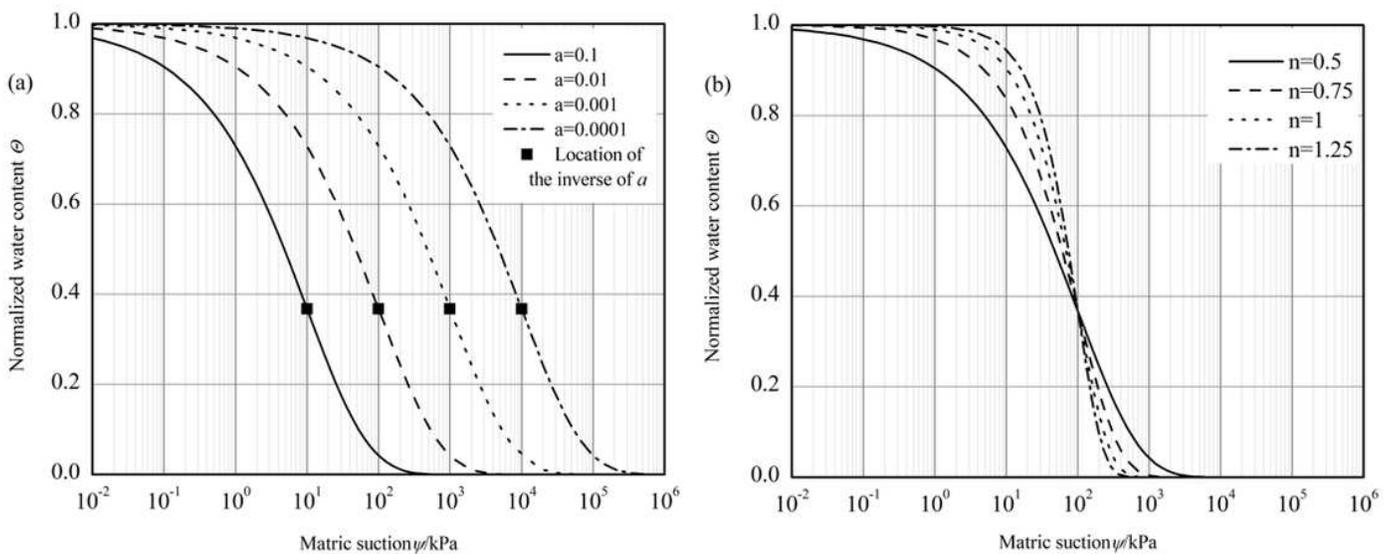


Figure 2

Plots of Weibull model with one varying parameter while the other is constant: (a) Constant $n=0.5$ and varying a ; (b) Constant $a=0.01$ and varying n .

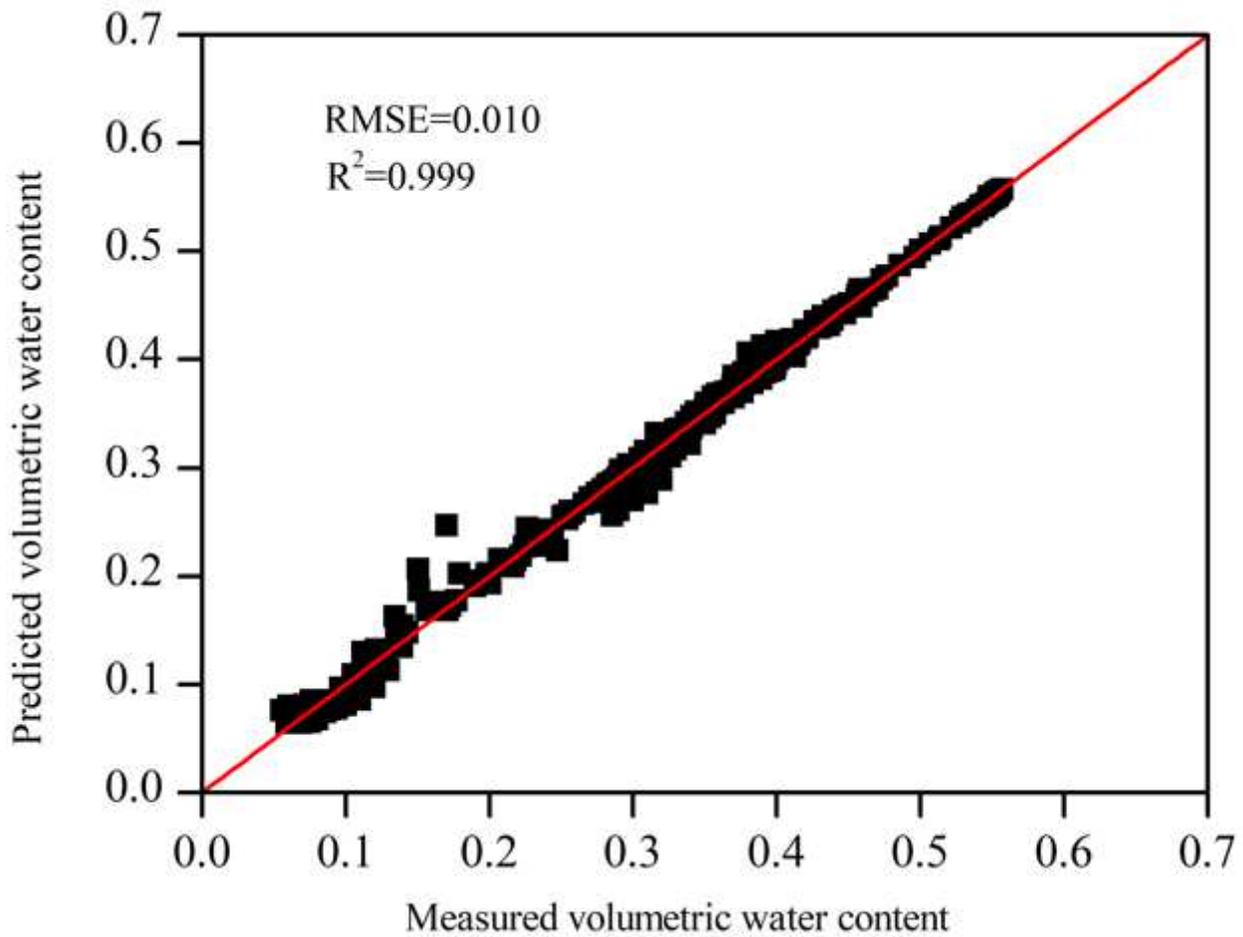
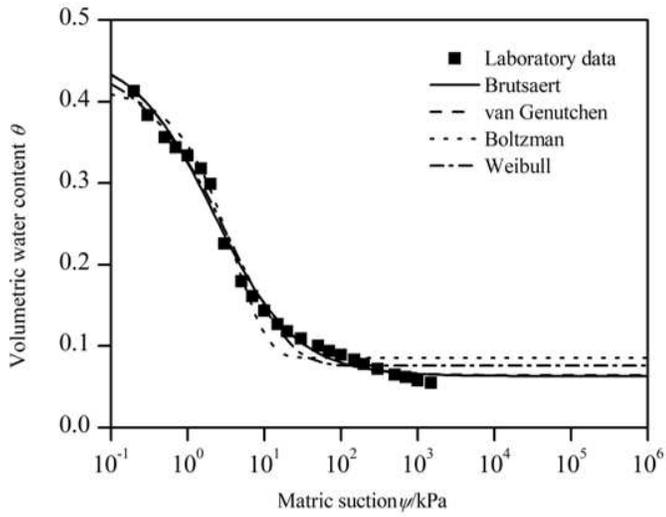
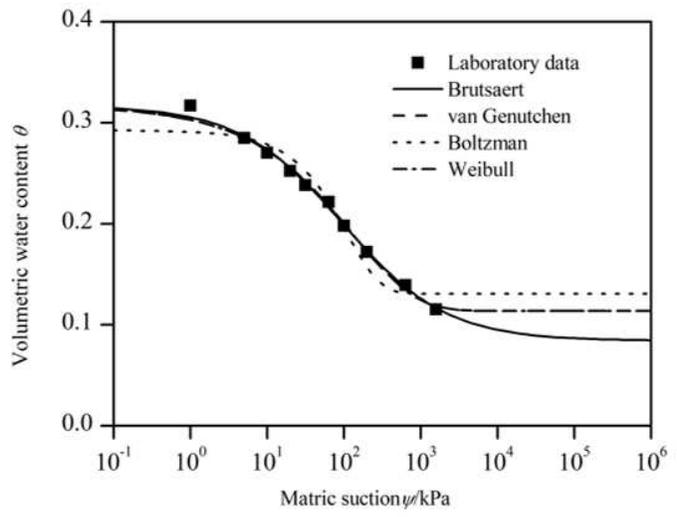


Figure 3

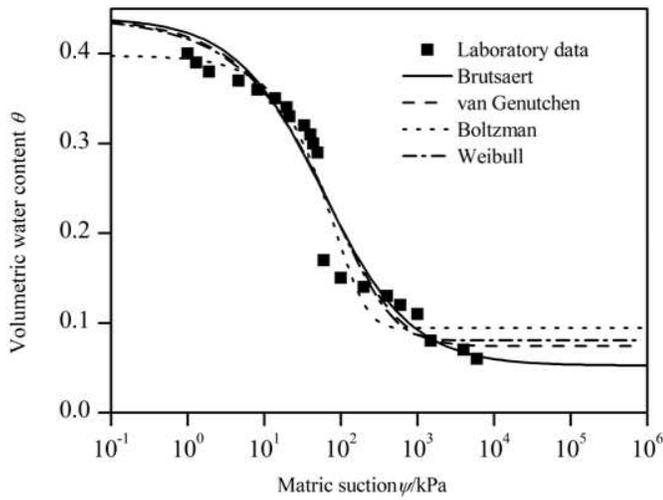
Comparison of measured versus predicted volumetric water content using Weibull model for 330 data points of 24 soil samples.



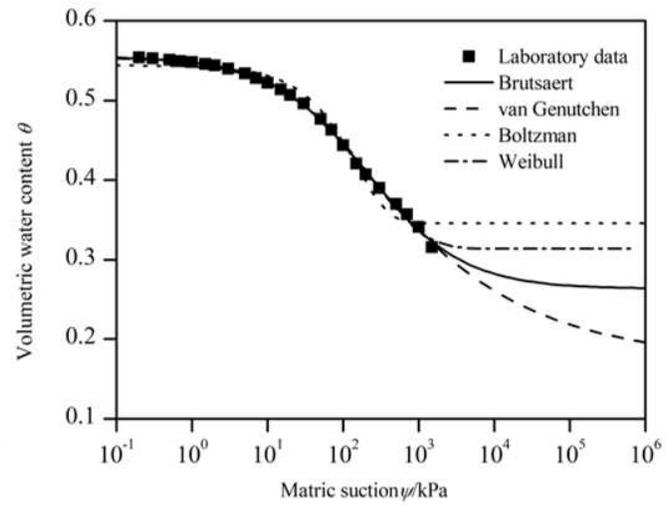
a (S2)



b (S5)



c (S10)



d (S24)

Figure 4

Best-fitted SWCCs to the measured data for sand (S2), sandy loam (S5), silt (S10), clay (S24).

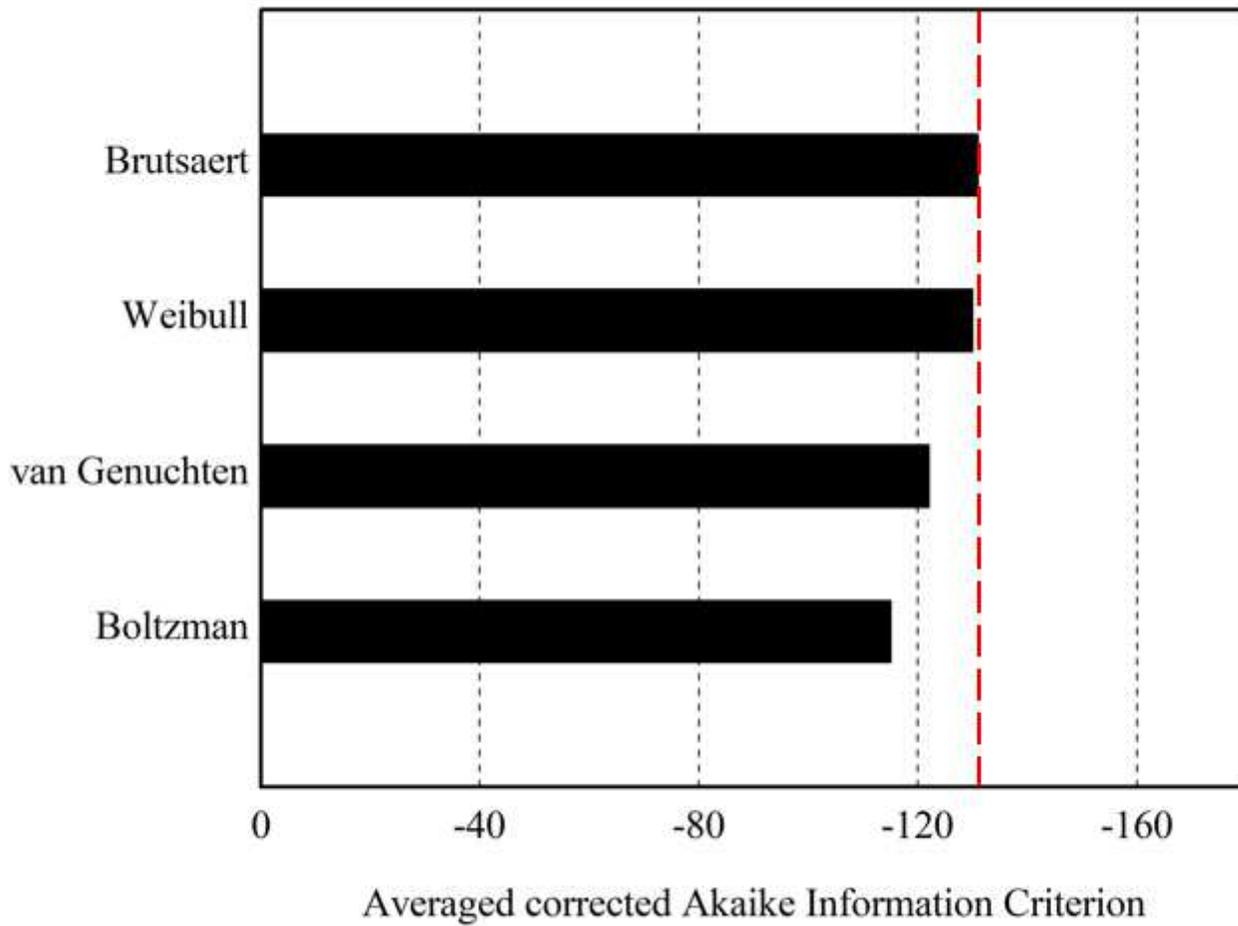


Figure 5

Averaged corrected Akaike Information Criterion for each model.

Supplementary Files

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