Multi Criteria Group Decision Making Based On Q-Rung Orthopair Trapezoidal Hesitant Dombi Fuzzy Aggregation Operators

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MULTI CRITERIA GROUP DECISION MAKING BASED ON Q-RUNG ORTHOPAIR TRAPEZOIDAL HESITANT DOMBI FUZZY AGGREGATION OPERATORS

ALIYA FAHMI, MUHAMMAD ASLAM, AND REHAN AHMAD

ABSTRACT. The paper determinations to present the impression of dombi aggregation operators for the q-rung orthopair trapezoidal hesitant dombi fuzzy numbers. The q-rung orthopair trapezoidal hesitant dombi fuzzy statistics is an imperative idea. On the other indicator, the association between the diverse combines of the qualities are well chronicled in rapports of dombi operators. Thus, possession these compensations of q-rung orthopair trapezoidal hesitant dombi fuzzy number, the impartial of this work is to describe numerous weighted averaging and geometric aggregation operators. The numerous possessions and the extraordinary cases of them are also derived. Further, the consequences of proposed new dombi aggregation operators are studied in view of some constraints. Finally, a multiple attribute decision making algorithm, based on the proposed operators, is established to solve the problems with uncertain data and exemplify with arithmetical instances. A relative assignment, domination check and discussion of the intentional approach are provided to authorize the methodology.

1. Introduction

Multiple attribute decision making (MADM) procedure comprises the inspection of an incomplete preparation of choices and putting them as far as the detail that they are so dependable to decision-maker(s) when all the rubrics are supposed of at the similar time. In this procedure, the score approximations of every choice join together careful data and authorities' personal facts. However, generally, it is predictable that the information provided by them are renewed in nature.

In any case, because of the randomness of the outline stage by stage, the honest covers frequent MADM subjects where the data is moreover vague, lose or uncertain in wildlife. To achieve it, Atanassov (1986) industrialized the outline of intuitionistic fuzzy set (IFS) which labelled the degree of fuzziness in footings of two meanings known as membership and non-membership functions with a complaint that their entirety must not surpass 1. Atanassov's intuitionistic fuzzy typical heightened the thought of Zadeh's fuzzy set (FS) (Zadeh 1965) which designated the inaccuracy in an indefinite incident with the support of only solitary function recognized as membership function on a gage of 0 to 1. Atanassov's IFS limits decision-makers in two habits; first, it limits the sum of membership and non- membership marks up to component interval owing to which decision makers cannot allocate these marks by their individual consent.

Key words and phrases. fuzzy set, hesitant set, dombi, qROTrHFN, MCDM, numerical application.

The multiple measures of all alternatives providing by assembly of area connoisseurs, development of finding best alternatives [1]. This process two serious errands. The morals of different principles precisely and effectively to designate in treacherous commission. For case, fuzzy set is a recognized sympathetic [2-12], there are countless of obtainable diverse, fuzzy sets introduced [13]. Yager's generalized OFS [14], qROFS is usually recognized. The qROFS is used to the degrees of membership and non-membership. The qROFS can be stared as the simplification of Zadeh's fuzzy set [15], IFS [16], and Yager's Pythagorean fuzzy set [17]. The qROFS will decrease to FS when $q = 1$ and the sum of of membership and non-membership is equal to 1, will reduce to IFS when $q = 1$, and will reduce to PFS when $q = 2$. In extra, the soulfulness of a qROFS stay to strengthening as q-upsurges, which deliver the clarification of fuzzy data. In the field of qROFSs conventional wide of GDM throughout the historical centuries. The qROFSs for MCGDM concerning investigation themes, such association coefficient of qROFSs [18], qROFS of distance actions [19], qROFS of similarity measures [20], qROFSs used to real life application of GDM issues [21, 22], operational rules of qROFNs [23-25], and qROFNs of aggregation operator [26-38].

Hesitant fuzzy set can be practical in countless decision making difficulties. To grow the optimal substitute in a decision making problematic with multiple attributes and multiple peoples, there are typically two conducts: (1) collective the decision makers' sentiments underneath each attribute for alternatives, then collective the mutual values of attributes for respectively alternative; (2) collective the quality morals assumed by the decision makers for each alternative, and then total the decision makers' sentiments for respectively alternative. Torra [52] defined the idea of hesitant fuzzy set.

The MCDM is labelled to standard data and ranking of all alternatives. Liu and Wang [26] introduced both the WA and the WG operators, Liu and Liu [27] introduced the WBM operator and WGBM operator. Liu and Wang [28] presented the WABM operators, Yang and Pang [29] introduced the WPBM operator and WPGBM operator, Wei et al. [30] introduced the WHM operator and the WGHM operator, Wei et al. [30] presented the WHM operator. Liu et al. [31] presented the WPHM operator. Wei et al. [32] given the the WMSM operator and the WGMSM operator, the WPMSM operator presented by Liu et al. [33], Bai et al. [34] presented the WPPMSM operator, the WMM operator and Wang et al. [35] presented the WGMM operator, Liu et al. [36] presented the weighted extended BM operator, Peng et al. [37] introduced the WE operator, and Xing et al. [38] introduced the WP operators. Each operator has its different works and sound for its exact purpose.

It is also of rank imprisonment the multifaceted interrelationships of criteria to make sensible aggregation consequences [28]. Further, the criteria are normally measured by specialists. It is frequently total objectivity, specialists will give unfair valuation values [33]. To way of sensible aggregation consequences, decrease the bad effect of biased criterion in the aggregation. Hashemi et al. [45] introduced the model depends on a new integration of IFSS theory, ELECTRE and VIKOR along with grey relational analysis (GRA). Ziemba [46] introduced the a new fuzzy MCDM method NEAT F-PROMETHEE. Hwang et al. [47] introduced the different technique. It is use different techniques of MCDM [48-51]. In 1982, Dombi [52] introduced some operations of Dombi T- norm and T-conorm. These operational

parameters show the benefits of good flexibility. Liu et al. [53] precede the Dombi operations IFSs and introduced some IF Dombi Bonferroni mean operator and applied them to MAGDM.

But there is yet an operator that is the three qualities at the same time:

- (1) Deliver the simplification in the aggregation of qROTrHFNs;
- (2) There are interrelationships diverse criteria with the condition, in which alienated into numerous shares of apiece criteria in dissimilar are sovereign.
- (3) The aggregation consequences decrease the result of excessively criterion.

In MCDM problems, favorites of aggregation of standard ethics multifaceted process. A perfect aggregation operator must be general to such alteration. Furthermore, there are multifaceted relations dissimilar criteria in the difficulties.

The article is systematized as follows. Section 2, we offer approximately properties of elementary notions. Section 3, we exhibit of q-ROTrHFNs and operational laws. Section 4, we define twelve aggregation operators founded on the q-ROTrHFN. Section 5, progress an method to MCDM. Section 6, the request of the industrialized method in GDM is exposed example. Section 7, we deliberate in virtual study. Lastly, we offer the conclusions in Section 8.

2. PRELIMINARIES

Definition 1. Let us consider that $\Phi \neq X$ and by a fuzzy set $\gamma = \left\{ \begin{array}{l} \langle x, \mu_{\gamma}(x) \rangle \\ : x \in X \end{array} \right\}$,

$\mu_{\gamma}(x)$ is a mapping from X to $[0, 1]$ represent membership function of an element x in X .

2.1. q_{Ω} ROHFN and operational laws.

Definition 2. The fixed set $\tilde{X}$ and the q_{Ω} ROHFN A is defined in $A = \{\langle H_A(x), N_A(x) \rangle : x \in X\}$, where $H_A(x)$ and $N_A(x)$ represent the MED and NOMED, and $H_A(x) \in [0, 1], N_A(x) \in [0, 1]$ and $0 \leq H_A(x)^{\tilde{q}} + N_A(x)^{\tilde{q}} \leq 1$. The degree of indeterminacy is defined as $\pi_A(x) = (H_A(x)^{\tilde{q}} + N_A(x)^{\tilde{q}} - H_A(x)^{\tilde{q}}N_A(x)^{\tilde{q}})^{\frac{1}{\tilde{q}}}$. The q_{Ω} ROHFN is denoted as $A = \langle H_A, N_A \rangle$.

Definition 3. Let $a_1 = \{\varsigma_1, \chi_1\}$ and $a_2 = \{\varsigma_2, \chi_2\}$ be two q_{Ω} ROHFNs, $\lambda > 0$, then

$$\begin{aligned} a_1 \oplus a_2 &= \bigcup_{\gamma_1 \in \varsigma_1, \gamma_2 \in \varsigma_2, \Delta_1 \in \chi_1, \Delta_2 \in \chi_2} \left[(\gamma_1^{\tilde{q}} + \gamma_2^{\tilde{q}} - \gamma_1^{\tilde{q}}\gamma_2^{\tilde{q}})^{\frac{1}{\tilde{q}}}, \Delta_1\Delta_2 \right]; \\ a_1 \otimes a_2 &= \bigcup_{\gamma_1 \in \varsigma_1, \gamma_2 \in \varsigma_2, \Delta_1 \in \chi_1, \Delta_2 \in \chi_2} \left[\gamma_1\gamma_2, (\Delta_1^{\tilde{q}} + \Delta_2^{\tilde{q}} - \Delta_1^{\tilde{q}}\Delta_2^{\tilde{q}})^{\frac{1}{\tilde{q}}} \right]; \\ \lambda a_1 &= \bigcup_{\gamma_1 \in \varsigma_1, \Delta_1 \in \chi_1} \left[(1 - (1 - \gamma_1^{\tilde{q}})^{\lambda})^{\frac{1}{\tilde{q}}}, \Delta_1^{\lambda} \right]; \\ a_1^{\lambda} &= \bigcup_{\gamma_1 \in \varsigma_1, \Delta_1 \in \chi_1} \left[\gamma_1^{\lambda}, (1 - (1 - \Delta_1^{\tilde{q}})^{\lambda})^{\frac{1}{\tilde{q}}} \right]. \end{aligned}$$

3. Q_{Ω} ROTrHFN AND OPERATIONAL LAWS

Definition 4. Let $\ddot{X}$ be an ordinary fixed set, a q_{Ω} ROTrHFS A in $\ddot{X}$ defined by

$$A = \left\{ \begin{array}{l} \langle [\mu_A(x), T_A(x), U_A(x), Y_A(x)], \\ [\nu_A(x), F_A(x), M_A(x), N_A(x)] \rangle \\ : x \in \ddot{X} \end{array} \right\}, \text{ where } [\mu_A(x), T_A(x), U_A(x), Y_A(x)] \text{ and } [\nu_A(x), F_A(x), M_A(x), N_A(x)] \text{ present the MED and NOMED, and } \mu_A(x) \in [0, 1], T_A(x) \in [0, 1], U_A(x) \in [0, 1], Y_A(x) \in [0, 1], \nu_A(x) \in [0, 1], F_A(x) \in [0, 1], M_A(x) \in [0, 1]$$

$[0, 1], N_A(x) \in [0, 1]$ and $0 \leq \mu_A(x)^{\check{q}} + \nu_A(x)^{\check{q}} \leq 1, 0 \leq T_A(x)^{\check{q}} + F_A(x)^{\check{q}} \leq 1, 0 \leq U_A(x)^{\check{q}} + M_A(x)^{\check{q}} \leq 1$ and $0 \leq Y_A(x)^{\check{q}} + N_A(x)^{\check{q}} \leq 1$. The degree of indeterminacy is given as $\pi_A(x) = (\mu_A(x)^{\check{q}} + \nu_A(x)^{\check{q}} - \mu_A(x)^{\check{q}}\nu_A(x)^{\check{q}})^{\frac{1}{\check{q}}}$, $\pi_A(x) = (T_A(x)^{\check{q}} + F_A(x)^{\check{q}} - T_A(x)^{\check{q}}F_A(x)^{\check{q}})^{\frac{1}{\check{q}}}$ and $\pi_A(x) = (U_A(x)^{\check{q}} + M_A(x)^{\check{q}} - U_A(x)^{\check{q}}M_A(x)^{\check{q}})^{\frac{1}{\check{q}}}$ and $\pi_A(x) = (Y_A(x)^{\check{q}} + N_A(x)^{\check{q}} - Y_A(x)^{\check{q}}N_A(x)^{\check{q}})^{\frac{1}{\check{q}}}$. To simplify, the $q_\Omega ROTrHFN$ is denoted as $A = \{[\mu_A, T_A, U_A, Y_A], [\nu_A, F_A, M_A, N_A]\}$.

Definition 5. Let $a_1 = \left\{ \begin{array}{l} [\eta_1, \kappa_1, \varsigma_1, \beta_1], \\ [\Lambda_1, \Upsilon_1, \vartheta_1, \varkappa_1] \end{array} \right\}$ and $a_2 = \left\{ \begin{array}{l} [\eta_2, \kappa_2, \varsigma_2, \beta_2], \\ [\Lambda_2, \Upsilon_2, \vartheta_2, \varkappa_2] \end{array} \right\}$ be two $q_\Omega ROTrHFN$ s and based on dombi, $\lambda > 0$, then

$$a_1 \oplus a_2 = \bigcup_{\substack{\mu_1 \in \eta_1, \mu_2 \in \eta_2, T_1 \in \kappa_1, \\ T_2 \in \kappa_2, G_1 \in \varsigma_1, G_2 \in \varsigma_2, \\ H_1 \in \beta_1, H_2 \in \beta_2, \\ \nu_1 \in \Lambda_1, \nu_2 \in \Lambda_2, F_1 \in \Upsilon_1, \\ F_2 \in \Upsilon_2, M_1 \in \vartheta_1, M_2 \in \vartheta_2, \\ N_1 \in \varkappa_1, N_2 \in \varkappa_2}} \left[\left(\begin{array}{l} \left(1 - \frac{1}{1 + \left\{ \left(\frac{\mu_{a_1}}{1 - \mu_{a_1}} \right)^\rho + \left(\frac{\mu_{a_2}}{1 - \mu_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{T_{a_1}}{1 - T_{a_1}} \right)^\rho + \left(\frac{T_{a_2}}{1 - T_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{G_{a_1}}{1 - G_{a_1}} \right)^\rho + \left(\frac{G_{a_2}}{1 - G_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{H_{a_1}}{1 - H_{a_1}} \right)^\rho + \left(\frac{H_{a_2}}{1 - H_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{1 - \nu_{a_1}}{\nu_{a_1}} \right)^\rho + \left(\frac{1 - \nu_{a_2}}{\nu_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{1 - F_{a_1}}{F_{a_1}} \right)^\rho + \left(\frac{1 - F_{a_2}}{F_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{1 - M_{a_1}}{M_{a_1}} \right)^\rho + \left(\frac{1 - M_{a_2}}{M_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{1 - N_{a_1}}{N_{a_1}} \right)^\rho + \left(\frac{1 - N_{a_2}}{N_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}} \end{array} \right) \right];$$

$$\begin{aligned}
 a_1 \otimes a_2 = & \bigcup_{\substack{\mu_1 \in \eta_1, \mu_2 \in \eta_2, T_1 \in \kappa_1, \\ T_2 \in \kappa_2, G_1 \in \varsigma_1, G_2 \in \varsigma_2, \\ H_1 \in \beta_1, H_2 \in \beta_2, \\ \nu_1 \in \Lambda_1, \nu_2 \in \Lambda_2, F_1 \in \Upsilon_1, \\ F_2 \in \Upsilon_2, M_1 \in \vartheta_1, M_2 \in \vartheta_2, \\ N_1 \in \varkappa_1, N_2 \in \varkappa_2}} \left[\left(\begin{array}{c} \frac{1}{1 + \left\{ \left(\frac{1 - \mu_{a_1}}{\mu_{a_1}} \right)^\rho + \left(\frac{1 - \mu_{a_2}}{\mu_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - T_{a_1}}{T_{a_1}} \right)^\rho + \left(\frac{1 - T_{a_2}}{T_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - G_{a_1}}{G_{a_1}} \right)^\rho + \left(\frac{1 - G_{a_2}}{G_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - H_{a_1}}{H_{a_1}} \right)^\rho + \left(\frac{1 - H_{a_2}}{H_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \end{array} \right)^{\frac{1}{q}}, \left(\begin{array}{c} \left(1 - \frac{1}{1 + \left\{ \left(\frac{\nu_{a_1}}{1 - \nu_{a_1}} \right)^\rho + \left(\frac{\nu_{a_2}}{1 - \nu_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{F_{a_1}}{1 - F_{a_1}} \right)^\rho + \left(\frac{F_{a_2}}{1 - F_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{M_{a_1}}{1 - M_{a_1}} \right)^\rho + \left(\frac{M_{a_2}}{1 - M_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{N_{a_1}}{1 - N_{a_1}} \right)^\rho + \left(\frac{N_{a_2}}{1 - N_{a_2}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{q}} \end{array} \right)^{\frac{1}{q}} \right]; \\
 \lambda a_1 = & \bigcup_{\substack{\mu_1 \in \eta_1, T_1 \in \kappa_1, G_1 \in \varsigma_1, H_1 \in \beta_1, \\ \nu_1 \in \Lambda_1, F_1 \in \Upsilon_1, M_1 \in \vartheta_1, N_1 \in \varkappa_1}} \left[\left(\begin{array}{c} \left(1 - \frac{1}{1 + \left\{ \left(\frac{\mu_{a_1}}{1 - \mu_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{T_{a_1}}{1 - T_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{G_{a_1}}{1 - G_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \left(\frac{H_{a_1}}{1 - H_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}} \end{array} \right)^{\frac{1}{q}}, \left(\begin{array}{c} \frac{1}{1 + \left\{ \left(\frac{1 - \nu_{a_1}}{\nu_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - F_{a_1}}{F_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - M_{a_1}}{M_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - N_{a_1}}{N_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \end{array} \right)^{\frac{1}{q}} \right];
 \end{aligned}$$

$$a_1^\lambda = \bigcup_{\substack{\mu_1 \in \eta_1, T_1 \in \kappa_1, G_1 \in \varsigma_1, H_1 \in \beta_1, \\ \nu_1 \in \Lambda_1, F_1 \in \Upsilon_1, M_1 \in \vartheta_1, N_1 \in \varkappa_1}} \left[\begin{pmatrix} \frac{1}{1 + \left\{ \left(\frac{1 - \mu_{a_1}}{\mu_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - T_{a_1}}{T_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - G_{a_1}}{G_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ \frac{1}{1 + \left\{ \left(\frac{1 - H_{a_1}}{H_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{q}}, \\ \left(\begin{pmatrix} 1 - \frac{1}{1 + \left\{ \left(\frac{\nu_{a_1}}{1 - \nu_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ 1 - \frac{1}{1 + \left\{ \left(\frac{F_{a_1}}{1 - F_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ 1 - \frac{1}{1 + \left\{ \left(\frac{M_{a_1}}{1 - M_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}}, \\ 1 - \frac{1}{1 + \left\{ \left(\frac{N_{a_1}}{1 - N_{a_1}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{q}} \end{pmatrix} \right].$$

Definition 6. The $q_\Omega R O T r H F N s$ are $a = \left\{ \begin{bmatrix} \eta, \kappa, \varsigma, \phi \\ \Lambda, \Upsilon, \vartheta, F \end{bmatrix} \right\}$, then the score function $\check{H}$ is define as: $\check{H} = \bigcup_{\substack{I \in \eta, O \in \kappa, J \in \varsigma, L \in \phi \\ N \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \frac{\{[I+O+J+L]-[N+F+M+N]\}}{8}$.

Definition 7. The $q_\Omega R O T r H F N s$ are $a = \left\{ \begin{bmatrix} \eta, \kappa, \varsigma, \phi \\ \Lambda, \Upsilon, \vartheta, F \end{bmatrix} \right\}$, then the accuracy function $\check{K}$ is define as: $\check{K} = \bigcup_{\substack{I \in \eta, O \in \kappa, J \in \varsigma, L \in \phi \\ N \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \frac{\{[I+O+J]+[N+F+M]\}}{8}$.

4. $Q_\Omega R O T r H D F$ AGGREGATION OPERATORS

In this section, the twelve aggregation operators are defined.

4.1. $q_\Omega R O T r H D F W A$ operator.

Definition 8. The gathering of $q_\Omega R O T r H F N s$ are $c_j = \left\{ \begin{bmatrix} \eta, \kappa, \varsigma, \phi \\ \Lambda, \Upsilon, \vartheta, F \end{bmatrix} \right\}$ and the weight vector is $\Delta = (\Delta_1, \Delta_2, \dots, \Delta_n)^T$ with $\Delta_j \in [0, 1]$ and $\sum_{j=1}^n \Delta_j = 1$. Then $q_\Omega R O T r H D F W A(c_1, c_2, \dots, c_n) = \bigoplus_{j=1}^n \Delta_j c_j$ is said $q_\Omega R O T r H D F W A$ operator.

Theorem 1. The collection of $q_\Omega R O T r H D F N s$ are $a_j = \left\{ \begin{bmatrix} \eta, \kappa, \varsigma, \phi \\ \Lambda, \Upsilon, \vartheta, F \end{bmatrix} \right\}$ and the weight vector is $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ with $\xi_j \in [0, 1]$ and $\sum_{j=1}^n \xi_j = 1$. Then it is said $q_\Omega R O T r H D F W A$ operator and $q_\Omega R O T r H D F W A(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\begin{pmatrix} 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{\mu_{a_j}}{1 - \mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \begin{pmatrix} 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{T_{a_j}}{1 - T_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \begin{pmatrix} 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{G_{a_j}}{1 - G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \begin{pmatrix} 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{H_{a_j}}{1 - H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \begin{pmatrix} \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - \nu_{a_j}}{\nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - F_{a_j}}{F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - M_{a_j}}{M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - N_{a_j}}{N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \end{pmatrix} \right)^{\frac{1}{q}} \right].$$

Theorem 2. (Idempotency): If $L_k = \left\{ \begin{matrix} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{matrix} \right\}$ for all $k = 1, 2, 3, \dots, n$, then $q_\Omega \text{ROTrHDFWA}(L, L, L, \dots, L) = L$.

Theorem 3. (Commutativity). If $(L'_1, L'_2, \dots, L'_n)$ is any permutation of $(L_1, L_2, \dots, L_n)$, then $q_\Omega \text{ROTrHDFWA}(L'_1, L'_2, \dots, L'_n) = q_\Omega \text{ROTrEHDFWA}(L_1, L_2, \dots, L_n)$.

Theorem 4. (Boundedness). If $\eta^- = \min(L_1, L_2, \dots, L_n)$, $\eta^+ = \max(L_1, L_2, \dots, L_n)$, then $\eta^- \leq q_\Omega \text{ROTrHDFWA}(L_1, L_2, \dots, L_n) \leq \eta^+$.

4.2. $q_\Omega \text{ROTrHDFOWA}$ operator.

Definition 9. The family of $q_\Omega R O T r H F N s$ are $b_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then $q_\Omega R O T r H D F O W A(b_1, b_2, \dots, b_n) = \bigoplus_{j=1}^n \Upsilon_j b_j$ is said $q_\Omega R O T r H F O W A$ operator.

Theorem 5. Let $a_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_\Omega R O T r H F N s$ and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then it is said $q_\Omega R O T r H D F O W A$ operator and $q_\Omega R O T r H D F O W A(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\begin{array}{c} \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{\mu_{a_j}}{1 - \mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{T_{a_j}}{1 - T_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{G_{a_j}}{1 - G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{H_{a_j}}{1 - H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - \nu_{a_j}}{\nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - F_{a_j}}{F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - M_{a_j}}{M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - N_{a_j}}{N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \end{array} \right) \right],$$

Theorem 6. (Idempotency): If $R_k = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ for all $k = 1, 2, 3, \dots, n$, then $q_\Omega R O T r H D F O W A(R, R, R, \dots, R) = R$.

Theorem 7. (Commutativity). *If $(d'_1, d'_2, \dots, d'_n)$ is any permutation of $(d_1, d_2, \dots, d_n)$, then $q_\Omega R O T r H D F O W A(d'_1, d'_2, \dots, d'_n) = q_\Omega R O T r H D F O W A(d_1, d_2, \dots, d_n)$.*

Theorem 8. (Boundedness). *If $\eta^- = \min(f_1, f_2, \dots, f_n)$, $\eta^+ = \max(f_1, f_2, \dots, f_n)$, then $\eta^- \leq q_\Omega R O T r H D F O W A(f_1, f_2, \dots, f_n) \leq \eta^+$.*

4.3. $q_\Omega R O T r H D F H W A$ operator.

Definition 10. *The family of $q_\Omega R O T r H F N$ s are $c_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ and the weight vector is $\Delta = (\Delta_1, \Delta_2, \dots, \Delta_n)^T$ with $\Delta_j \in [0, 1]$ and $\sum_{j=1}^n \Delta_j = 1$. Then $q_\Omega R O T r H D F H W A(c_1, c_2, \dots, c_n) = \bigoplus_{j=1}^n \Delta_j c_j$ is said $q_\Omega R O T r H D F H W A$ operator.*

Theorem 9. *Let $a_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_\Omega R O T r H F N$ s and the associate vector is $\Delta = (\Delta_1, \Delta_2, \dots, \Delta_n)^T$ with $\Delta_j \in [0, 1]$ and $\sum_{j=1}^n \Delta_j = 1$. Then it is said $q_\Omega R O T r H D F H W A$ operator and $q_\Omega R O T r H D F H W A(a_1, a_2, \dots, a_n) =$*

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\begin{pmatrix} 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{\mu_{a_j}}{1 - \mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \right. \\ \left(\begin{pmatrix} 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{T_{a_j}}{1 - T_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(\begin{pmatrix} 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{G_{a_j}}{1 - G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(\begin{pmatrix} 1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{H_{a_j}}{1 - H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(\begin{pmatrix} \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - \nu_{a_j}}{\nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(\begin{pmatrix} \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - F_{a_j}}{F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(\begin{pmatrix} \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - M_{a_j}}{M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(\begin{pmatrix} \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - N_{a_j}}{N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}} \end{pmatrix} \right].$$

Theorem 10. (Idempotency): If $L_k = \left\{ \begin{matrix} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{matrix} \right\}$ for all $k = 1, 2, 3, \dots, n$, then $q_\Omega \text{ROTrHDFHWA}(L, L, L, \dots, L) = L$.

Theorem 11. (Commutativity). If $(L'_1, L'_2, \dots, L'_n)$ is any permutation of $(L_1, L_2, \dots, L_n)$, then $q_\Omega \text{ROTrHDFHWA}(L'_1, L'_2, \dots, L'_n) = q_\Omega \text{ROTrHDFHWA}(L_1, L_2, \dots, L_n)$.

Theorem 12. (Boundedness). If $\eta^- = \min(e_1, e_2, \dots, e_n)$, $\eta^+ = \max(e_1, e_2, \dots, e_n)$, then $\eta^- \leq q_\Omega \text{ROTrHDFHWA}(e_1, e_2, \dots, e_n) \leq \eta^+$.

4.4. $\mathbf{Gq_\Omega \text{ROTrHDFHWA}}$ operator.

Definition 11. The gathering of $q_\Omega ROTrHFNs$ are $c_j = \left\{ \begin{array}{l} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ and the weight vector is $\Delta = (\Delta_1, \Delta_2, \dots, \Delta_n)^T$ with $\Delta_j \in [0, 1]$ and $\sum_{j=1}^n \Delta_j = 1$. Then $G_{q_\Omega ROTrHDFWA}(c_1, c_2, \dots, c_n) = \left(\bigoplus_{j=1}^n c_j \right)^{\frac{1}{\Delta_j}}$ is said $G_{q_\Omega ROTrHDFWA}$ operator.

Theorem 13. The collection of $q_\Omega ROTrHFNs$ are $a_j = \left\{ \begin{array}{l} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ and the weight vector is $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ with $\xi_j \in [0, 1]$ and $\sum_{j=1}^n \xi_j = 1$. Then it is said $G_{q_\Omega ROTrHDFWA}$ operator and $G_{q_\Omega ROTrHDFWA}(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\begin{pmatrix} \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{\mu_{a_j}}{1 - \mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{S_{a_j}}{1 - S_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{G_{a_j}}{1 - G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{H_{a_j}}{1 - H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - \nu_{a_j}}{\nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - F_{a_j}}{F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - M_{a_j}}{M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - N_{a_j}}{N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \end{pmatrix} \right) \right].$$

Theorem 14. (Idempotency): If $L_k = \left\{ \begin{matrix} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{matrix} \right\}$ for all $k = 1, 2, 3, \dots, n$, then $Gq_\Omega R OTr H DFWA(L, L, L, \dots, L) = L$.

Theorem 15. (Commutativity). If $(L'_1, L'_2, \dots, L'_n)$ is any permutation of $(L_1, L_2, \dots, L_n)$, then $G_{q_\Omega}ROTrHDFWA(L'_1, L'_2, \dots, L'_n) = G_{q_\Omega}ROTrHDFWA(L_1, L_2, \dots, L_n)$.

Theorem 16. (Boundedness). If $\eta^- = \min(L_1, L_2, \dots, L_n)$, $\eta^+ = \max(L_1, L_2, \dots, L_n)$, then $\eta^- \leq G_{q_\Omega}ROTrHDFWA(L_1, L_2, \dots, L_n) \leq \eta^+$.

4.5. $G_{q_\Omega}ROTrHDFOWA$ operator.

Definition 12. The family of $q_\Omega ROTrHFNs$ are $b_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then $G_{q_\Omega}ROTrHDFOWA(b_1, b_2, \dots, b_n) = \bigoplus_{j=1}^n \Upsilon_j b_j$ is said $G_{q_\Omega}ROTrHDFOWA$ operator.

Theorem 17. Let $a_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_\Omega ROTrHFNs$ and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then it is said $G_{q_\Omega}ROTrHDFOWA$ operator and $G_{q_\Omega}ROTrHDFOWA(a_1, a_2, \dots, a_n) =$

$$\left[\begin{array}{c} \left(\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{\mu_{a_j}}{1 - \mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right) \\ \left(\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{S_{a_j}}{1 - S_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right) \\ \left(\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{G_{a_j}}{1 - G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right) \\ \left(\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{H_{a_j}}{1 - H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right) \\ \left(\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - \nu_{a_j}}{\nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right) \\ \left(\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - F_{a_j}}{F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right) \\ \left(\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - M_{a_j}}{M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right) \\ \left(\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - N_{a_j}}{N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right) \end{array} \right]$$

Theorem 18. (Idempotency): If $R_k = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ for all $k = 1, 2, 3, \dots, n$, then $Gq_\Omega R OTrHDFOWA(R, R, R, \dots, R) = R$.

Theorem 19. (Commutativity). If $(d'_1, d'_2, \dots, d'_n)$ is any permutation of $(d_1, d_2, \dots, d_n)$, then $G_{q\Omega}ROTrHDFOWA(d'_1, d'_2, \dots, d'_n) = G_{q\Omega}ROTrDFOWA(d_1, d_2, \dots, d_n)$.

Theorem 20. (Boundedness). If $\eta^- = \min(f_1, f_2, \dots, f_n)$, $\eta^+ = \max(f_1, f_2, \dots, f_n)$, then $\eta^- \leq G_{q\Omega}ROTrHDFOWA(f_1, f_2, \dots, f_n) \leq \eta^+$.

4.6. $G_{q\Omega}ROTrHDFHWA$ operator.

Definition 13. The family of $q_{\Omega}ROTrHFNs$ are $c_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ and the weight vector is $\Delta = (\Delta_1, \Delta_2, \dots, \Delta_n)^T$ with $\Delta_j \in [0, 1]$ and $\sum_{j=1}^n \Delta_j = 1$. Then $G_{q\Omega}ROTrHDFHWA(c_1, c_2, \dots, c_n) = \bigoplus_{j=1}^n \Delta_j c_j$ is said $G_{q\Omega}ROTrHDFHWA$ operator.

Theorem 21. Let $a_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_{\Omega}ROTrHFNs$ and the associate vector is $\Delta = (\Delta_1, \Delta_2, \dots, \Delta_n)^T$ with $\Delta_j \in [0, 1]$ and $\sum_{j=1}^n \Delta_j = 1$. Then it is said $G_{q\Omega}ROTrHDFHWA$ operator and $G_{q\Omega}ROTrHDFHWA(a_1, a_2, \dots, a_n) =$

$$\left[\begin{array}{c} \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{\mu_{a_j}}{1 - \mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right), \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{S_{a_j}}{1 - S_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right), \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{G_{a_j}}{1 - G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right), \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{H_{a_j}}{1 - H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right)^{\frac{1}{q}} \right), \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - \nu_{a_j}}{\nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right), \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - F_{a_j}}{F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right), \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - M_{a_j}}{M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right), \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - N_{a_j}}{N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\xi_j}} \right) \end{array} \right].$$

Theorem 22. (Idempotency): If $L_k = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ for all $k = 1, 2, 3, \dots, n$, then $Gq_\Omega R OTrHDFHWA(L, L, L, \dots, L) = L$.

Theorem 23. (Commutativity). If $(L'_1, L'_2, \dots, L'_n)$ is any permutation of $(L_1, L_2, \dots, L_n)$, then $G_{q_\Omega}ROTrHDFHWA(L'_1, L'_2, \dots, L'_n) = G_{q_\Omega}ROTrHDFHWA(L_1, L_2, \dots, L_n)$.

Theorem 24. (Boundedness). If $\eta^- = \min(e_1, e_2, \dots, e_n)$, $\eta^+ = \max(e_1, e_2, \dots, e_n)$, then $\eta^- \leq G_{q_\Omega}ROTrHDFHWA(e_1, e_2, \dots, e_n) \leq \eta^+$.

4.7. q_Ω ROTrHDFWG operator.

Definition 14. Let $L_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of q_Ω ROTrHFNs and the weight vector is $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$ with $\lambda_j \in [0, 1]$ and $\sum_{j=1}^n \lambda_j = 1$. Then $q_\Omega ROTrEHDFWG(L_1, L_2, \dots, L_n) = \bigotimes_{j=1}^n L_j^{\lambda_j}$ is said to be q_Ω ROTrHDFWG operator.

Theorem 25. Let $a_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of q_Ω ROTrHFNs and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then it is introduced q_Ω ROTrHDFWG operator and $q_\Omega ROTrHDFWG(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu_{a_j}}{\mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - T_{a_j}}{T_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G_{a_j}}{G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{\rho}}, \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H_{a_j}}{H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}}} \right)^{\frac{1}{\rho}}, \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu_{a_j}}{1 - \nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}} \right)^{\frac{1}{q}}, \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F_{a_j}}{1 - F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}} \right)^{\frac{1}{q}}, \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M_{a_j}}{1 - M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}} \right)^{\frac{1}{q}}, \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N_{a_j}}{1 - N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}} \right)^{\frac{1}{q}} \right)$$

Theorem 26. (Idempotency): If $L_k = \left\{ \begin{bmatrix} \eta, \kappa, \varsigma, \phi \\ \Lambda, \Upsilon, \vartheta, F \end{bmatrix} \right\}$ for all $k = 1, 2, 3, \dots, n$, then $q_\Omega \text{ROTrHDFWG}(L, L, L, \dots, L) = L$.

Theorem 27. (Commutativity). If $(L'_1, L'_2, \dots, L'_n)$ is any permutation of $(L_1, L_2, \dots, L_n)$, then $q_\Omega \text{ROTrHDFWG}(L'_1, L'_2, \dots, L'_n) = q_\Omega \text{ROTrHDFWG}(L_1, L_2, \dots, L_n)$.

Theorem 28. (Boundedness). If $\eta^- = \min(L_1, L_2, \dots, L_n)$, $\eta^+ = \max(L_1, L_2, \dots, L_n)$, then $\eta^- \leq q_\Omega \text{ROTrHDFWG}(L_1, L_2, \dots, L_n) \leq \eta^+$.

4.8. $\check{q}_L \text{ROTrHDFOWG}$ operator.

Definition 15. Let $R_j = \left\{ \begin{bmatrix} \eta, \kappa, \varsigma, \phi \\ \Lambda, \Upsilon, \vartheta, F \end{bmatrix} \right\}$ be a family of $q_\Omega \text{ROTrHFNs}$ and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then

$q_{\Omega}ROTrHDFOWG(R_1, R_2, \dots, R_n) = \bigotimes_{j=1}^n R_j^{\Upsilon_j}$ is given in $q_{\Omega}ROTrHDFOWG$ operator.

Theorem 29. Let $a_j = \left\{ \begin{bmatrix} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{bmatrix} \right\}$ be a family of $q_{\Omega}ROTrHFNs$ and $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ be the weight vector with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then it is said to $q_{\Omega}ROTrHDFOWG$ operator and $q_{\Omega}ROTrHDFOWG(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\begin{pmatrix} \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu_{a_j}}{\mu_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - T_{a_j}}{T_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G_{a_j}}{G_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H_{a_j}}{H_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}} \right)^{\frac{1}{q}}, \\ \left(\frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu_{a_j}}{1 - \nu_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F_{a_j}}{1 - F_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}} \right)^{\frac{1}{q}}, \\ \left(\frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M_{a_j}}{1 - M_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N_{a_j}}{1 - N_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}} \right)^{\frac{1}{q}}, \\ \left(\frac{1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{S_{a_j}}{1 - S_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{T_{a_j}}{1 - T_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}} \right)^{\frac{1}{q}} \end{pmatrix} \right].$$

Theorem 30. (Idempotency): If $L_j = \left\{ \begin{bmatrix} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{bmatrix} \right\}$ for all $j = 1, 2, 3, \dots, n$, then $q_{\Omega}ROTrHDFOWG(L, L, L, \dots, L) = L$.

Theorem 31. (Commutativity). If $(L'_1, L'_2, \dots, L'_n)$ is any permutation of $(L_1, L_2, \dots, L_n)$, then $q_{\Omega}ROTrHDFOWG(L'_1, L'_2, \dots, L'_n) = q_{\Omega}ROTrHDFOWG(L_1, L_2, \dots, L_n)$.

Theorem 32. (Boundedness). If $\eta^- = \min(L_1, L_2, \dots, L_n)$, $\eta^+ = \max(L_1, L_2, \dots, L_n)$,

then $\eta^- \leq q_\Omega R O Tr H D F O W G(L_1, L_2, \dots, L_n) \leq \eta^+$.

4.9. $q_\Omega R O Tr H D F H W G$ operator.

Definition 16. Let $R_j = \begin{Bmatrix} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{Bmatrix}$ be a family of $q_\Omega R O Tr H F N$ s and the weight vector is $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ with $\xi_j \in [0, 1]$ and $\sum_{j=1}^n \xi_j = 1$. Then $q_\Omega R O Tr E H D F H W G(R_1, R_2, \dots, R_n) = \bigotimes_{j=1}^n R_j^{\xi_j}$ is said $q_\Omega R O Tr H D F H W G$ operator.

Theorem 33. Let $a_j = \begin{Bmatrix} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{Bmatrix}$ be a family of $q_\Omega R O Tr H F N$ s and the weight vector is $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ with $\xi_j \in [0, 1]$ and $\sum_{j=1}^n \xi_j = 1$. Then it is said $q_\Omega R O Tr H D F H W G$ operator and $q_\Omega R O Tr H D F H W G(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\begin{pmatrix} \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu a_j}{\mu a_j} \right)^\rho \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - T a_j}{T a_j} \right)^\rho \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G a_j}{G a_j} \right)^\rho \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H a_j}{H a_j} \right)^\rho \right\}^{\frac{1}{\rho}}}}^{\frac{1}{\rho}} \right], \\ \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu a_j}{1 - \nu a_j} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \right. \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F a_j}{1 - F a_j} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M a_j}{1 - M a_j} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left. \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N a_j}{1 - N a_j} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}} \right) \right].$$

Theorem 34. (Idempotency): If $R_j = \left\{ \begin{array}{l} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ for all $j = 1, 2, 3, \dots, n$, then $q_{\Omega}ROTrHDFHWG(R, R, R, \dots, R) = R$.

Theorem 35. (Commutativity). If $(R'_1, R'_2, \dots, R'_n)$ is any permutation of $(R_1, R_2, \dots, R_n)$, then $q_{\Omega}ROTrHDFHWG(R'_1, R'_2, \dots, R'_n) = q_{\Omega}ROTrHDFHWG(R_1, R_2, \dots, R_n)$.

Theorem 36. (Boundedness). If $\eta^- = \min(R_1, R_2, \dots, R_n)$, $\eta^+ = \max(R_1, R_2, \dots, R_n)$, then $\eta^- \leq q_{\Omega}ROTrHDFHWG(R_1, R_2, \dots, R_n) \leq \eta^+$.

4.10. $Gq_{\Omega}ROTrHDFWG$ operator.

Definition 17. Let $L_j = \left\{ \begin{array}{l} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_{\Omega}ROTrHFNs$ and the weight vector is $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$ with $\lambda_j \in [0, 1]$ and $\sum_{j=1}^n \lambda_j = 1$. Then $Gq_{\Omega}ROTrHDFWG(L_1, L_2, \dots, L_n) = \bigotimes_{j=1}^n L_j^{\lambda_j}$ is said to be $Gq_{\Omega}ROTrHDFWG$ operator.

Theorem 37. Let $a_j = \left\{ \begin{array}{l} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_{\Omega}ROTrHFNs$ and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then it is introduced $Gq_{\Omega}ROTrHDFWG$ operator and $Gq_{\Omega}ROTrHDFWG(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu_{a_j}}{\mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \right. \\
\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - S_{a_j}}{S_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\
\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G_{a_j}}{G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\
\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H_{a_j}}{H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\
\left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu_{a_j}}{1 - \nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \right. \\
\left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F_{a_j}}{1 - F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \right. \\
\left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M_{a_j}}{1 - M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \right. \\
\left. \left. \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N_{a_j}}{1 - N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \right) \right) \right]$$

Theorem 38. (Idempotency): If $L_k = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ for all $k = 1, 2, 3, \dots, n,$,
then $Gq_\Omega ROTrHDFWG(L, L, L, \dots, L) = L$.

Theorem 39. (Commutativity). If $(L'_1, L'_2, \dots, L'_n)$ is any permutation of $(L_1, L_2, \dots, L_n)$, then $G_{q\Omega}ROTrHDFWG(L'_1, L'_2, \dots, L'_n) = G_{q\Omega}ROTrHDFWG(L_1, L_2, \dots, L_n)$.

Theorem 40. (Boundedness). If $\eta^- = \min(L_1, L_2, \dots, L_n)$, $\eta^+ = \max(L_1, L_2, \dots, L_n)$, then $\eta^- \leq G_{q\Omega}ROTrHDFWG(L_1, L_2, \dots, L_n) \leq \eta^+$.

4.11. $G_{q\mathcal{L}}ROTrHDFOWG$ operator.

Definition 18. Let $R_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_{\Omega}ROTrHFNs$ and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then $G_{q\Omega}ROTrHDFOWG(R_1, R_2, \dots, R_n) = \bigotimes_{j=1}^n R_j^{\Upsilon_j}$ is given in $G_{q\Omega}ROTrHDFOWG$ operator.

Theorem 41. Let $a_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_{\Omega}ROTrHFNs$ and $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ be the weight vector with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then it is said to $G_{q\Omega}ROTrHDFOWG$ operator and $G_{q\Omega}ROTrHDFOWG(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\begin{array}{c} \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu_{a_j}}{\mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \\ \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - S_{a_j}}{S_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \\ \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G_{a_j}}{G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \\ \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H_{a_j}}{H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \end{array} \right), \left(\begin{array}{c} \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu_{a_j}}{1 - \nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \\ \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F_{a_j}}{1 - F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \\ \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M_{a_j}}{1 - M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \\ \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N_{a_j}}{1 - N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \end{array} \right) \right].$$

Theorem 42. (Idempotency): If $L_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ for all $j = 1, 2, 3, \dots, n$, then $Gq_\Omega ROTrHDFOWG(L, L, L, \dots, L) = L$.

Theorem 43. (Commutativity). If $(L'_1, L'_2, \dots, L'_n)$ is any permutation of $(L_1, L_2, \dots, L_n)$, then $G_{q_\Omega}ROTrHDFOWG(L'_1, L'_2, \dots, L'_n) = G_{q_\Omega}ROTrHDFOWG(L_1, L_2, \dots, L_n)$.

Theorem 44. (Boundedness). If $\eta^- = \min(L_1, L_2, \dots, L_n)$, $\eta^+ = \max(L_1, L_2, \dots, L_n)$, then $\eta^- \leq G_{q_\Omega}ROTrHDFOWG(L_1, L_2, \dots, L_n) \leq \eta^+$.

4.12. $G_{q_\Omega}ROTrHDFHWG$ operator.

Definition 19. Let $R_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_\Omega ROTrHFNs$ and the weight vector is $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ with $\xi_j \in [0, 1]$ and $\sum_{j=1}^n \xi_j = 1$. Then $G_{q_\Omega}ROTrHDFHWG(R_1, R_2, \dots, R_n) = \bigotimes_{j=1}^n R_j^{\xi_j}$ is said $G_{q_\Omega}ROTrHDFHWG$ operator.

Theorem 45. Let $a_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ be a family of $q_\Omega ROTrHFNs$ and the weight vector is $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)^T$ with $\Upsilon_j \in [0, 1]$ and $\sum_{j=1}^n \Upsilon_j = 1$. Then it is said $G_{q_\Omega}ROTrHDFHWG$ operator and $G_{q_\Omega}ROTrHDFHWG(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\begin{array}{c} \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu_{a_j}}{\mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \\ \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - S_{a_j}}{S_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \\ \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G_{a_j}}{G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \\ \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H_{a_j}}{H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \end{array} \right), \left(\begin{array}{c} \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu_{a_j}}{1 - \nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \\ \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F_{a_j}}{1 - F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \\ \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M_{a_j}}{1 - M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \\ \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N_{a_j}}{1 - N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \end{array} \right) \right].$$

Theorem 46. (Idempotency): If $R_j = \left\{ \begin{array}{c} [\eta, \kappa, \varsigma, \phi], \\ [\Lambda, \Upsilon, \vartheta, F] \end{array} \right\}$ for all $j = 1, 2, 3, \dots, n$, then $Gq_\Omega ROTrHDFHWG(R, R, R, \dots, R) = R$.

Theorem 47. (Commutativity). If $(R'_1, R'_2, \dots, R'_n)$ is any permutation of $(R_1, R_2, \dots, R_n)$, then $Gq_{\Omega}ROTrHDFHWG(R'_1, R'_2, \dots, R'_n) = Gq_{\Omega}ROTrHDFHWG(R_1, R_2, \dots, R_n)$.

Theorem 48. (Boundedness). If $\eta^- = \min(R_1, R_2, \dots, R_n)$, $\eta^+ = \max(R_1, R_2, \dots, R_n)$, then $\eta^- \leq Gq_{\Omega}ROTrHDFHWG(R_1, R_2, \dots, R_n) \leq \eta^+$.

5. PROPOSED TECHNIQUE OF QROTrHF DATA

In this section, we introduce six operators based on proposed technique of qROTrHF data.

5.1. $q_{\mathcal{L}}ROTrHDFWA$ operator. Step 1: Describe the $q_{\Omega}ROTrHF$ decision matrix

Step 2: Describe the $q_{\Omega}ROTrHDFWA$ operator and $\xi = (\xi_1, \xi_2, \dots, \xi_n)$.

$q_{\Omega}ROTrHDFWA(a_1, a_2, \dots, a_n) =$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\begin{pmatrix} \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{\mu_{a_j}}{1 - \mu_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{T_{a_j}}{1 - T_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{G_{a_j}}{1 - G_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{H_{a_j}}{1 - H_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{q}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - \nu_{a_j}}{\nu_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - F_{a_j}}{F_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - M_{a_j}}{M_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \xi_j \left(\frac{1 - N_{a_j}}{N_{a_j}} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \end{pmatrix} \right) \right]$$

Step 3: The score function is

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \frac{\{[\mu + S + G + H] - [\nu + F + M + N]\}}{8}$$

Step 4: Given the ranking.

5.2. **q_{Ω} ROTrHDFWG operator.** Step 1: Given the q_{Ω} ROTrHF decision matrix

Step 2: Deliberate the q_{Ω} ROTrHDFWG operator and $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)$.

$$q_{\Omega}ROTrHDFWG(a_1, a_2, \dots, a_n) = \bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\begin{array}{c} \left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu a_j}{\mu a_j} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - T a_j}{T a_j} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G a_j}{G a_j} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\rho}}, \\ \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H a_j}{H a_j} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\rho}}, \\ \left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu a_j}{1 - \nu a_j} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\rho}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F a_j}{1 - F a_j} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\rho}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M a_j}{1 - M a_j} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\rho}} \right)^{\frac{1}{q}}, \\ \left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N a_j}{1 - N a_j} \right)^{\rho} \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\rho}} \right)^{\frac{1}{q}} \end{array} \right]$$

Step 3: Calculate the score function

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \frac{\{[\mu + S + G + H] - [\nu + F + M + N]\}}{8}$$

Step 4: Given the ranking.

5.3. **Gq_{Ω} ROTrHDFWA operator.** Step 1: Given the q_{Ω} ROTrHF decision matrix

Step 2: Deliberate the Gq_{Ω} ROTrHDFWA operator.

$$q_{\Omega}ROTrHDFWA(a_1, a_2, \dots, a_n) =$$

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \left[\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu_{a_j}}{\mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \right. \\
\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - S_{a_j}}{S_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\
\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G_{a_j}}{G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\
\left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H_{a_j}}{H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\
\left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu_{a_j}}{1 - \nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \right. \\
\left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F_{a_j}}{1 - F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \right. \\
\left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M_{a_j}}{1 - M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \right. \\
\left. \left. \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N_{a_j}}{1 - N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \right) \right) \right]$$

Step 3: The score function is

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \frac{\{[\mu + S + G + H] - [\nu + F + M + N]\}}{8}$$

Step 4: Find the ranking.

5.4. **G_{q_Ω}ROTrHDFWG operator.** Step 1: Given the q_ΩROTrHF decision matrix

Step 2: Deliberate the G_{q_Ω}ROTrHDFWG operator and $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n)$.

G_{q_Ω}ROTrHDFWG($a_1, a_2, \dots, a_n$) =

$$\left[\begin{array}{c} \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - \mu_{a_j}}{\mu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - S_{a_j}}{S_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - G_{a_j}}{G_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\ \left(\left(\frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{1 - H_{a_j}}{H_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\ \bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi \\ \nu \in \Lambda, F \in \Upsilon, M \in \varnothing, N \in F}} \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{\nu_{a_j}}{1 - \nu_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{F_{a_j}}{1 - F_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{M_{a_j}}{1 - M_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}}, \\ \left(\left(\left(1 - \frac{1}{1 + \left\{ \sum_{j=1}^n \Upsilon_j \left(\frac{N_{a_j}}{1 - N_{a_j}} \right)^\rho \right\}^{\frac{1}{\rho}}} \right)^{\frac{1}{\Upsilon_j}} \right)^{\frac{1}{q}} \right) \end{array} \right]$$

Step 3: Calculate the score function

$$\bigcup_{\substack{\mu \in \eta, S \in \kappa, G \in \varsigma, H \in \phi, \\ \nu \in \Lambda, F \in \Upsilon, M \in \vartheta, N \in F}} \frac{\{[\mu + S + G + H] - [\nu + F + M + N]\}}{8}$$

Step 4: Find the ranking

6. NUMERICAL APPLICATION OF QROTrHF DATA

In this section, we introduce six operators based on proposed technique of qROTrHF data.

Material and clothing industry is imperative in financial and social terms for advancement and growth of diverse nations. The ability of planning dress, adornments, and foot wears according to display patterns may be a key instrument for driving businesses. The potential of the textile industries to contribute to long-run advancement depend not as it were on the criteria of the investors but too on the quality of their items. The texture drift changes from time to time and diverse at distinctive places concurring to culture of that specific put A recently graduated fashion architect is arranging to open her boutique in town. As the foremost necessarily portion is the fabric itself subsequently on the premise of fine texture she considers four material businesses that are doing well in advertise. She counsels a explanatory material technologist c to select the best option among all the businesses in restricted time and exertion. The choice producer compares five industries with regard to four criteria which are as take after: KH_1 = Toughness of fabric; LH_2 = Cost of fabric; FS = Dampness assimilation and warm conductivity; RW_4 = Appearance and fashion of the fabric with comparing weight vector $S = [0.30, 0.22, 0.25, 0.23]$ given by choice creator. The decision network is communicated within the taking after unthinkable frame table 1. To supply adequate adaptability inside the evaluation of the values of the four criteria of each elective industry, masters were allowed to utilize qROTrHFNs. The evaluation comes around of the four pros are exclusively recorded inside the taking after four cross sections.

6.1. q_Ω ROTrHDFWA operator. Step 1: Describe the q_Ω ROTrHF decision matrix table 1

	C_1	C_2	C_3	C_4
KH_1	[0.11, 0.13, 0.15, 0.16], [0.12, 0.14, 0.16, 0.18]	[0.12, 0.14, 0.16, 0.18], [0.13, 0.15, 0.17, 0.19]	[0.19, 0.21, 0.22, 0.24], [0.23, 0.24, 0.26, 0.28]	[0.11, 0.31, 0.51, 0.53], [0.21, 0.41, 0.61, 0.71]
LH_2	[0.11, 0.31, 0.51, 0.53], [0.21, 0.41, 0.61, 0.71]	[0.19, 0.21, 0.22, 0.24], [0.23, 0.24, 0.26, 0.28]	[0.11, 0.31, 0.51, 0.53], [0.21, 0.41, 0.61, 0.71]	[0.12, 0.14, 0.16, 0.18], [0.13, 0.15, 0.17, 0.19]
FS_3	[0.12, 0.14, 0.16, 0.18], [0.13, 0.15, 0.17, 0.19]	[0.19, 0.21, 0.22, 0.24], [0.23, 0.24, 0.26, 0.28]	[0.1, 0.3, 0.5, 0.7], [0.2, 0.4, 0.6, 0.8]	[0.19, 0.21, 0.22, 0.24], [0.23, 0.24, 0.26, 0.28]
RW_4	[0.11, 0.31, 0.51, 0.53], [0.21, 0.41, 0.61, 0.71]	[0.12, 0.14, 0.16, 0.18], [0.13, 0.15, 0.17, 0.19]	[0.11, 0.31, 0.51, 0.53], [0.21, 0.41, 0.61, 0.71]	[0.12, 0.14, 0.16, 0.18], [0.13, 0.15, 0.17, 0.19]

Step 2: Describe the q_Ω ROTrHDFWA operator and $\xi = (0.22, 0.25, 0.24, 0.29)$.

	q_Ω ROTrHDFWA operator table 2
KH_1	[0.1984, 0.3015, 0.0231, 0.2345], [0.0156, 0.4356, 0.6441, 0.4578]
LH_2	[0.3134, 0.1256, 0.2512, 0.2345], [0.2511, 0.4432, 0.3445, 0.5623]
FS_3	[0.3001, 0.4353, 0.1935, 0.1222], [0.3892, 0.7894, 0.0936, 0.2312]
RW_4	[0.3231, 0.3343, 0.5005, 0.2312], [0.3652, 0.8764, 0.9806, 0.2134]

Step 3: The score function is

$$\eta_1 = 0.1915, \eta_2 = 0.1155, \eta_3 = 0.2666, \eta_4 = 0.1101.$$

Step 4: Given the ranking $\eta_3 > \eta_1 > \eta_2 > \eta_4$ and η_3 is the best one.

Figure 1, score function of q_Ω ROTrHDFWA operator.

6.2. q_Ω ROTrHDFWG operator. Step 1: Given the q_Ω ROTrHF decision matrix table 3

	C_1	C_2	C_3	C_4
KH_1	[0.11, 0.31, 0.51, 71], [0.21, 0.41, 0.61, 0.81]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.09, 0.011, 0.012, 0.013], [0.01, 0.012, 0.013, 0.014]	[0.01, 0.03, 0.05, 0.07], [0.02, 0.04, 0.06, 0.08]
LH_2	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.12, 0.32, 0.52, 0.54], [0.22, 0.42, 0.62, 0.64]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.12, 0.32, 0.52, 0.54], [0.22, 0.42, 0.62, 0.64]
FS_3	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.93, 0.113, 0.123, 0.125], [0.114, 0.123, 0.126, 0.131]	[0.12, 0.32, 0.52, 0.54], [0.22, 0.42, 0.62, 0.64]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]
RW_4	[0.09, 0.011, 0.012, 0.014], [0.01, 0.012, 0.013, 0.015]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.9, 0.11, 0.12, 0.14], [0.11, 0.12, 0.13, 0.15]	[0.1, 0.3, 0.5, 0.7], [0.2, 0.4, 0.6, 0.8]

Step 2: Describe the q_Ω ROTrHDFWG operator and $\xi = (0.24, 0.49, 0.25)$.

	q_Ω ROTrHDFWG operator table 4
KH_1	[0.0693, 0.3929, 0.3478, 0.2321], [0.0709, 0.5125, 0.8241, 0.2312]
LH_2	[0.3004, 0.7258, 0.0113, 0.3412], [0.2511, 0.4188, 0.3445, 0.2141]
FS_3	[0.1358, 0.4353, 0.9035, 0.5612], [0.3032, 0.7004, 0.5936, 0.1328]
RW_4	[0.7631, 0.3923, 0.5345, 0.2145], [0.3952, 0.9764, 0.6806, 0.2176]

Step 3: The score function is

$$\eta_1 = 0.2267, \eta_2 = 0.1452, \eta_3 = 0.5919, \eta_4 = 0.0288.$$

Step 4: Given the ranking $\eta_3 > \eta_1 > \eta_2 > \eta_4$ and η_3 is the best one.

Figure 2, score function of q_Ω ROTrHDFWG operator.

6.3. G_{q_Ω} ROTrHDFWA operator. Step 1: Given the q_Ω ROTrHF decision matrix table 5

	C_1	C_2	C_3	C_4
KH_1	[0.11, 0.31, 0.51, 0.61], [0.21, 0.41, 0.61, 0.71]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.09, 0.011, 0.012, 0.014], [0.01, 0.012, 0.013, 0.015]	[0.01, 0.03, 0.05, 0.07], [0.02, 0.04, 0.06, 0.08]
LH_2	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.11, 0.31, 0.51, 0.61], [0.21, 0.41, 0.61, 0.71]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.11, 0.31, 0.51, 0.61], [0.21, 0.41, 0.61, 0.71]
FS_3	[0.11, 0.31, 0.51, 0.61], [0.21, 0.41, 0.61, 0.71]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.11, 0.31, 0.51, 0.61], [0.21, 0.41, 0.61, 0.71]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]
RW_4	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.11, 0.31, 0.51, 0.61], [0.21, 0.41, 0.61, 0.71]	[0.99, 0.119, 0.129, 0.131], [0.119, 0.129, 0.139, 0.141]	[0.13, 0.33, 0.53, 0.64], [0.23, 0.43, 0.63, 0.65]

Step 2: Describe the Gq_{Ω} ROTrHDFWA operator and $\xi = (0.24, 0.49, 0.23, 0.04)$.

	Gq_{Ω} ROTrHDFWA operator table 6
KH_1	[0.1693, 0.3029, 0.3078, 0.1245], [0.2709, 0.5125, 0.8241, 0.3423]
LH_2	[0.3104, 0.7258, 0.0113, 0.1267], [0.2511, 0.4188, 0.3145, 0.6311]
FS_3	[0.1458, 0.4353, 0.9035, 0.1254], [0.3332, 0.7004, 0.5936, 0.4512]
RW_4	[0.1631, 0.1923, 0.1345, 0.0971], [0.1952, 0.9764, 0.1806, 0.0021]

Step 3: The score function is

$$\eta_1 = 0.2534, \eta_2 = 0.1734, \eta_3 = 0.5678, \eta_4 = 0.1298.$$

Step 4: Given the ranking $\eta_3 > \eta_1 > \eta_2 > \eta_4$ and η_3 is the best one.

Figure 3, score function of $G_{q_{\Omega}}\text{ROTrHDFWA}$ operator.

6.4. **$G_{q_{\Omega}}\text{ROTrHDFWG}$ operator.** Step 1: Given the $q_{\Omega}\text{ROTrHF}$ decision matrix table 7

	C_1	C_2	C_3	C_4
KH_1	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.3331, 0.3333, 0.3335, 0.3337], [0.3332, 0.3334, 0.3336, 0.3338]	[0.3331, 0.3333, 0.3335, 0.3337], [0.3332, 0.3334, 0.3336, 0.3338]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]
LH_2	[0.3331, 0.3333, 0.3335, 0.3337], [0.3332, 0.3334, 0.3336, 0.3338]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.3331, 0.3333, 0.3335, 0.3337], [0.3332, 0.3334, 0.3336, 0.3338]	[0.3331, 0.3333, 0.3335, 0.3337], [0.3332, 0.3334, 0.3336, 0.3338]
FS_3	[0.1, 0.3, 0.5, 0.7], [0.2, 0.4, 0.6, 0.8]	[0.3331, 0.3333, 0.3335, 0.3337], [0.3332, 0.3334, 0.3336, 0.3338]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.09, 0.011, 0.012, 0.014], [0.01, 0.012, 0.013, 0.015]
RW_4	[0.3331, 0.3333, 0.3335, 0.3337], [0.3332, 0.3334, 0.3336, 0.3338]	[0.3331, 0.3333, 0.3335, 0.3337], [0.3332, 0.3334, 0.3336, 0.3338]	[0.102, 0.104, 0.106, 0.108], [0.103, 0.105, 0.107, 0.109]	[0.1001, 0.3001, 0.5001, 0.6002], [0.2001, 0.4001, 0.6001, 0.7812]

Step 2: Describe the q_Ω ROTrHDFWG operator and $\xi = (0.21, 0.23, 0.25, 0.31)$.

	Gq_Ω ROTrHDFWG operator table 8
KH_1	[0.0612, 0.3987, 0.3412, 0.4519], [0.0749, 0.5105, 0.5281, 0.0453]
LH_2	[0.3114, 0.1258, 0.0134, 0.3411], [0.2111, 0.4188, 0.3145, 0.0567]
FS_3	[0.4358, 0.1353, 0.0035, 0.2312], [0.3939, 0.7204, 0.0936, 0.5634]
RW_4	[0.1631, 0.1923, 0.1345, 0.3434], [0.3922, 0.1764, 0.3806, 0.6541]

Step 3: The score function is

$$\eta_1 = 0.2092, \eta_2 = 0.1882, \eta_3 = 0.6208, \eta_4 = 0.0459.$$

Step 4: Give the ranking $\eta_3 > \eta_1 > \eta_2 > \eta_4$ and η_3 is the best one.

Figure 4, score function of $G_{q_{\Omega}}\text{ROTrHDFWG}$ operator.

The comparison technique with proposed technique table 9.

Proposed technique	Score function of ranking
$q_{\Omega}\text{ROTrHDFWA}$	$\eta_3 > \eta_1 > \eta_2 > \eta_4$
$G_{q_{\Omega}}\text{ROTrHDFWA}$	$\eta_3 > \eta_1 > \eta_2 > \eta_4$
$q_{\Omega}\text{ROTrHDFWG}$	$\eta_3 > \eta_1 > \eta_2 > \eta_4$
$G_{q_{\Omega}}\text{ROTrHDFWG}$	$\eta_3 > \eta_1 > \eta_2 > \eta_4$

7. COMPARSION TECHNIQUE WITH EXISTING METHOD

To approve and set up viability of the recommended strategy, its comparison with an other strategy subordinate upon IFSs [22] and TIFWG data [15] is displayed with cases. Other strategies are extraordinary cases of our technique strategy that's founded on $q_{\Omega}\text{ROTrHFN}$ to the same illustrative case.

7.1. IFN with existing method. Step 1: The IF decision matrix is table 10 in given below

	CHI_1	CHI_2	CHI_3	CHI_4
KH_1	0.1, 0.3	0.12, 0.14	0.9, 0.11	0.1, 0.3
LH_2	0.12, 0.13	0.4, 0.6	0.31, 0.33	[0.112, 0.313]
FS_3	0.34, 0.36	0.12, 0.13	0.4, 0.6	0.554, 0.556
RW_4	0.3004, 0.3006	0.34, 0.36	0.12, 0.13	0.4, 0.6

Step 2: Define the IFWA operator and (0.1, 0.2, 0.3).

Now applying given formula

	IFWA operator table 11
KH_1	[0.1024, 0.3145]
LH_2	[0.0134, 0.2156]
FS_3	[0.0991, 0.3353]
RW_4	[0.0201, 0.3043]

Step 3: The score function is

$$\eta_1 = 0.3101, \eta_2 = 0.0198, \eta_3 = 0.0871, \eta_4 = 0.0101.$$

Step 4: Given the ranking $\eta_1 > \eta_3 > \eta_2 > \eta_4$ and the η_1 is the best.

Figure 5, score function of IFWA operator.

7.2. TIF number with existing method. Step 1: The TIF decision matrix is given in table 12

	CHI_1	CHI_2	CHI_3	CHI_4
KH_1	[0.1, 0.3, 0.5], [0.2, 0.4]	[0.12, 0.14, 0.16], [0.13, 0.15]	[0.9, 0.11, 0.12], [0.11, 0.12]	[0.1, 0.3, 0.5], [0.2, 0.4]
LH_2	[0.9, 0.11, 0.12], [0.11, 0.12]	[0.1, 0.3, 0.5], [0.2, 0.4]	[0.31, 0.33, 0.35], [0.32, 0.34]	[0.112, 0.313, 0.515], [0.211, 0.411]
FS_3	[0.31, 0.33, 0.35], [0.32, 0.34]	[0.9, 0.11, 0.12], [0.11, 0.12]	[0.1, 0.3, 0.5], [0.2, 0.4]	[0.551, 0.553, 0.555], [0.552, 0.554]
RW_4	[0.3001, 0.3003, 0.3005], [0.3002, 0.3004]	[0.31, 0.33, 0.35], [0.32, 0.34]	[0.9, 0.11, 0.12], [0.11, 0.12]	[0.1, 0.3, 0.5], [0.2, 0.4]

Step 2: The TIFW operator and (0.2, 0.3, 0.5)

Now applying given formula

	TFWA operator table 13
KH_1	[0.1124, 0.3145, 0.5231], [0.2156, 0.4456]
LH_2	[0.9134, 0.1156, 0.1512], [0.1111, 0.1132]
FS_3	[0.3991, 0.9353, 0.8935], [0.3892, 0.7894]
RW_4	[0.3231, 0.3343, 0.5005], [0.3652, 0.8764]

Step 3: The score function is

$$\eta_1 = 0.0014, \eta_2 = 0.7705, \eta_3 = 0.5806, \eta_4 = 0.6101.$$

Step 4: Find the ranking $\eta_2 > \eta_4 > \eta_3 > \eta_1$ and η_2 is the best

Figure 6, score function of TIFWG operator.

The ranking values of the above discussion are given in table 14.

Method	Ranking
$q_{\Omega}ROTrDHFWA$	$\eta_2 > \eta_4 > \eta_3 > \eta_1$
$q_{\Omega}ROTrDHFOWA$	$\eta_2 > \eta_4 > \eta_3 > \eta_1$
$q_{\Omega}ROTrDHFHWA$	$\eta_2 > \eta_4 > \eta_3 > \eta_1$
$q_{\Omega}ROTrDHFHWG$	$\eta_2 > \eta_4 > \eta_3 > \eta_1$
$q_{\Omega}ROTrDHFOWG$	$\eta_2 > \eta_4 > \eta_3 > \eta_1$
$q_{\Omega}ROTrDHFHFWG$	$\eta_2 > \eta_4 > \eta_3 > \eta_1$

Generally, a subjective judgment among divergent MCDM approaches can be affirmed by comparing. For the existing approaches and the proposed technique, the outline and suppleness within the accumulation of $q_{\Omega}ROTFNs$, the competence to contract with the interrelationships among divergent criteria, and the competence to diminish the undesirable impact of the unreasonably small standard values on the accumulation results are assigned as the differentiate physiognomies. The results of the differentiate are appeared in Table 15. The particulars of the comparison are clarified.

When all criteria are self-governing of each extra: It is no faltering that all of the enrolled approaches holder contract with this circumstance.

Table 15

Methods	Generality	Independent	Multiply	Heterogeneous	Capability
WA [26]	Restricted	True	False	False	False
WBM [27]	Restricted	True	False	False	False
WABM [28]	Restricted	True	False	False	False
WPBM [29]	Restricted	True	False	False	False
WHM [30]	Restricted	True	False	False	False
WPHM [31]	Restricted	True	False	False	False
WMSM [32]	Restricted	True	False	False	False
WPMSM [33]	Restricted	True	False	False	False
WPPMSM [34]	Restricted	True	False	False	False
WMM [35]	Restricted	True	False	False	False

Figure 7, different criteria

When criteria is any interrelationships. The WE, WP, WA, and WG techniques are only appropriate for the autonomous situation. All extra approaches consume the competence of commerce with the circumstance in which there are interrelationships any criteria.

Table 16

Methods	0	1	2	3	4	5	6	7	8
Greco et al. [1]	-0.1245	0.0253	0.2901	0.3008	0.0621	0.3498	0.8456	0.7634	0.8256
Liu [2]	0.1259	0.0909	0.4056	0.3087	0.4085	0.0095	0.0405	0.2906	0.4856
Wan et al. [3]	0.1501	0.1537	0.1881	-0.8601	-0.1916	0.0483	0.0823	0.0348	-0.5036
Jamkhaneh [4]	-0.5457	-0.5243	0.3041	0.3007	0.9721	0.6098	0.0651	0.0334	0.4556

Figure 8, comparsion technique

Table 17

Methods	0	1	2	3	4	5	6	7	8
Liu et al. [5]	-0.6248	0.8203	0.6031	0.1625	0.0387	0.3081	0.3091	0.3045	0.0098
Dong et al. [6]	-0.4205	-0.6248	-0.0299	0.3088	0.1995	-0.1919	0.9611	0.5635	0.3459
Wan et al. [7]	0.0218	0.0267	0.0369	0.3088	0.0801	0.1002	0.0026	0.1029	0.1002

Figure 9, comparsion technique

Table 18

Methods	0	1	2	3	4	5	6	7	8
Wang et al. [8]	0.2939	0.0278	0.2481	0.0564	0.9085	0.0095	0.0069	0.0044	0.6091
Yang et al. [9]	0.3069	0.1045	0.4982	0.5038	0.2007	0.4035	0.5003	0.4065	0.3884
Garg et al. [10]	0.0259	0.0373	0.5971	0.3078	0.0523	0.0457	0.2059	0.0548	0.7264

Figure 10,comparsion technique

Table 19

Methods	0	1	2	3	4	5	6	7	8
WA [26]	0.1201	0.0203	0.0001	0.3204	0.1121	0.1098	0.1451	0.1634	0.1256
WBM [27]	0.1505	-0.0903	-0.4001	0.3007	-0.1924	0.1995	-0.3451	0.2936	-0.4056
WABM [28]	0.3201	0.0833	0.0781	0.9621	0.3915	0.3498	0.7851	0.5334	0.5556
WPBM [29]	0.1451	0.0243	0.0041	0.3304	0.1421	0.1598	0.1651	0.1934	0.4556

Figure 11, comparsion technique.

Table 20

Methods	0	1	2	3	4	5	6	7	8
WHM [30]	0.1201	0.0203	0.0031	0.5623	0.0317	0.3921	0.3871	0.3965	0.1938
WPHM [31]	0.3205	0.0245	0.0281	-0.3282	0.1195	-0.1117	-0.3011	-0.0039	-0.1059
WMSM [32]	0.0217	0.0243	0.0331	0.3278	0.1901	0.1072	0.1926	0.1925	0.1712

Figure 12, comparsion technique.

Table 21

Methods	0	1	2	3	4	5	6	7	8
WPMSM [33]	-0.2234	0.0203	0.2081	0.0004	0.9003	0.0235	0.0456	0.0782	0.0921
WPPMSM [34]	-0.3239	0.1905	-0.4002	-0.5634	0.2377	0.1034	0.5453	0.4004	0.3891
WMM [35]	0.0201	0.0342	0.0221	-0.3004	0.0121	0.1038	0.2354	0.1943	-0.0256

Figure 13, comparsion technique.

8. CONCLUSION

In the work, we have given an intensive study on q_Ω ROTrHF data and their application in decision making. We define q_Ω ROTrHF average and geometric operators based on MCDM. The idempotency and boundedness of the q_Ω ROTrHFNs. The different strategy for understanding the MCDM issues based on q_Ω ROTrHFNs are proposed. We stated several dombi weighted aggregation operators namely, weighted and geometric operators. Several desirable relations are derived to study their properties. Further, we extend them to geometric operators also and study their features. The major advantage of using such power aggregation operators is that it takes into account the relationship of the information being aggregated as compared to several other existing operators. The comes about of the comparisons propose that the proposed strategy is common and adaptable at both accumulation of measure values and capture of measure interrelationships, and concurrently is the capability to handle the heterogeneous interrelationships of criteria and diminish the negative impact of the one-sided measure values. The MADM algorithm based on the stated operators is explained, which is more generalized and flexible with the parameter q to the decision-maker. The applicability of the algorithm is demonstrated through a numerical example, our approach is more suitable and widely applicable to solve the MADM problem under the diverse fuzzy environment.

In the future, we will expand our study and able to tackle the real-problems under the different fuzzy environment. In group decision-making problems, because the experts usually come from different speciality fields and have different backgrounds and levels of knowledge, they usually have diverging opinions. Thus, in future work,

we will present a consensus model for group decision-making with trapezoidal soft cubic neutrosophic hesitant fuzzy information.

9. COMPLIANCE WITH ETHICAL STANDARDS

The authors declare that there is no conflict of interests regarding the publication of this paper. Compliance with Ethical Standards: This study is not supported by any source or any organizations. Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

Supplementary Information & ESM:

No electronic supplementary material (animations, movies, audio, large original data, etc.) are re-quired for this study.

Competing Interest:

It is declared that the authors have no competing interests.

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Consent to participate:

All authors participated in this study with their consent. (OR download consent form from net and send it to the journal after getting signed from all authors, if asked by the journal).

Data availability statement:

All the data used in this study is artificial and can be used in any research project by just citing this article.

REFERENCES

- [1] Greco S, Figueira J, Ehrgott M. Multiple Criteria Decision Analysis: State of the Art Surveys. New York: Springer-Verlag New York; 2016.
- [2] Liu P. Some Hamacher aggregation operators based on the interval-valued intuitionistic fuzzy numbers and their application to group decision making. IEEE Transactions on Fuzzy systems 2013; 22(1): 83-97.
- [3] Wan S, Zhu Y. Triangular intuitionistic fuzzy triple Bonferroni harmonic mean operators and application to multi-attribute group decision making. Iranian Journal of Fuzzy Systems 2016; 13(5): 117-145.
- [4] Jamkhaneh EB, Garg H. Some new operations over the generalized intuitionistic fuzzy sets and their application to decision-making process. Granular Computing 2018; 3(2): 111-122.
- [5] Liu P, Tang G. Some intuitionistic fuzzy prioritized interactive Einstein Choquet operators and their application in decision making. IEEE Access 2018; 6: 72357-72371.
- [6] Dong J, Chen Y, Wan S. A cosine similarity based QUALIFLEX approach with hesitant fuzzy linguistic term sets for financial performance evaluation. Applied Soft Computing 2018; 69: 316-329.
- [7] Wan S, Jin Z, Dong J. Pythagorean fuzzy mathematical programming method for multi-attribute group decision making with Pythagorean fuzzy truth degrees. Knowledge and Information Systems 2018; 55(2): 437-466.

- [8] Wang J, Gao H, Wei G, Wei Y. Methods for multiple-attribute group decision making with q-rung interval-valued orthopair fuzzy information and their applications to the selection of green suppliers. *Symmetry* 2019; 11(1): 56.
- [9] Yang W, Wang C, Liu Y, Sun Y. Hesitant Pythagorean fuzzy interaction aggregation operators and their application in multiple attribute decision-making. *Complex & Intelligent Systems* 2019; 5(2): 199-216.
- [10] Garg H, Kaur G. TOPSIS based on nonlinear-programming methodology for solving decision-making problems under cubic intuitionistic fuzzy set environment. *Computational and Applied Mathematics* 2019; 38(3): 114.
- [11] Garg H, Kumar K. Linguistic interval-valued Atanassov intuitionistic fuzzy sets and their applications to group decision-making problems. *IEEE Transactions on Fuzzy Systems* 2019; <https://doi.org/10.1109/TFUZZ.2019.2897961>
- [12] Wan S, Jin Z, Dong J. A new order relation for Pythagorean fuzzy numbers and application to multi-attribute group decision making. *Knowledge and Information Systems* 2019; <https://doi.org/10.1007/s10115-019-01369-8>.
- [13] Bustince H, Barrenechea E, Pagola M, Fernandez J, Xu Z, Bedregal B, Montero J, Hagrais H, Herrera F, De Baets B. A Historical Account of Types of Fuzzy Sets and Their Relationships. *IEEE Transactions on Fuzzy Systems* 2016; 24(1): 179-194.
- [14] Yager RR. Generalized Orthopair Fuzzy Sets. *IEEE Transactions on Fuzzy Systems* 2017; 25(5): 1222-1230.
- [15] Zadeh LA. Fuzzy Sets. *Information and Control* 1965; 8(3): 338-353.
- [16] Atanassov KT. Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems* 1986; 20(1): 87-96.
- [17] Yager RR. Pythagorean Membership Grades in Multicriteria Decision Making. *IEEE Transactions on Fuzzy Systems* 2014; 22(4): 958-965.
- [18] Du WS. Correlation and correlation coefficient of generalized orthopair fuzzy sets. *International Journal of Intelligent Systems* 2019; 34(4): 564-583.
- [19] Du WS. Minkowski-type distance measures for generalized orthopair fuzzy sets. *International Journal of Intelligent Systems* 2018; 33(4): 802-817.
- [20] Wang P, Wang J, Wei G, Wei C. Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications. *Mathematics* 2019; 7(4): 340.
- [21] Wang R, Li Y. A Novel Approach for Green Supplier Selection under a q-Rung Orthopair Fuzzy Environment. *Symmetry*, 2018; 10(12): 687.
- [22] Peng X, Dai J. Research on the assessment of classroom teaching quality with q-rung orthopair fuzzy information based on multiparametric similarity measure and combinative distance-based assessment. *International Journal of Intelligent Systems* 2019; <https://doi.org/10.1002/int.22109>.
- [23] Du WS. Research on arithmetic operations over generalized orthopair fuzzy sets. *International Journal of Intelligent Systems* 2019; 34(5): 709-732.
- [24] Gao J, Liang Z, Shang J, Xu Z. Continuities, Derivatives and Differentials of q-Rung Orthopair Fuzzy Functions, *IEEE Transactions on Fuzzy Systems* 2018; <https://doi.org/10.1109/TFUZZ.2018.2887187>
- [25] Ye J, Ai Z, Xu Z. Single variable differential calculus under q-rung orthopair fuzzy environment: Limit, derivative, chain rules, and its application. *International Journal of Intelligent Systems* 2019; <https://doi.org/10.1002/int.22100>.
- [26] Liu P, Wang P. Some q-rung orthopair fuzzy aggregation operators and their applications to multiple attribute decision making. *International Journal of Intelligent Systems* 2018; 33(2): 259-280.
- [27] Liu P, Liu J. Some q-rung orthopair fuzzy Bonferroni mean operators and their application to multi-attribute group decision making. *International Journal of Intelligent Systems* 2018; 33(2): 315-347.
- [28] Liu P, Wang P. Multiple-Attribute Decision Making based on Archimedean Bonferroni Operators of qRung Orthopair Fuzzy Numbers. *IEEE Transactions on Fuzzy Systems* 2019; 27(5): 834-848.
- [29] Yang W, Pang Y. New q-rung orthopair fuzzy partitioned Bonferroni mean operators and their application in multiple attribute decision making. *International Journal of Intelligent Systems* 2019; 34(3): 439-476.

- [30] Wei G, Gao H, Wei Y. Some q-rung orthopair fuzzy Heronian mean operators in multiple attribute decision making. *International Journal of Intelligent Systems* 2018; 33(7): 1426-1458.
- [31] Liu Z, Wang S, Liu P. Multiple attribute group decision making based on q-rung orthopair fuzzy Heronian mean operators. *International Journal of Intelligent Systems* 2018; 33(12): 2341-2363.
- [32] Wei G, Wei C, Wang J, Gao H, Wei Y. Some q-rung orthopair fuzzy Maclaurin symmetric mean operators and their applications to potential evaluation of emerging technology commercialization. *International Journal of Intelligent Systems* 2019; 34(1): 50-81.
- [33] Liu P, Chen SM, Wang P. Multiple-Attribute Group Decision-Making Based on q-Rung Orthopair Fuzzy Power Maclaurin Symmetric Mean Operators. *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 2018; <https://doi.org/10.1109/TSMC.2018.2852948>
- [34] Bai K, Zhu X, Wang J, Zhang R. Some partitioned Maclaurin symmetric mean based on q-rung orthopair fuzzy information for dealing with multi-attribute group decision making. *Symmetry* 2018; 10(9): 383.
- [35] Wang J, Zhang R, Zhu X, Zhou Z, Shang X, Li W. Some q-rung orthopair fuzzy Muirhead means with their application to multi- attribute group decision making. *Journal of Intelligent & Fuzzy Systems* 2019;36(2): 1599-1614.
- [36] Liu Z, Liu P, Liang X. Multiple attribute decision-making method for dealing with heterogeneous relationship among attributes and unknown attribute weight information under q-rung orthopair fuzzy environment. *International Journal of Intelligent Systems* 2018; 33(9): 1900-1928.
- [37] Peng X, Dai J, Garg H. Exponential operation and aggregation operator for q-rung orthopair fuzzy set and their decision-making method with a new score function. *International Journal of Intelligent Systems* 2018; 33(11): 2255-2282.
- [38] Xing Y, Zhang R, Zhou Z, Wang J. Some q-rung orthopair fuzzy point weighted aggregation operators for multi-attribute decision making. *Soft Computing* 2019; <https://doi.org/10.1007/s00500-018-03712-7>.
- [39] Muirhead RF. Some methods applicable to identities and inequalities of symmetric algebraic functions of n letters. *Proceedings of the Edinburgh Mathematical Society* 1902; 21: 144-162.
- [40] Garg H, Arora R. Bonferroni mean aggregation operators under intuitionistic fuzzy soft set environment and their applications to decision-making. *Journal of the Operational Research Society* 2018; 69(11):1711-1724.
- [41] Kaur G, Garg H. Multi-attribute decision-making based on Bonferroni mean operators under cubic intuitionistic fuzzy set environment. *Entropy* 2018; 20(1): 65.
- [42] Yang W, Pang Y. New Pythagorean fuzzy interaction Maclaurin symmetric mean operators and their application in multiple attribute decision making. *IEEE Access* 2018; 6: 39241-39260.
- [43] Wei G, Garg H, Gao H, Wei C. Interval-valued Pythagorean fuzzy Maclaurin symmetric mean operators in multiple attribute decision making. *IEEE Access* 2018; 6: 67866-67884.
- [44] Liu P, Li D. Some Muirhead mean operators for intuitionistic fuzzy numbers and their applications to group decision making. *PLOS ONE*. 2017; 12(1): e0168767. <https://doi.org/10.1371/journal.pone.0168767> PMID: 28103244.
- [45] Hashemi, H.; Mousavi, S.M.; Zavadskas, E.K.; Chalekaee, A.; Turskis, Z. A New Group Decision Model Based on Grey-Intuitionistic Fuzzy-ELECTRE and VIKOR for Contractor Assessment Problem. *Sustainability* 2018, 10, 1635.
- [46] Ziemba, P. NEAT F-PROMETHEE—A new fuzzy multiple criteria decision making method based on the adjustment of mapping trapezoidal fuzzy numbers. *Expert Syst. Appl.* 2018, 110, 363-380.
- [47] Hwang, C.L.; Yoon, K. Multiple Attribute Decision Making. *Economics and Mathematical Systems*; Springer:Berlin, Germany, 1981.
- [48] Chen, C.T.; Lin, C.T.; Huang, S.F. A fuzzy approach for supplier evaluation and selection in supply chain management. *Int. J. Prod. Econ.* 2006, 102, 289-301.
- [49] Wei, G.W. Grey relational analysis method for intuitionistic fuzzy multiple attribute decision making with preference information on alternatives. *Int. J. Comput. Intell. Syst.* 2011, 4, 164-173.
- [50] Wei, G.W. Grey relational analysis model for dynamic hybrid multiple attribute decision making. *Knowl. Syst.* 2011, 24, 672-679.

- [51] Wei, G.W. Gray relational analysis method for intuitionistic fuzzy multiple attribute decision making. Expert Syst. Appl. 2011, 38, 11671-11677.
- [52] Torra. V. Hesitant fuzzy sets, International Journal of Intelligent Systems, 2010, 25(6), 529-539.
- [53] Dombi, J. A general class of fuzzy operators, the DeMorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators. Fuzzy sets and systems, 1982, 8(2), 149-163.
- [54] Liu, P., Liu, J and Chen, S. M. Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making. Journal of the Operational Research Society, 2017, 1-26.

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Figures

Figure 1

score function of q ROTrHDFWA operator

Figure 2

score function of q ROTrHDFWG operator

Figure 3

score function of Gq ROTrHDFWA operator

Figure 4

score function of GqROTrHDFWG operator

Figure 5

score function of IFWA operator

Figure 6
score function of TIFWG operator.

Figure 7

different criteria

Figure 8

comparsion technique

Figure 9

comparsion technique

Figure 10

comparsion technique

Figure 11

comparsion technique.

Figure 12

comparsion technique.

Figure 13

comparsion technique.