

Developmental trajectories of number line estimations in math anxiety: evidence from bounded and unbounded number line estimation

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Abstract

In the number line estimation task, participants are instructed to place a number, spatially, on a number line. In the present study, 2nd, 3rd and 5th grade children (n = 94) participated in bounded and unbounded number line estimation tasks, half with low math anxiety (LMA) and half with high MA (HMA).

The spatial theory views MA as resulting from weakness in spatial abilities, subsequent to deficits in basic numerical abilities. Accordingly, due to number space associations, weakness in estimations are expected in HMA individuals. Accordingly, young children with HMA show non-mature numerical estimations compared to participants with LMA. Specifically, HMA participants showed higher logarithmic tendency than LMA peers, and showed indications for usage of 2 reference points rather than 3 reference points in number line estimations (bounded and unbounded). However, for older HMA children, estimations were normalized and group differences were eliminated. Finally, we found that estimations (linear fits and errors) in the bounded but not the unbounded tasks, predicted usage of advance memory-based strategies in simple addition operations. These results indicated that bounded and unbounded number line estimations are dissociable in 1) developmental trajectories, 2) in relation to MA and 3) in relation to math performances.

Introduction

It is widely believed that quantity manipulation and understanding is an innate human ability, which is preverbal and based upon mapping between space and quantities¹⁻⁵. Infants are born with preverbal approximate number sense (ANS). Later, with maturation and schooling, an exact symbolic quantity system develops, supported by the preverbal ANS as well as verbal abilities^{6,7}.

The preverbal representation of quantity (ANS) is believed to be needed mostly across the life span for estimation tasks. During numerical estimation, a quantity (symbolic or non-symbolic)⁸ is translated into an abstract code of that quantity in the form of the mental number line^{3,4,9}. It has been suggested that the mental number it is initially logarithmic, and becomes linear with maturation and schooling^{10,11}. Logarithmic representation of the number line includes over-estimation of small numbers and under-estimation of larger numbers towards the end of the scale. By contrast, linear representation includes equal distances between quantities, regardless of their size.

Number line estimation tasks

One popular estimation task, aimed at examining the number space association, is the number line estimation task¹¹. In the bounded number line estimation task, a number is presented above a number line, with 0 at one end and 100 or 1,000 at its other end. The participant is instructed to place a number spatially on the line. Then their estimation is plotted against the real spatial location of the number. The relations between estimation and real locations in adults can be described by a linear function (estimation location = real location). However, these relations can be better described by a logarithmic

curve in younger ages. It has been suggested that corresponding to the development of the mental number line (see the previous section), with maturation, children's estimation shifts from logarithmic to linear^{10,11}. The shift between logarithmic and linear representations is related to familiarity with the range (i.e., a child can present a linear representation for a small and familiar range, and a logarithmic representation for a larger range) and age^{10,11}. However, there are difficulties with the conclusions about mental representation drawn from the performances on the number line estimation task. For example, variations in number-line estimation are assumed to be based upon numerical processing and numerical representations, provided that the spatial components of the task do not introduce their own variations¹².

Do bounded number line tasks reflect pure non-verbal numerical estimations?

In recent years, researchers have debated whether the bounded classical number line task reflects pure nonverbal spatial estimation or specific task related strategies¹³⁻¹⁹.²⁰ were the first to suggest that that performances in the bounded number line estimation task are best explained by the usage of proportion-judgment strategies in addition to numerical properties.²⁰ examined estimations in different age groups and found that , one-cycle and two-cycle functions explain the data better (in relation to percentage of explained variance), compared to logarithmic and linear functions²⁰. A one-cycle function indicates that a child used two reference points (beginning and end of the line), while a two-cycle function indicates that a child used three reference points (beginning, middle and end of the line) to estimate locations. Since then, several research groups have used a psychophysical model of proportion judgment with the number line estimations data¹². Specifically, a popular estimation model predicts that estimate of proportions will take the form of S- shaped or reverse S-shaped curves. Developmental changes in numerical estimation according to proportion-judgment account can be described by two changes: 1) the value of β parameter (as it becomes higher it improves). 2) learning can improve the usage of reference points, resulting in the shift between one or two cyclical power model.

Cohen and Blanc-Goldhammer²¹ presented a new unbounded version of the number line estimation task, and argued that this version provides a purer measurement of numerical estimation than the original bounded version. In the unbounded number line estimation task, only the beginning of the number line is presented with a scaling unit, while no end-points are presented. This version of the number line estimation task has very distinct response estimation patterns compared to the classical bounded number line task. For example, there are differences in the developmental trajectories of bounded and unbounded number line estimations. Specifically, for children in the 1st to 4th grades²² and in the 5th to 7th grades¹³, it was found that the bounded number line estimation task changes developmentally, more than the unbounded task. Accordingly, the present study will examine similarities and dissimilarities between bounded and unbounded number line estimation tasks.

Number line estimation and math achievements

There are multiple indications that estimation in the number line estimation task is related to math performance (for review, see ²³. First, linear tendency in the bounded number line task was related to individual differences in numerical abilities. Specifically, participants with better numerical abilities will show preference for linear representation over logarithmic representation in the bounded number line task ²⁴. Only a few studies tested the differential relations between the bounded or unbounded task and arithmetical performance. For example, it was found that estimation in bounded but not unbounded tasks was strongly associated with basic arithmetic ¹³.

1.4 Math anxiety (MA) and number line estimation

MA is a feeling of tension and anxiety that interferes with the manipulation of numbers and the solution of mathematical problems in a wide variety of daily life and academic situations ²⁵. There is growing evidence that high MA individuals experience difficulties at different levels and domains of mathematics. Ashcraft and colleagues showed that while HMA individuals do not show difficulties in basic addition and multiplication of whole-number arithmetic problems, there is evidence that they have trouble with more complicated addition problems, such as those that require a carry-over operation (e.g., $37+96$, or $5+8$). These results led them to assume that high MA individuals have difficulties especially in arithmetical operations that require working memory (WM). Hence, HMA individuals have limited WM resources available while solving math problems, due to anxiety related ruminations, thus resulting in poor math performance ^{26,27}. Ashcraft and colleagues' findings supported the dominant theory regarding MA, the WM theory, arguing that MA disrupts WM availability, and thus has a negative effect only on high level, WM demanding mathematics. Hence, deficits in the number line estimation task, that requires minimal WM demand, are not expected in high MA individuals.

Newer theories regarding MA ²⁸, have shown that MA is accompanied by difficulties in lower numerical abilities, with minimal WM demands. Specifically, high MA individuals had difficulties in enumeration and number comparisons ²⁹. Results indicated that high MA individuals showed a steeper numerical distance effect than LMA individuals, resulting presumably from less accurate representations of the mental number line. Combining results of the two experiments, Maloney and colleagues concluded that MA not only affects complex and high-level mathematics, but rather is also accompanied by difficulties in basic numerical processing.

The preverbal representation of numbers is spatial in nature ⁹, and basic numerical abilities are linked to number-space associations ^{30,31}. Spatial skills predict the formation and improvement of linear number line representation, which is known to have an important role in the development of math abilities and is a predictor of performance in an approximate symbolic calculation ³². Hence, in a recent study with the number line estimation task, estimation abilities in the bounded task mediated the relation between spatial skills and math performance (words, problems and calculations ³³. Hence, the weakness in basic numerical abilities (based upon number space associations) found in high MA individuals ^{28,29,34} can be based upon deficits in spatial skills.

The Bounded Number line estimation task was tested in high MA adults³⁵. Estimations were examined both in familiar ranges (e.g., 0–100, or 0–1,000) as well as unfamiliar ranges (e.g., 0–100,000, and 267–367). Results indicated that high MA participants showed deficits in the 267–367 ranges, but not in the other ranges. In line with the WM theory of MA, the authors concluded that high MA individuals have difficulties in complex but not simple arithmetic^{26,27}.

The present study

The present study will test the effect of development and MA on bounded and unbounded number line estimations. We will also look at the relations between estimations and math achievement by predicting advance strategy use during mental addition.

Advance strategy use normally develops during elementary school. During the initial stages of elementary school, children usually count addends using backup calculating strategies, when solving addition problems. Repetitive use of backup, non-advance counting strategies, results in formation of memories for basic fact. Once formed, these memory representations result in the use of advance memory based strategies such as direct retrieval and decomposition (based upon calculating the answer by partial sum retrieval, and then counting the rest of the answer)^{36,37}.

We will examine three age groups, second, third and fifth grade children, half with low and half with high MA. The range of 1-100 will be used in estimations. In that familiar range, all our age groups should show linear tendency in the bounded task. However, we believe that linear fit, in the bounded task will increase with age. Moreover, developmental changes should be greater in the bounded task than the unbounded task. Similarly, we believe that the relation between estimations and advance strategy use will be stronger in the bounded than the unbounded number line tasks. Last, HMA participants will show less precise estimations in number line estimation task than LMA.

Results

Table 1 represents the social demographic background information of the two MA groups as a function of age group. As can be seen, all the MA groups in all the grades were similar in age, handedness, number of participants, percentage of females, visuospatial memory span, and presented advance strategy use. However, as expected the high MA group had higher MA than the low MA group in all the grades.

PAE mean

A three-way analysis of variance (ANOVA) was performed on percentage of absolute error, with task (bounded or unbounded) as the within subjects factor and MA group (Low or High) and grade as the between subjects factor. The main effect of grade reached significance, $F(2, 88) = 17.79$, partial $\eta^2 = .29$, $p < .001$, observed power = 1. As expected, errors were larger in the second grade ($M = 7.68$, $SD = 3.77$), compared to the third grade ($M = 5.42$, $SD = 1.45$), $t(60) = 3.49$, $p < .01$. Errors were also greater in the third

grade compared to the fifth grade ($M = 4.48$, $SD = 1.56$), $t(60) = 3.48$, $p < .01$. The main effect of MA or interactions with MA were not significant.

PAE STD

A three-way analysis of variance (ANOVA) was performed on the percentage of absolute error standard deviation, with task (bounded or unbounded) as the within subjects factor and MA group (Low or High) and grade as the between subjects factor. The main effect of grade reached significance, $F(2, 88) = 16.76$, partial $\eta^2 = .28$, $p < .001$, observed power = 1. As expected, standard deviations of the errors were larger in the second grade ($M = 5.93$, $SD = 2.42$), compared to the third grade ($M = 4.31$, $SD = 0.98$), $t(60) = 3.45$, $p < .01$. STD's were also greater in the third grade compared to the fifth grade ($M = 3.66$, $SD = 0.84$), $t(61) = 2.84$, $p < .01$. The effect of task also reached significance ($F(2, 88) = 10.95$, partial $\eta^2 = .11$, $p < .01$, observed power = .90). Standard deviations of unbounded number line tasks were larger ($M = 4.95$, $SD = 2.15$) than bounded number lines ($M = 4.30$, $SD = 1.98$) (see Figure 1). The main effect of MA or interactions with MA were not significant.

Linearity and logarithmic representation, model fits.

According to classical views, the mental number line possesses a few documented characteristics, including its arrangement: it is initially logarithmic, and becomes linear with maturation and schooling^{10,11}. Hence, in the current analysis we compared linear and logarithmic fits, where logarithmic fits represent non-mature estimations. A four-way analysis of variance (ANOVA) was performed on the R^2 of goodness of fit with task (bounded or unbounded) and representation (linear/ logarithmic) as the within subjects factor and MA group (Low or High) and grade as the between subjects factor. The main effect of task reached significance, $F(2, 86) = 4.39$, partial $\eta^2 = .05$, $p < .05$, observed power = .54. The function fits were larger for the unbounded task (.94, $SD = .08$) compared to the bounded task (.92, $SD = .12$). The main effect of representation was significant as well $F(1, 86) = 37.78$, partial $\eta^2 = .3$, $p < .01$, observed power = .54. The goodness of fits for the logarithmic function (.96, $SD = .05$) was higher than for the linear function (.91, $SD = .13$). The interaction between representation and task was significant, $F(1, 86) = 4.4$, partial $\eta^2 = .05$, $p < .05$, observed power = .54. The interaction between representation and grade was significant, $F(2, 86) = 3.94$, partial $\eta^2 = .09$, $p < .01$, observed power = .70. This interaction was modulated by MA group $F(2, 86) = 7.7$, partial $\eta^2 = .15$, $p < .01$, observed power = .94. In the second grade LMA participants had better linear fits (.90, $SD = .12$) than HMA participants (.78, $SD = .15$). However, logarithmic fit was similar for the two groups (.92, $SD = .08$ and .91, $SD = .10$). Moreover, logarithmic function had better fit than linear function in the second grade for HMA, $t(15) = -7.51$, $p < .01$, but not for LMA, $p = .21$. LMA and HMA groups did not differ on logarithmic or linear fits in the 3rd and 5th grades (see Figure 2).

Goodness of fits for one cycle and two cycle.

β value. A four-way analysis of variance (ANOVA) was performed on the β of the 1 cycle cyclical power model and the 2 cycle cyclical power model, with task (bounded or unbounded) and function (1 cycle / 2

cycle) as the within subjects factor and MA group (Low or High) and grade as the between subjects factor. The main effect of task reached significance, $F(1, 88) = 9.37$, partial $\eta^2 = .1$, $p < .01$, observed power = .86. The main effect of representation fit was significant as well, $F(1, 88) = 21.7$, partial $\eta^2 = .2$, $p < .01$, observed power = .99. The interaction between task, representation, grade and MA group reached significance, $F(2, 88) = 10.72$, partial $\eta^2 = .2$, $p < .01$, observed power = .99. As can be seen in Figure 3, on the bounded task, in the 2nd grade LMA individuals had higher β values than HMA individuals in the 1 cycle function $t(29) = 2.98$, $p < .01$. However, for the 2 cycle function this pattern was reversed. HMA individuals had higher β values than LMA individuals, $t(29) = -6.09$, $p < .01$. Moreover, the observed β parameter improved with age for LMA for the bounded task and the 2 cycle function between 2nd and 3rd grades, $t(33) = -8.54$, $p < .01$, and between 3rd and 5th grades, $t(35) = -2.69$, $p < .05$. For the HMA group 2 cycle function in the bounded task, only between 2nd to 3rd grades, $t(26) = -3.66$, $p < .01$. The HMA group also improved between 2nd and 3rd grades for the 1 cycle function, in the bounded task, $t(26) = -3.66$, $p < .01$, and the unbounded task, $t(26) = -3.12$, $p < .01$ (see Figure 3).

1 cycle vs. 2 cycle goodness of fits. A four-way analysis of variance (ANOVA) was performed on fits of the 1 cycle cyclical power model and 2 cycle cyclical power model, with task (bounded or unbounded) and function (1 cycle / 2 cycle) as the within subjects factor, and MA group (Low or High) and grade as the between subjects factor. None of the effects reached significance. Higher fits for the 1 cycle over the 2 cycle function indicate usage of 2 reference points (beginning and end of the line) rather than 3 reference points. Accordingly, in order to determine the function that best fit the data, we compared fits of 1 cycle / 2 cycle in each of the age groups, MA groups and tasks. The results indicate that the data fits better with the 1 cycle function than the 2 cycle function for the HMA group in the bounded, $t(15) = 2.31$, $p < .05$, and in the unbounded task, $t(15) = 2.18$, $p < .05$ for the 2nd grade only. There were no other significant differences for the LMA in all grades or for the HMA in the 3rd and 5th grades (see Figure 4).

Regression Analysis

Usage of advance strategies

We ran multiple regression analyses to predict the usage of advance strategies, anxiety level (as a continuous variable) and mean PAE of bounded and unbounded or linear fit for bounded and unbounded.

Advance strategy PAE. The model predicted significantly the usage of advance strategy ($R^2 = .17$, $p < .01$). First, the effect of mean PAE for the bounded task was highly significant ($\beta = -.38$, $t(87) = -3.4$, $p < .01$), high error predicted lower usage of advance strategy. All other effects were not significant (see Table 5A).

Advance strategy linear fit. The model predicted significantly the usage of advance strategy ($R^2 = .13$, $p < .05$). The effect of linear fit for the bounded task was highly significant ($\beta = -.31$, $t(87) = 2.64$, $p < .05$), high linear fit predicted higher usage of advance strategy. All the other effects were not significant (see Table 5B).

Discussion

The development trajectories of the bounded number line task have been tested multiple times ^{10,11,24}. However, the development trajectories of the unbounded number line task, as well as the effect of MA on number line estimations, have rarely been tested. Hence, the present study mainly aimed at testing the influence of MA on number line estimations.

Across MA groups, we found a developmental trend in errors and standard deviations of errors. As expected, errors and standard deviations of errors decreased as grade increased. Moreover, standard deviations of errors were larger in the unbounded than in the bounded task.

Classically, in number line estimation, with development, linear fit should increase while logarithmic fit should decrease ^{10,11}. MA affected linear / logarithmic fits: for our youngest group, HMA participants' responses were best explained by a logarithmic function rather than by a linear function. A similar tendency was not observed in the LMA groups. There were no group differences among older children (3rd and 5th graders).

Moreover, it was suggested that estimation in the number line estimation task reflects proportion estimation strategies rather than pure estimations, and hence best fits with 1-cycle or 2 cycle functions ^{14,15,19}. Accordingly, we tested both functions fits and free β values. Similar to the analysis of linear / logarithmic fits, we found that 2nd graders in the HMA group showed less mature number line estimations than LMA children. First, the fits for the 1 cycle function were larger than for the 2 cycle function, for both bounded and unbounded tasks. This indicates usage of 2 rather than 3 reference points in numerical estimations of young HMA participants. Second, the β values of the HMA group for 1 cycle function was lower than for the 2nd grade LMA group. Again, performances of older children (3rd and 5th graders) were not modulated by MA level. Hence, MA estimations were normalized from the third grade on.

The last step of the analysis was to test the relations between number line estimation performances, MA and math abilities. We tested the predictive role of MA and number line estimations in the usage of advance, memory based strategies during the solution of addition problems. Here we found that linear fit and errors in the bounded task predicted advance memory-based strategies use, but not in the unbounded task. This result further emphasizes the dissociation between the bounded and unbounded tasks.

The spatial theory of MA ^{28,31,32} views MA as resulting from weakness in spatial abilities subsequent to deficits in basic numerical abilities such as symbolic number comparisons ^{28,31,32}. Quantities are believed to be pre-verbally mapped on a mental number line ^{2,3,5}. The mental number line estimation task requires number space associations. Hence, according to the spatial theory of MA, we expected to find weakness in all the mental number line tasks, with specific deficits in the unbounded number line (that reflects purer numerical estimation than the bounded number line) ^{17,21,22}.

In line with the spatial theory of MA, we found few indications for weakness in number line representations, regardless of task. First, linear or logarithmic fits: for our youngest group, high, but not low MA showed a logarithmic tendency, regardless of task. Estimations from all the older HMA groups and all the LMA groups were not best explained by logarithmic fits. Similarly, for our youngest group, HMA estimations were best explained by the 1 cycle function over the 2 cycle function, as found for the bounded and unbounded task. Moreover, only in the bounded task was the β free parameter lower for the 1 cycle function for HMA participants compared to LMA participants. This indicated that HMA participants have non-precise mental number line representations in younger ages that normalize in older ages.

Only one other study tested numerical estimation in HMA participants. However, it is very difficult to compare the results of the two studies, due both to age differences and methodological differences. [Núñez-Peña, et al.](#) ³⁵ looked at bounded number line estimations in students with high or low MA. They found group differences between low and high MA in non-familiar ranges (267–367), but not in familiar ranges (0–100, 0–1,000, 0–100,000). Contrary to the present results, they concluded that MA results from WM reductions rather than spatial deficits ([Ashcraft & Faust, 1994](#); [Ashcraft & Kirk, 2001](#)).

In the present study, we found positive relations between advance strategy use and bounded but not unbounded number line estimations. The relations were found in regression analysis, including 1) age, 2) MA, 3) bounded and 4) unbounded estimations. Similar results were found in predicting errors (beta = -.38) and linear fit (beta = .33). Specifically, better estimations in the bounded task were associated with more advance strategy use. Moreover, in the two analyses the contribution of the other variables (age and MA) was not significant.

The correlations between bounded number line but not unbounded number line estimation and math performance is in line with several previous studies ^{13,22}. First, [Link, et al.](#) ²² found correlations between basic arithmetic and bounded but not unbounded tasks in 4th graders. The authors suggested that estimation in the bounded number line required a division of the number line to guide the application of proportion-based strategies by reference points. Subsequently, a subtraction or addition is needed in order to understand the relation between the target number and a reference point. Explicit addition or subtraction is not required in the unbounded number line estimation. Similarly, [Jung, et al.](#) ¹³ looked at the correlations between bounded and unbounded number line estimations and addition, subtraction, multiplication and division in 5th to 7th graders. They found correlations between bounded number line estimation and math performance, but not between unbounded number line and math performance. The correlations changed developmentally, and only by the 7th grade were performances in addition, subtraction, multiplication and division related to estimations. In the 5th grade only subtraction was related to bounded estimations. Lastly, the present result regarding correlation between bounded estimation and strategy use is in line with a recent meta-analysis. Specifically, [Schneider, et al.](#) ²³ performed a meta-analysis to reveal the correlation between bounded number line estimations and broader mathematical competence. They found positive medium correlations across tasks between math

performance and bounded estimations. The correlation increased with age and remained stable across a wide range of task variants and mathematical competence measures (i.e., counting, arithmetic, school achievement).

Hence, multiple studies have also suggested that the unbounded number line reflects pure nonverbal estimation, and should accordingly be associated with mature math performance^{1-3,9}. The results of the present study suggest that pure numerical estimation is not an essential part in mature verbal computational mathematics. However, other abilities required in the bounded number line, such as base ten understanding and addition or subtraction, are needed in verbal computational mathematics.

Conclusion

The present study examined developmental trajectories of bounded and unbounded number lines in 2nd, 3rd and 5th grade children. The effect of MA on estimations and the relations between estimations and strategy use was also tested in the present study. First, children with high MA showed non-mature estimations more than children with low MA; they performed overestimations in smaller numbers and underestimations in larger numbers. In addition, their data was best explained by a one cycle function rather than a two cycle function. These patterns of estimation were similar in the bounded and unbounded number line estimation task, indicating that the MA weakness is associated with number space associations. However, later in development (3rd and 5th graders), the high MA group estimation normalized, and group differences were eliminated.

Dissociable developmental trends were found in the bounded and unbounded task. Similarly, bounded, but not unbounded, number line estimations explained individual differences in advance strategy use during mental arithmetic and future dissociated bounded and unbounded tasks.

Method

Participants

Ninety-four students participated in the study, including 31 second graders (average age 7.8, 52% girls), 31 third graders (average age 8.9, 52% girls) and 32 fifth graders (average age 10.7, 53% girls) recruited from six local elementary schools in the Jerusalem area. All participants were native Hebrew speakers, with no reported learning disabilities or severe learning deficits (according to teachers' reports). Students with learning disabilities, psychiatric and neurological disorders were excluded.

Procedure

All participants performed the experiment in the morning at school. Every student performed the task individually in a quiet room beginning with the 1) Bounded number line 2) Unbounded number line 3) Corsi Task to test VSWM, 4) Strategy assessment, and 4) concluded with the Math Anxiety Questionnaire.

The full details regarding strategy assessment of simple and complex addition and future analysis related to strategy assessment can be found in (Ashkenazi and Cohen, in press).

This study was approved by the local ethics committee of the Seymour Fox School of Education at the Hebrew University of Jerusalem, as well as the ethics committee of the Ministry of Education. All experiments were performed in accordance with the guidelines and regulations of the ethics committee of the Seymour Fox School of Education. Informed consent was obtained from all the legal guardians of our participants.

All the participants began with the bounded task and then performed the unbounded task. Set orders in the bounded and unbounded task (A or B) were counterbalanced, so that each of the four possible combinations was presented to a subgroup of the participants of each age. Each task included 52 numbers between 1 and 99.

For the unbounded task, the experimenter began by saying, "Now we are going to play with another number line. In this number line there is no end point, but a unit that equals ten numerical units. As before, I'm going to ask you to show me where some numbers are on the number line. When you decide where the number goes, I want you to make a line through the number line like this [making a vertical hatch mark]." Before each item, the experimenter said, "If this is 10 numerical units, where would you put N?" (with N being the number specified in the particular trial). A demonstration was provided prior to the beginning of each task.

Assessment of Visual-Spatial Working Memory: Corsi Block Tapping Task Forward ³⁸.

This task was conducted via an online computer program - PEBL the Psychology Experiment Building Language; <http://pebl.sourceforge.net>) Mueller & Piper, 2014(. During this task nine blocks were presented on the computer screen in varying locations. Each trial consisted of a presentation phase in which a series of blocks were illuminated one at a time – the sequence length (number of illuminated blocks within each trial) increased in every two trials, ranging from two blocks to seven blocks. During the response phase of each trial, the participant was required to click a mouse on the blocks in the same order as they were presented. A practice block of three trials was administered prior to the experimental block – each practice trial consisted of a sequence length of three illuminated blocks. If the participant answered correctly on at least one of the two trials within each sequence length, the experiment continued. If not, the task was terminated. At the end of the task, the participant's visual-spatial working memory score was computed.

Assessment of Math Anxiety

Children's math anxiety was evaluated with a Hebrew translated and adjusted form of the well-known Abbreviated Math Anxiety Scale (AMAS; ³⁹. The instrument was translated into Hebrew by Prof. Orly Rubinsten's lab (forward translation) and then from Hebrew back into English (back translation) to ensure the validity of the translation ⁴⁰. The AMAS is a nine-item self-report questionnaire found to be as

effective as the longer Math Anxiety Rating Scale⁴⁰. Each item consists of a statement describing an event, and participants indicate how anxious it would make them on a five-point Likert scale (1 = never, 2 = rarely, 3 = sometimes, 4 = usually, 5 = always). The modification of the AMAS for primary-school students involved substituting certain wordings, such as “fear” instead of “anxious.” Scores on the AMAS range from 9 to 45, with a higher score indicating a higher level of math anxiety. Cronbach’s alpha for the AMAS in the current sample was 0.77. The sum of the 9 item score in the current sample ranged between 9 and 41. The score divided the participants into two groups: low and high math anxiety groups. For the partition, we used the median split procedure³¹, i.e., participants were assigned to either a low (LMA) or a high (HMA) math anxiety group if their score was below or above the group median score, respectively. We calculated the group median for all the age groups of the participants: the group median was 17. Hence, every participant with a score of 17 or below was assigned to the LMA group, while every participant with a score of 18 or above was assigned to the HMA group. Accordingly, 16 second graders were assigned to the low MA group (mean anxiety = 12.56, S.D., 2.39) and 15 were assigned to the high MA group (mean anxiety = 21.73, S.D., 2.37). 20 of the third graders were assigned to the low MA group (mean anxiety = 13.5, S.D., 2.70) and 11 were assigned to the high MA group (mean anxiety = 25.63, S.D., 6.31). 17 of the fifth graders were assigned to the low MA group (mean anxiety = 13.17, S.D., 2.65) and 16 were assigned to the high MA group (mean anxiety = 21.93, S.D., 3.42).

Number line estimation Bounded number line estimation. Each problem involved a 25 cm line, with the left end labeled “0” and the right end labeled “100”. The number to be estimated appeared 2 cm above the center of the line. Two sets of numbers-positions with similar distributions of numbers were created for each scale. Set A included 26 numbers; Set B included 26 other numbers. These numbers were chosen to maximize discriminability of logarithmic and linear functions and to minimize the influence of specific knowledge, such as that 50 is halfway between 0 and 100.

Unbounded number line estimation. Each problem involved a 25-cm line, with a section to the left labeled “10”. The number to be estimated appeared 2 cm above the center of the line. Two sets of numbers-positions with similar distributions of numbers were created for each scale. Set A included 52 numbers; Set B included 52 other numbers. These numbers were chosen to maximize discriminability of logarithmic and linear functions and to minimize the influence of specific knowledge, such as that 50 is halfway between 0 and 100.

Analysis Number line estimation. Followed [Slusser, et al.](#)¹² we calculate β value and function fit for 1 cycle cyclical power model (0-100 range) with the following function $Y=100*(x^A/(x^A+(100-x)^A))$ and with 2 cycle cyclical power model (0-100 range) with the following function $Y=IF(X<50,100*(((X^A)/((X^A)+(50-X)^A))*0.5), 100*(((X-50)^A)/(((X-50)^A)+(100-X)^A))*0.5 + 0.5))$ with β as a free parameter. Development according to the proportion judgment theory can be discovered by the β free parameter value, which reflects the degree of bias associated with estimation of individual magnitudes. The observed β parameter may change gradually with age or experience, such that estimates are more accurate for older and more experienced observers where β is equal to 1 $X = Y$.

Strategy assessment.

Each child's strategy use for single-digit addition problems (e.g. $2 + 4 = ?$) was first assessed using standardized, well-validated measures that classify strategies based on reaction time (RT) patterns, experimenter's observation, and child's report³⁷. The problems were presented one at a time on a computer monitor. The experiment started with 6 practice trials. Then, there were 20 simple problems with random pairs of integers from 2 to 9 (e.g., $2 + 4 = ?$) and sums ranging from 5 to 16. Finally, the following 6 problems were classified as complex (e.g., $9+15=?$) comprising one two-digit integer (from 14 to 19) and one single digit integer (from 3 to 9), summing from 21 to 25.

Problems with identical addends (e.g., $2 + 2$, $5 + 5$). 0 or 1 were excluded because they evince less strategy variability (Siegler, 1987). No repetition of either addend was allowed across consecutive problems. Children were instructed to state the answer as soon as they arrived at it. The experimenter then probed the child on which strategy was used during problem solving. Responses were categorized as: retrieval (e.g., 'just knew it'), finger count (e.g., 'counted on my fingers') or manual count ('counted in my head'), decomposition (e.g., $9 + 5 = 9 + (1 + 4) = (9 + 1) + 4 = 10 + 4 = 14$), fingers (i.e., the child looked at his or her fingers but did not count them) and other/multiple strategies. Trials in which the experimenter noted overt signs of counting even when the child reported retrieval were classified as counting. A voice key, triggered by the child's response to the answer, measured RT. We calculated the presence of advance strategy use by testing the number of trials in which a participant used retrieval strategy plus the number of trials that a participant used decomposition, divided by the number of all trials. This was done once for simple problems and once for complex problems, and we used the average score of simple and complex.

Declarations

Author contributions statement

SA: Conceptualization, Methodology, Software, Validation Writing - Original Draft

NC: was Project administration and Writing - Review & Editing.

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Tables

Table 1. Social demographic background information of the two MA groups as a function of age group.

	Second grade			Third grade			Fifth grade		
	HMA	LMA	Sig	HMA	LMA	sig	HMA	LMA	sig
Age in months	94.15	94	.92	107.41	106.57	.599	129.53	127.87	.33
N	11	20		20	14		15	17	
% Females	54.5	55		35.7	58.8		53.3	52.9	
VSWM	4.48	4.18	.25	4.62	4.86	.255	4.94	4.86	.72
MA	20.3	11.36	.00	21.06	12.24	.000	23.12	12.27	.00
Advance strategy use (%)	71%	67%	.72	85%	87%	.79	90%	87%	.64

Table 2. Multiple regression analysis predicting the usage of advance strategy with age, bounded estimations, unbounded estimations and MA level. A) testing absolute errors, B) testing linear fits

	<u>B</u>	<u>SE</u>	<u>B</u>	<u>I</u>	<u>p-value</u>
A) Age	0.00	0.00	0.04	0.36	.72
PAE bounded	-0.04	0.01	-0.38	-3.35	.00
PAE unbounded	0.00	0.01	-0.00	-0.02	.98
MA	0.00	0.00	0.08	0.78	.44
B) Age	0.00	0.00	0.07	0.60	0.55
Linear fit bounded	4.03	1.53	0.31	2.64	0.01
Linear fit unbounded	-0.12	0.54	-0.02	-0.22	0.83
MA	0.0	0.00	0.07	0.67	0.51

Dependent variable: percentage of usage of advanced strategies, MA=Math Anxiety.

Figures

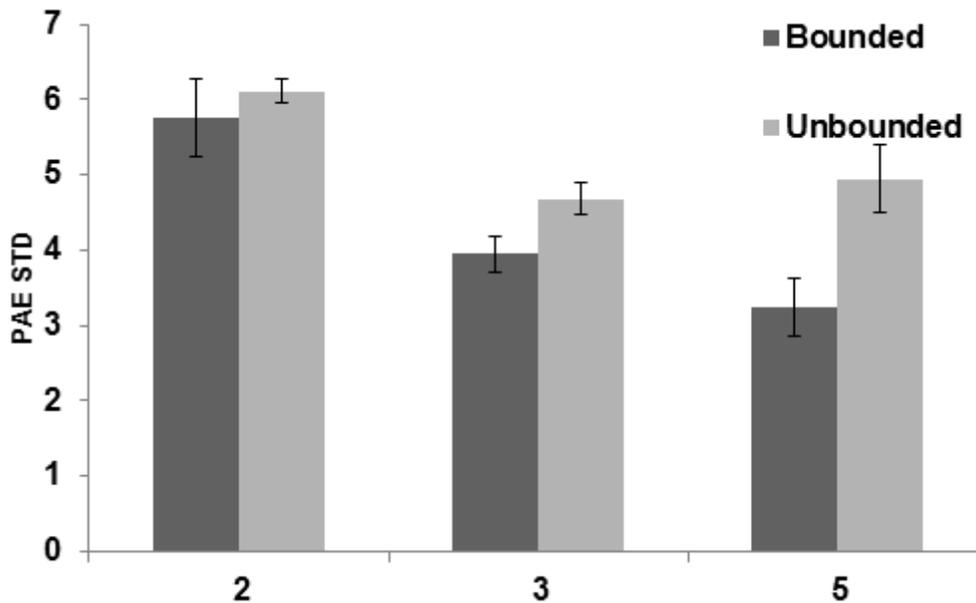


Figure 1

Absolute error standard deviations as a function of grade and task. Absolute error standard deviation decreased as grade increased, and was smaller in the bounded compared to the unbounded task.

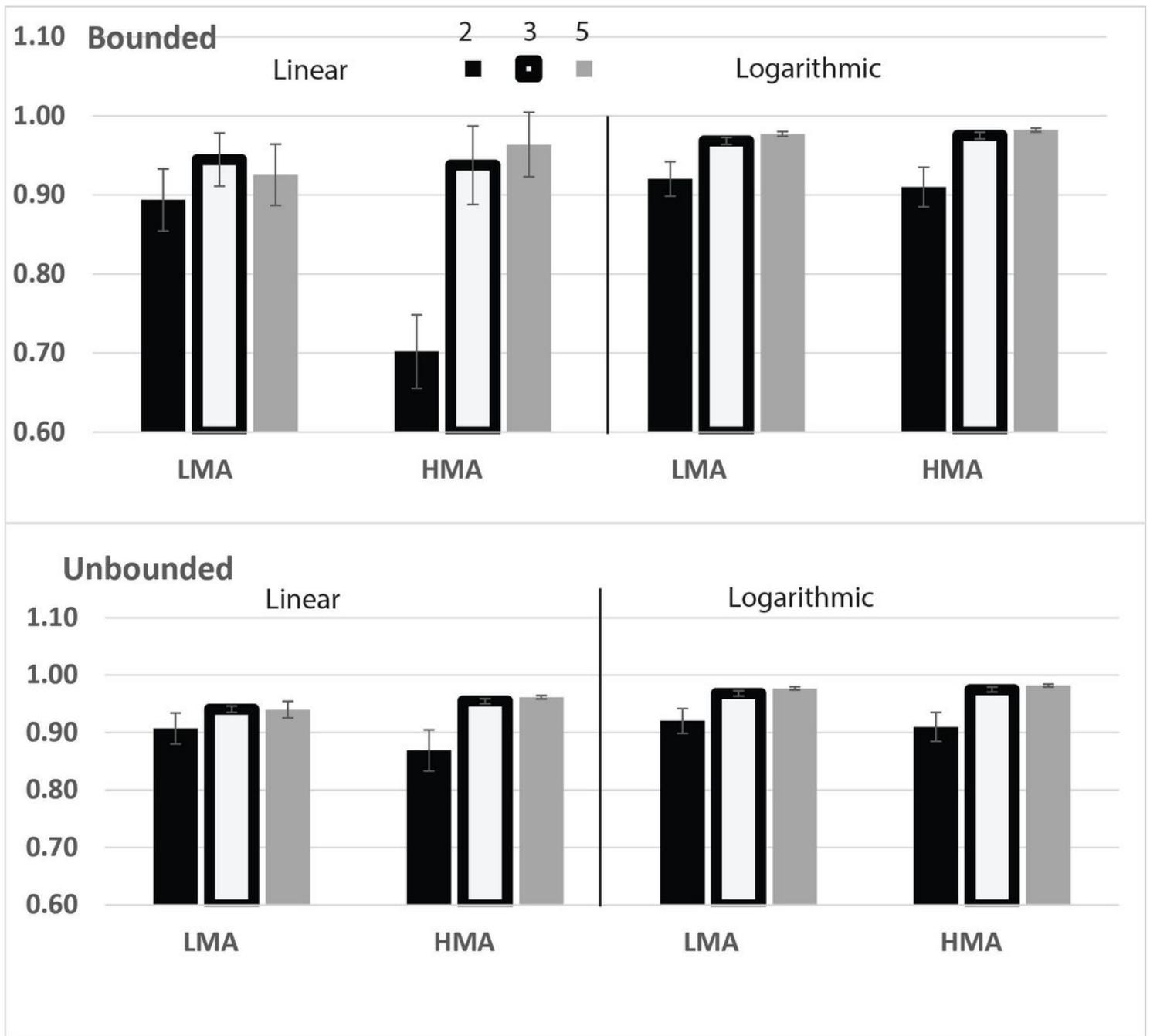


Figure 2

Functions goodness of fit for linear (left side) and logarithmic (right side) for the bounded (upper panel) and unbounded (lower panel) tasks. The results are presented as a function of MA group. Estimations of young HMA participants had better fits to logarithmic functions than linear function, regardless of task. However, preferences for logarithmic fits over linear fits were not observed in LMA participants.

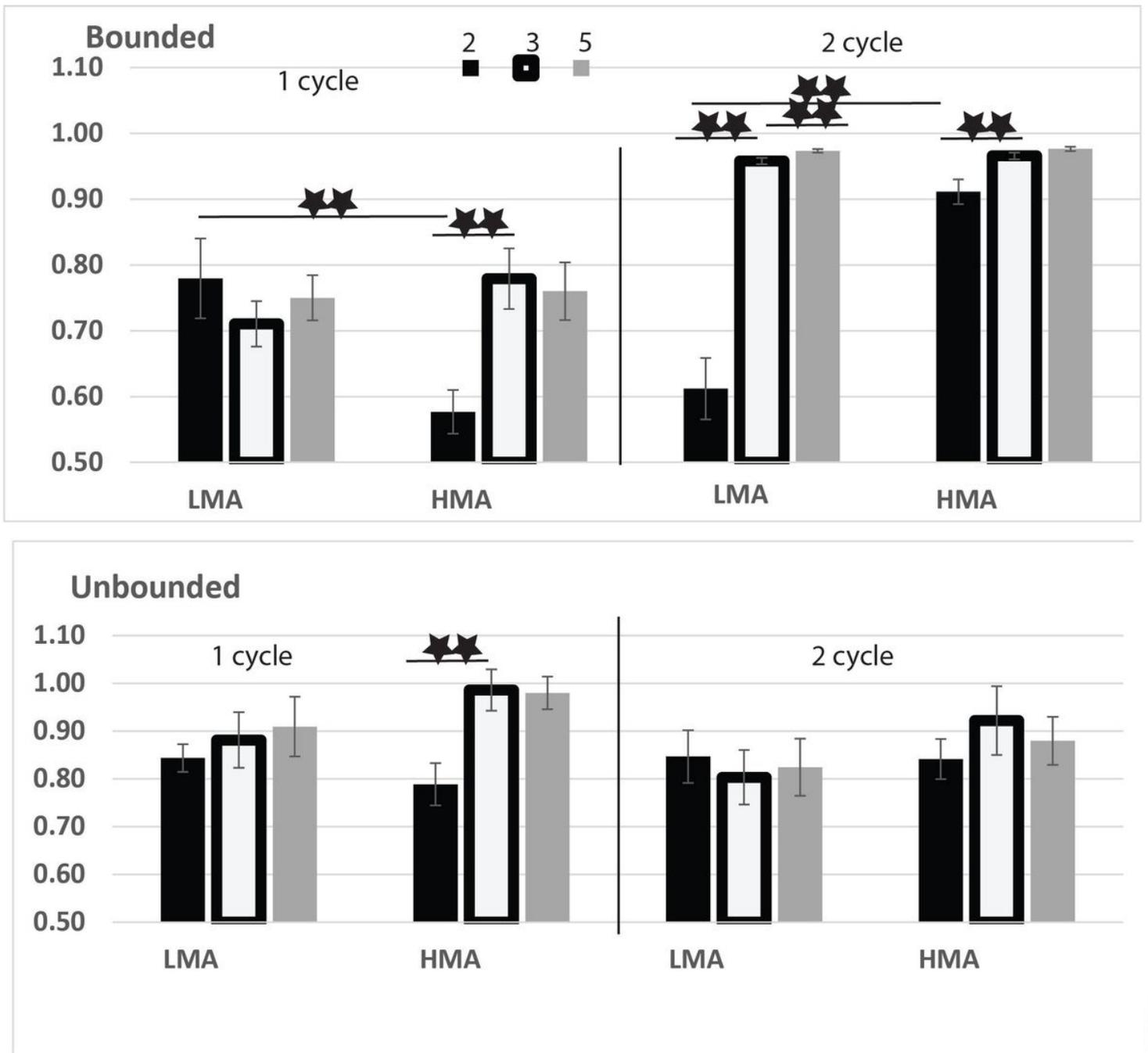


Figure 3

β parameter value for one cycle (left side) and 2 cycle (right side) for the bounded (upper panel) and unbounded (lower panel) tasks. The results are presented as a function of MA group.

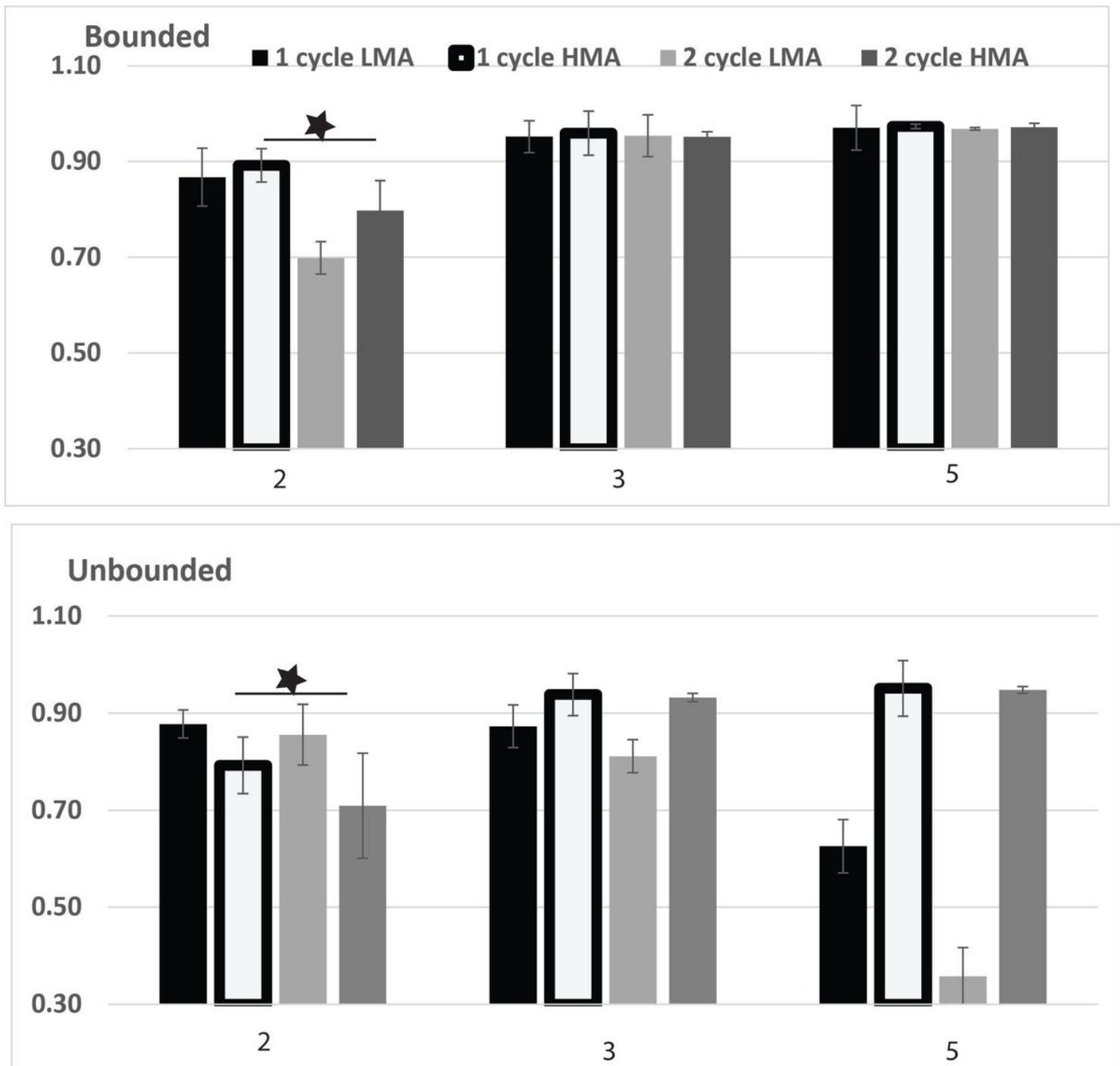


Figure 4

Goodness of fits value for one cycle and 2 cycle as function of task: the bounded (upper panel) and unbounded (lower panel) tasks. Results are presented as a function of MA group. In the second grade, results indicated that the data fits better with the 1 cycle function than the 2 cycle function for the HMA group.