On Non-Spherical Models for Spiral Galaxy Gravitation Potential Yielding Flat Rotation Curve

Swagatam Sen (swagatam.x.sen@gmail.com)
N/A https://orcid.org/0000-0002-4336-9196

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Swagatam Sen

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Abstract

A two component model of gravitation potential for spiral galaxies has been proposed which couples a spherically symmetric component with a second component that observes planar radial symmetry on the galactic plane and vanishes outside an annular disk beyond the edge of galaxy's effective radius. It is shown that such a model for potential satisfying Poisson Equation would produce rotation velocity curve towards the edge of the galaxy which is flat over distance from the galactic centre. This relationship, which is experimentally observed in many spiral galaxies, is shown as a consequence of classical understanding of gravity and specific symmetry of the gravitational potential without any extrinsic requirement of dark matter. It is also demonstrated that this potential directly yields a relationship between inner mass of the galaxy and terminal rotation velocity, which has been empirically observed and known as Baryonic Tully-Fisher relations. Furthermore a direct test has been proposed for experimental verification of the proposed theory.

Keywords: Galaxies: kinematics and dynamics, galaxies: spiral, cosmology: dark matter, cosmology: theory

1 Introduction

The problem of the anomaly observed in spiral galaxy rotation curves is a longstanding one (Babcock, 1939; van de Hulst et al., 1957; Schmidt, 1957; Oort, 1940; Rubin and Ford, 1970). Rotation velocities of stars at the edge of most spiral galaxies display indifference to distance from galactic centres, while predictions from Newtonian gravity require such velocities to be inversely proportional to distance (Persic et al., 1996). Existing literature predominantly focuses on expected density profiles to match such empirical rotation curves (Navarro et al., 1996; Ashman et al., 1993; Broeils, 1992; Carignan et al., 1990). While many such density profiles assume spherically symmetric distribution of mass and gravitational potential (Hernquist, 1990; Jaffe, 1983; Merritt, 2006; Merritt et al., 2005) which is perhaps influenced by earlier physical models of spherical and globular galaxies (de Vaucouleurs, 1959; Eddington, 1916), there has been significant work in modelling aspherical as well as asymmetric density profiles. (Binney and Tremaine, 2008) provides a detailed account of such aspherical (axisymmetric and disk) potentials.

However such expected profiles do not match the derived density distribution based on luminosity profiles of galaxies (Rubin et al., 1980). thus resulting in a 'mass gap' which is conjectured to be explained by the dark matter halo around the galaxy(Luminet, 2021), which only interacts gravitationally with other Baryonic matter (Zwicky, 1937; de Swart et al., 2017; Burbidge and Sargent, 1969; Emerson and Baldwin, 1973). But such theories have additional onus to experimentally validate existence of such non-Baryonic matter but hasn't yet been corroborated in direct (Aprile, E. et al., 2017; Cui, 2017) and indirect (Ackermann, 2017) detection attempts.

Furthermore, even though the rotation curves appears to show a 'mass gap' between gravitational mass and luminous Baryonic mass of the galaxies, observations across multiple spiral galaxies show that terminal stellar velocities are strongly related to the Baryonic galactic mass through Tully-Fisher relationship (Yegorova and Salucci, 2007; Tully et al., 1975; McGaugh et al., 2000; Torres-Flores et al., 2011). This offers some amount of difficulty for dark matter theories as ideally terminal velocities should couple with the total gravitational mass of galaxies including baryonic as well as dark matter (Lelli et al., 2017).

On the other hand, there is also considerable literature that suggest a modified understanding of how non-relativistic dynamics works and deviates from Newtonian dynamics at large scale and/or slow acceleration regime (Milgrom, 1983; Bekenstein and Milgrom, 1984). While such theories are able to explain a number of phenomena unexplained or partially explained by dark matter theories (McGaugh, 2015; Kroupa et al., 2012; Boylan-Kolchin et al., 2011; McGaugh, 2012; Famaey and McGaugh, 2012*a*; Romanowsky et al., 2003; Milgrom and Sanders, 2003), there are concerns around well-posedness of the theories (Yunes and Siemens, 2013), Stability of the spherically symmetric models (Seifert, 2007) among others (Famaey and McGaugh, 2012*b*). Some of these are addressed through non-relativistic and relativistic extensions of the original modified Newtonian gravity (Bekenstein, 2004; Buchdahl, 1970; Jacobson and Mattingly, 2001) even if not all conceptual issues are resolved (Contaldi et al., 2008). But the most significant challenge for modified dynamics theories come from the observations around non-spherically-symmetric systems like Bullet Clusters (Clowe et al., 2006; Aguirre et al., 2001; Angus et al., 2006). There have also been attempts to leverage non-classical yet low-energy solutions to Einstein Field Equations from General Relativity to explain the anomaly (Cooperstock and Tieu, 2005; Cooperstock and Tieu, 2007; Balasin and Grumiller, 2008; Brownstein and Moffat, 2006).

One insight from existing literature is that it's important to consider aspherical or asymmetric gravitational potential to model gravitational effects of galaxies (Zhang, 2019) as most galaxies and galaxy clusters are far from spherically symmetric (Trevese and Vignato, 1977; Limousin et al., 2013; Piffaretti, R. et al., 2003; Ferreras, 2019), and indeed most of the observational challenges to existing theories come from more aspherical structures (Madejski, 2005; Navarro and Benz, 1991). To do that we'd start from the Poisson equation which is the most general form of Newtonian gravity. Interestingly, for an aspherical system Poisson equation is usually thought of as a continuous approximation for a system composed of many small, spherical or point masses all following inverse-square-law of gravity. However, Poisson equation is also the direct consequence of General relativity for a low-energy composite, aspherical system. This distinction is important as the latter interpretation allows us to free ourselves from the inverse-square-law and help understand the gravitation potential purely in terms of how much space-time is curved around a massive gravitational system like a galaxy. In fact, within the classical setup of Newtonian gravity and Poisson equation $\nabla^2\phi = 4\pi G\rho$, canonical potential $\phi_N = \frac{GM}{r}$ is simply one of the solutions. Curiously it only becomes unique solution under the condition of spherical symmetry and under different physical symmetries and boundary conditions, Poisson equation can lead to very different solutions all of which are consistent with Newtonian gravity (Nyambuya, 2015).

Following this line of thought, we intend to explore a specific non-spherically-symmetric solution to Poisson equation that can help explain the flatness of rotation curves without requiring exotic particles or changing the laws of nature. Such a solution, in general, won't render itself to be expressed as a combination of many inverse-square-law potentials, but would present itself as an expression of the underlying space-time metric around the galaxy. In the present work we'd illustrate that such potentials can exist, and freed up from the inverse-square-law potentials, can explain the rotation curves of many spiral galaxies. We would restrain ourselves from claiming uniqueness of such solutions and would only tentatively intend to lay down groundwork that too by particularly focusing on spiral galaxies. Such galaxies usually consist of a number of structural components including a spherical bulge and a planar disc with spiral arms (Sparke and Gallagher, 2007; Seigar and James, 1998). To accommodate the non-spherical structure of the system we would introduce an *Effective Radius* r_0 for the galaxy so that most of the galactic baryonic matter resides within it and would simultaneously propose an annular region D outside the effective radius with a constant width of h where planar radial symmetry along the galactic disk plane takes over from spherical symmetry. In other words, we would be looking for a potential Φ that is

1. spherically symmetric within the effective radius
2. observes planar radial symmetry on an annulus along the galactic disk just outside effective radius
3. is continuous at the effective radius
4. has continuous derivative at the effective radius

In the next section we would use these constraints to derive an alternative gravitation potential and resulting dynamic.

2 Gravitational Potential

2.1 Two component model

Our approach to build a potential function Φ that satisfies the conditions (1-4) of the preceding section, would entail first constructing one spherically symmetric component Φ_0 to account for the galactic matter within the Effective Radius and a second component Φ_D that is only defined on the external annular disk D and is radially symmetric on the plane. Eventually on D where both components are defined, the composite potential Φ would be an affine combination of the two components, while everywhere else Φ

would simply coincide with Φ_0. Since both the components would be required to satisfy Poisson equation, so would the composite model.

Since Φ_0 is spherically symmetric, Poisson Equation

$$\nabla^2 \Phi_0 = 4\pi G\rho \tag{1}$$

would produce the Newtonian potential,

$$\Phi_0 = \frac{Gm(r)}{r} \tag{2}$$

where $m(r)$ denotes the total mass inside the sphere of radius $r < r_0$.

However, we'd see that the disk component of the potential, Φ_D would need to assume a different functional form driven by its own symmetry.

2.2 Disk Potential

Let us start by setting up a cylindrical reference frame (r, θ, z) where the coordinates stand for radial distance from galaxy centre, inclination and normal distance from the disk plane respectively.

As discussed we'd assume that Φ_D is radially symmetric on the disk plane (rather than spherically symmetric) and $\partial_z \Phi$ vanishes within the width of the disc.

In other words, the desired disk potential Φ_D would need to satisfy the following conditions -

1. $\Phi_D(r, \theta, z) = 0 \ \forall \phi \notin [-\frac{h}{2}, \frac{h}{2}], \forall r, \theta$
2. $\Phi_D(r_0, \theta, z) = \Phi_0(r_0, \theta, z) \ \forall \theta, z \in [-\frac{h}{2}, \frac{h}{2}]$
3. $\partial_r \Phi_D(r_0, \theta, z) = \partial_r \Phi_0(r_0, \theta, z) \ \forall \theta, z \in [-\frac{h}{2}, \frac{h}{2}]$
4. $\partial_\theta \Phi_D = 0 \ \forall r, \theta, z$
5. $\partial_z \Phi_D = 0 \ \forall z \in [-\frac{h}{2}, \frac{h}{2}]$

Additionally this non-relativistic potential function Φ on the disk would need to satisfy Poisson equation,

$$\nabla^2 \Phi_D = 4\pi G\rho \tag{3}$$

But conditions 4 and 5 would reduce the equation to

$$\nabla^2 \Phi_D = \partial_r^2 \Phi_D + \frac{1}{r}\partial_r \Phi_D = 4\pi G\rho \tag{4}$$

for $r > r_0$ and $z \in [-\frac{h}{2}, \frac{h}{2}]$.

Solving the equation for ϕ we get

$$\Phi_D = \lambda_0 + \lambda_1 \ln\frac{r}{r_0} + 4\pi G \int_{r_0}^{r} \frac{1}{s'} \int_{r_0}^{s'} s\rho(s) ds ds' \tag{5}$$

where λ_0, λ_1 are constants.

Now if we use conditions 2 and 3, we can pin down the values for the constants

$$\begin{aligned} \lambda_0 &= \frac{Gm_0}{r_0} \\ \lambda_1 &= -\frac{Gm_0}{r_0} \end{aligned} \tag{6}$$

where $m_0 = m(r_0)$ is the Baryonic mass of the galaxy within the effective radius r_0.

That'd finally give us the unique form for Φ_D as

$$\Phi_D = \frac{Gm_0}{r_0}(1 - \ln\frac{r}{r_0}) + 4\pi G \int_{r_0}^{r} \frac{1}{s'} \int_{r_0}^{s'} s\rho(s) ds ds' \tag{7}$$

2.3 Rotation Curve and Terminal velocity

We can immediately observe that an object positioned at a distance r from the centre on the disk would experience a radial acceleration

$$\frac{d^2 r}{dt^2} = \Phi'_D(r) = -\frac{Gm_0}{r_0 r} + \frac{4\pi G}{r}\int_{r_0}^{r} s\rho(s) ds \tag{8}$$

Now for this radial acceleration to exactly counterbalance the opposing centrifugal acceleration so that the stellar object remains on a stable orbit, we would need the orbital velocity to satisfy,

$$v^2 = \frac{Gm_0}{r_0} - 4\pi G \int_{r_0}^{r} s\rho(s)ds \tag{9}$$

From Eq. (9), it is clearly evident that the potential we derived based on the stated symmetries and continuity conditions yield orbital velocity of stars on the disk with a dominant constant term. In fact, if the density of the disk is considered to be negligible outside the effective radius, we would get a perfectly flat rotation curve,

$$v_0 = \sqrt{\frac{Gm_0}{r_0}} \tag{10}$$

However, most interesting feature of Eq, (9) is that any matter outside the effective radius would have a dampening effect on the 'flatness' of the rotation curve. This observation is completely contrary to any predictions from standard Newtonian gravity which requires a halo of additional mass around the edges of the galaxies to explain flat rotation curves.

It would be possible to establish a relationship between terminal velocity v_0 and the baryonic mass in the galaxy if we assume ρ_0 to be the average density of matter on the disk plane covered within the the effective radius. Then total baryonic mass of the galactic disk could be expressed as $m_0 = \pi\rho_0 r_0^2 h$, which when plugged into the expression for v_0 yields,

$$m_0 = \frac{v_0^4}{\pi\rho_0 h G^2} \tag{11}$$

which is eerily close to observed Baryonic Tully-Fisher relation (BTFR) with the exception of the density ρ_0 and width h which can potentially vary across galaxies. To derive the exact form of BTFR, we would need to introduce the general potential combining the two components.

2.4 Baryonic Tully-Fisher Relation

In general - as discussed before - even though Φ_D would dominate over Φ_0 on D, the general solution for a potential that satisfies Poisson equation, nevertheless would be,

$$\Phi = w_0\Phi_0 + w_D\Phi_D \tag{12}$$

where $w_0, w_D \geq 0$ and $w_0 + w_D = 1$.

In plain terms w_D serves the same role dark-to-normal matter ratio does in dark matter theories. It captures the relative strength of the halo created by Φ_D outside the edge of the galaxy in contrast to the matter distributed within the Effective radius. In this section, we'd try to uniquely establish w_D for a galaxy as a function of its baryonic mass distribution. For that we would need to re-calculate terminal velocity under the general composite potential.

Such a composite potential would produce a radial acceleration of

$$\frac{d^2r}{dt^2} = \Phi'(r) = -\frac{w_0 Gm_0}{r^2} - \frac{w_D Gm_0}{r_0 r} \tag{13}$$

Here, and from here on, we'd assume mass density outside of Effective Radius to be negligible.

Again similar calculations as to those of last section would produce a rotation curve,

$$v_0 = \sqrt{w_0\frac{Gm_0}{r} + w_D\frac{Gm_0}{r_0}} \tag{14}$$

with a constant and a diminishing term effectively providing an upper ($\sqrt{\frac{Gm_0}{r_0}}$) and a lower bound ($\sqrt{\frac{w_D Gm_0}{r_0}}$) for the orbital velocity.

Following the same argument as in previous section, terminal velocity in this general set up would be given by

$$v_0 = \sqrt{\frac{w_D G m_0}{r_0}} \tag{15}$$

as $r \to \infty$.

Hence calculations that gave us Eq. (11) would yield,

$$m_0 = \frac{v_0^4}{w_D^2 \pi \rho_0 h G^2} \tag{16}$$

Finally if we parametrise w_D, such that

$$w_D^2 = \frac{\kappa}{\rho_0 h} \tag{17}$$

where κ is a universal constant which we would refer as critical density, then we would arrive at the desired Baryonic Tully-Fisher relations.

$$m_0 = \frac{v_0^4}{\kappa \pi G^2} \tag{18}$$

Since the logarithmic intercept ($\mathscr{A}$) in BTFR

$$m_0 = \mathscr{A} v_0^4 \tag{19}$$

is experimentally verified(McGaugh, 2012), one can directly calculate κ as,

$$\kappa = \frac{1}{\pi \mathscr{A} G^2} \tag{20}$$

The parametrisation represented by Eq (17) is significant enough to merit some discussion. It is well known, within Dark Matter literature, that smaller and sparser galaxies contain relatively more DM halo compared to normal matter (Kravtsov et al., 1998; Carignan and Purton, 1998; Carignan and Freeman, 1988; de Blok and McGaugh, 1997). In Eq (17), we have a more quantified expression for that, of course, without explicitly requiring any dark matter. But more significantly it presents us an opportunity to define a test for the theoretical framework introduced here, by tracking the rotation curve gradient at the effective radius.

Rotation Curve gradient, we would define as

$$R_{grad} = \frac{dv_0^2}{dr}\bigg|_{r=r_0} = -w_0 \frac{G m_0}{r_0^2} \tag{21}$$

Since, $w_0 = 1 - w_D = 1 - \sqrt{\frac{\kappa}{\rho_0 h}}$ and $m_0 = \pi \rho_0 r_0^2 h$, we can rewrite Eq. (21) as

$$R_{grad} = -\pi G h \rho_0 \Big(1 - \sqrt{\frac{\kappa}{\rho_0 h}}\Big) \tag{22}$$

Thus it would be possible to test the Eq (22) empirically by calculating R_{grad} for multiple spiral galaxies and mapping them against known mass density of those galaxies as we already know the value of κ. Also it is to be noted that if $\rho_0 < \frac{\kappa}{h}$, then the rotation curve would have a positive slope and would continue to move up even after the effective radius. On the other hand, denser galaxies ($\rho_0 \geq \frac{\kappa}{h}$) would show a slightly downward slope in their rotation curve at the effective radius.

3 Conclusion

Spiral disk galaxies and their terminal rotation velocity curves offer one of the most prominent instances of a 'mass gap' in the universe. However, that's by no means the only such ramifications. Current theoretical landscape addresses such anomalies either through Dark Matter (whose variants are as yet undetected) or through modifications of fundamental laws of dynamics.

In this work we've suggested an alternative gravity potential, which is consistent with the general understanding of classical gravity and doesn't require New Physics. Key conceptual leap in considering such a potential is around the fact that we haven't restricted the solution to a continuous combination of smaller point or spherical potentials, but rather as an expression for the spacetime metric in the empty space around the galaxy. Admittedly the current work is based on some simplifying assumptions. For example, we have assumed that for every point on the 'shadow' disk outside the effective radius of the galaxy, potential remains static across narrow width of the disk. Also the aspherical component is assumed to be wholly concentrated on D, vanishing everywhere. In reality, it is perhaps more likely that the aspherical component would tend to diffuse further from the 'shadow' disk. However, such corrections can be made to the calculations without impacting the broader outcome.

Another potential concern for the proposed model could be around the physical reality of the model. Particularly since we are proposing to include an aspherical component concentrated on and around an annular region just outside the galaxy's effective radius along the 2-dimensional fundamental plane, it might raise question around whether there is any physical meaning to such a structure. To understand and clarify such concerns, we need to note that the annular 'shadow' disk is relatively empty of Baryonic matter and a specific form of potential shouldn't be interpreted to have any bearing on 'matter distribution' there. Most general understanding of classical gravity does allow potential to shape up in the form we have described in this work, even without having to resort to additional 'matter content'. In that sense, the proposed model doesn't breach any known physical laws of reality.

Also, from the perspective of symmetries of the underlying system, fundamental plane does represent a structural symmetry of the spiral galaxy. In fact most of the characteristic factors describing a spiral galaxy (central surface brightness, disk scale length and rotation velocity), are definitive properties of the fundamental plane (Pharasyn et al., 1997; Shen et al., 2002; Han et al., 2001). Hence it's perhaps not unreasonable to consider gravitation potentials partially concentrated on part of the fundamental plane. One might indeed reason that by avoiding reliance on undetected particles or new physical laws, the work presented here provides us an opportunity to tie our understanding of galactic dynamics more strongly with physical reality.

Admittedly the work here doesn't fully address the entirety of the known problems concerning 'mass gap' anomaly. For example, the framework posited here needs to be extended beyond spiral galaxies to other aspherical systems. It needs to be able to reconcile with observations from gravitational lensing - particularly those from Bullet cluster. We would also need to develop similar solutions for Elliptic and Globular systems, in subsequent work. In general, we would need to expand the work further to explain wider kinematics and dynamics of the galaxies beyond that of rotation curve only (Lopez-Corredoira, 2019; Binney et al., 2014). Particularly, detailed effects from extrinsic events like mergers need to be understood in the context of how the proposed gravitation potential would transform under such process. This would be useful to make falsifiable predictions around dynamic systems like Bullet clusters (Nipoti et al., 2007)

Having said that, the present work already reproduces two of the most prominent features of spiral galaxies i.e. Flat rotation curve and Baryonic Tully-Fisher Relations. Additionally we are able to bring insight into why smaller and sparser galaxies tend to have relatively large ‘mass gap’. As a side observation, we have also been able to characterise that sparse galaxies can produce rotation curves that actually continues to grow even after the effective radius. Using that fact, we have shown that it’s possible to formulate a clear and falsifiable test which can determine whether the fundamental structure of the theory established here holds ground.

While all of that can potentially constitute a future body of work - both theoretical and experimental - we conclude with gentle confidence that the work presented here can hopefully create an alternative stream of thinking in our efforts to understand the dynamics of galaxies and other more asymmetric astronomical systems.

4 Data Availability

There were no direct use of any observational data for the purpose of the work presented here.

5 Conflict of Interest

There are no commercial or other conflicts of interest in wider circulation or publication of the work held by the author or anyone associated with the author.

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