Tool Profile Modification of Hypoid Gear Machined by Duplex Helical Method

Shunxing Wu

Central South University

Hongzhi Yan (✉ 397805132@qq.com)

Central South University <https://orcid.org/0000-0003-4144-5279>

Zhiyong Wang

Central South University of Forestry and Technology

Rengui Bi

Central South University

Jia Li

Central South University

Research Article

Keywords: Duplex helical method, Hypoid gear, Edge contact, Tool profile modification, Loaded tooth contact analysis, Finite element analysis

Posted Date: March 23rd, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-338846/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Version of Record: A version of this preprint was published at The International Journal of Advanced Manufacturing Technology on January 9th, 2022. See the published version at

<https://doi.org/10.1007/s00170-021-08461-w>.

Tool profile modification of hypoid gear machined by duplex helical method

Shunxing Wu^{1,2} • Hongzhi Yan^{1,*} • Zhiyong Wang³ • Rengui Bi² • Li Jia¹

¹ State Key Laboratory for High Performance Complex Manufacturing, Central South University, Changsha 410083, China

² College of Physics and Electromechanical Engineering, Jishou University, Jishou 416000, China

³ College of Mechanical and Electrical Engineering, Central South University of Forestry and Technology, Changsha 410004, China

* Corresponding author: Hongzhi Yan, PhD, Professor; Tel: +86-13637315291; E-mail: yhzcsu@163.com; ORCID: <https://orcid.org/0000-0003-4144-5279>.

Abstract: For the hypoid gear pair of the heavy-duty vehicle drive axle machined by the duplex helical method, in order to avoid edge contact and stress concentration on the tooth surface, a four-segment tool profile is designed to modify the concave and convex surfaces simultaneously. First, the geometric model of the four-segment tool profile is established. Second, the mathematical model of the duplex helical method based on the four-segment tool profile is established, and the method of solving the tooth surface generated by the connecting points of the four-segment tool profile is given. Finally, the finite element method of loaded tooth contact analysis is used to analyze the meshing performance of the gear pair obtained by the four-segment tool profile modification, and the results are compared with the original gear pair. The results show that after the tooth surfaces are modified, the edge contact of the tooth surfaces are avoided, the stress distribution of the tooth surfaces are improved, the maximum contact stress of the tooth surfaces are reduced, and the fatigue and wear life of the tooth surface are improved.

Keywords: Duplex helical method • Hypoid gear • Edge contact • Tool profile modification • Loaded tooth contact analysis • Finite element analysis

1. Introduction

The hypoid gear pair of the heavy-duty vehicle drive axle has the characteristics of heavy load and high speed when working. Due to the axial force generated by the heavy load and meshing, the tooth surface contact slip and deformation are caused. Even a well-designed hypoid gear pair may have edge contact, which will affect the vibration, noise and service life of the gear pair. Therefore, it is necessary to redesign or optimize the tooth surface of the gear pair with edge contact. Tooth surface modification is an effective method to improve the load meshing performance of the gear pair [1-5], which can avoid edge contact and stress concentration during the working of the gear pair. In the research on tooth surface modification with curved tool profile or multi-segment tool profile instead of straight tool profile, some scholars have done some research on the spiral bevel gears with tapered tooth depth machined by the five-cut method [6-9] and the spiral bevel gears with uniform tooth depth machined by the hobbing method [10-11]. In addition, there are related studies on optimizing the geometric parameters of the tool profile to improve the meshing performance of the spiral bevel gear pair with uniform tooth depth [12-15]. For the spiral bevel gear pair with tapered tooth depth machined by the duplex helical method with simultaneous machining of concave and convex surfaces, Zhang et al. [16] used a circular arc profile to modify the tooth surface to avoid edge contact and severe contact stress. However, the use of a circular arc tool profile to modify the tooth surface cannot satisfy the demand for accurate modification of the entire tooth surface, and may cause the problem of excessive partial modification of the tooth surface. When using a multi-segment tool profile to modify

the tooth surface, the corresponding modification quantity of each segment tool profile can be selected according to the edge contact and stress concentration of the tooth tip and the tooth root, which provides better flexibility.

Compared with the five-cut method, the duplex helical method has the advantages of high processing efficiency and quality consistency, making it gradually replace the five-cut method as the mainstream processing method for hypoid gears. However, because the duplex helical method involves simultaneous processing of concave and convex surfaces, it is more difficult to obtain a high-precision geometry of the tooth surface than the five-cut method. And there is less research literature on duplex helical method. Therefore, for the hypoid gear machined by duplex helical method, in order to avoid the problems of edge contact and stress concentration caused by heavy load, a new tool profile is designed to modify the tooth surface. Finally, the finite element method of loaded tooth contact analysis [17-22] is used to discuss the modification effect of the new tool.

2. Solving the tooth surface of the hypoid gear machined by duplex helical method

2.1. Derivation of pinion tooth surface

For the hypoid gear pair machined by the duplex helical method, the gear is machined by the forming method, and the detailed modeling process is given by in reference [6]. The pinion is machined by the duplex helical method, the detailed modeling process of cutting a pinion is described below.

The geometrical schematic diagram of the four-segment tool profile for machining the pinion tooth surface is shown in Fig. 1. The four-segment tool profile consists of the a-segment, b-segment, c-segment and d-segment tool profile, and the four-segment tool profile is tangent at the connecting point respectively. Among them, the a-segment tool profile is a parabola, which is used to generate the top of tooth surface. The b-segment tool profile is a straight line, which is used to generate the working area of tooth surface. The c-segment tool profile is a parabola, which is used to generate the transition surface of tooth root. The d-segment tool profile is an arc line of the tool tip, which is used to generate the transition surface of tooth root arc. Points A and B are the parabolic vertices of the a-segment tool profile and c-segment tool profile, respectively, and the distances h_a and h_c are the heights of points A and B in the cutter head, respectively. Point D is the connection point of the c-segment and d-segment tool profiles, DF is the common tangent of the c-segment and d-segment tool profiles, a_c is the pressure angle of the tangent DF , DG is the vertical line of the tool tip plane, x_G is the x-axis coordinate of point G in the S_p coordinate system. u_{pa} is the distance from point A to the conjugate contact point along the generatrix of the cutting edge. u_{pc} is the distance from point B to the conjugate contact point along the generatrix of the cutting edge. α_p is the blade angle, p_{w1} is the point width, p_f is the arc radius of the d-segment tool profile, λ_p is the arc angle of the a-segment tool profile, R_{c1} is the center radius of the cutter head, R_f is the radius of the cutter head at the intersection of the a-segment tool profile and blade plane, and R_p is the tip radius of the straight tool profile. Fig. 2 shows the generating surfaces of head-cutter for pinion generation, θ_p and u_p are the tooth surface coordinates of the generating surfaces.

the same meaning as in Eqs. (1) and (2), and will not be explained again.

The vector function $\mathbf{r}_p^b$ and unit normal vector $\mathbf{n}_p^b$ of the generating cone generated by the b-segment tool profile (straight line) can be expressed as

$$\mathbf{r}_p^b(u_p, \theta_p) = \begin{bmatrix} (\mathbf{R}_p m u_p \sin \alpha_p) \cos \theta_p \\ (\mathbf{R}_p m u_p \sin \alpha_p) \sin \theta_p \\ -s_p \cos \alpha_p \\ 1 \end{bmatrix} \quad (4)$$

$$\mathbf{n}_p^b = \begin{bmatrix} \cos \alpha_p \cos \theta_p \\ \cos \alpha_p \sin \theta_p \\ m \sin \alpha_p \end{bmatrix} \quad (5)$$

The vector function $\mathbf{r}_p^c$ and unit normal vector $\mathbf{n}_p^c$ of the generating cone generated by the c-segment tool profile (parabola) can be expressed as

$$\mathbf{r}_p^c(u_p, \theta_p) = \begin{bmatrix} (\mathbf{R}_p m u_p \sin \alpha_p m a_{pc} (u_p - h_c / \cos \alpha_p)^2 \cos \alpha_p) \cos \theta_p \\ (\mathbf{R}_p m u_p \sin \alpha_p m a_{pc} (u_p - h_c / \cos \alpha_p)^2 \cos \alpha_p) \sin \theta_p \\ -u_p \cos \alpha_p + a_{pc} (u_p - h_c / \cos \alpha_p)^2 \sin \alpha_p \\ 1 \end{bmatrix} \quad (6)$$

$$\mathbf{n}_p^c = \begin{bmatrix} (\cos \alpha_p - 2a_{pc} (u_p - h_c / \cos \alpha_p) \sin \alpha_p) \cos \theta_p \\ (\cos \alpha_p - 2a_{pc} (u_p - h_c / \cos \alpha_p) \sin \alpha_p) \sin \theta_p \\ m \sin \alpha_p m 2a_{pc} (u_p - h_c / \cos \alpha_p) \cos \alpha_p \end{bmatrix} \div \sqrt{1 + 4a_{pc}^2 (u_p - h_c / \cos \alpha_p)^2} \quad (7)$$

Where a_{pc} in Eqs. (6) and (7) is the parabolic coefficient of the d-segment tool profile,

The vector function $\mathbf{r}_p^d$ and unit normal vector $\mathbf{n}_p^d$ of the generating cone generated by the d-segment tool profile (arc line) can be expressed as

$$\mathbf{r}_p^d(\lambda_f, \theta_p) = \begin{bmatrix} (\mathbf{R}_f m p_f \sin \lambda_f) \cos \theta_p \\ (\mathbf{R}_f m p_f \sin \lambda_f) \sin \theta_p \\ -p_f (1 - \cos \lambda_f) \\ 1 \end{bmatrix} \quad 0 \leq \lambda_f \leq \frac{\pi}{2} - \alpha_c \quad (8)$$

$$\mathbf{n}_p^d = \begin{bmatrix} \sin \lambda_f \cos \theta_p \\ \sin \lambda_f \sin \theta_p \\ m \cos \lambda_f \end{bmatrix} \quad (9)$$

$$\mathbf{R}_f = x_G \pm \rho_f (1 - \sin \alpha_c) / \cos \alpha_c \quad (10)$$

The positions of the connecting points A and B of the four-segment tool profile shown in Fig. 1 in the S_p coordinate system can be determined by the parameters h_a , h_c and a_p . The position of the connecting point G (common tangent point) of the c-segment tool profile (parabola) and the d-segment tool profile (arc line), x_G and α_c can be obtained by the following methods.

Taking the coordinates of point G in the S_p coordinate system as (x_G, z_G) , then, the expression of (x_G, z_G) can be obtained according to Eq. (6)

$$\begin{cases} x_G = r_p m u_{pG} \sin \alpha_p m a_{pc} (u_{pG} - h_c / \cos \alpha_p)^2 \cos \alpha_p \\ z_G = -u_{pG} \cos \alpha_p + a_{pc} (u_{pG} - h_c / \cos \alpha_p)^2 \sin \alpha_p \end{cases} \quad (11)$$

where u_{pG} is the position parameter of the c-segment tool profile (parabola) at point G. Then, the slope of the tangent DF is

$$k_{DF} = \left(\frac{\partial x_{D-}}{\partial u_{pG}} \right) / \left(\frac{\partial z_{D-}}{\partial u_{pG}} \right) \quad (12)$$

Since the pressure angle of the tangent DF is α_c , there is

$$k_{DF} = \tan \alpha_c \quad (13)$$

In addition, it can be seen from Fig. 1

$$z_G = DG \quad (14)$$

Finally, Eqs. (11), (12), (13) and (14) can be combined to determine u_{pG} , x_G and α_c .

The coordinate systems applied for the pinion generation are shown in Fig. 3. S_p is the coordinate system rigidly fixed to the cutter head, S_a is the reference coordinate system of the tilt angle and the swivel angle, S_{m1} , S_c and S_d are the fixed coordinate systems rigidly fixed to the machine tool; S_b and S_1 are the moving coordinate systems rigidly fixed with the cradle and the pinion, respectively. The installation and adjustment parameters of cutter head and pinion: q_1 , s_{r1} , i , j , e_1 , x_{b1} , γ_{m1} , x_{g1} , and hl represent the center roll position, radial distance, tilt angle, swivel angle, work offset, sliding base, machine root angle, machine center to the cross point, and the velocity coefficient of helical motion, respectively. When the pinion is machined by duplex helical method, the cradle rotates around the axis z_{m1} while moving as a helix along the axis z_{m1} . At the same time, the pinion makes a rotary motion around the axis x_d . φ and m_{b1} are the rotation angle of the pinion and roll ratio of the machine tool, respectively.

Fig. 3. Coordinate systems applied for the pinion generation

According to the coordinate systems applied for the pinion generation shown in Fig. 3, the r_p and n_p is converted to the S_1 coordinate system. As a result, the tooth surface equation r_1 and its unit normal vector n_1 of the pinion in the S_1 coordinate system can be obtained as follows:

$$\begin{cases} r_1(u_p, \theta_p, \varphi) = \mathbf{M}_{1m1} \mathbf{M}_{m1p} r_p(u_p, \theta_p) \\ n_1(u_p, \theta_p, \varphi) = \mathbf{L}_{1m1} \mathbf{L}_{m1p} n_p(u_p, \theta_p) \end{cases} \quad (15)$$

where matrixes $\mathbf{M}_{m1p}$ and $\mathbf{M}_{1m1}$ represents the transformation matrix from the coordinate systems S_p to S_{m1} and S_{m1} to S_1 , respectively. And matrixes $\mathbf{L}_{m1p}$ and $\mathbf{L}_{1m1}$ are the third-order submatrix of the matrixes $\mathbf{M}_{m1p}$

and $\mathbf{M}_{1m1}$, respectively. Matrixes $\mathbf{M}_{m1p}$, $\mathbf{L}_{m1p}$, $\mathbf{M}_{1m1p}$ and $\mathbf{L}_{1m1}$ can be expressed as

$$\mathbf{M}_{m1p} = \begin{bmatrix} \cos q & \sin q & 0 & 0 \\ -\sin q & \cos q & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} -\sin j & -\cos j & 0 & s_{r1} \\ \cos j & -\sin j & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \cos i & 0 & \sin i & 0 \\ 0 & 1 & 0 & 0 \\ -\sin i & 0 & \cos i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

$$\mathbf{M}_{1m1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi & 0 \\ 0 & \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \bullet \begin{bmatrix} \cos \gamma_{m1} & 0 & \sin \gamma_{m1} & -x_b \sin \gamma_{m1} - x_{g1} \\ 0 & 1 & 0 & e_1 \\ -\sin \gamma_{m1} & 0 & \cos \gamma_{m1} & 0 \\ 0 & 0 & 0 & -x_b \cos \gamma_1 \end{bmatrix} \quad (17)$$

$$\mathbf{L}_{m1p} = \mathbf{M}_{m1p}(1:3, 1:3) \quad (18)$$

$$\mathbf{L}_{1m1} = \mathbf{M}_{1m1}(1:3, 1:3) \quad (19)$$

In Eq. (16), $q=q_1+\varphi_1 \bullet m_{b1}$. When the duplex helical method is used to machine the pinion, the cradle has axial movement. Therefore, the x_b in Eq. (17) can be expressed as

$$x_b = x_{b0} - hl \bullet \varphi \bullet m_{b1} \quad (20)$$

In addition, in the of generating a pinion, the meshing equation is satisfied [20]

$$f = \mathbf{n}_1(u_p, \theta_p, \varphi) \mathbf{g}(\partial \mathbf{r}_1(u_p, \theta_p, \varphi)(1:3) / \partial \varphi) \quad (21)$$

Incorporating meshing Eq. (21) into Eq. (15) can eliminate the parameter φ , and the pinion tooth surface equation can be expressed as:

$$\begin{cases} \mathbf{r}_1 = \mathbf{r}_1(u_p, \theta_p, \xi_l) \\ \mathbf{n}_1 = \mathbf{n}_1(u_p, \theta_p, \xi_l) \end{cases} \quad (22)$$

where ξ_l represents the machining parameters of the pinion.

2.2 The Solution of tooth surface generated by connecting points of four-segment tool profile

In order to obtain the tooth surface space coordinate points, the tooth surface needs to be discretized according to the geometric parameters of the gear pair, that is, the tooth surface is divided into uniform grid points. Fig. 4 shows the discrete diagram of tooth surface. The relationship between the coordinates (L, R) of the discrete points of the tooth surface on the rotating projection surface and the tooth surface equation $\mathbf{r}_1$ can be expressed as Eq. (23):

Fig. 4. Discrete diagram of tooth surface

$$\begin{cases} L = |\mathbf{r}_1 \times \mathbf{p}| \\ R = -\mathbf{r}_1 \bullet \mathbf{p} \end{cases} \quad (23)$$

where $\mathbf{p}$ is the unit vector in the axial direction of the pinion. Given the coordinate points (L, R) , the $\mathbf{r}_1$ and $\mathbf{n}_1$ of the corresponding points on the tooth surface can be obtained by Eq. (22) and (23).

When a four-segment tool profile is used to modify the tooth surface, the appropriate amount of modification can be selected according to the edge contact and stress concentration at different positions of the tooth surface, which is more flexible than the modification with only one tool profile. However, when using the four-segment tool profile to modify the tooth surface, it is necessary to separately solve the tooth surface generated by the four-segment tool profile, and connect the tooth surface obtained by the four-segment tool profile into a complete tooth surface. Therefore, it is necessary to solve the tooth surface boundary line generated by the connection points between the tool profiles.

The tooth surface boundary line is the set of conjugate contact points generated by the tool profile connection points on the tooth surface. Fig. 1 shows that the four-segment blade has 3 connection points, which are the connection point A of the a-segment and b-segment tool profiles, the connection point B of the b-segment and c-segment tool profiles, and the connection point D of the c-segment and the d-segment tool profiles.

For connection point A and connection point B , both can be solved by the tooth surface equation corresponding to the b-segment tool profile. Among them, the equations for solving the boundary line generated by connecting point A are

$$\begin{cases} u_p = h_a / \cos \alpha_p \\ L = |\mathbf{r}_1^b \bullet \mathbf{p}| \end{cases} \quad (24)$$

Similarly, the equations for solving the boundary line generated by connecting point B are

$$\begin{cases} u_p = h_c / \cos \alpha_p \\ L = |\mathbf{r}_1^b \bullet \mathbf{p}| \end{cases} \quad (25)$$

For the connection point D , it can be solved by the tooth surface equation corresponding to the c-segment tool profile. Therefore, the equations for solving the boundary line generated by connecting point D are

$$\begin{cases} u_p = u_{pG} \\ L = |r_1^c \bullet p| \end{cases} \quad (26)$$

Since h_a , h_c and α_p are known parameters, the u_p in Eqs. (23), (24), (25) and (26) are also determined. And L can also be determined according to the geometric parameters of the gear pair. Therefore, the conjugate contact points on the boundary line can be obtained by solving the Eqs. (24), (25) and (26) on the premise of keeping u_p unchanged and taking equal points of L . The boundary line can be obtained by connecting the obtained conjugate contact points, the schematic diagram of the boundary line of the four-segment tool profile are shown in Fig. 5. The tooth surface is discretized according to the boundary line to obtain discrete points (L , R), and then the tooth surface developed by each cutting edge can be obtained by combining the tooth surface Eqs. (22) and (23). Finally, the entire tooth surface can be obtained by connecting the tooth surface developed by the four segment tool profile.

Fig. 5. Boundary line of the connecting point for the four-segment tool profile.

3. Example analysis

3.1. Geometric parameter design

The hypoid gear pair used in the heavy-duty drive axle is taken as the research object, in which the gear is machined by the forming method, and the pinion is modified. The main geometrical parameters of hypoid gear pair are shown in Table 1. The Machining parameters of hypoid gear pair are shown in Table 2. The design parameter of the four-segment tool profile for machined pinion are shown in Table 3.

Table 1 Main geometric parameters of hypoid gear pair

Items	Gear	Pinion
Face width (mm)	70	77.1228
Pinion offset	/	44.450
Module	12.35	/
Pitch angle (deg)	71.762	17.825
Face angle of blank (deg)	72.4885	20.717
Root angle (deg)	68.8122	17.1131
Addendum (mm)	3.2982	15.6413
Dedendum (mm)	18.3265	5.9834

Table 2 Machining parameters of hypoid gear pair

Items	Gear		Pinion	
	Convex	Concave	Convex	Concave
Blade angle (deg)	22.5	22.5	30	15
Point radius (mm)	149.860	154.940	148.471	153.332
Radial distance (mm)	175.072		178.113	
Tilt angle (deg)	/		20.027	
Swivel angle (deg)	/		-49.588	
Work offset (mm)	/		54.277	
Machine root angle (deg)	68.761		-5.421	
Machine center to back (mm)	1.327		3.670	
Sliding base (mm)	/		48.065	
Ratio of roll	/		4.187	
Center roll position (deg)	/		59.581	
The first order coefficient of helix motion	/		10.096	
Blade edge radius (mm)	2.400		1.270	

Table 3 Design parameter of the four-segment tool profile for machined pinion

Items	Convex	Concave
Parabolic coefficient a_{pa}	0.0003	0.0005
Position parameter of the parabola vertex h_a (mm)	12.5	14
Parabolic coefficient a_{pc}	0.003	0.0045
Position parameter of the parabola vertex h_c (mm)	5	5.5
Arc radius of the d-segment tool profile p_f (mm)	2.5	2.5

The contact pattern and transmission error curve of the hypoid gear pair corresponding to the parameters in Tables 1 and 2 under no load can be obtained by the tooth surface contact analysis method (TCA)[23]. Among them, the contact pattern of the gear are shown in Fig. 6, and the transmission error of the gear pair are shown in Fig. 7. After modifying the tooth surface of the pinion according to the four-segment blade shown in Table 3, the deviation diagram between the modified and original tooth surfaces can be obtained as shown in Fig. 8.

Fig. 6. Results of TCA: contact pattern of the gear

Fig. 7. Results of TCA: Transmission error curve of the gear pair

Fig. 8. Deviation diagram between the modified and original tooth surface

3.2. Loaded tooth contact analysis (LTCA)

In order to study the effect of the four-segment tool profile on the tooth surface modification, the (LTCA) based on the finite element analysis (FEA) is used to analyze the loaded meshing performance of the modified and original gear pairs. First, according to the principle of tooth surface cutting, the gear and the pinion are obtained. Second, the assembly relation of gear pair is obtained by TCA, the gear and the pinion are imported into Pro/E software for assembly and interference rolling inspection, and cut out the single tooth of the assembled gear and pinion. Third, the single-tooth gear pair is imported into the HyperMesh software to divide the finite element mesh of the tooth surface. Finally, import the gear pair that has been divided into finite element meshes into the Abaqus software, and set the material properties, contact conditions and loading conditions for (LTCA). Fig. 9 shows the Five-tooth FEA model of LTCA. The parameter settings for LTCA are shown in Table 4. During the simulation, the stress values and stress cloud diagrams of 80 contact points are taken in total, and the meshing performance of the third pair of teeth of the original and modified gear pair is analyzed. Fig. 10 shows the original contact stress cloud diagram on the tooth surface of the gear. Fig. 11 shows the contact stress cloud diagram on the tooth surface of the gear after modification. Fig. 12 shows the contact stress value of the contact point on the tooth surface of the original and modified gear. Fig. 13 shows the contact stress value of the contact point on the tooth surface of the original and modified pinion. Fig. 14 shows the transmission error curve of the original and modified gear pair obtained by the LTCA.

Fig. 9. Five-tooth FEA model of LTCA

Table 4 Parameter settings for LTCA

parameter	value	parameter	value
Material density (kg/m ³)	7.85×10^3	Load of the gear (kN•m)	1.1
Elastic modulus (MP)	1.88×10^5	Speed of the pinion (r/min)	1200
Poisson's ratio	0.3	Running time (s)	0.24

Fig. 10. Original contact stress cloud diagram

Fig. 11. Contact stress cloud diagram after modification

Fig. 12. Contact stress value of the contact point on the tooth surface of the original and modified gear

Fig. 13. Contact stress value of the contact point on the tooth surface of the original and modified pinion

(a) Gear convex (Drive)

(b) Gear concave (Coast)

Fig. 14. Transmission error curve of the original and modified gear pair obtained by the LTCA

It can be seen from Fig. 10 that under heavy load, the original convex and concave surfaces of the gear appear edge contact, and there is a stress concentration on the convex and the concave surfaces near the tooth root. By comparing Figs. 10 and 11, it can be seen that after tooth surface modification, tip edge contact and root stress concentration of convex and concave surfaces of gear are avoided, and the contact stress distribution of tooth surface is more uniform. Further analyze the contact stress of all contact points on a tooth surface, It can be seen from Fig. 12 that after modification: the maximum contact stress of the convex surface of the gear is reduced from 1717 MP to 1470 MP, which is reduced by 14.4%, and the maximum contact stress of the concave surface of the gear is reduced from 1806 MP to 1447 MP, which is reduced by 19.8%. Similarly, as shown in Fig. 13, after the modification, the maximum contact stress of the convex surface of the pinion is reduced from 1917 MP to 1403 MP, which is reduced by 26.8%, and the maximum contact stress of the concave surface of the pinion is reduced from 1793 MP to 1397 MP, which is reduced by 22.1%. And it can be seen from Figs. 12 and 13 that the contact stress of the entry and exit points of the gear and the pinion are reduced after modification. In addition, as can be seen from Fig. 14, the transmission error on the concave and convex surfaces of the gear pair has a tendency to increase after modification. Therefore, in order to avoid the transmission error from increasing too much during the modification, the appropriate modification can be selected according to the actual working conditions.

Considering that the tooth surface of the pinion has been modified, it is necessary to check the influence of the modification on the bending stress of the tooth root for the pinion. Fig. 15 shows the tooth root bending stress cloud diagram of the concave and convex surfaces for the pinion. Fig. 16 shows the tooth root bending stress values corresponding to the contact points of the original and modified pinion.

Fig. 15. Root bending stress cloud diagram of the convex and concave surfaces for the pinion

(a) Pinion convex

(b) Pinion concave

Fig. 16. Root bending stress values corresponding to the contact points of the original and modified pinion.

As can be seen from Fig. 16, the tooth root bending stress on both the convex and concave surfaces of the pinion increases after modification. The maximum tooth root bending stress of the convex surface increased from 327 MP to 335 MP, and the maximum tooth root bending stress of the concave surface increased from 320 MP to 332 MP. And the maximum bending stress of the tooth root is the middle part of the tooth surface close to the tooth root. Therefore, when modifying the pinion, the appropriate modification parameters can be selected according to the actual working conditions.

3.3 Tooth surface contact pattern rolling test

In order to further compare the original and modified contact pattern on the tooth surface of the gear, the eight-axis five-link CNC spiral bevel gear processing machine H650GC (see Fig. 17) produced by Changsha Haliang Kaishuai Co., Ltd. is used to perform gear processing experiments on the original and modified gear pairs. And under light load, the tooth surface is tested by rolling inspection machine (see Fig. 18). Fig.19 shows the original contact pattern of the gear. Fig. 20 shows the contact pattern of the modified gear.

Fig. 17. Spiral bevel gear machine tool **Fig. 18.** Tooth surface contact pattern rolling inspection

(a) Gear convex

(b) Gear concave

Fig. 19. Original contact pattern of the gear

(a) Gear convex

(b) Gear concave

Fig. 20. Contact pattern of the gear after modification

By comparing Fig. 19 and 20, it can be seen that after modification, the contact pattern of the tooth surface tends to shrink to the middle of the tooth surface, so as to avoid contact with the edge of the tooth surface, which is consistent with the simulation results in Figs. 10 and 11. Furthermore, the effectiveness of the proposed design of four-segment tool profile modification is verified.

5. Conclusions

In this paper, the hypoid gear pair machined by double helix method is studied. A new four-segment tool profile is designed to modify the tooth surface to avoid the problem of tooth surface edge contact under heavy load. Compared with the modification of the tooth surface with a single tool profile, the four-segment cutter profile modification is more flexible. The meshing performance of the original and modified gear pairs is analyzed by combining the LTCA based on FEA. The main conclusions are as follows:

- (1) The geometric model of the four-segment tool profile and the mathematical model of hypoid gear machined by duplex helical method are established, and the method of solving the boundary line corresponding to the connecting point of the tool profile is given.
- (2) Under heavy load, the original and modified hypoid gear pairs are subjected to loading tooth surface contact analysis. The results show that after modification, the edge contact of the tooth surfaces are avoided, the stress distribution of the tooth surfaces are improved, the maximum contact stress of the tooth surfaces are reduced, and the contact stress of the entry and exit points of the gear and the pinion

are reduced. Specifically, after the modification, the maximum contact stress of the convex and concave surfaces of the gear are reduced by 14.4% and 19.8%, respectively, and the maximum contact stress of the convex and concave surfaces of the pinion are reduced by 26.8% and 22.1%, respectively.

- (3) After the modification, the tooth root bending stress on the convex and concave surfaces of the pinion increased, and the maximum value of the tooth root bending stress is the middle part of the tooth surface close to the tooth root. Therefore, the influence of the modification on the tooth root bending stress should be considered when selecting the modification parameters, so as to better improve the meshing performance of the gear pair and enhance the bearing capacity of the gear pair.

Ethical Approval and Consent to Participate

Not applicable (This article does not contain any studies with human participants or animals performed by any of the authors).

Authors Contributions

WU Shun-xing, YAN Hong-zhi contributed to the conception of the study. WU Shun-xing, YAN Hong-zhi established the mathematical model of hypoid gear machining by duplex helical method and four-segment tool profile. WANG Zhi-yong and BI Ren-gui established the finite element analysis model. WU Shun-xing, Li jia performed the experiment, data analyses and wrote the manuscript.

Funding

This work was supported by National Natural Science Foundation of China (Grants No. 51575533, 52075552 and 51805555).

Competing Interests

The authors declare that they have no competing interests.

Availability of data and materials

All data generated or analysed during this study are included in this published article.

References

- [1] Kawasaki K, Tamura H (1996) Method for cutting hypoid gears. *Jsmc Int J C-Mech Sy* 40(4):768-775. doi: 10.1299/jsmec.40.768.
- [2] Kawasaki K (2005) Effect of cutting edge profile on meshing and contact of spiral bevel gears in cyclo-paloid system. *Mech Based Des Struc* 33(3):343-357. doi: 10.1080/15367730500374415.
- [3] Zhao N, Guo H, Fang ZD, Wei BY (2008) Modification and loaded contact analysis of spur face gears. *J Aerospace Power* 23(11): 196-200. doi :10.13224/j.cnki.jasp.2008.11.007.
- [4] Zhang XJ, Jiang JK, Jia C, Hou XY, Yang Q (2018) Topological modification of a helical gear with circular tooth profile in gear cutter and tooth drum-shaped. *J Mech Transm* 42(1):7-10. doi: 10.16578/j.issn.1004.2539.2018.01.002.
- [5] Wang C (2021) Study on 3-D modification for reducing vibration of helical gear based on TCA technology, LTCA technology and system dynamics. *Mech Syst Signal Pr* 146:106991. doi: 10.1016/j.ymssp.2020.106991.
- [6] Litvin FL, Fuentes A, Hayasaka K (2006) Design, manufacture, stress analysis, and experimental tests of low-noise high endurance spiral bevel gears. *Mech Mach Theory* 41(1):83-118. doi: 10.1016/j.mechmachtheory.2005.03.001.
- [7] Zhang WQ, Wen YK, Guo XD, Huang G (2019) Tooth surface relief method of spiral bevel gear based on high-order cutting motion. *J Aerospace Power* 34(1):245-254. doi: 10.13224/j.cnki.jasp.2019.01.027.
- [8] Mu YM, Fang ZD, Zhang XJ (2019) Tooth surface design of spiral bevel gears based on tool profile modification.

J Huazhong Univ Sci Techno: Na Sci Ed 47(7):97-101. doi: 10.13245/j.hust.190718.

- [9] Mu YM, Li WL, Fang ZD (2020) Tooth surface modification method of face-milling spiral bevel gears with high contact ratio based on cutter blade profile correction. *Int J Adv Manuf Tech* 106(7): 3229-3237. doi: 10.1007/s00170-019-04738-3.
- [10] Nie SW, Deng XZ, Li TX, Zhang H, Deng J (2012) Tooth meshing contact analysis for bevel gears by face hobbing method based on arc blade profile. *J Aerospace Power*, 27(6):1424-1431. doi:10.13224/j.cnki.jasp.2012.06.004. (in chinese)
- [11] Guo WC, Mao SM, Yang Y, Kuang YH (2016) Optimization of cutter blade profile for face-hobbed spiral bevel gears. *Int J Adv Manuf Tech* 85:209-216. doi: 10.1007/s00170-015-7893-5.
- [12] Simon V (2019) Head-cutter for optimal tooth modifications in spiral bevel gears. *Mech Mach Theory* 44(7):1420-1435. doi: 10.1016/j.mechmachtheory.2008.11.007.
- [13] Simon V (2013) Design of face-hobbed spiral bevel gears with reduced maximum tooth contact pressure and transmission errors. *Chinese J Aeronaut* 26(3):777-790. doi: 10.1016/j.cja.2013.05.005.
- [14] Simon V (2014) Optimal tooth modifications in face-hobbed spiral bevel gears to reduce the influence of misalignments on elastohydrodynamic lubrication. *J Mech Design* 136(7):071007. doi: 10.1115/1.4026264.
- [15] Simon V (2014) Optimal machine-tool Settings for the manufacture of face-hobbed spiral bevel gears. *J Mech Design* 136(8):081004. doi: 10.1115/1.4027635.
- [16] Zhang Y, Yan HZ, Zeng T, Zeng YY (2016) Tooth surface geometry optimization of spiral bevel and hypoid gears generated by duplex helical method with circular profile blade. *J Cent South Univ* 23(3):544-554. doi: 10.1007/s11771-016-3101-5.
- [17] Wang X, Fang ZD, Li SJ, Cui YM (2015) Precise modeling of HGT hypoid gear and loaded tooth contact analysis. *J Sichuan Univ* 47(4):181-185. doi: 10.15961/j.jsuese.2015.04.027.
- [18] Liu C, Shi WK, He W, Guo NC (2016) Meshing analysis of hypoid gears based on complete drive axle model. *Auto Eng* 38(9):1153-1157. doi: 10.3969/j.issn.1000-680X.2016.09.018.
- [19] Hou XY, Fang ZD, Fu XZ, Zhang XJ (2019) Meshing performance of spiral bevel gear with different loads and modules considering edge contact by finite element method. *T Can Soc Mech Eng* 43(3):322-332. doi: 10.1139/tcsme-2017-0029.
- [20] Alfonso FA, Ramon RO, Ignacio GP (2017) Numerical approach for determination of rough-cutting machine-tool settings for fixed-setting face-milled spiral bevel gears. *Mech Mach Theory* 112:22-42. doi: 10.1016/j.mechmachtheory.2017.01.010.
- [21] Ding H, Tang JY (2019) Machine-tool settings driven high-order topology optimization to grinding tooth flank by considering loaded tooth contact pattern for spiral bevel gears. *Int J Mech Sci* 172:105397. doi: 10.1016/j.ijmecsci.2019.105397.
- [22] Qu W, Ding H, Tang JY (2020) An innovative semi-analytical determination approach to numerical loaded tooth contact analysis (NLTCa) for spiral bevel and hypoid gears. *Adv Eng Softw* 149:102892. doi: 10.1016/j.advengsoft.2020.102892.
- [23] An LQ, Zhang LH, Qin SL, Lan G, Chen BK (2021) Mathematical design and computerized analysis of spiral bevel gears based on geometric elements. *Mech Mach Theory* 156:104131. doi:10.1016/j.mechmachtheory.2020.104131.

Figures

Figure 1

Geometrical schematic diagram of the four-segment tool profile for machining the pinion

(a) Convex generating cone

(b) concave generating cone

Figure 2

Generating surfaces of cutter head with four-segment tool profile for pinion generation

Figure 4

Discrete diagram of tooth surface

Figure 5

Boundary line of the connecting point for the four-segment tool profile.

Figure 6

Boundary line of the connecting point for the four-segment tool profile.

(a) Gear convex (Drive)

(b) Gear concave (Coast)

Figure 7

Results of TCA Transmission error curve of the gear pair

(a) Pinion convex

(b) Pinion concave

Figure 8

Deviation diagram between the modified and original tooth surface

Figure 9

Five-tooth FEA model of LTCA

Figure 10

Original contact stress cloud diagram

(a) Gear convex

(b) Gear concave

Figure 11

Contact stress cloud diagram after modification

(a) Gear convex

(b) Gear concave

Figure 12

Contact stress value of the contact point on the tooth surface of the original and modified gear

(a) Pinion Convex

(b) Pinion concave

Figure 13

Contact stress value of the contact point on the tooth surface of the original and modified pinion

Figure 14

Transmission error curve of the original and modified gear pair obtained by the LTCA

(a) Pinion convex

(b) Pinion concave

Figure 15

Root bending stress cloud diagram of the convex and concave surfaces for the pinion

(a) Pinion convex

(b) Pinion concave

Figure 16

Root bending stress values corresponding to the contact points of the original and modified pinion.

Figure 17

Spiral bevel gear machine tool

Figure 18

Tooth surface contact pattern rolling inspection

(a) Gear convex

(b) Gear concave

Figure 19

Original contact pattern of the gear

(a) Gear convex

(b) Gear concave

Figure 20

Contact pattern of the gear after modification