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A Firefly Algorithm Based Hybrid Method for Structural Topology Optimization

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Abstract

In this paper a firefly algorithm based hybrid algorithm through retaining global convergence of firefly algorithm and ability of generating connected topologies of optimality criteria (OC) method is proposed as an alternative method to solve stress-based topology optimization problems. Lower and upper limit of design variables (0 and 1) were used to find initial material distribution to initialize firefly algorithm based section of the hybrid algorithm. Input parameters, number of fireflies and number function evaluations were determined before implementation of firefly algorithm to solve formulated problems. Since direct application of firefly algorithm cannot generate connected topologies, outputs from firefly algorithm were used as an initial input material distribution for OC method. The proposed method was validated using two-dimensional benchmark problems and the results were compared with results using OC method. Weight percentage reduction, maximum stress induced, optimal material distribution and compliance were used to compare results. Results from the proposed method showed that the proposed method can generate connected topologies and generated topologies are free from interference of end users, which only dependence on boundary conditions or the design variables. From the results, the objective function (weight of the design domain) can be further reduced in the range of 5% to 15% compared to OC method.

Key words: Firefly algorithm, FEA, Stress based topology optimization

1. Introduction

Continuum structural topology optimization as a generalized shape optimization problem has received extensive attention and considerable progress over the past few years. Different methods have been developed based on this concept. One of the most established methods is a homogenization method where a structure is represented by micro scale void materials. Evaluation and orientation of optimal microstructures is one of the major challenges in homogenization. Another approach named SIMP method which was originally proposed by Bendsoe [1] has got a general acceptance due to its conceptual simplicity and computational efficiency. The existence of transition elements and local convergence has been a major challenge in SIMP method. Filtering techniques and parameter constraints have been used to address these challenges. Evolutionary structural topology optimization/Bi-directional structural topology optimization (ESO/BESO) methods are other varieties of optimization methods proposed to solve structural topology optimization [2-4]. Due to truss like optimal layouts it generates, ESO based methods have been recommended to solve problems having pin-jointed connections. The other variation is level set method which is based on the optimization of implicit interfaces. The boundaries move according to the stress on the boundaries. This method is very sensitive to the initial guess and easily caught in local minima [5]. For real world problems globally optimum solution among alternates is preferable and those methods which can generate globally optimal solutions are getting more attention in solving optimization problems. Metaheuristic algorithm-based methods have been getting attention in solving structural topology optimization problems [5-9].

Firefly algorithm (FA) is one of nature based optimization algorithms developed by Xin-She Yang in late 2007 [10, 11]. It is one of stochastic, nature- based, Meta-heuristic algorithm which has been implemented for solving different optimization problems. The algorithm is inspired by flashing behavior of fireflies. This lighting behavior has a function of attracting mating partners and reminder to a potential predator. This algorithm has been used to solve different numerical problems [12-16] and shows an outstanding performance in terms of convergence and efficiency over the other nature inspired algorithms including genetic algorithm and particle-swarm optimization algorithm [14, 17-19]. Firefly algorithm and its variants have been applied for different optimization problems and different engineering problems as shown in Figure 1.

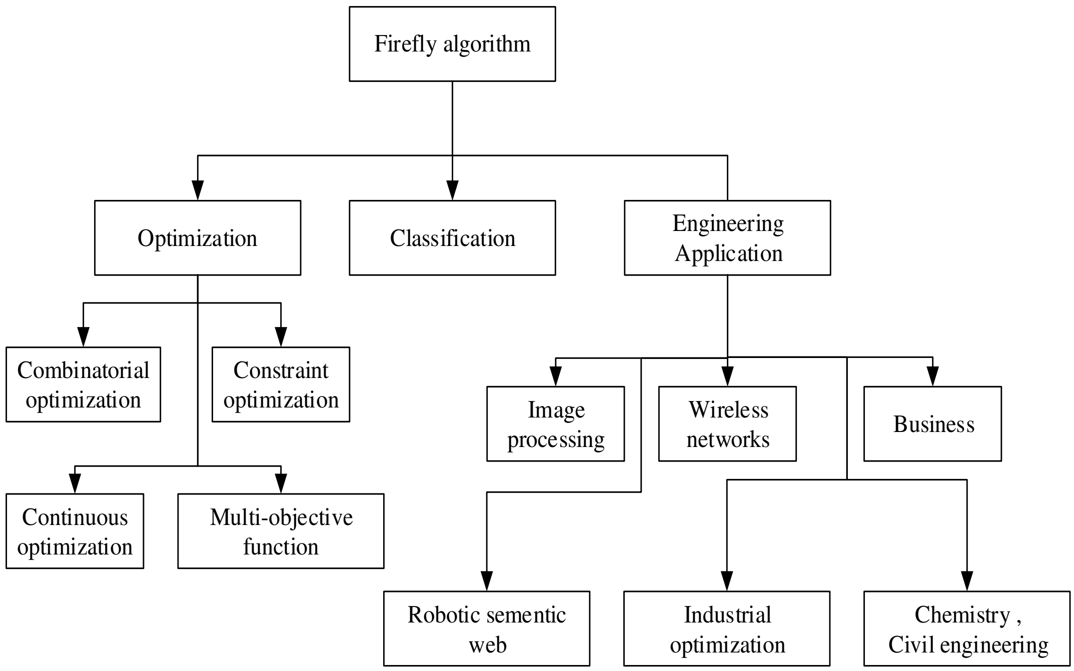


Figure 1 Taxonomy of firefly application [17]

2. Principle of Firefly Algorithm

The FA idealizes several aspects of fireflies in nature. A real firefly flashes in discrete patterns, whereas the modeled fireflies will be treated as always glowing. Then, three rules are used to model fireflies behavior and govern the algorithm [10]

1. The fireflies are unisex which leads fireflies to be attracted to another firefly irrespective of their sex.

2. Attractiveness is proportional to the brightness, where both decreases as the distance between two fireflies increase. For any two fireflies within the domain considered, less bright fireflies always will be attracted to a brighter one. If there is no brighter firefly within the domain, the firefly will move randomly.

3. The brightness of a firefly is proportional to the value of the function being maximized and can be considered as the value of the objective function

Based on these three rules, the basic firefly algorithm (FA) can be summarized by a pseud code as

Objective function $f(x),\ x = (x_1, x_2, \dots, x_d)^T$
Generate an initial population on n firflies $x_i\ (i = 1,2, \dots, n)$
Light intensity I_i *at* x_i*is determined by* $f(x_i)$
Define light absorption coefficient γ
While $t < MaxGeneration$
For $i = 1{:}n\ (all\ n\ firflies)$
For j=1: n (all n fireflies, inner loop)
If $I_i < I_j$
Move firefly i *towards* j
End if
Vary attractiveness with distance r *via* $exp\ [\gamma r^2]$
Evaluate the new solution and update light intensity
End for j
End for i
Rank the fireflies and find the current global best g_*
End while
Post process of results and visualization

Figure 2 Pseudo code of firefly algorithm [10]

Step1. (Generating initial population of solution)

The firefly algorithm generates randomly initial population of solutions, $x_{ik} =,$ where $i = 1,2, \dots SP\ and\ k = 1,2,3 \dots D.$

$$\boldsymbol{x_{ik} = l_k + (u_b - l_b) * rand(size(l_b))} \qquad (1)$$

Where SP is population size, D is dimension of the problem, u_b and l_b are the lower and upper limit of the parameter or design variable x_{ik}. after the generation of initial population, the objective function values for all solutions x_i will be calculated and variable t which is the number of iteration, is set to 1.

Step2. (Calculate the new population)

Each solution of the new population is created from the appropriate solution $x_i$as follows:

For each solution x_i, algorithm examines every solution$x_j, j = 1,2,3, \ldots i$, iteratively, starting from$j = 1$. If solution x_j has higher objective function value than x_i ($x_j$is brighter than x_i), the parameter values $x_{ik}, k = 1,2,3, \ldots, D$ are updated by

$$x_{ik} = x_{ik} + \beta\left(x_{ik} - x_{jk}\right) + \ \alpha S_k(rand - 0.5) \quad (2)$$

Where the first, second and thirds term are the Cartesian distance, is due to the attraction and randomization term. In the second term, β is a monotonically decreasing exponential function which describes firefly's attractiveness as expressed as follows:

$$\boldsymbol{\beta = \beta_0 e^{\gamma r_{ij}}} \quad (3)$$

Where r_{ij} is the distance between firefly i and firefly j, while β_0 and γ are predetermined algorithm parameters, maximum attractiveness value and absorption coefficient, respectively. Distance r_{ij} between fireflies i and j is obtained by Cartesian distance by

$$\boldsymbol{r_{ij} = \left|x_i - x_j\right| = \sqrt{\sum_{k=1}^{d}(x_{i,k} - x_{j,k})^2}} \quad (4)$$

Control parameter β_0 describes attractiveness when two fireflies are found at the same point of search space, i.e. at $r = 0$ the variation of attractiveness with increasing distance from a communicated firefly is determined by the control parameter γ.

$\alpha \in [0,1]$ in the third term in Eq.2 is a randomization parameter, S_k and $rand_k$ are scaling parameter and random number uniformly distributed between 0 and 1, respectively. The scaling parameters S_k is calculated by

$$\boldsymbol{S_k = |u_k - l_k|} \quad (5)$$

In addition, whenever the values of the solution $\boldsymbol{x_{ii}}$ are changed, the FA controls the boundary conditions of created solutions and memorizes the new objective function value instead of the old one. The boundary constraint–handling mechanism used in the FA is given by:

$$\boldsymbol{x_{ik}} = \begin{cases} \boldsymbol{l_k, if\ \ x_{ik} < \ l_k} \\ \boldsymbol{u_k, if\ \ x_{ik} > \ l_k} \end{cases} \tag{6}$$

Last solution obtained by Eq.6 will be taken as the final solution of the new population which will be transformed the next iteration of the FA.

Step3. (Reduce the randomization parameter)

The solution quality can be enhanced by reducing the randomization parameter α with a geometric progression reduction scheme which can be expressed as

$$\boldsymbol{\alpha(t) = \alpha(t-1).\theta^{\frac{1}{MCN}}} \tag{7}$$

Where MCN is the maximum cycle number, t is the current iteration number, $\theta \in (0,1]$ is the randomness reduction constant. In most applications, we can use $\theta = 0.95 \sim 0.99$ and $\alpha_0 = 1$ [10]

Step4. (Record the best solution)

Rank the fireflies by their light intensity /objectives and memorizes the best solution so far $\boldsymbol{x_{best}}$ and increase the variable $\boldsymbol{t}$ by one.

Step5. (Check the termination criterion)

If the $\boldsymbol{t}$ is equal to the maximum number of iterations then finish the algorithm, else go to step 2.

In the simplest case for maximum optimization problem, the brightness I of a firefly at a location x can be chosen $I(x) \propto f(x)$ However, the attractiveness $\boldsymbol{\beta}$ will be judged by other fireflies. Thus, it varies with the distance r_{ij} between firefly i and j. The distance between any two fireflies i and j at x_i and x_j respectively, is the Cartesian distance

$$\boldsymbol{r_{ij} = |x_i - x_j| = \sqrt{\textstyle\sum_{k=1}^{d}(x_{i,k} - x_{j,k})^2}} \tag{8}$$

Thus, the light intensity decreases with the distance from its source and the light is also absorbed in the media. The light intensity $I(\gamma)$ varies according to the inverse square law,

$$\boldsymbol{I(\gamma)} = \frac{\boldsymbol{I_s}}{\boldsymbol{r^2}} \tag{9}$$

Where I_s is the intensity at the source. For a given medium with a fixed light absorption coefficient γ, the light intensity I varies with the distance r which can be expressed as

$$\boldsymbol{I = I_0 e^{-\gamma r}} \tag{10}$$

Where I_0 is the original light intensity at zero distance $r \approx 0$. Since a firefly's attractiveness is proportional to the light intensity seen by the adjacent fireflies, which can be defined as

$$\boldsymbol{\beta = \beta_0 e^{\gamma r^2}} \tag{11}$$

Where β_0 is the attractiveness at$r = 0$. The movement of a firefly i to another more attractive (brighter) firefly j is determined as;

$$\boldsymbol{x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r^2}\left(x_j^t - x_i^t\right) + \alpha \epsilon_i^t} \tag{12}$$

Where the first term is the Cartesian distance between two fireflies , the second term is due to the attraction. The third term is randomization, with α being the randomization parameter having a value of [0, 1] and ϵ_i is a vector of random numbers drawn from a Gaussian distribution or uniform distribution.

4. Optimization process

4.1. Problem formulation

Based on von Mises stress failure theory a ductile material will fail when the von-Mises stress induced in the material is higher than the yield strength of the material. Taking this failure into account, a generalized stress constrained topology optimization problem for two- and three-dimensional mass minimization can be defined as

$$\min_X V = \sum_{e=1}^{N} x_e{}^P v_e$$

$$\text{Subjected to : } \boldsymbol{g(x_e)} = \frac{\boldsymbol{\sigma^{vm}}}{\boldsymbol{\sigma_{yield}}} < \boldsymbol{1} \tag{13}$$

$$: KU = F$$

$$0 < x_{min} \leq x_e \leq 1$$

Where, $\boldsymbol{V}$ is the volume (objective function), $\boldsymbol{N}$ total number of elements which defines the design domain, $\boldsymbol{e}$ elements within the design domain, $\boldsymbol{ve}$ is volume of each element in the design domain, $\boldsymbol{\sigma_{vm}}$ Von-mises stress, $\boldsymbol{\sigma_{yield}}$ is maximum (yield stress). $\boldsymbol{K}$ is global stiffness matrix, $\boldsymbol{U}$ is global displacement vector, $\boldsymbol{F}$ global force vector, $\boldsymbol{x_e}$ is relative density/Design variable, $\boldsymbol{x_{min}}$ is the minimum relative density to control the singularity phenomenon associated with the design variable.

4.2. Finite Element Analysis

For 2D problems, all the design domains are assumed to be rectangular and discretized by a square finite element as shown in Figure 3. Element numbering and degree of freedoms for each node are also defined in the Figure.

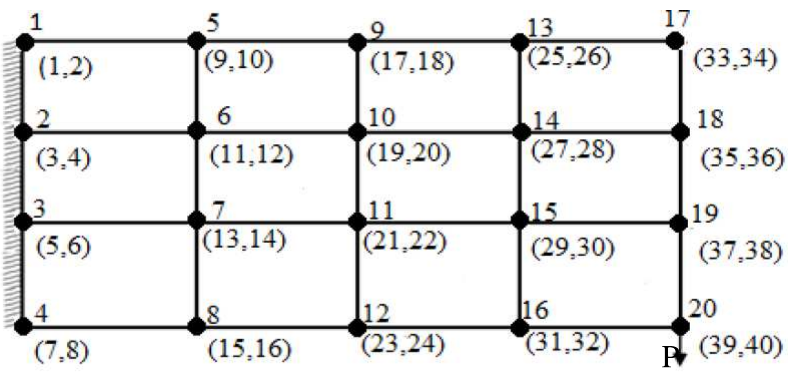


Figure 3 Discretized design domain

4.2.1. Stress and Strain Analysis

A two-dimensional stress state consists of three different stress components as shown in Figure 4, which are the normal stresses $\boldsymbol{\sigma_{yy}}$ and $\boldsymbol{\sigma_{xx}}$ and the shear stress $\boldsymbol{\sigma_{xy}} = \boldsymbol{\sigma_{yx}}$.

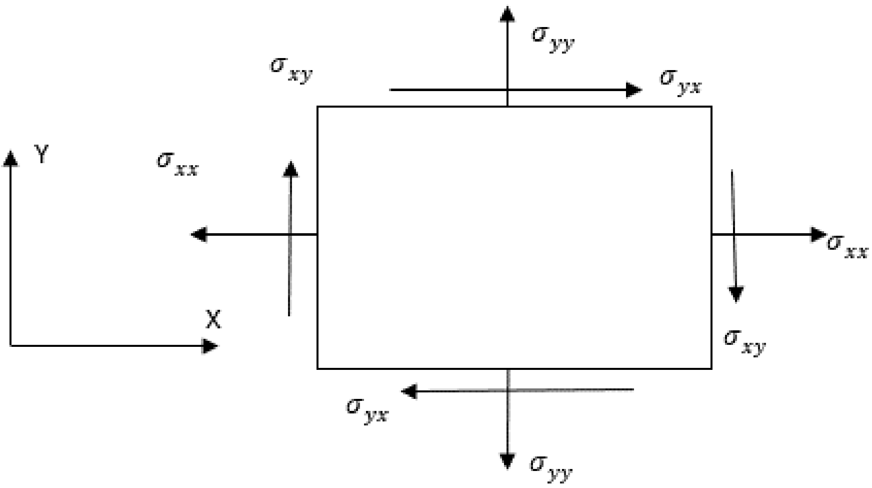


Figure 4 Two-dimensional element stress state

Like the stress state, there are three strain components which are directly proportional to the displacements in the respective direction can be expressed as

$$\boldsymbol{\varepsilon}_{\boldsymbol{xx}}=\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}, \boldsymbol{\varepsilon}_{\boldsymbol{yy}}=\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{y}}, \boldsymbol{\varepsilon}_{\boldsymbol{xy}}=\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}}+\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{y}} \tag{14}$$

Where $\varepsilon_{xx,}\varepsilon_{xx}$ and ε_{xy} are the strains in X , Y and XY plane, respectively. u and v are the displacement in X and Y direction, respectively. The stress induced in the material can be relate with the strain as

$$\{\boldsymbol{\sigma}\} = [\boldsymbol{D}]\{\boldsymbol{\varepsilon}\} \tag{15}$$

Where D is the constitutive matrix, which can be related with the Young's modulus and Poisson's ratio as

$$[\boldsymbol{D}] = \frac{\boldsymbol{E}}{\boldsymbol{1-v^2}}\begin{bmatrix} \boldsymbol{1} & \boldsymbol{v} & \boldsymbol{0} \\ \boldsymbol{v} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \frac{\boldsymbol{1-v}}{\boldsymbol{2}} \end{bmatrix} \tag{16}$$

Where, E the Young's modulus which is a measure of ratio of axial tress to axial strain in uniaxial tension. Poisson's ratio, respectively. v is the Poisson's ratio which is the negative ratio of lateral strain to axial strain a value ranges within 0.25 and 0.35 [20].

4.2.2. Shape Function

Shape functions are used to interpolate the displacement field within the design domain. For a linear displacement, the shape functions in a local coordinate as shown Figure 5 can be expressed as shown in Eq.17. To calculate the stress induced and elemental stiffness matrix the displacement is approximated using shape functions as:

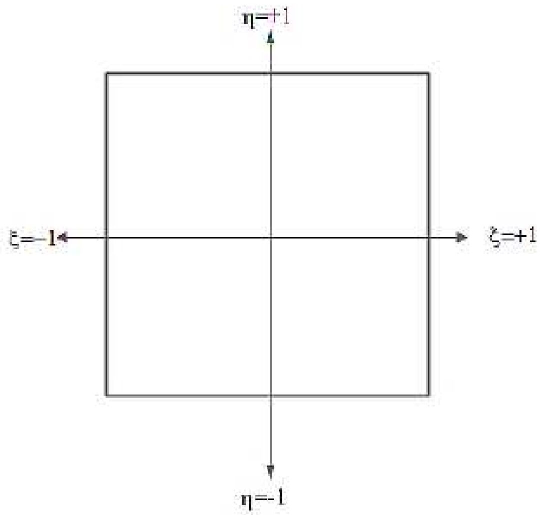


Figure 5 Square finite element in the Natural coordinate system

$$[\boldsymbol{N}] = \begin{bmatrix} \boldsymbol{N1} & \boldsymbol{0} & \boldsymbol{N2} & \boldsymbol{0} & \boldsymbol{N3} & \boldsymbol{0} & \boldsymbol{N4} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{N1} & \boldsymbol{0} & \boldsymbol{N2} & \boldsymbol{0} & \boldsymbol{N3} & \boldsymbol{0} & \boldsymbol{N4} \end{bmatrix} \quad (17)$$

Where

$$N1 = \frac{1}{4}(1-\xi)(1-\eta) \qquad N3 = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N2 = \frac{1}{4}(1+\xi)(1-\eta) \qquad N4 = \frac{1}{4}(1-\xi)(1+\eta)$$

4.2.3. Strain-Displacement Matrix

The displacement vector can be approximated in terms of the shape functions as

$$\{\boldsymbol{u}\} = [\boldsymbol{N}]\{\boldsymbol{d}\}$$

$$\{\boldsymbol{\varepsilon}\} = [\boldsymbol{\partial}]\{\boldsymbol{u}\} \quad (18)$$

$$\{\boldsymbol{\varepsilon}\} = [\boldsymbol{\partial}][\boldsymbol{N}]\{\boldsymbol{d}\}$$

Where $\{\boldsymbol{d}\}$ is nodal displacement vector, $\{\boldsymbol{u}\}$ is displacement vector and $[\boldsymbol{N}]$ is vector of shape function. For computing the stress, strain and stiffness the matrix the strain-displacement matrix having a size of *3X8*.

The term $[\boldsymbol{\partial}]\{\boldsymbol{N}\}$ is described as a strain displacement matric which can be expressed as

$$[\boldsymbol{B}] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} \end{bmatrix} \tag{19}$$

4.2.4. Solving Formulated Problems

The proposed hybrid method has two-stage in solving proposed problem, the first stage will generate the values for design variables using FA algorithm and generated values of design variables will be used as an input for the second stage OC based method. The design variables at the second stage of proposed method are updated using the scheme shown in Eq.20.

$$\begin{gathered} \boldsymbol{if} \;\; x_e\beta_e^{\eta} \leq \max\,(x_{min}, x_e - m) \\ x_e^{new} = \max\,(x_{min}, x_e - m) \\ \textbf{if } \max(x_{min}, x_e - m) < x_e\beta_e^{\eta} \leq \min\,(1, x_e + m) \\ x_e^{new} = x_e\beta_e^{\eta} \\ \boldsymbol{if} \min(1, x_e + m) < x_e\beta_e^{\eta} \\ x_e^{new} = \min(1, x_e + m) \end{gathered} \tag{20}$$

Where $\boldsymbol{m}$ is a positive limit, which usually takes a value of 0.2, $\boldsymbol{\eta}$ is a numerical damping coefficient with a value of 0.5 [21-23] , $\boldsymbol{\beta}$ which will be dependent on the type of problems defined in Eq.21.

$$\boldsymbol{\beta}_e = \frac{\frac{\partial v}{\partial x}}{\lambda\frac{\partial g}{\partial x}} \tag{21}$$

Where, $\boldsymbol{\lambda}$ is a Lagrangian multiplier, where the value can be found from a bi-sectioning algorithm. The sensitivity analysis for problems defined in Eq.21 can be calculated as

$$\frac{\frac{\partial v}{\partial x}}{\frac{\partial g}{\partial x}} = \frac{1}{\frac{\partial}{\partial x}(\frac{\sigma^{vm}}{\sigma_{yield}} - 1)} \tag{22}$$

In this paper, the stress is calculated at the centroid of an element [23-25]. To relate the macro and micro stress levels, local stress interpolation scheme proposed by Duysinx and Sigmund [26] is used as

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \frac{\boldsymbol{D}_e(x)\bar{\varepsilon}(x)}{x^q} \tag{23}$$

Where, $\sigma(\mathrm{x})$ is local stress at a material point, $\boldsymbol{D}_e(\boldsymbol{x})$ is macroscopic elastic tensor which can be related to the constitutive elasticity tensor $\boldsymbol{D}_0$ by a power law approach as shown in Eq. 24, $\bar{\boldsymbol{\varepsilon}}(\boldsymbol{x})$ is the average strain of a material point which can be expressed in terms of strain displacement matrix $\boldsymbol{B}_e$ and elemental displacement vector $\boldsymbol{u}_e$. The exponent $\boldsymbol{q} > \mathbf{1}$ is a constant to preserve physical consistency in material model.

$$\boldsymbol{D}_e(\boldsymbol{x}) = \boldsymbol{x}^{\boldsymbol{P}}\boldsymbol{D}_0 \tag{24}$$

$$\boldsymbol{\varepsilon}(\boldsymbol{x}) = \boldsymbol{B}\boldsymbol{u}_e \tag{25}$$

Substituting Eq.24 and Eq.25 into Eq.23, the stress at any material point with the given design domain can be expressed as;

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \boldsymbol{x}^{\boldsymbol{P}-\boldsymbol{q}}\boldsymbol{D}_0\boldsymbol{B}_e\boldsymbol{u}_e \tag{26}$$

From Eq.26 the partial derivative of the constraint function in Eq.22 can be expressed as

$$\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}} = (\boldsymbol{p}-\boldsymbol{q})\boldsymbol{x}^{\boldsymbol{P}-\boldsymbol{q}-\mathbf{1}}\boldsymbol{D}_0\boldsymbol{B}_e\boldsymbol{u}_e + \boldsymbol{x}^{\boldsymbol{P}-\boldsymbol{q}}\boldsymbol{D}_0\boldsymbol{B}_e\frac{\partial \boldsymbol{u}_e}{\partial \boldsymbol{x}} \tag{27}$$

From equilibrium equation we have, $\boldsymbol{K}\boldsymbol{U} = \boldsymbol{F}$, differentiating both sides of the equilibrium equation with respect to the design variable yields

$$\frac{\partial \boldsymbol{k}}{\partial \boldsymbol{x}}\boldsymbol{u} + \boldsymbol{k}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} = \mathbf{0} \Rightarrow \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} = \frac{\boldsymbol{k}}{\boldsymbol{u}}\frac{\partial \boldsymbol{k}}{\partial \boldsymbol{x}} \tag{28}$$

Substituting Eq.20 for partial derivative of the displacement vector in Eq. 27 yields the sensitivity analysis for stress constraint defined in Eq.22 as

$$\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}} = (\boldsymbol{p}-\boldsymbol{q})\boldsymbol{x}^{\boldsymbol{P}-\boldsymbol{q}-\mathbf{1}}\boldsymbol{D}_0\boldsymbol{B}_e\boldsymbol{u}_e - \boldsymbol{x}^{\boldsymbol{P}-\boldsymbol{q}}\boldsymbol{D}_0\boldsymbol{B}_e\frac{\partial \boldsymbol{k}}{\partial \boldsymbol{x}}\frac{\boldsymbol{u}}{\boldsymbol{k}} \tag{29}$$

Therefore, the sensitivity analysis for stress-based topology optimization becomes

$$\frac{\frac{\partial v}{\partial x}}{\frac{\partial g}{\partial x}} = \frac{\mathbf{1}}{(p-q)x^{P-q-1}D_0B_eu_e - x^{P-q}D_0B_e\frac{\partial ku}{\partial xk}} \tag{30}$$

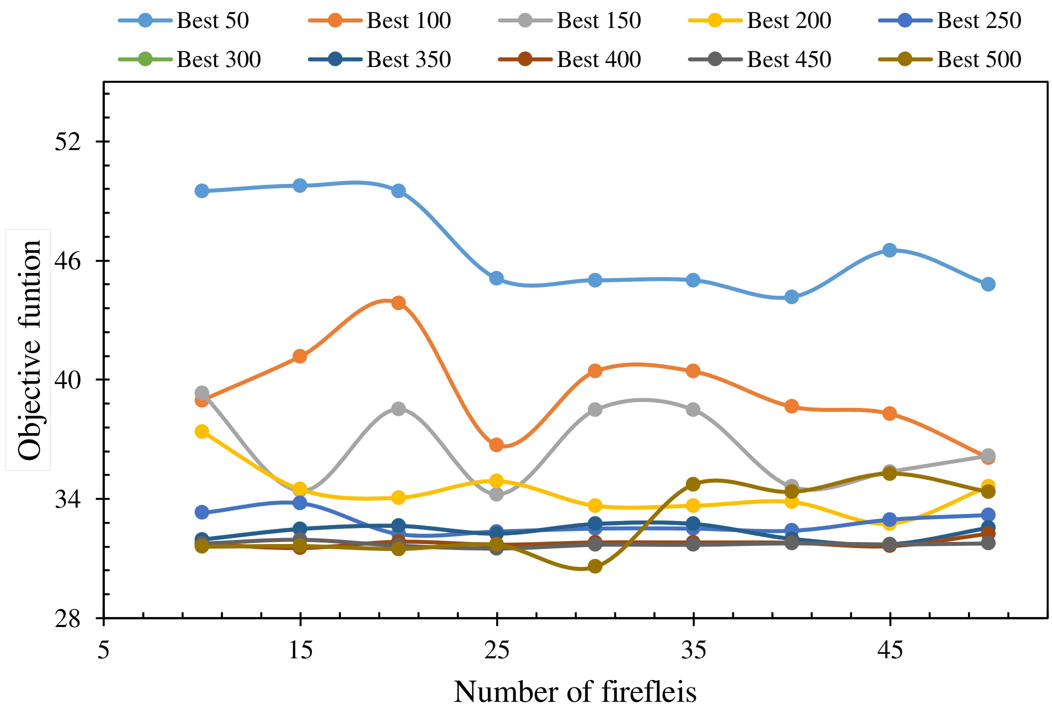
Once the sensitivity analysis for both formulation was completed a Matlab code is written for the stress based topology optimization and an existing Matlab code was used for compliance based [23, 24].

4.3. Determination input parameters

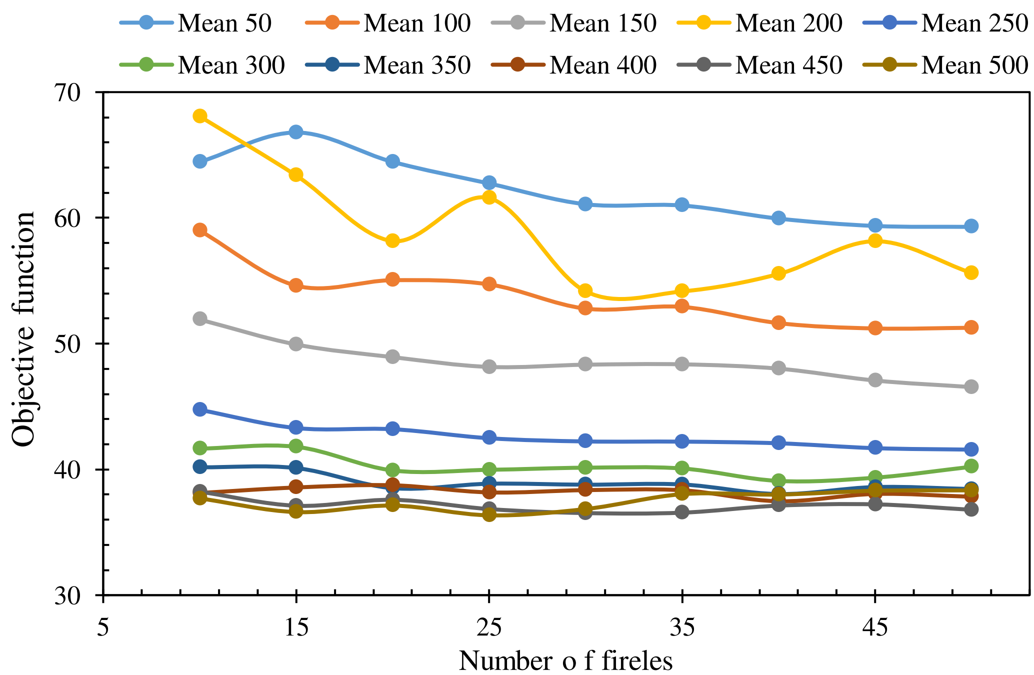
Before using FA to solve stressed based topology optimization problem, input parameters must be determined first. These parameters include number of fireflies, maximum number of iterations, randomness parameter $\boldsymbol{\alpha}$ and initial brightness value $\boldsymbol{\beta}$. Among these parameters the effect of number of fireflies and number of iterations were studied, and the best combination of these parameter was selected for further implementation of the algorithm. The following sections will discuss on the determination of these parameters (number of iteration and number of fireflies). The value of other parameters were directly adopted from other application of firefly algorithm [12, 14].

4.3.1. Number of Fireflies

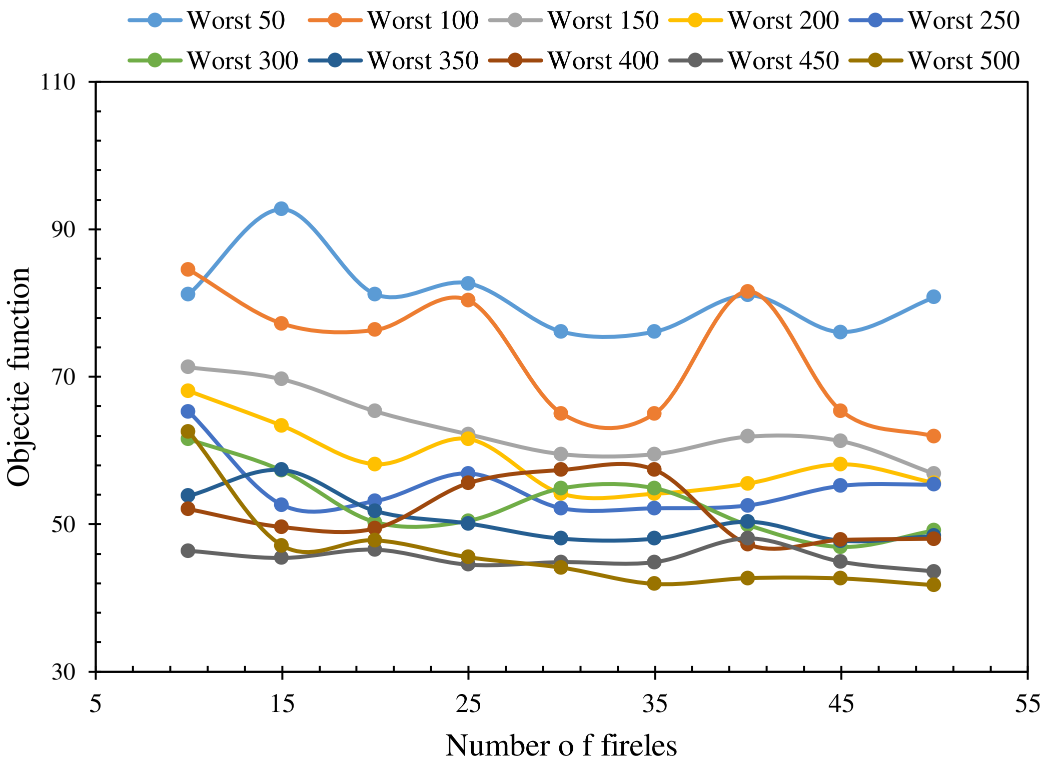
In order to assess effect of number of function evaluations on efficiency of FA algorithm a range between [50 and 500] function evaluations were considered. A design domain was formulated and solved using a FA algorithm for the range of function evaluations for 100 test runs. Then, variation of best, mean and worst values of objective function with function evaluation was studied through plotting these values of the objective function with number of fireflies as shown in Figure 6. From the figure, the objective function is minimum when the number of function evaluation is 200 and number of fireflies is 30 as shown in Figure 6a but the variation of the objective is not stable with the variation of number of fireflies as shown in Figure 6b and Figure 6c. From a figure, it can be noticed that variation of objective function is less sensitive for number of fireflies greater than 35. For the range of fireflies greater than 35 and function evaluations greater than 250 are the best range for best values of the objective function.



(a) Best values of objective function



(b) Mean values of objective function

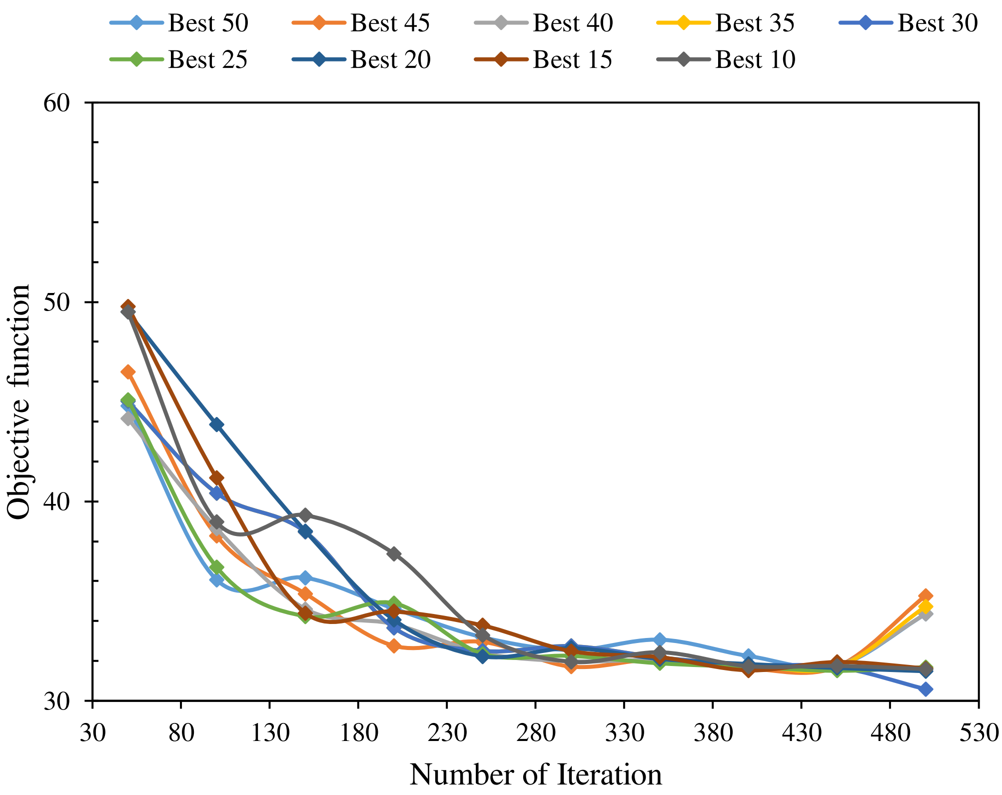


(c) Worst values of objective function

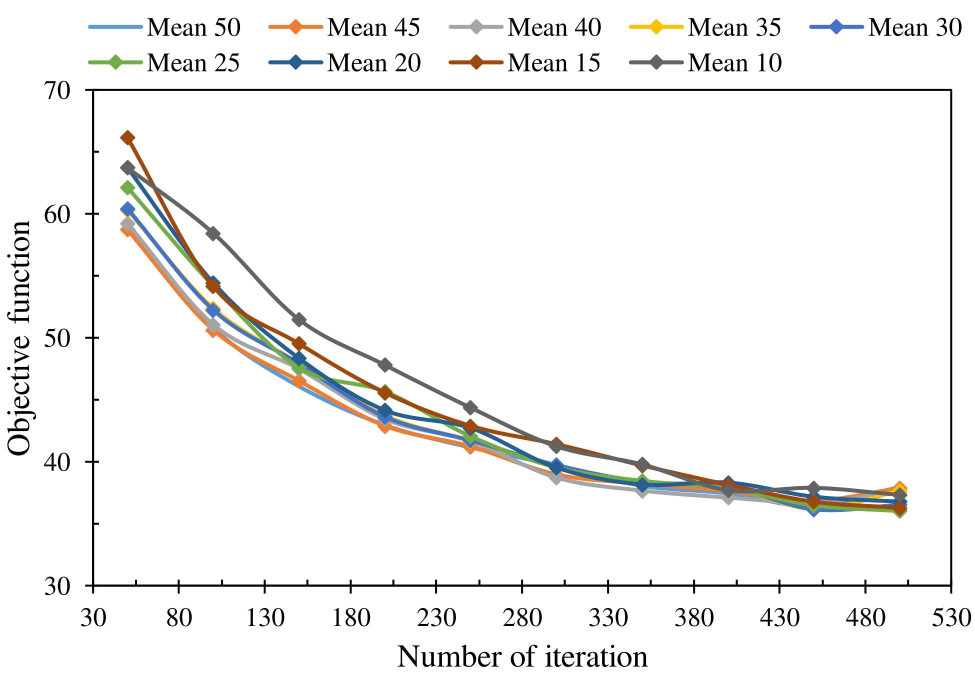
Figure 6 Effect of variation of function evaluation on objective function

4.3.2. Number of Iteration

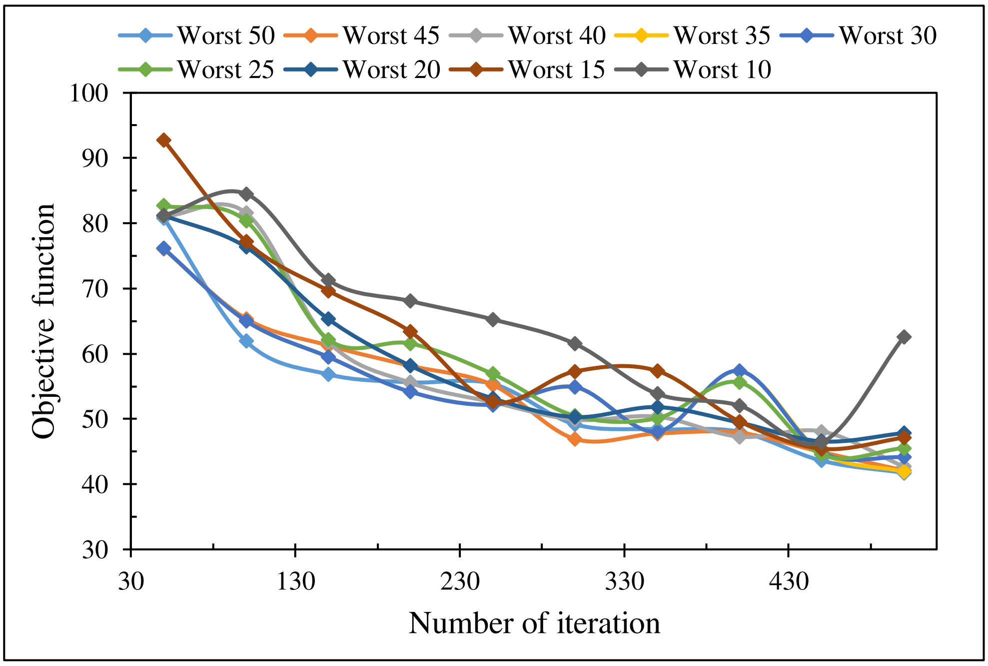
An optimization problem based on Eq.13 was formulated and solved using a firefly algorithm for the range of function evaluations for 100 test runs. Then, variation of best, mean and worst values of objective function with variation of number of fireflies was studied through plotting these values of the objective function with number of fireflies as shown in Figure 7. From the figure, it can be noticed that variation of objective function is less sensitive to number of function evaluations greater than 350. For the range of function evaluations greater than 350, range of number of fireflies greater than 30 is the best range for best values of the objective function.



a) Best values of objective function



b) Mean values of objective function

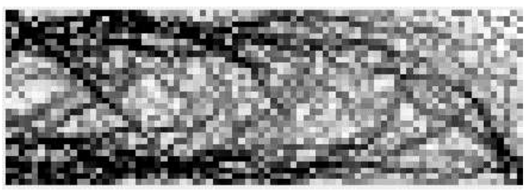


c) Worst values of objective function

Figure 7 Effect of number of fireflies on objective function

Since direct implementation of firefly algorithm yields a topology full of transition elements and highly affected by checkerboard effect as shown Figure 8. A hybrid method was proposed to overcome this issue as described in Figure 9. One of the challenges in the currently available methods is dependency of initial material distributions [5]. From the initial topology generated from firefly algorithm we can have the best values for the design variables which can lead to the best values of the objective function. Even if it generates values of design variables, generated topologies are full of disconnected elements which is unwanted from engineering perspective. To address this issue in direct implementation of firefly algorithm and dependence of OC method on initial material, the outputs of design variable values using firefly are used as an input for OC method. From the convergence history shown in Figure 10, the proposed method can generate connected optimal plots.

(a)



(b)

Figure 8 (a) Design domain and (b) generated material distribution

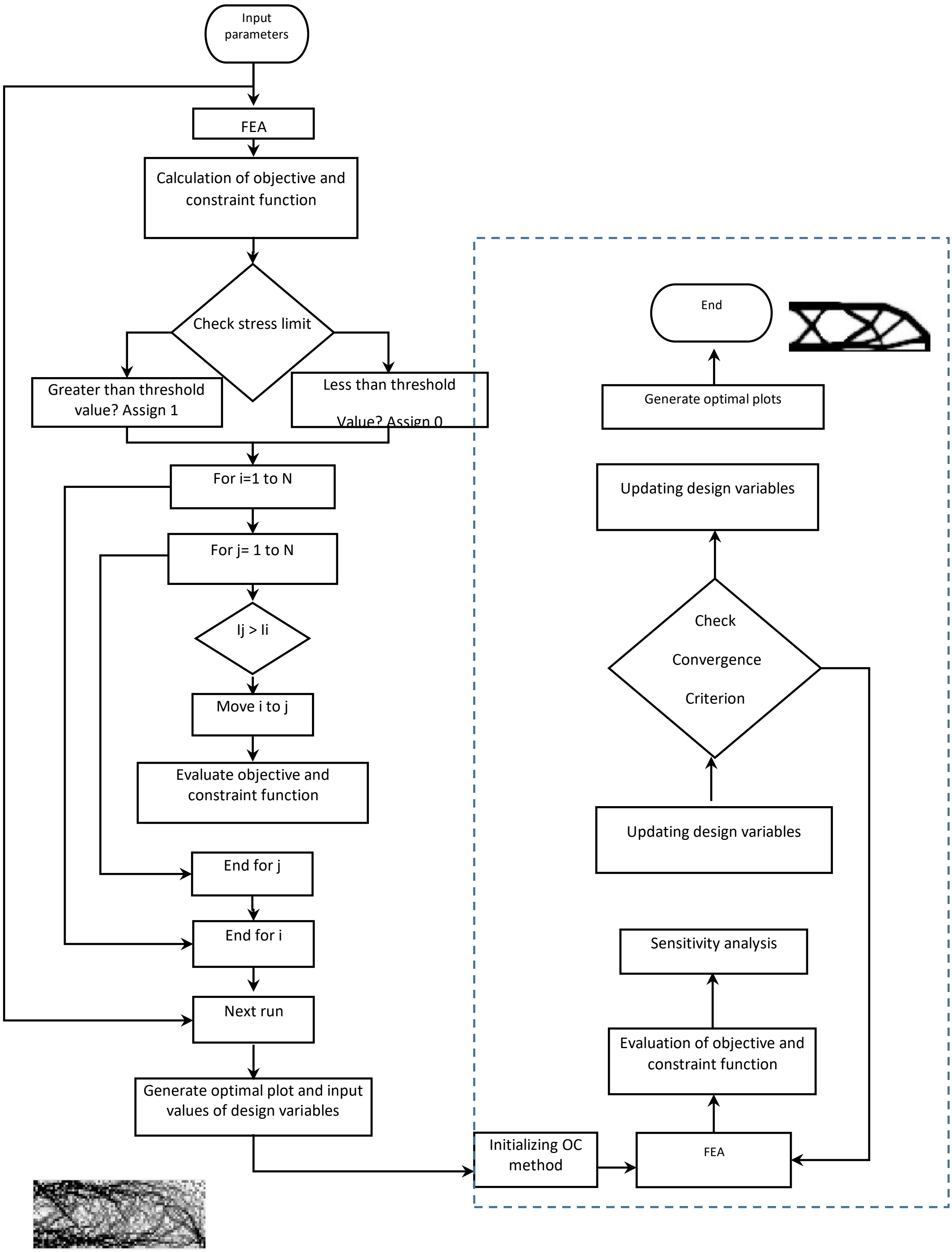


Figure 9 A schematic diagram of structural topology optimization using discrete firefly

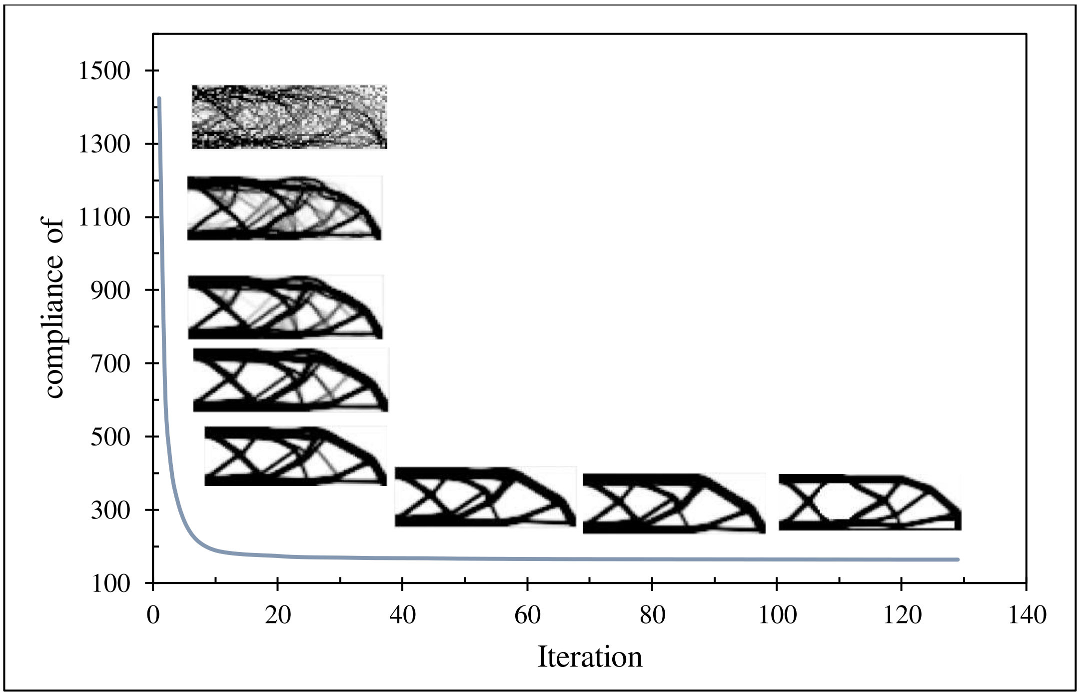


Figure 10 Convergence history of hybrid method

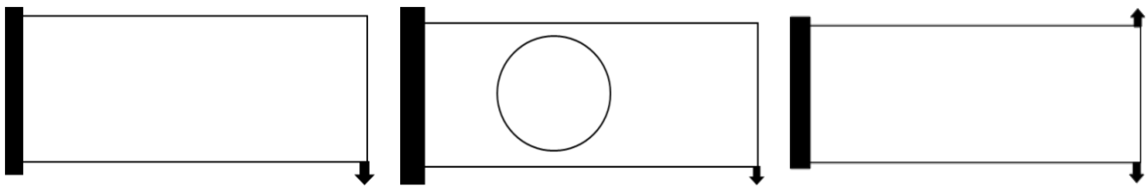
5. Result and discussion

The propose method was used to solve benchmark problems under different discretization size and the results are compared with solutions using an OC method.

5.1.Numerical Results

5.1.1. Cantilever Beam

The first case studies considered was a cantilever beam under loading and boundary conditions defined in Figure 11.



(a) Classical cantilever beam

(b) Cantilever beam with pre-defined shape

(c) Cantilever beam with multiple loading

Figure 11 Boundary and Loading condition

The first case study solved using proposed method was a design domain and loading conditions defined in Figure 11a and the generated topologies for different discretization size of the design domain is presented as shown in Figure 12.

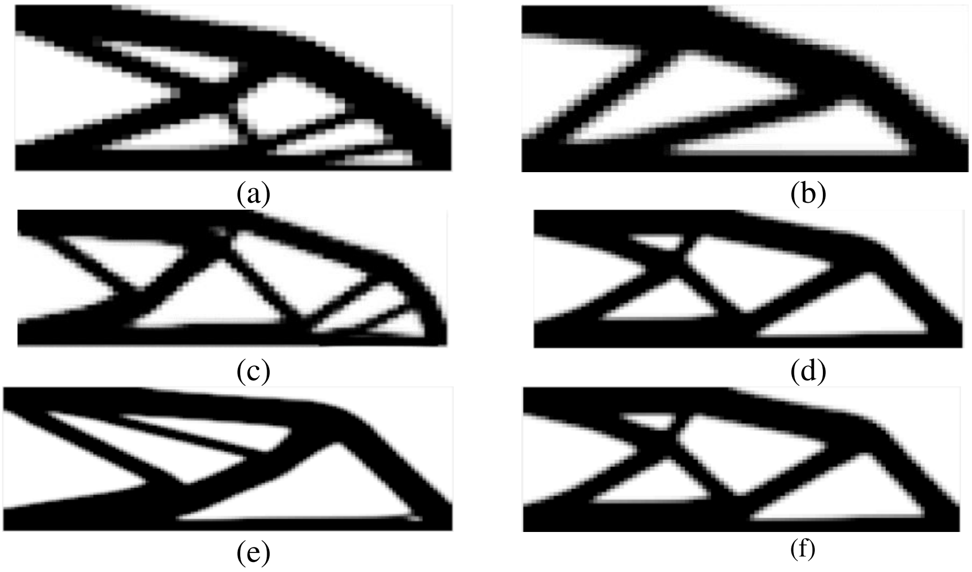


Figure 12 Optimal material plots for numerous sizes of design domain (a) *45 X 32* (b) *60 X80* (c) *80 X* 30 (d) 100X40 (e) 120X60 and (f) 200X70

The generated topologies for Cantilever beam under loading and boundary conditions presented in Figure 11b and Figure 11c are presented as shown in Figure 13 and Figure 14, respectively.

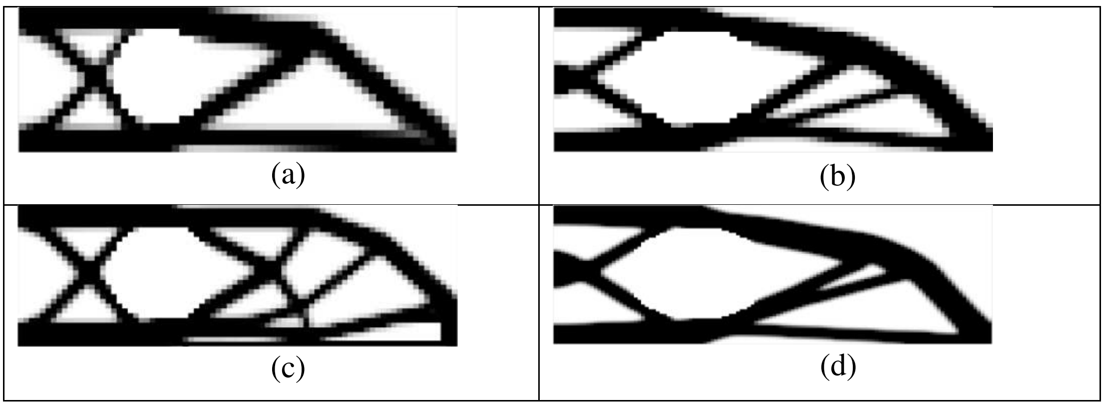
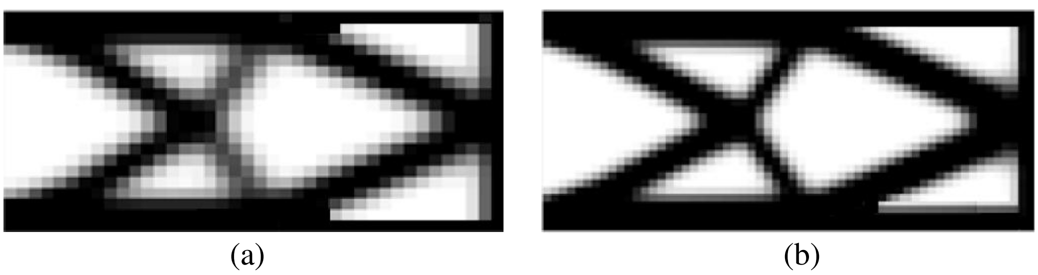
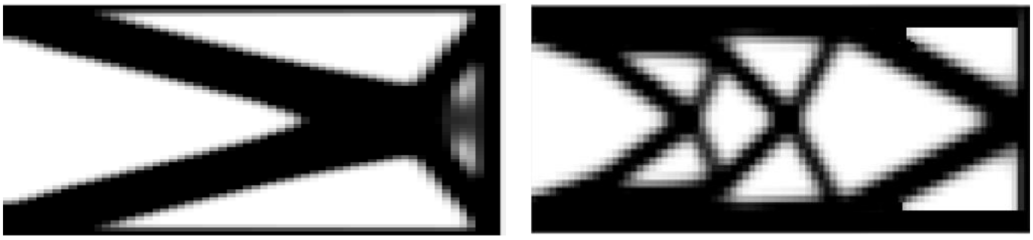


Figure 13 Optimal material plots for numerous sizes of design domain (a) 40 *X 40* (b) 60 *X 30* (c) *80 X 30 and* (d) *120X60*





(c) (d)

Figure 14 Optimal material plots for numerous sizes of design domain (a) 40 *X 20, (b) 60 X30,* (c) *60 X 60* and (d) *80X30*

5.1.2. L-shape Beam

The other bench mark problem considered to be solved using proposed method is an L-shape beam under different boundary and loading conditions. Respective generated topologies for the design domains and generated topologies using the proposed method are show in Table 1 and Table 2.

Table 1 Boundary and loading condition with respective topologies under various discretization size L-shape variant 1

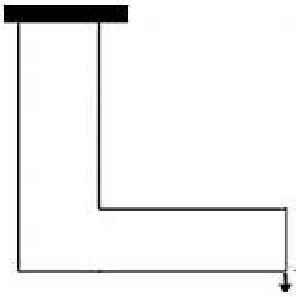
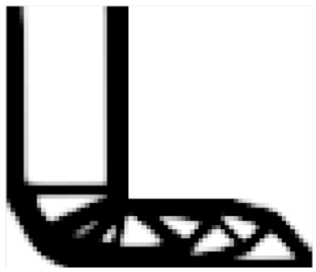
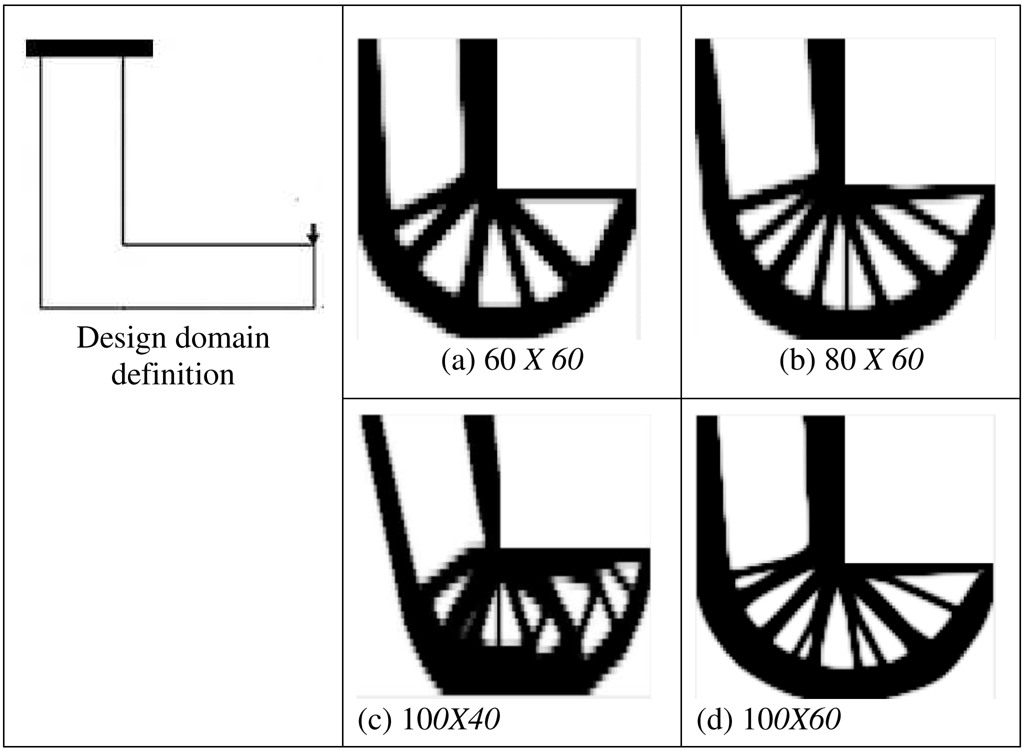
 Design domain definition	 (a) 60 *X 40*	 (b) *60 X 60*
	 (c) *50X50*	 (d) *70X60*

Table 2 Boundary and loading condition with respective topologies under various discretization size L-shape variant 2



5.1.3. Simply Supported

The other benchmark problem used for validation of the proposed method was a simply supported beam under the loading and boundary condition defined in Figure 15 and generated topologies are shown in Figure 16 under different discretization size.

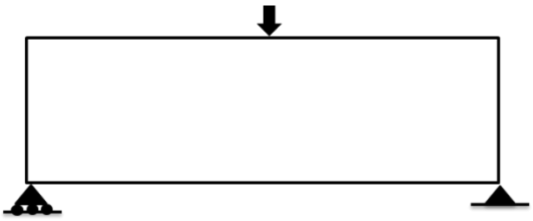


Figure 15 Definition of boundary and loading conditions

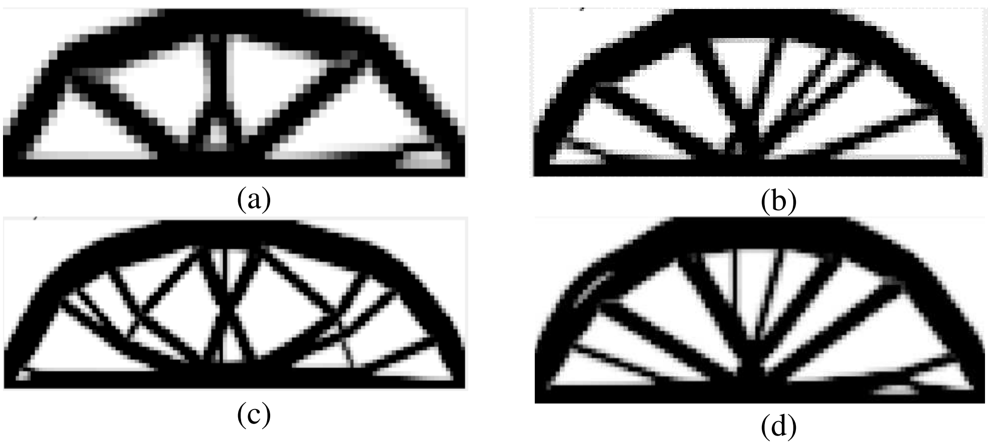


Figure 16 Optimal material plots for different sizes of design domain (a) 60 *X 20,* (b) *80 X 30*, (c) *120X 60* and (d) *100X40*

6. Comparison of results

The optimization results using proposed method were compared with results from OC method topologies. Table 3 shows composition of generated optimal topologies using proposed method and optimal topologies using OC based method for different benchmark problems. Optimal topologies using proposed method have less weight reduction percentage than that topologies generated using OC method as shown in

Table **4**. Even if the weight reduction percentage is less for respective design domains, generated topologies using the proposed method are more complex than OC method.

Table 3 Generated optimal topologies using proposed method and Optimality criteria (OC) method

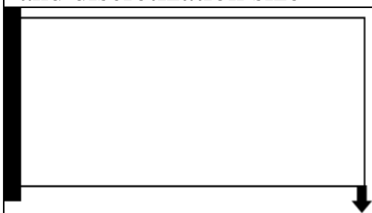
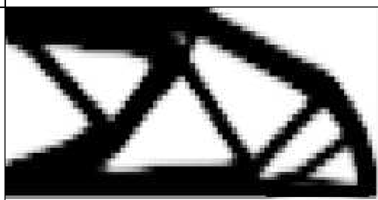
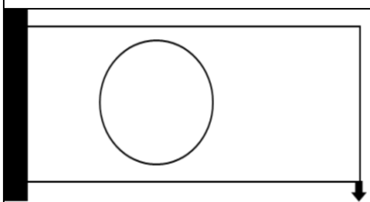
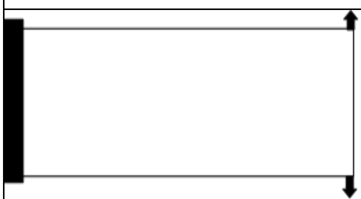
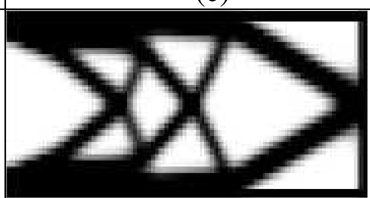
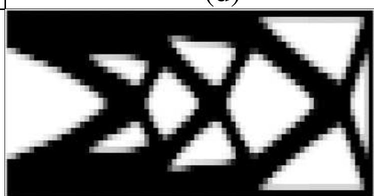
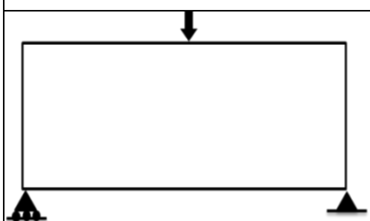
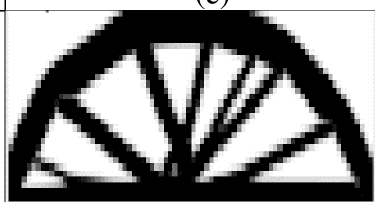
Design domain definitions and discretization size	Proposed method	OC based
	 (a)	 (b)
	 (c)	 (d)
	 (e)	 (f)
	 (g)	 (h)

Table 4 Performance evaluation of proposed method on

Optimal plots / Design domain	Void material (%) proposed	Void material (%) OC based	Solid material (%) proposed	Solid material (%) OC based	Transition material (%) proposed	Transition material (%) OC based
CCA	51.13	36.71	37.54	55.00	12.63	8.29
CCB	54.50	44.67	7.96	8.50	37.54	46.83
CCC	44.50	38.75	13.46	9.33	42.04	51.92
SS	44.67	37.13	14.38	8.96	40.96	53.92
LLL	60.52	54.6	5.04	4.8	34.44	40.6
LLU	47.96	44.05	6.96	5.6	45.07	50.35

Where CCA, CCB, CCC, SS,LLL and LLU indicates classical cantilever beam, Cantilever beam with pre-defined shape, cantilever beam under multiple loading, simply supported beam and loading and boundary conditions of L-shaped beam, respectively.

7. Conclusion

This paper presented a firefly algorithm-based hybrid method for stress-based topology optimization of 2D structures. The advantage of generating globally convergent solutions from FA algorithm and ability to generate connected topologies of OC method is the crucial elements in the proposed method. In the proposed method, initial parameters for SIMP method were determined using Firefly algorithm and used as an input for OC based method then optimal topologies were generated. The proposed method was validated using different benchmark problems to address global convergence and dependence of optimal material distribution on initial values of design variables of OC method. Generated topologies Simulation results shows the objective function which is weight of the design domain can be further minimized in the range of 5-15%.

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